Current Status of the Nucleon Spin Decomposition Problem

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1. **Introduction to the nucleon spin decomposition problem**

   - "canonical" or "mechanical" decomposition ? -

2. **Does the "canonical" OAM really satisfy the SU(2) C.R. ?**
3. **What is potential angular momentum ?**
4. **On the relation with deep-inelastic-scattering observables ?**
5. **Summary and conclusion**

1. Introduction to nucleon spin decomposition problem

To get a complete decomposition of nucleon spin is a fundamentally important homework of QCD.

In fact, if our researches end up without being able to accomplish this task, a tremendous efforts since the 1st EMC discovery would lead nowhere.

To reach this goal, we must answer the following two questions :

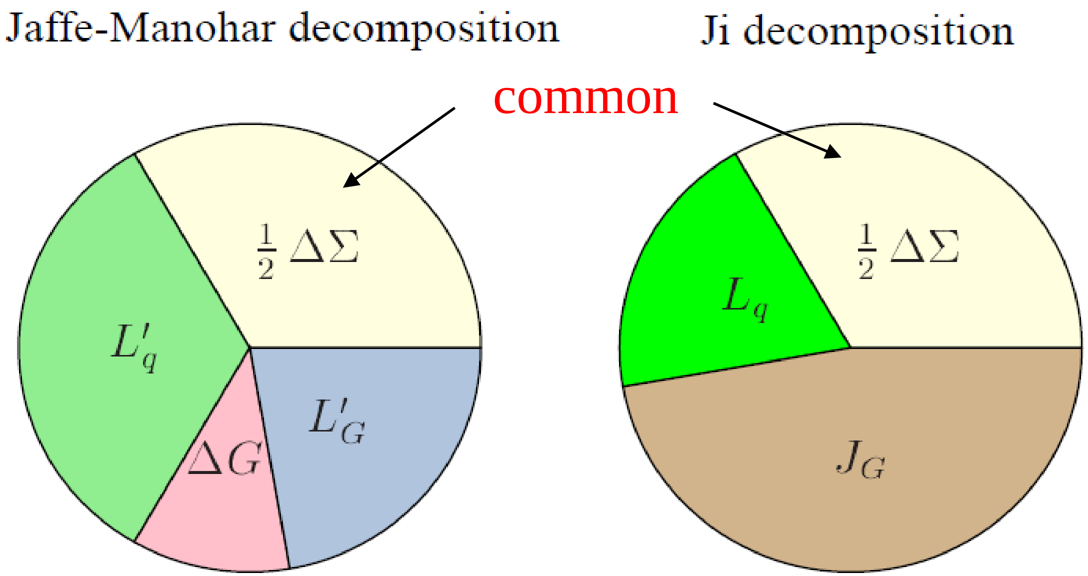
- What is a **precise (QCD) definition** of each term of the decomposition ?
- How can we extract individual term by means of **direct measurements** ?

Since QCD is a **color SU(3) gauge theory**, and since the **general principle of physics** dictates that **gauge-invariance** is a **necessary condition** of **observability** the **color gauge-invariance** plays a crucially important role in this problem.

Recently, two reviews appeared to summarize controversial status of the problem :

- E. Leader and C. Lorcé, arXiv : 1309.4235 [hep-ph]
- M. Wakamatsu, arXiv : 1402.4193 [hep-ph]

two popular decompositions of the nucleon spin



$$
\begin{aligned}
\boldsymbol{J}_{QCD} \;=\; & \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma}\, \psi\, d^3x \\
+ & \int \psi^\dagger\, \boldsymbol{x} \times \frac{1}{i} \nabla\, \psi\, d^3x \\
+ & \int \boldsymbol{E}^a \times \boldsymbol{A}^a\, d^3x \\
+ & \int E^{ai}\, \boldsymbol{x} \times \nabla\, A^{ai}\, d^3x
\end{aligned}
$$

