Recent progress of FF global analysis with the B factory data ($\sqrt{s}$=10.5 GeV)

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HKNS07: PRD75, 094009 (2007) [$\sqrt{s}$=12-91.2 GeV]

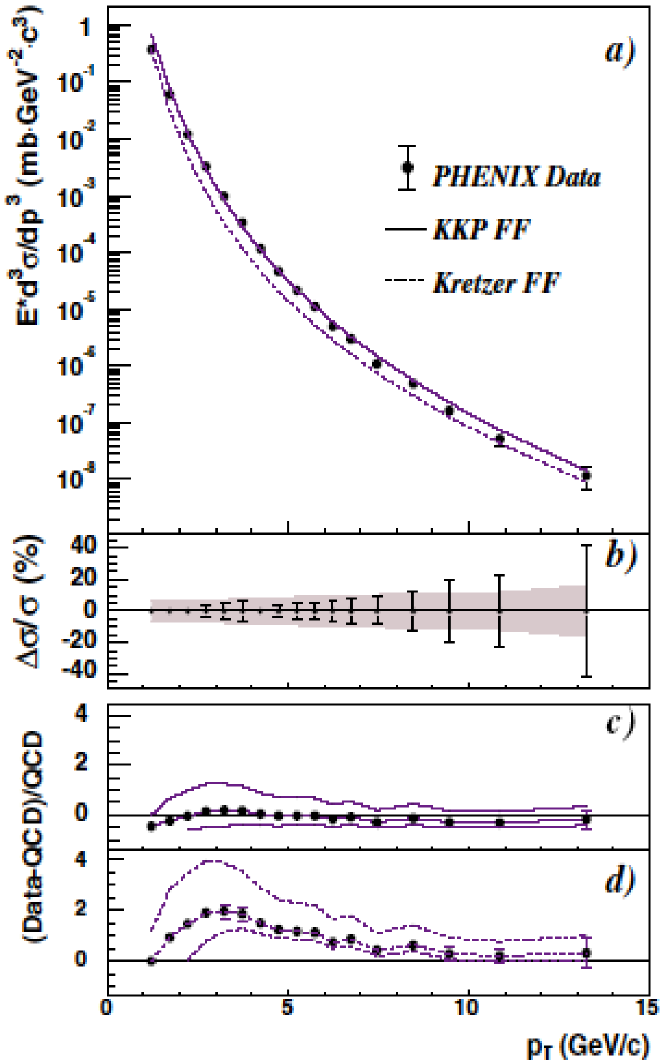
http://research.kek.jp/people/kumanos/ffs.html

http://www2.pv.infn.it/~radici/FFdatabase/

Introduction

- **Increasing needs for the FFs**
  - Hadron production process by collider experiments: $p + p \rightarrow h + X$
    - X-section: $\sigma = \sum_{a,b,c} f_a(x_a, Q^2) \otimes f_b(x_b, Q^2)$

      $\otimes \hat{\sigma}(ab \rightarrow cX) \otimes D_c^h(z, Q^2)$
    - Consistent with NLO pQCD calculation in 8th order for pion production X-sect !
    - Reference for the QGP at RHIC and LHC
    - QCD back ground for searching beyond SM
    - Determination of gluon spin component Δg
  - Semi-inclusive DIS : $\vec{e} + \vec{p} \rightarrow e + h + X$
    - Flavor decomposition of longitudinal polarized anti-quark distributions
    - SSA(Single spin asymmetry) coming from the Sivers functions

