Gluon helicity ΔG from a universality class of operators on a lattice

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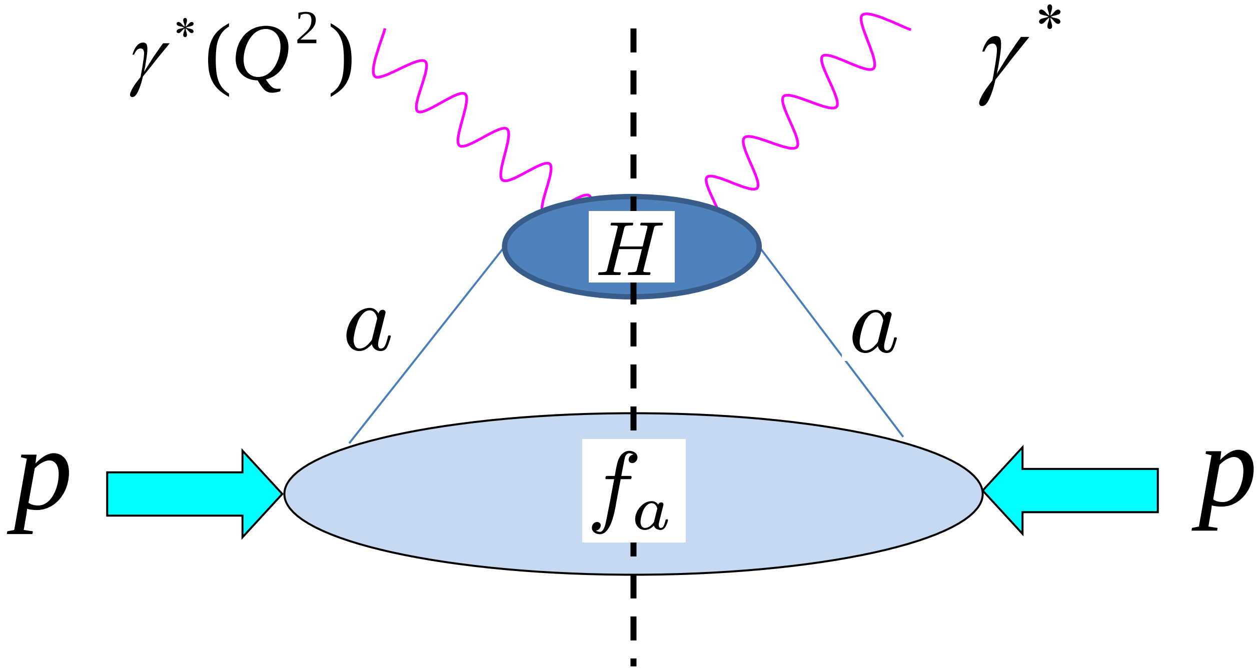
Outline

- Parton distribution function in QCD
- Gluon helicity ΔG
- Difficulty and strategy to measure ΔG on a lattice
- Axial gauges

QCD factorization in DIS



$$F_2(x, Q^2) = \sum_a \int_x^1 d\xi \, f_a(\xi, \mu^2) H_a\left(\frac{x}{\xi}, \frac{Q}{\mu}\right) + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

parton distribution function, nonperturbative, universal

hard scattering coefficient, perturbative

Operator definition

$$q_f(x,Q^2) = \frac{1}{4\pi}\int dy^- e^{ixP^+y^-}\langle P|\bar{q}_f(0)W[0,y^-]\gamma^+ q_f(y^-)|P\rangle_{Q^2}$$

Wilson line

$$W[0,y^-] = P\exp\left(ig\int_0^{y^-} A^+(z^-)dz^-\right) \qquad y^\pm = \frac{1}{\sqrt{2}}(y^0 \pm y^3)$$

Physical interpretation in the light-cone gauge $A^+ = \frac{1}{2}(A^0 + A^z) = 0$

Probability to find a parton of flavor f and momentum fraction x inside a fast-moving proton.