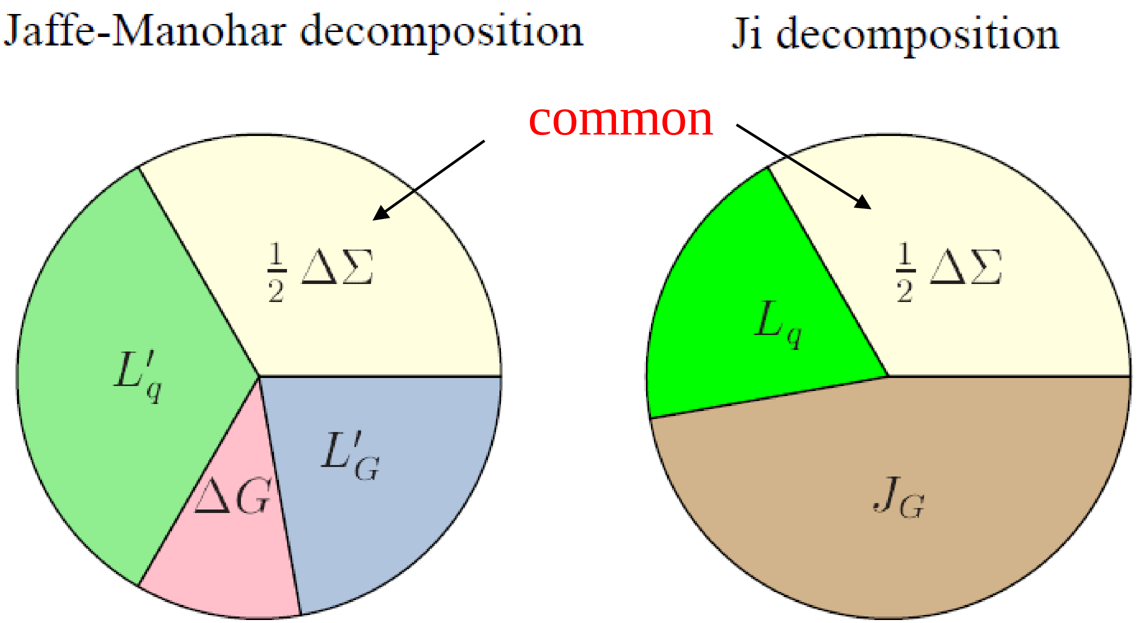
$$
\begin{aligned}
\boldsymbol{J}_{QCD} \;=\; & \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma}\, \psi\, d^3x \\
+ & \int \psi^\dagger\, \boldsymbol{x} \times \frac{1}{i} \boldsymbol{D}\, \psi\, d^3x \\
+ & \int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a)\, d^3x
\end{aligned}
$$

J_G

Each term is not separately gauge-invariant !

No further GI decomposition !

two popular decompositions of the nucleon spin (- continued -)



An especially annoying observation here was that, since

$$L'_q \neq L_q$$

one must inevitably conclude that

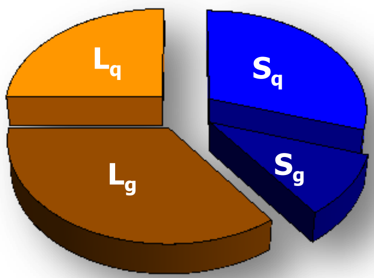
$$\Delta G + L'_G \neq J_G \ !$$

Current versions of gauge-invariant decomposition of the nucleon spin

- based on Lorcé @ JLab2013 -

Canonical family

Jaffe-Manohar (1990)

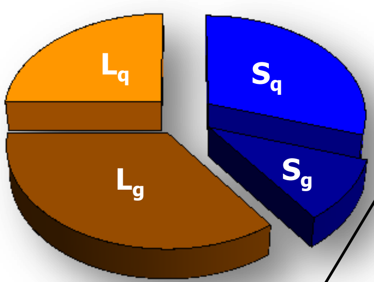


$$S_q = \frac{1}{2}\int \psi^\dagger \Sigma \psi \, d^3r$$
$$L_q = \int \psi^\dagger \boldsymbol{r} \times \frac{1}{i}\nabla \psi \, d^3r$$
$$S_g = \int \boldsymbol{E}^a \times \boldsymbol{A}^a \, d^3r$$
$$L_g = \int E^{ai} \boldsymbol{r} \times \nabla A^{ai} \, d^3r$$

key ingredient

Chen *et al.* (2008)

$$A = A_{phys} + A_{pure}$$

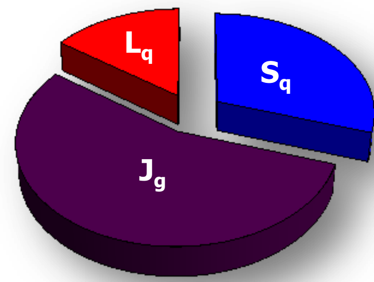


$$S_q = \frac{1}{2}\int \psi^\dagger \Sigma \psi \, d^3r$$
$$L_q = \int \psi^\dagger \boldsymbol{r} \times \frac{1}{i}\boldsymbol{D}_{pure} \psi \, d^3r$$
$$S_g = \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3r$$
$$L_g = \int E^{ai} \boldsymbol{r} \times \boldsymbol{D}_{pure} A^{ai}_{phys} \, d^3r$$

"generalized" canonical OAM

Mechanical family

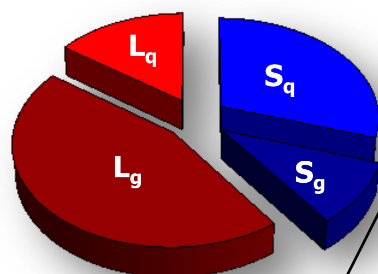
Ji (1997)



$$S_q = \frac{1}{2}\int \psi^\dagger \Sigma \psi \, d^3r$$
$$L_q = \int \psi^\dagger \boldsymbol{r} \times \frac{1}{i}\boldsymbol{D} \psi \, d^3r$$
$$J_g = \int \boldsymbol{r} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \, d^3r$$

Wakamatsu (2010)

$$A = A_{phys} + A_{pure}$$



$$S_q = \frac{1}{2}\int \psi^\dagger \Sigma \psi \, d^3r$$
$$L_q = \int \psi^\dagger \boldsymbol{r} \times \frac{1}{i}\boldsymbol{D} \psi \, d^3r$$
$$S_g = \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3r$$
$$L_g = \int E^{ai} \boldsymbol{r} \times \boldsymbol{D}_{pure} A^{ai}_{phys} \, d^3r + \int \rho^a (\boldsymbol{r} \times \boldsymbol{A}^a_{phys}) \, d^3r$$

mechanical OAM

potential OAM

The following two issues are still under debate.