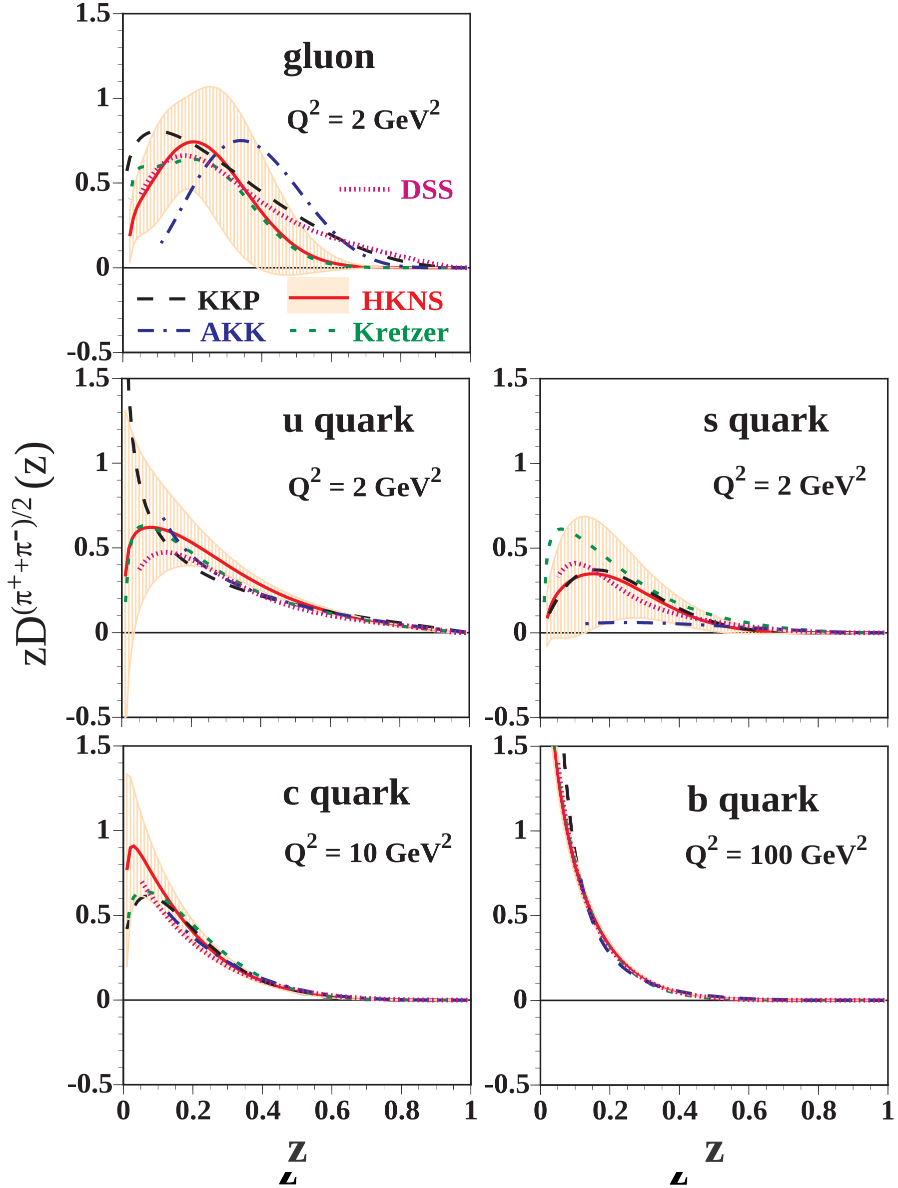


PRL91, 241803 (2003)

Fragmentation Functions & their uncertainties

Fragmentation Functions(FFs)

- Information of hadronization
  - Containing non-perturbative object
- Global analysis with experimental data of e^+e^- annihilation process
- Uncertainties of FFs?
  - Gluon FF: large uncertainty



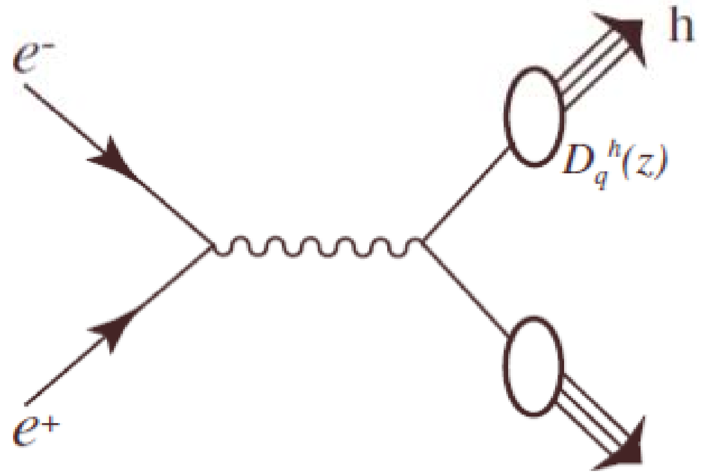
- **Model dependence of analyses?**
  - HKNS: PRD75, 09400 (2007)
  - B. A. Kniehl, G. Kremer, B. Potter: NPB582, 514 (2000)
  - Kretzer: PRD62,054001 (2000)
  - S. Albino, B. A. Kniehl, G. Kremer : NPB725, 181 (2005), NPB803, 42 (2008)
  - D. Florian, R. Sassort, M. Stratmann: PRD75, 114010 (2007)