Measuring PDF on a lattice?

$$q_f(x,Q^2) = \frac{1}{4\pi}\int dy^- e^{ixP^+y^-}\langle P|\bar{q}_f(0)W[0,y^-]\gamma^+ q_f(y^-)|P\rangle_{Q^2}$$

Nonlocal operator along the light-cone

→ Real-time problem

Local operator after taking the moment

$$\begin{aligned} q_f(j,Q^2) &= \int_0^1 dx x^{j-1} q_f(x,Q^2) \\ &= \frac{1}{2(P^+)^j}\langle P|\bar{q}_f(0)\gamma^+(iD^+)^{j-1}q_f(0)|P\rangle \end{aligned}$$

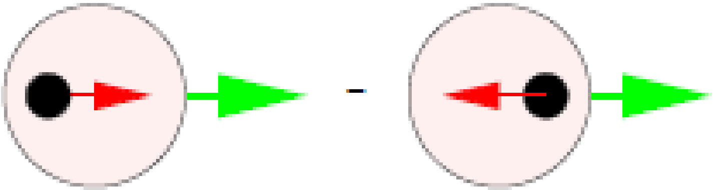
For a recent calculation, see, T. Doi, et al., 1312.4816

Polarized gluon distribution and ΔG

$$\Delta G = \int dx \Delta G(x)$$
$$= \int dx \frac{i}{2xP^+} \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle PS|F^{+\mu}(y^-)W[y^-,0]\tilde{F}_\mu{}^+(0)|PS\rangle$$

$$iF^{+\mu}\tilde{F}_\mu{}^+ = F^{+\mu}F^{+\nu}\varepsilon_\mu^R\varepsilon_\nu^{R*} - F^{+\mu}F^{+\nu}\varepsilon_\mu^L\varepsilon_\nu^{L*}$$

Number density of right-handed gluons minus left-handed gluons inside the longitudinally polarized nucleon.



Note that ΔG is nonlocal even though it is a x-moment.

Why is ΔG interesting?

Decomposition of the nucleon spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

In the naïve quark model, $\Delta\Sigma = \sum_f \int dx \Delta q_f(x) = 1$

Polarized quark distribution

The latest value from experiment $\Delta\Sigma = 0.25 \sim 0.3$

``Spin crisis”

