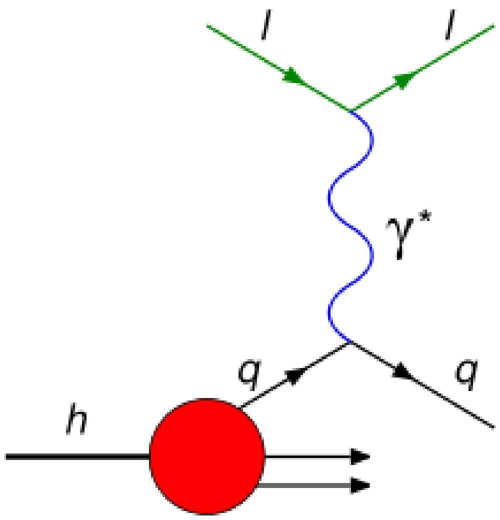
Transverse-spin gluon distribution function

Kazuhiro Tanaka (Juntendo U/KEK)

DIS



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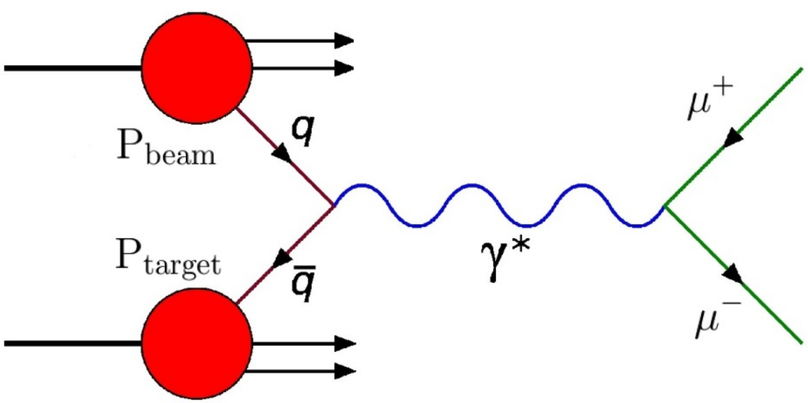
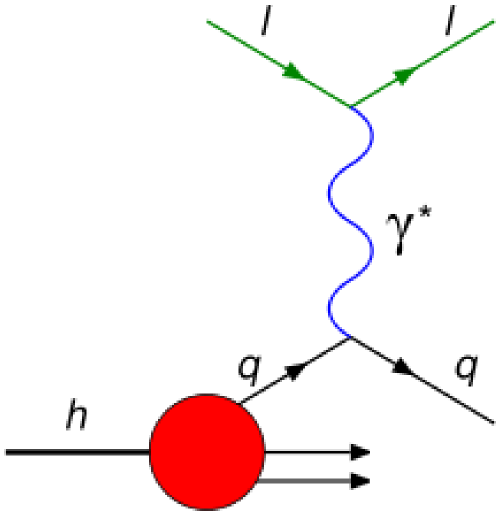
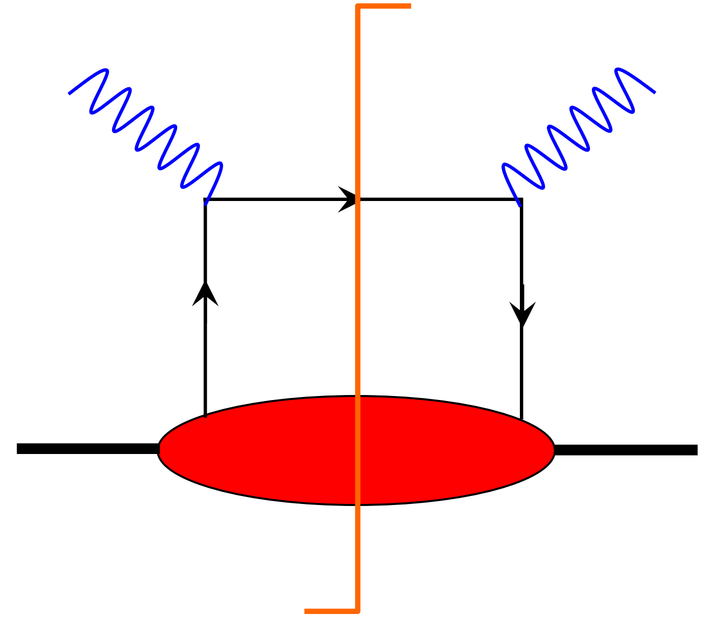