Global analysis of FF $e^+e^- \to h^\pm X$

- **Cross sections: observable**

$$F_{theory} = \frac{1}{\sigma_{tot}}\frac{d\sigma^h}{dz}, \quad \sigma_{tot} = \sum_q \sigma_0^q \left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$

$$\frac{d\sigma^h}{dz} = \sum_i \int_z^1 \frac{d\xi}{\xi} C_i(\xi, Q^2, \mu_{F,R}^2)\, D_i^h\left(\frac{z}{\xi}, \mu_F^2\right)$$



Coefficient Function
calculable in pQCD

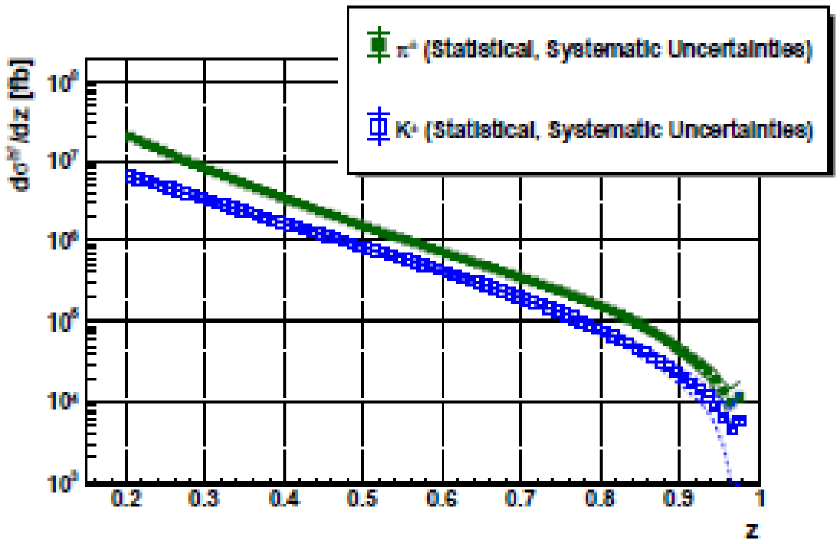
Fragmentation Function
extracted from experiments

$z \equiv \frac{2P_h \cdot q}{Q^2} = \frac{2E_h}{Q}$: scaling variable

- **DGLAP equation:**
  - $$\frac{\partial}{\partial \ln Q^2} D_j(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{z}{\xi}, \alpha_s\right) D_i^h(\xi, Q^2),$$ P_{ij} : $j \to i$ splitting function
  - Scale: $Q=\mu_F=\mu_R=\sqrt{s}$: center of mass energy

Belle experimental data

- **Final Belle data ($\sqrt{s}$ =10.52GeV)**
  - Phys. Rev. Lett. 111, 062002 (2013)
  - $Z(=2E_h/\sqrt{s})$: 0.205-0.975
  - 78 points
  - Correction factor for total x-section
    - C_{ISR}=0.65 (estimated by PYTHIA)



$$[\sigma(\mathrm{QCD}, e^+e^- \to q\bar{q}) \otimes \rho_{ISR}]\Big|_{E_{ISR}<0.5\%\cdot\sqrt{s}} \cong [\sigma(\mathrm{QCD}, e^+e^- \to q\bar{q}) \otimes \rho_{ISR}]\cdot 0.65$$