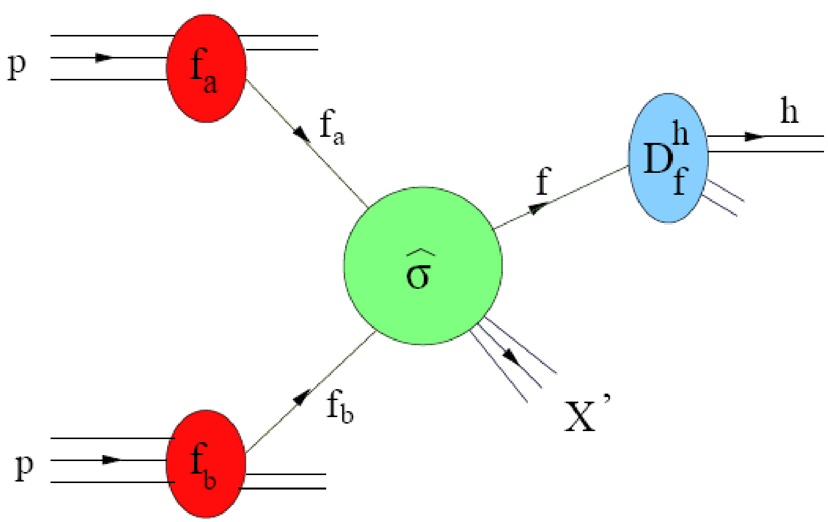
Determination of ΔG

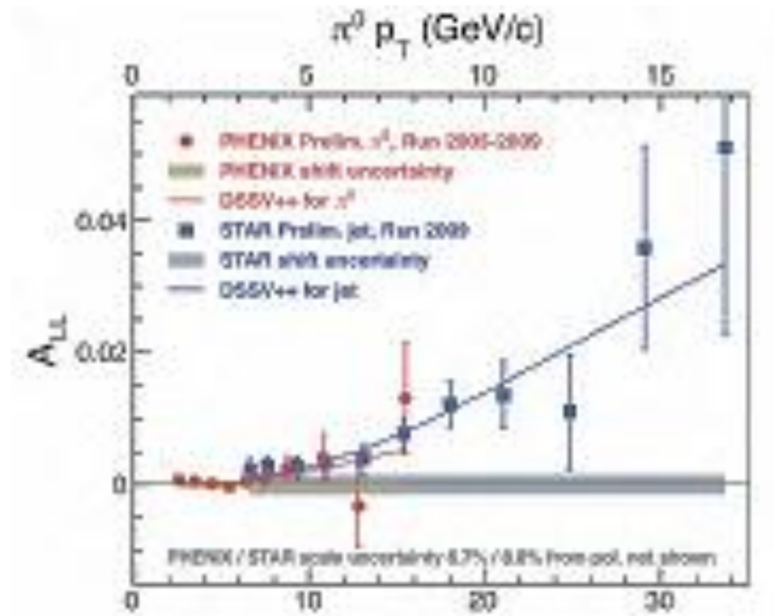
PHENIX, STAR, COMPASS, HERMES…

Longitudinal double spin asymmetry

$$A_{LL}^{\pi} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

$$\sim \sum_{a,b} \Delta f_a \quad \Delta f_b \quad \Delta\sigma_{a,b} \quad D^{\pi}$$





Latest value

$$\int_{0.05}^{0.2} dx \, \Delta G(x) = 0.1 \pm 0.06$$

DeFlorian, Sassot
Stratmann, Vogelsang

Towards measuring ΔG on a lattice

$$\Delta G(\mu) = \int dx \frac{i}{2xP^+} \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle PS|F^{+\alpha}(y^-) W \tilde{F}^+_\alpha(0)|PS\rangle$$

$$= \frac{1}{2P^+} \langle PS|\epsilon^{ij} F^{i+}(0) A^j_{phys}(0)|PS\rangle$$

The `physical part' of the gauge field Y.H. (2011)

$$A^\mu_{phys}(x) \equiv \frac{1}{D^+} F^{+\mu}$$

In the light-cone gauge, $A^\mu_{phys} = A^\mu$

$\epsilon^{ij} F^{i0} A^j = (\vec{E} \times \vec{A})^z$ is the (textbook) gluon spin operator used by Jaffe and Manohar (1990).

$$\Delta G(\mu) = \frac{1}{2P^+} \langle PS | \epsilon^{ij} F^{i+}(0) A^j_{phys}(0) | PS \rangle$$

infinite momentum frame,
gauge invariant, but nonlocal,
impossible to calculate on a lattice.

??

$$\Delta G(P_z, \mu) = \frac{1}{2P^0} \langle PS | \epsilon^{ij} F^{i0} A^j | PS \rangle$$

finite momentum frame,
gauge variant, but local
calculable on a lattice after gauge fixing.

Jaffe-Manohar in the Coulomb gauge

Ji, Zhang, Zhao (2013)

Calculate the naive gluon helicity

$$\Delta G(P_z,\mu) = \frac{1}{2P^0}\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle$$

in the Coulomb gauge $\vec{\nabla}\cdot\vec{A}=0$ → $A^\mu = A^\mu_{Coul}$

Then consider the infinite momentum frame (IMF) limit *

$$A^\mu_{Coul} \xrightarrow{\text{IMF}} A^\mu_{phys} = \frac{1}{D^+}F^{+\mu}$$

* For any 4-vector, $V^+ \to \Lambda V^+, \quad V^- \to V^-/\Lambda, \quad \vec{V}_\perp \to \vec{V}_\perp \quad (\Lambda\to\infty)$

A subtlety

In the infinite momentum limit,

$$A^{\mu}_{Coul} \longrightarrow A^{\mu}_{phys}$$

valid at the operator level

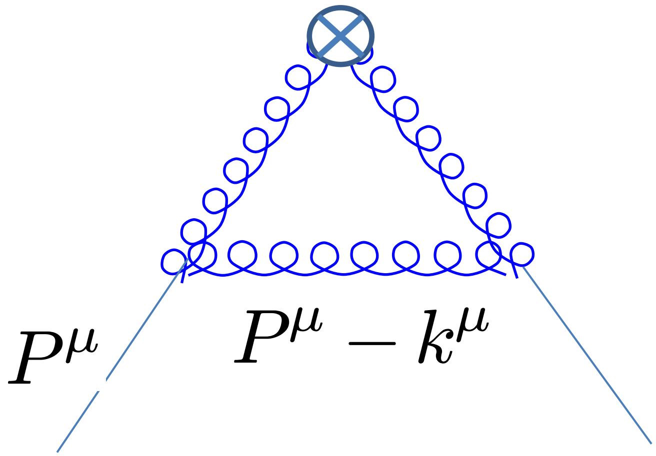
BUT

??

$$\langle \epsilon^{ij} F^{i0} A^{j}_{Coul} \rangle \not\longrightarrow \langle \epsilon^{ij} F^{i+} A^{j}_{phys} \rangle$$

The order of limits

$$\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle \sim$$



The limits $P^z \to \infty$ and $k^\mu \to \infty$
do not commute due to UV divergences.