DIS





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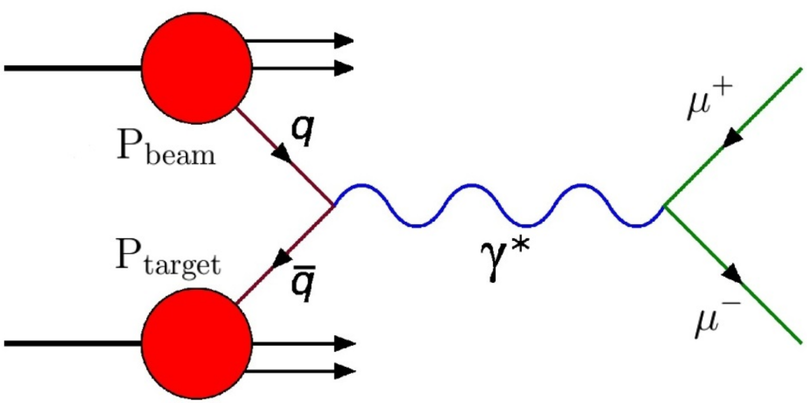
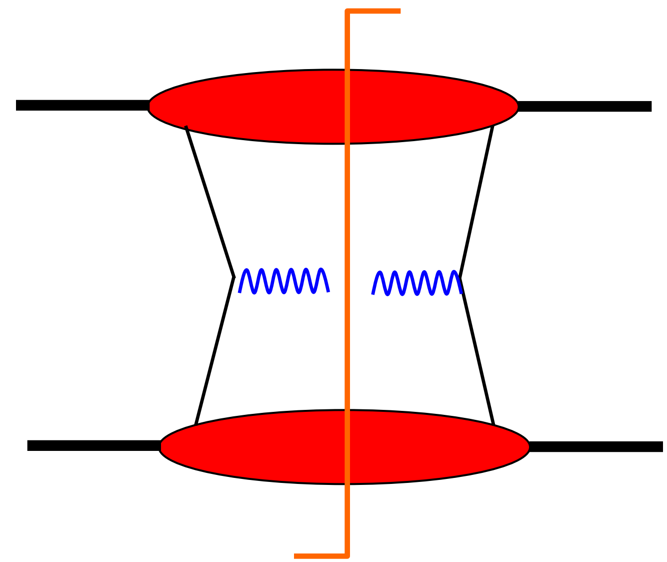
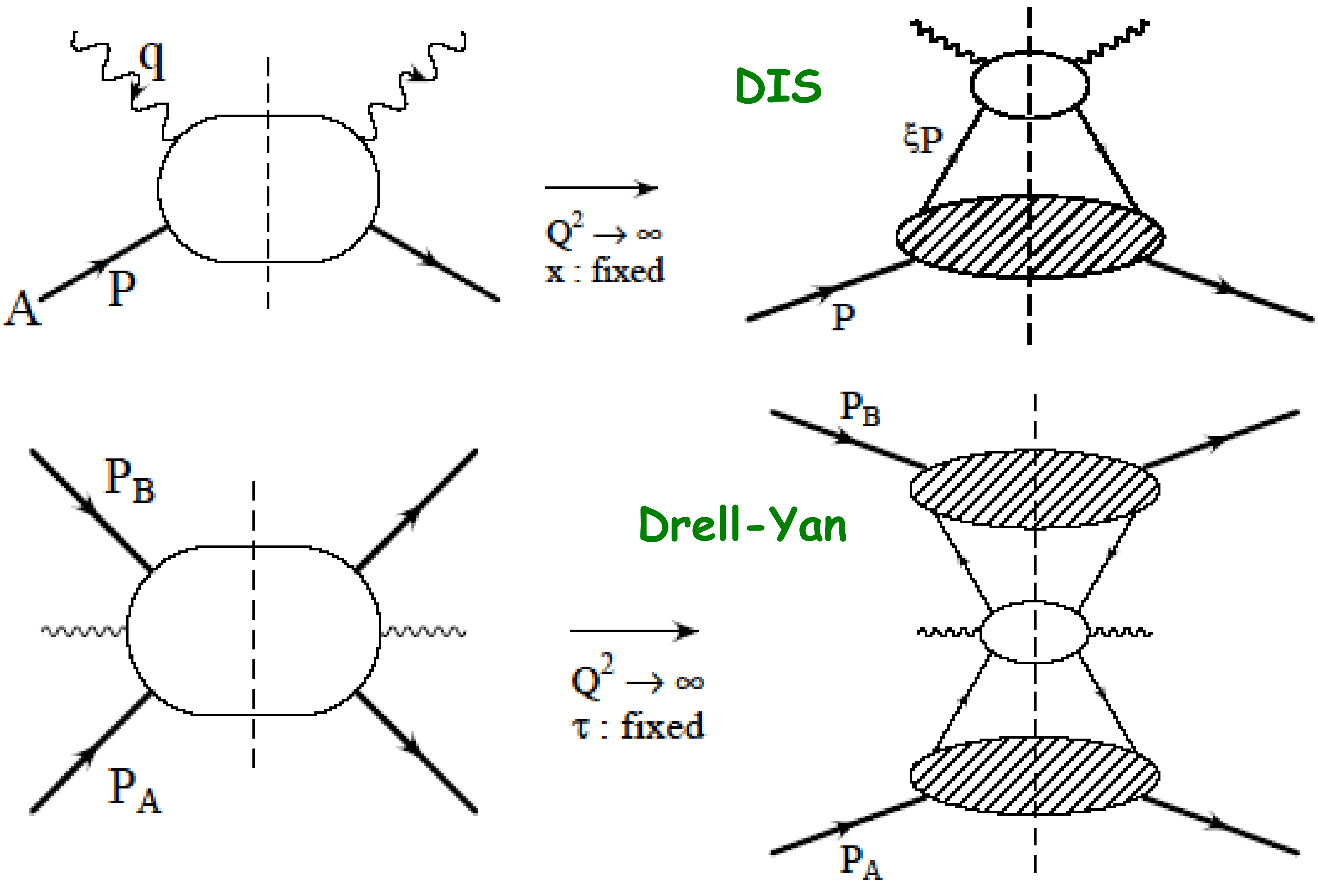
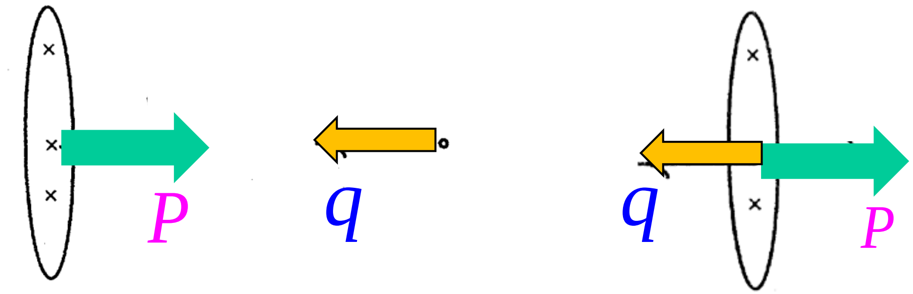