  - Differential x-section

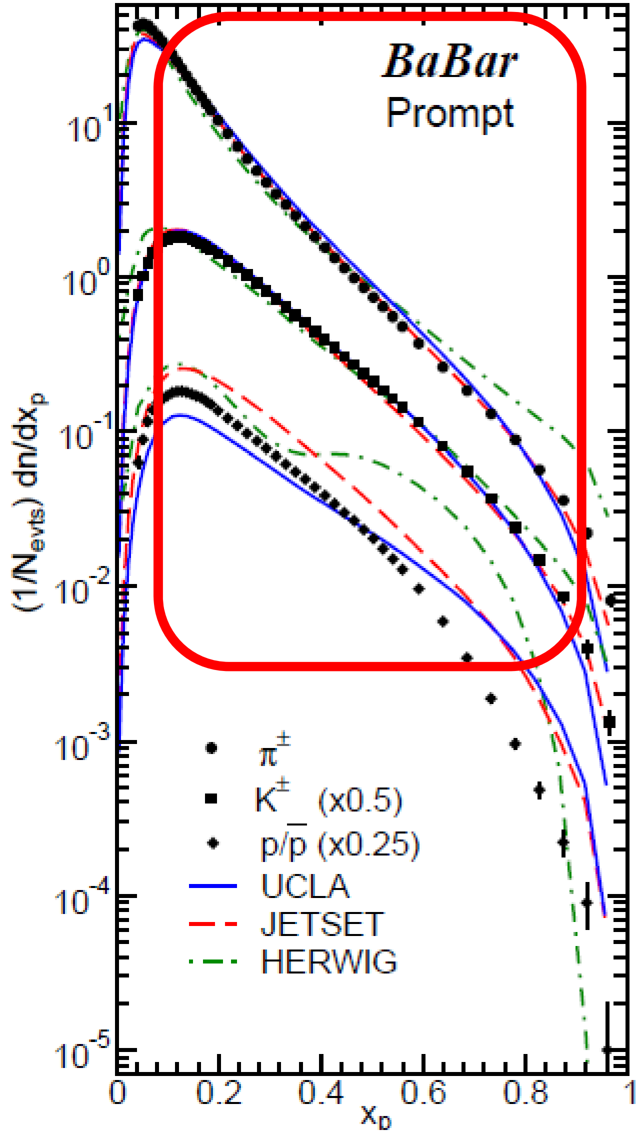
$$F_{Belle} = \frac{1}{\sigma_{\mathrm{tot.Belle}}}\left(\frac{d\sigma}{dz}\right)_{Belle} = \frac{1}{[\sigma(\mathrm{QCD}, e^+e^- \to q\bar{q}) \otimes \rho_{ISR}]\Big|_{E_{ISR}<0.5\%\cdot\sqrt{s}}} \frac{d[\sigma(\mathrm{QCD}, e^+e^- \to q\bar{q} \to \pi/\mathrm{K}+X) \otimes \rho_{ISR}]}{dz}\Bigg|_{E_{ISR}<0.5\%\cdot\sqrt{s}}$$

$$F = \frac{1}{\sigma_{\mathrm{tot.theory}}}\left(\frac{d\sigma}{dz}\right)_{theory} \overset{assumed}{=} \frac{1}{\sigma_{\mathrm{tot.theory}}\cdot 0.65}\left(\frac{d\sigma}{dz}\right)_{Belle}$$

z dependence for ISR correction is included in systematic errors

BaBar experimental data

- **Final BaBar data ($\sqrt{s}$ =10.54GeV)**
  - Phys. Rev. D88, 032011 (2013)
  - $x_p(=2p_h/\sqrt{s})$, p_h=0.225 - 5.135 (GeV)
    - z=0.051-0.972 (45 points)
    - 36 points (0.1< z < 0.83)
      - Modified leading logarithm approximation (MLLA) at the small-z (z<0.1)
      - Cutting the data for the small and large-z (unknown resummation effects)
  - Reported two data sets
    - Prompt (strongly or electromagnetic)
      - Primary hadrons or products of a decay cha in in which all particles have lifetime shorter than 10^{-11} s
    - Conventional (adding weakly decay)
      - Adding the decay daughters of particles with lifetime in the range 1-3 $\times 10^{-11}$s



Fragmentation functions of pions

- **Functional form of initial distributions at Q_0^2**

- $D_q^{\pi^+}(z,Q_0^2) = N_q^{\pi^+} z^{\alpha_q^{\pi^+}} (1-z)^{\beta_q^{\pi^+}}$,

$$\begin{cases} D_u^{\pi^+}(z,Q_0^2) = D_{\bar{d}}^{\pi^+}(z,Q_0^2), \\ D_{\bar{u}}^{\pi^+}(z,Q_0^2) = D_{d,s,\bar{s}}^{\pi^+}(z,Q_0^2), \\ D_{c=\bar{c}}^{\pi^+}(z,m_c^2),\ D_{b=\bar{b}}^{\pi^+}(z,m_b^2), \\ D_g^{\pi^+}(z,Q_0^2),\ \left(\beta_g^{\pi^+} = 8.0 \text{ fixed !}\right) \end{cases}$$

$$D_q^{\pi^-}(z) = D_{\bar{q}}^{\pi^+}(z), \quad D_q^{\pi^0}(z) = \frac{D_q^{\pi^+}(z) + D_q^{\pi^-}(z)}{2}$$