One-loop matching

$$\Delta G = \frac{1}{2P^+}\langle PS|\epsilon^{ij}F^{i+}A^j|PS\rangle$$
$$= \frac{C_F\alpha_s}{4\pi}\left(\frac{3}{\varepsilon}+7\right)$$

Light-cone gauge $A^+ = 0$

$$\Delta G(P_z,\mu) = \frac{1}{2P^0}\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle$$

Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$

$$= \frac{C_F\alpha_s}{4\pi}\left(\frac{5}{3\varepsilon}-\frac{1}{9}+\frac{4}{3}\ln\frac{4P_z^2}{m^2}\right)+\mathcal{O}\left(\frac{m^2}{P_z^2}\right)$$

logarithmic dependence!

$$\Delta\tilde{G}(P^z,\mu) = Z_{gg}(P^z/\mu)\Delta G(\mu) + Z_{gq}(P^z/\mu)\Delta\Sigma(\mu)$$

$$Z_{gg}(P^z/\mu) = \frac{C_F\alpha_s}{4\pi}\left(\frac{4}{3}\ln\frac{4(P^z)^2}{\mu^2}-\frac{64}{9}\right)$$

Generalization

There is nothing special about the Coulomb gauge

For example, try the temporal axial gauge $A^0 = 0$

In this gauge, one can identically write

$$A^\mu = \frac{1}{D^0} F^{0\mu}$$

$$\xrightarrow{\text{IMF}} \quad \frac{1}{D^+} F^{+\mu} = A^\mu_{phys}$$

One-loop results

$$\Delta G(P_z,\mu) = \frac{1}{2P^0}\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle$$

$$= \frac{C_F\alpha_s}{4\pi}\left(\frac{5}{3\varepsilon} - \frac{1}{9} + \frac{4}{3}\ln\frac{4P_z^2}{m^2}\right)$$

Coulomb $\vec{\nabla}\cdot\vec{A} = 0$

$$= \frac{C_F\alpha_s}{4\pi}\left(\frac{3}{\varepsilon} + 7\right)$$

axial $A^0 = 0$

$$= \frac{C_F\alpha_s}{4\pi}\left(\frac{2}{\varepsilon} + 4 + \ln\frac{4P_z^2}{m^2}\right)$$