Hard processes in QCD

"hard $\otimes$ soft"

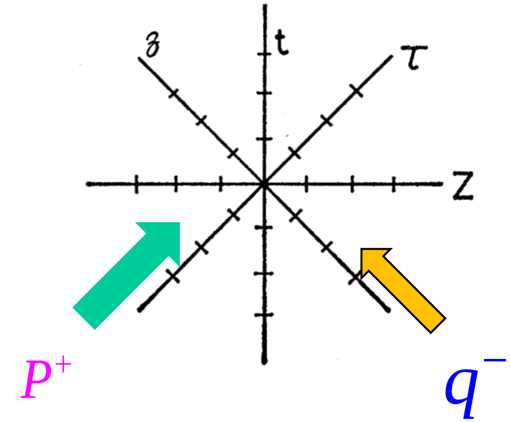


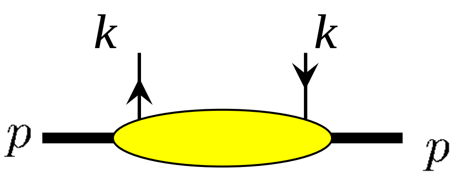


$$z^{\pm} = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$$P^{\pm} = \frac{P^0 \pm P^3}{\sqrt{2}}$$

$$\left(P^- \approx 0\right)$$





$$\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik^+ z^- - i\mathbf{k}_\perp \cdot \mathbf{z}_\perp} \left\langle P \right| \psi^\dagger(0) \;\; \psi(z^+ = 0, z^-, \mathbf{z}_\perp) \left| P \right\rangle$$

TMD

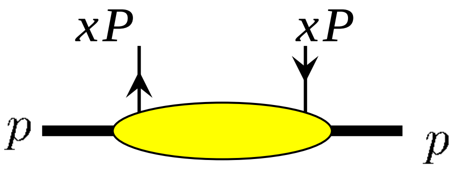
$$U(0; z^-, \mathbf{z}_\perp) = P \exp\left(ig \int_{(z^-, \mathbf{z}_\perp)}^{0} d\xi_\mu A^\mu(\xi) \right)$$

$$f(x) \sim \int \frac{dz^-}{2\pi} e^{i(xP^+) z^-} \left\langle P \right| \psi^\dagger(0) \;\; \psi(z^+ = 0, z^-, \mathbf{z}_\perp = \mathbf{0}_\perp) \left| P \right\rangle$$

PDF

$$U(0; z^-, \mathbf{z}_\perp = 0) = P \exp\left(ig \int_{z^-}^{0} d\xi^- A^+(\xi^-) \right)$$

Collins, Soper ('82)



$$f(x) \sim \int \frac{dz^-}{2\pi} e^{i(xP^+)z^-} \langle P | \psi^\dagger(0) \;\; \psi(z^-) | P \rangle$$ **PDF**

$$n^\mu = \left(0, n^-, \mathbf{0}_\perp\right) \qquad P \cdot n = P^+ n^- = 1 \qquad z^- = \lambda n^-$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S | \overline{\psi}(0) \gamma^\sigma \psi(\lambda n) | P\, S \rangle = q(x) P^\sigma$$

$$S^2 = -M_N^2$$

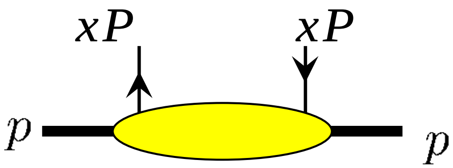
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S | \overline{\psi}(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | P\, S \rangle = \Delta q(x) \left(S \cdot n\right) P^\sigma + g_T(x) S_\perp^\sigma$$

$$S^\sigma = \left(S \cdot n\right) P^\sigma + S_\perp^\sigma$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S | \overline{\psi}(0) \sigma^{\mu\nu} i\gamma_5 \psi(\lambda n) | P\, S \rangle = \delta q(x) \frac{S_\perp^\mu P^\nu - S_\perp^\nu P^\mu}{M_N} + h_L(x) \left(P^\mu n^\nu - P^\nu n^\mu\right) M_N \left(S \cdot n\right)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S | \overline{\psi}(0) \psi(\lambda n) | P\, S \rangle = M_N e(x)$$

Jaffe, Ji ('92)



$$f(x) \sim \int \frac{dz^-}{2\pi} e^{i(xP^+)z^-} \left\langle P \middle| \psi^\dagger(0) \quad \psi(z^-) \middle| P \right\rangle$$ **PDF**

$$F^{+\alpha}(0)F^{+\beta}(z^-) \qquad F^{+\alpha} = \partial^+ A^\alpha$$

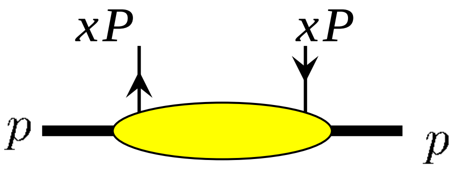
$$n^\mu = \left(0, n^-, \mathbf{0}_\perp\right) \qquad P\cdot n = P^+ n^- = 1 \qquad z^- = \lambda n^-$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S\,|\,\overline{\psi}(0)\gamma^\sigma \psi(\lambda n)\,|\,P\ S\rangle = q(x)P^\sigma$$