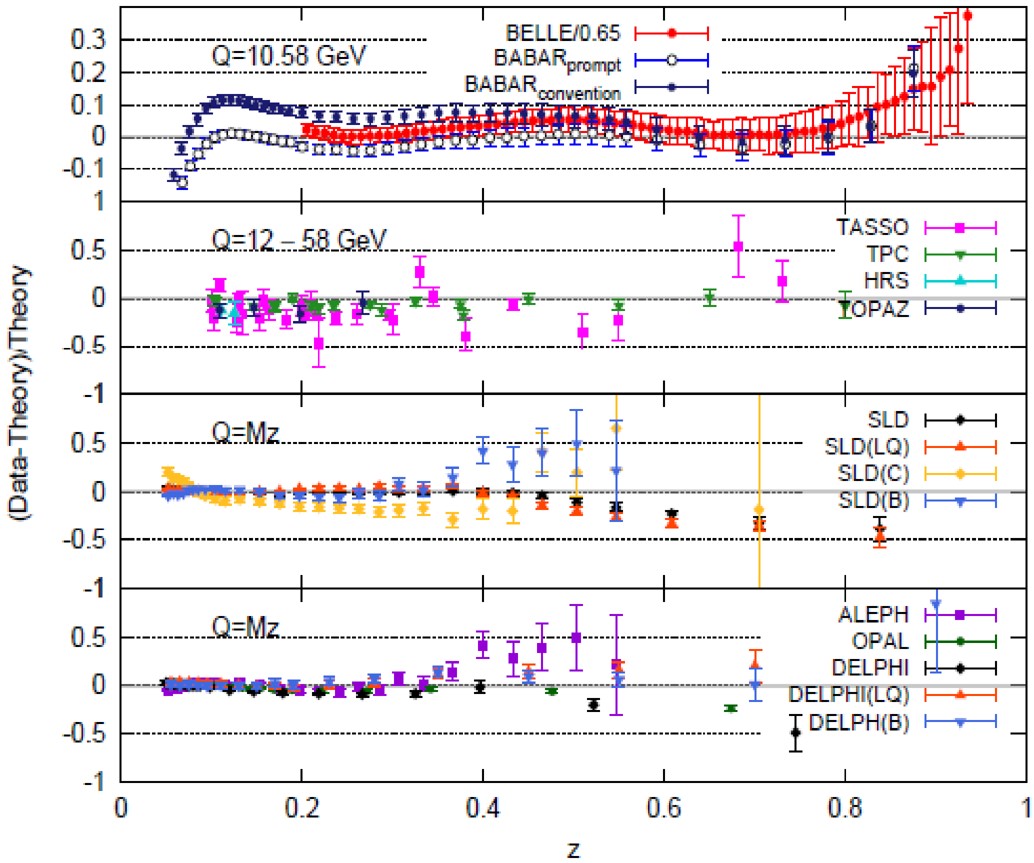
$$n_f = \begin{cases} 3,\ Q_0^2 < Q^2 < m_c^2 \\ 4,\ m_c^2 < Q^2 < m_b^2 \\ 5,\ m_b^2 < Q^2 < m_t^2 \\ 6,\ m_t^2 < Q^2 \end{cases}, \quad \begin{pmatrix} Q_0^2 = 1 \text{ GeV}^2 \\ m_c = 1.43 \text{ GeV} \\ m_b = 4.3 \text{ GeV} \end{pmatrix}$$

- **Constraint condition**
  - 2^{nd} moment should be finite and less than 1
    - $N = M^{\text{2nd}} \dfrac{\Gamma(\alpha+\beta+3)}{\Gamma(\alpha+2)\Gamma(\beta+1)}, \quad M^{\text{2nd}} \equiv \int_0^1 zD(z)dz,$
    - $\sum_h M_{q,h}^{\text{2nd}} = 1$ **(momentum conservation)**

→ $\alpha_i > -2, \quad \beta_i > 0, \quad 0 < M_q^{\text{2nd}} \left(= \int_0^1 zD_q^h(z)\, dz \right) \le 1$