axial $A^z = 0$

$$= \frac{C_F\alpha_s}{4\pi}\left(\frac{2}{\varepsilon} + 4\right)$$

covariant $\partial\cdot A = 0$

$$= \frac{C_F\alpha_s}{4\pi}\left(\frac{3}{2\varepsilon} + \frac{7}{2}\right)$$

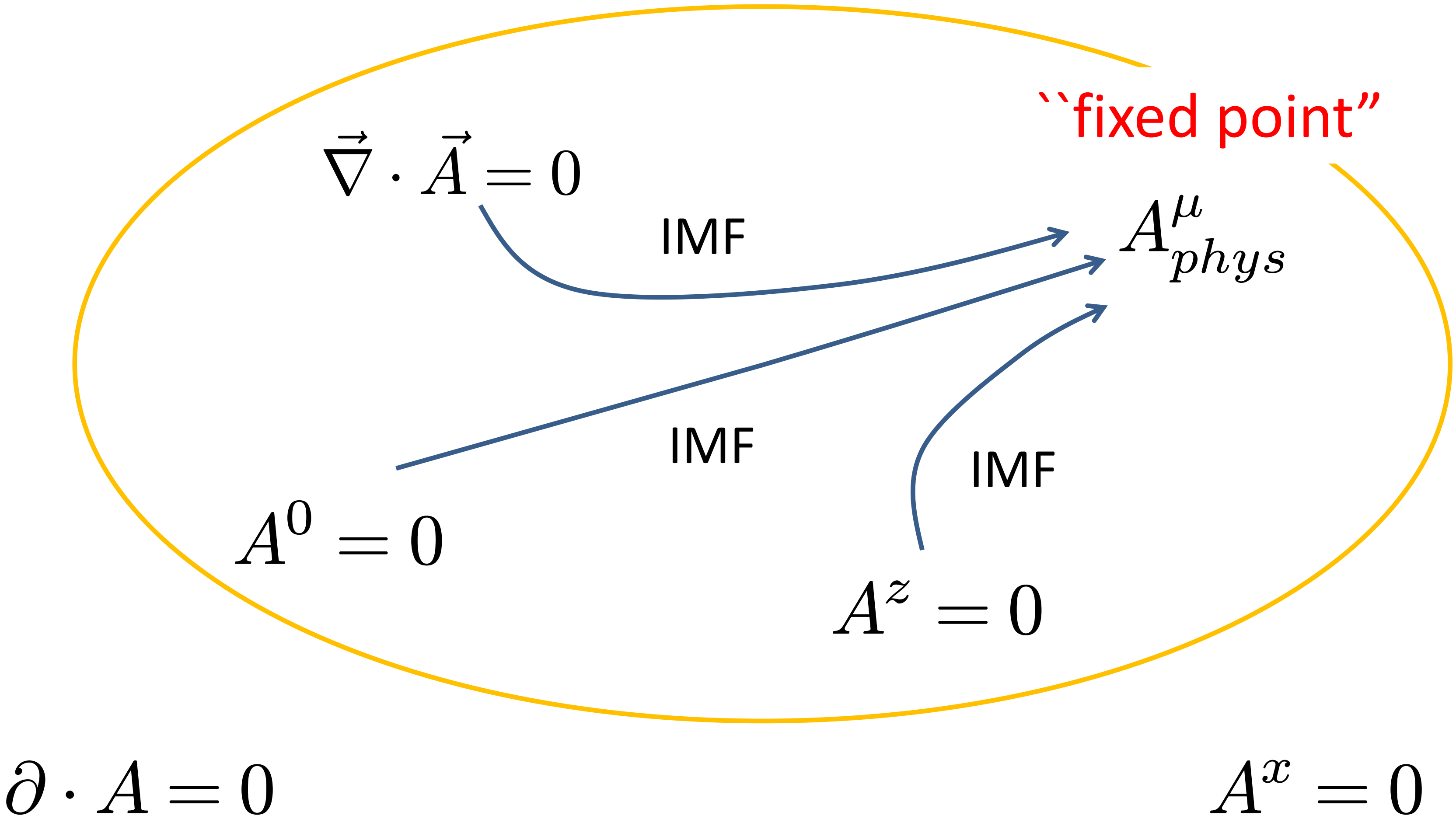
axial $A^x = 0$

One-loop results

$$\Delta G(P_z,\mu)=\frac{1}{2P^0}\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle$$

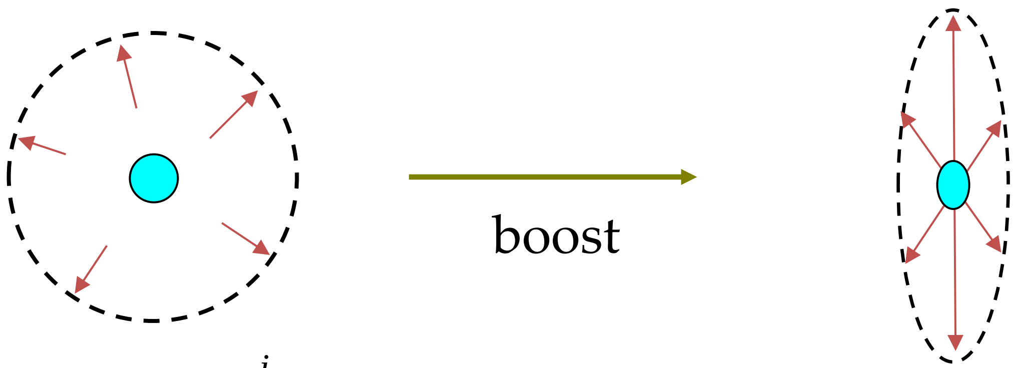
			Matching
$=\frac{C_F\alpha_s}{4\pi}\left(\frac{5}{3\varepsilon}-\frac{1}{9}+\frac{4}{3}\ln\frac{4P_z^2}{m^2}\right)$	Coulomb	$\vec{\nabla}\cdot\vec{A}=0$	✓
$=\frac{C_F\alpha_s}{4\pi}\left(\frac{3}{\varepsilon}+7\right)$	axial	$A^0=0$	✓
$=\frac{C_F\alpha_s}{4\pi}\left(\frac{2}{\varepsilon}+4+\ln\frac{4P_z^2}{m^2}\right)$	axial	$A^z=0$	✓
$=\frac{C_F\alpha_s}{4\pi}\left(\frac{2}{\varepsilon}+4\right)$	covariant	$\partial\cdot A=0$	✗
$=\frac{C_F\alpha_s}{4\pi}\left(\frac{3}{2\varepsilon}+\frac{7}{2}\right)$	axial	$A^x=0$	✗