$$S^2 = -M_N^2$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S\,|\,\overline{\psi}(0)\gamma^\sigma \gamma_5 \psi(\lambda n)\,|\,P\ S\rangle = \Delta q(x)\left(S\cdot n\right)P^\sigma + g_T(x) S_\perp^\sigma$$

$$S^\sigma = \left(S\cdot n\right)P^\sigma + S_\perp^\sigma$$



$$f(x) \sim \int \frac{dz^-}{2\pi} e^{i(xP^+)z^-} \langle P | \psi^\dagger(0) \;\; \psi(z^-) | P \rangle$$ **PDF**

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$$S^2 = -M_N^2$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S | \bar{\psi}(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | P\, S \rangle = \Delta q(x) \left(S \cdot n\right) P^\sigma + g_T(x) S_\perp^\sigma$$

$$S^\sigma = \left(S \cdot n\right) P^\sigma + S_\perp^\sigma$$

$$-\frac{4(n^-)^2}{x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S | F^{+\nu}(0) F^{+\sigma}(\lambda n) | P\, S \rangle$$

Ji, PLB289 ('92) 137

$$= G(x) g^{\nu\sigma} + \Delta G(x) i\epsilon^{\nu\sigma P n} \left(S \cdot n\right) + \mathcal{G}_3(x) i\epsilon^{\nu\sigma\alpha n} S_{\perp\alpha}$$

$$\epsilon^{\nu\sigma P n} \equiv \epsilon^{\nu\sigma\alpha\beta} P_\alpha n_\beta$$

$$-\frac{4(n^-)^2}{x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\,S\,|\,F^{+\nu}(0)F^{+\sigma}(\lambda n)\,|\,P\,S\rangle$$

$$=G(x)g^{\nu\sigma}+\Delta G(x)i\epsilon^{\nu\sigma Pn}\left(S\cdot n\right)+\mathcal{G}_3(x)i\epsilon^{\nu\sigma\alpha n}S_{\perp\alpha}$$

$$-\frac{4}{x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\,S\,|\,F^{\mu\nu}(0)F^{\xi\sigma}(\lambda n)\,|\,P\,S\rangle$$

$$L = L_q + L_g$$ Chen, et al.; Wakamatsu; Hatta; …

$$J_{\parallel} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{\sqrt{2}} \langle P\ S_{\parallel} | \bar{\psi}(0)\gamma^{+}\gamma_5\psi(0) | P\ S_{\parallel} \rangle$$

$$\Delta G = \int dx \Delta G(x)$$

$$J_T = \frac{1}{2} = L_T + \frac{1}{2}\Delta\Sigma_T + \Delta G_T$$

Hatta, KT, Yoshida (2013)

$$\Delta\Sigma_T = \int dx g_T(x) = \langle P\ S_{\perp} | \bar{\psi}(0)\gamma^{\perp}\gamma_5\psi(0) | P\ S_{\perp} \rangle = \Delta\Sigma$$

$$\Delta G_T = \int dx G_T(x)$$

$$\Delta G_T = \Delta G \quad !$$

$$\int dx \mathcal{G}_3(x) = \Delta G$$

Hatta, KT, Yoshida (2013)

$$\int dx \mathcal{G}_2(x) = 0$$

Summary

spin-operator representation for

bilocal operator definitions of PDFs

$q(x)$	$\Delta q(x)$	$g_T(x)$
$G(x)$	$\Delta G(x)$	$G_T(x)$
density	**helicity**	**flip** "transverse spin"

$$G_T(x) = \mathcal{G}_3(x) + \mathcal{G}_2(x)$$

$$\int dx G_T(x) = \int dx \Delta G(x) = \Delta G \qquad \int dx x^{n-1} G_T(x)$$

Hard processes with $G_T(x)$?