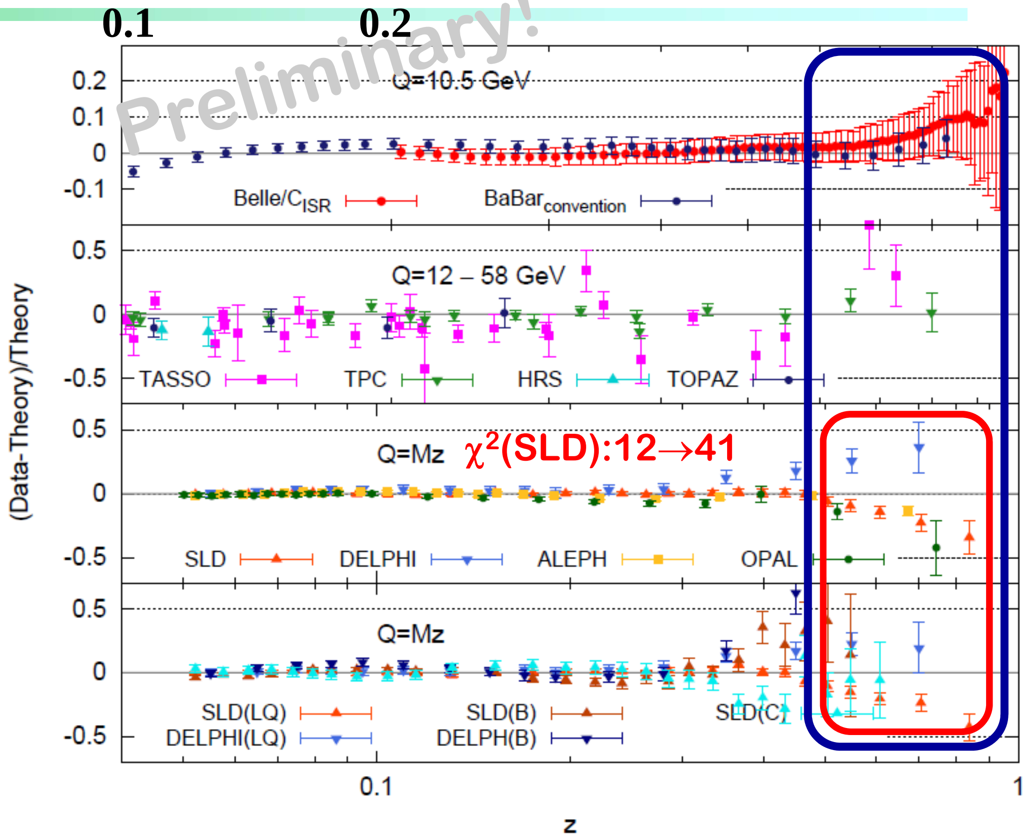
Trial analysis of FF for pion data

- **Fixing C_{ISR}=0.65 (Belle)**
- **Showing the similar behavior to the Belle and prompt data set in the range z=0.2-0.8**
- **Over shooting the $BaBar_{conventional}$ data**
- **However, appearing the critical difference between the BaBar data sets in the range z=0.2-0.5**
  - Small errors of the data ?
  - Using the conventional data set as the BaBar data for keeping consistency with other data sets