A universality class



Weiszacker-Williams interpretation

Boosted Coulomb field



$$E^i = \frac{e}{4\pi}\frac{x^i}{r^3}$$

$$\vec{E}_T = \frac{\sqrt{2}e}{4\pi}\frac{\vec{x}_\perp}{\vec{x}_\perp^2}\delta(x^-)$$

$$\vec{\nabla}\cdot\vec{A} = 0$$
$$A^0 = 0 \quad A^z = 0$$

$$A^i_{WW} = -2e\frac{x^i_\perp}{x^2_\perp}\theta(x^-)$$

$$A^x = 0 \quad \partial\cdot A = 0$$

$$A^+_{WW} = -e\ln x^2_\perp\,\delta(x^-)$$

More on the temporal axial gauge

In the $A^0 = 0$ gauge,

$$\Delta G(P_z, \mu) = \frac{C_F \alpha_s}{4\pi} \left(\frac{3}{\varepsilon} + 7 \right)$$

Same as in the LC gauge

cf. Wakamatsu (2013)

Why? Because $\epsilon^{ij} F^{i0} A^j$ in the $A^0 = 0$ gauge is a component of the topological current, which is a Lorentz vector.

$$\partial^\mu K_\mu = F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$$K^z \sim \epsilon^{ij} \left(\underline{2F^{i0} A^j} - F^{ij} A^0 - g f_{abc} A^0_a A^i_b A^j_c \right)$$

(Forward) Matrix element of K^μ perturbatively gauge invariant.

All-order generalization

Gauge dependence due to anomaly can be precisely determined in axial gauges

$$A \cdot n = 0$$

Balitsky, Braun (1991)

$$\langle PS|K^\mu|P+q,S\rangle_N \Big|_{A\cdot n=0}$$

$$\xrightarrow{q^\mu \to 0} 4\left(S^\mu - \frac{q\cdot S}{q\cdot n}n^\mu\right)\underline{\Delta G\,(n,P)} + \frac{in^\mu}{q\cdot n}\langle PS|F_a^{\mu\nu}\tilde{F}^a_{\mu\nu}|PS\rangle_N$$

$$\Delta G(n,P) \;=\; \frac{1}{2S^\mu}\int_0^\infty d\lambda \langle PS|n^\tau F_{\tau\nu}(\lambda n) W \tilde{F}^{\nu\mu}(0)|PS\rangle_N$$

OPE ⟶ $$= \Delta G + \mathcal{O}\left(\frac{n^2}{(P\cdot n)^2}\right)$$

Zoo of operators

$$\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle_N\,\Big|_{A^0=0} = 2S^z\Delta G + \mathcal{O}(1/P_z^2)$$

naïve gluon spin!

$$\langle PS|\epsilon^{ij}F^{iz}\partial^z A^j|PS\rangle_N\,\Big|_{A^z=0} = 2S^0\Delta G + \mathcal{O}(1/P_z^2)$$

$$\begin{aligned}\int_0^\infty d\xi^0\langle PS|F^0{}_\nu(\xi^0)\mathcal{L}\tilde{F}^{\nu 0}(0)|PS\rangle_N &= \langle PS|\vec{A}^a\cdot\vec{B}^a|PS\rangle_N\,\Big|_{A^0=0} \\ &= 2S^0\Delta G + \mathcal{O}(1/P_z^2)\end{aligned}$$

$$\begin{aligned}\int_0^\infty d\xi^z\langle PS|F^z{}_\nu(\xi^z)\mathcal{L}\tilde{F}^{\nu z}(0)|PS\rangle_N &= \langle PS|\epsilon^{ij}\left(F^{i0}A^j - \frac{1}{2}A^0F^{ij}\right)|PS\rangle_N\,\Big|_{A^z=0} \\ &= 2S^z\Delta G + \mathcal{O}(1/P_z^2)\end{aligned}$$