Including normalization factor in analysis

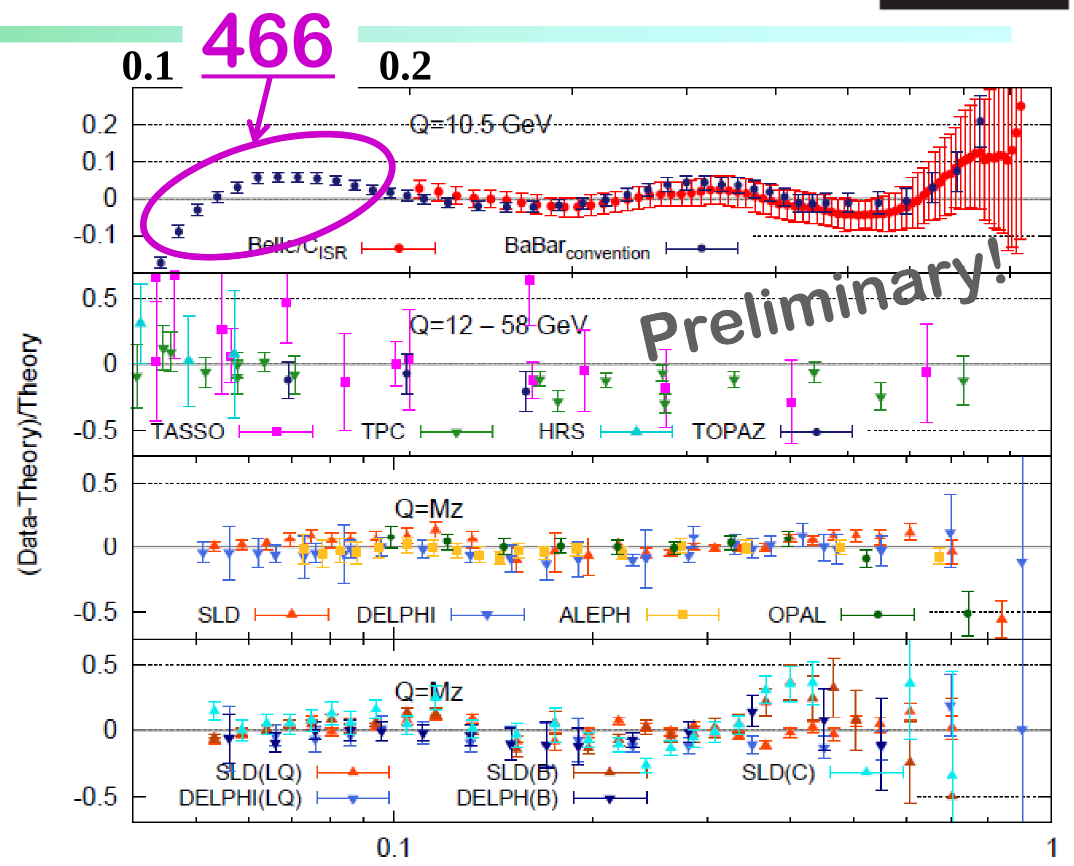
- **The d'Agostini bias**
  - $\langle T \rangle \sim T(1-f(\sigma_{Nor}^2))$
    - Optimized parameter <T> has a downward shift !
    - JHEP 1005 (2010) 075
- **Definition of χ^2 (Penalty trick)**
  - $$\chi^2 = \sum_j^n \left[\sum_i^m \frac{(T/N_j - D_i)^2}{\sigma_{\exp,i}^2} + \frac{(N_j - 1)^2}{\sigma_{Nor,j}^2} \right]$$
  - N is normalization factor which is determined by fitting
  - σ_{Nor} is normalization uncertainty of experimental data for π and k (Not include in the systematic errors in general)
    - Belle: 1.4%
    - BaBar: 0.98 %



(# of data)	χ^2 of data	χ^2 of N_{fit}	N_{fit}
Belle (78)	24.1	20.2	0.94
BaBar(36)	44.9	96.7	0.90

FFs of the k^+ from the Belle and BaBar data

- **Flavor dependence of the kaon FFs cannot be determined uniquely !**
  - Adding FF for the strange quark
  - Several solutions exist near minimum point of χ^2
  - MLLA behavior becomes pronounced
  - If these data are cut, $M_d > M_{sb}$
    - Strongly depending on a parameter of the gluon (α_g)
- **Obtained different values of the normalization factor N_{fit}**



(# of data)	χ^2 of data	χ^2 of N_{fit}	N_{fit}
Belle (78)	32.0	8.9	1.04
BaBar(42)	498	10.4	0.94

Summary

- Global analysis of FFs including the new data from the Belle and BaBar experiments
  - High precision measurements for the pion and kaon productions
  - Expect improvements for the determination of the gluon FFs via Q^2 evolution ($Q=\sqrt{s}$: 10.5 ~ 91 GeV)
- Reduction of the uncertainties for the u, d, c quarks and gluon
  - Changes of the FFs for the u, d, c quarks in the rather small-z region
    - The precise data covers in the region ($z > 0.1$)
  - Great impact on the determination of the gluon FF, especially !
  - However, increasing χ^2 of the SLD and ALEPH data for the light & charm quarks
    - Showing conflict behavior among the these data sets at the large-z, why?
- Why is so different normalization factors between pion and kaon data ?
  - Belle (σ_{nor}=1.4%) : N^{π}=0.94, N^{k}=1.04
  - BaBar (σ_{nor}=0.98%) : N^{π}=0.90, N^{k}=0.94