They all give ΔG , and they are all measurable on a lattice

One-loop matching

One-loop matrix elements

in lattice perturbation theory

$$\langle P|\mathcal{O}|P\rangle_{\text{lattice}} = 1 + \alpha_s \left(\gamma \ln a^2 P^2 + c\right)$$

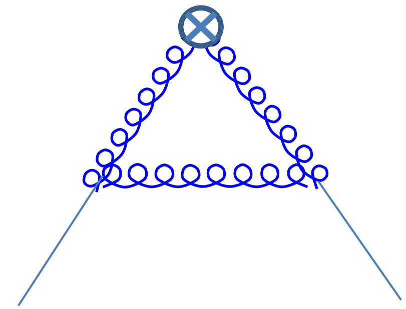
in MSbar scheme

$$\langle P|\mathcal{O}|P\rangle_{\overline{\text{MS}}} = 1 + \alpha_s \left(\gamma \ln P^2/\mu^2 + c'\right)$$

One-loop matching

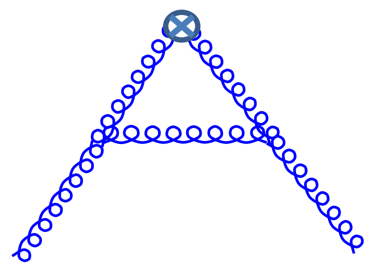
$$\langle P|\mathcal{O}|P\rangle_{\text{lattice}} = \left(1 + \alpha_s \gamma \ln a^2 \mu^2 + c - c'\right) \langle P|\mathcal{O}|P\rangle_{\overline{\text{MS}}}$$

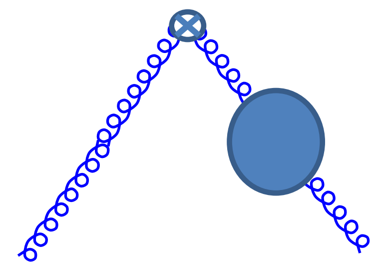
Quark matrix element



$$\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle_q \Big|_{A^0=0} = \frac{C_F\alpha_s}{4\pi}\left(\frac{3}{\varepsilon_v}+4\right)\langle PS|\bar{q}\gamma_5\gamma^z q|PS\rangle_q^{tree}$$

Gluon matrix element





$$\langle Ph|\epsilon^{ij}F^{i0}A^j|Ph\rangle_g\Big|_{A^0=0} = \left[1+\frac{\alpha_s}{4\pi}\left(\frac{\beta_0}{\varepsilon_v}+\frac{103N_c-10N_f}{9}\right)\right]\langle Ph|\epsilon^{ij}F^{i0}A^j|Ph\rangle_g^{tree}$$

We can work in the $A^+=0$ gauge to get this.

Use the Mandelstam-Leibbrandt prescription $\frac{1}{k^+}\to\frac{1}{k^+ + i\epsilon k^-}$

x -dependence?

Ji (2013)

Lin, Chen, Cohen, Ji (2014)

A proposed factorization formula

$$q(x,\mu^2,P^z) \equiv \int \frac{dz}{4\pi} e^{izxP^z} \langle P|\bar{q}(z)\gamma^z W q(0)|P\rangle$$

$$= \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \cdots$$

perturbatively calculable.

Similar formula for

$$\Delta G(x,\mu^2,P^z) = \frac{i}{2xP^+} \int \frac{dz}{2\pi} e^{izxP^z} \langle PS|F^{+\alpha}(z) W \tilde{F}^+_\alpha(0)|PS\rangle$$

Remarks

$q(x,\mu^2,P^z)$ has support for all values of x

Difficult to measure when x is small. (Need a large lattice.)

Conclusions

- ΔG can be calculated on a lattice in a universality class of gauges.
- Need matching and need large Pz
- Axial gauges are special. The argument generalizes nonperturbatively.