(1) Is the gluon spin ΔG a gauge-invariant quantity in a true or traditional sense ?

(2) Which is a favorable decomposition from a physical viewpoint ?

A word of caution :

(1) A satisfactory answer to the 1st question must clarify the contradiction to the text book statement that the total photon angular momentum cannot be gauge-invariantly decomposed into its spin and orbital parts.

(2) Often-claimed advantages of canonical decomposition.

- Each piece of the decomposition satisfies the SU(2) commutation relation or or angular momentum algebra, for example,

$$[L^i_{can}, L^j_{can}] = i\,\epsilon^{ijk}\,L^k_{can}$$

- L_{can} is compatible with free partonic picture of constituent orbital motion.

⇩

In the present talk, I confine to the **2nd issue**, and try to show that both claims above are not necessarily true ! (See my review, for the 1st delicate problem.)

2. Does the "canonical" OAM really satisfy the angular momentum C.R. ?

Many people claim that a greatest advantage of the canonical type decomposition of the nucleon spin is that each term satisfies the angular momentum C.R. , and that this is crucial for the interpretation of each term as an angular momentum.

I will show that this is a delusion for a massless particle like the photon.

See, for instance, S.J. Van Enk and G. Nienhuis, J. of Optics 41 (1994) 963.

total angular momentum of photon

$$\boldsymbol{J} \;=\; \int d^3 r \;\, \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B})$$

decomposition of the vector potential $\boldsymbol{A}$

$$\boldsymbol{A} \;=\; \boldsymbol{A}_{\perp} \;+\; \boldsymbol{A}_{\parallel} \;\equiv\; \boldsymbol{A}_{phys} \;+\; \boldsymbol{A}_{pure}$$

corresponding decomposition of $\boldsymbol{E}$

$$\boldsymbol{E} \;=\; \boldsymbol{E}_{\perp} \;+\; \boldsymbol{E}_{\parallel}$$

with

$$\boldsymbol{E}_{\perp} \;=\; -\,\frac{\partial \boldsymbol{A}_{\perp}}{\partial t}, \qquad \boldsymbol{E}_{\parallel} \;=\; -\,\nabla A^0 \;-\; \frac{\partial \boldsymbol{A}_{\parallel}}{\partial t}$$

This gives a corresponding decomposition of the total $\boldsymbol{J}$

$$\boldsymbol{J} = \int d^3r\, r\, \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}) + \int d^3r\, r\, \boldsymbol{r} \times (\boldsymbol{E}_{\perp} \times \boldsymbol{B})$$
$$\equiv \boldsymbol{J}_{long} + \boldsymbol{J}_{trans}$$

where

$$\boldsymbol{J}_{long} = \int d^3r\, \rho\, (\boldsymbol{r} \times \boldsymbol{A}_{\perp}) \quad : \quad \text{potential angular momentum}$$

$$\boldsymbol{J}_{trans} = \int d^3r\, E_l^{\perp}\, (\boldsymbol{r} \times \nabla)\, A_l^{\perp} + \int d^3r\, \boldsymbol{E}_{\perp} \times \boldsymbol{A}_{\perp} = \boldsymbol{L} + \boldsymbol{S}$$

canonical OAM intrinsic spin

We first point out that, for **free photons**, $\boldsymbol{J}_{long} = 0$, since $\rho = 0$.

Introduce transverse mode functions $\boldsymbol{F}_{\lambda}$ with polarization λ .

$$\nabla^2 \boldsymbol{F}_{\lambda} = -k^2 \boldsymbol{F}_{\lambda}, \quad \nabla \cdot \boldsymbol{F}_{\lambda} = 0$$
$$\langle \boldsymbol{F}_{\lambda} \,|\, \boldsymbol{F}_{\lambda'} \rangle \equiv \int d^3r\, \boldsymbol{F}_{\lambda} \cdot \boldsymbol{F}_{\lambda'} = \delta_{\lambda\lambda'}$$

The simplest choice is circularly polarized plane waves :

$$\boldsymbol{F}_{\lambda} = \frac{1}{\sqrt{V}}\, \boldsymbol{\varepsilon}_{\boldsymbol{k},s}\, e^{i\,\boldsymbol{k}\cdot\boldsymbol{r}} \quad (s = \pm 1)$$

but we can also take other choices like that of paraxial laser beam.

mode expansion of transverse electromagnetic field

$$\boldsymbol{A}_\perp = \sum_\lambda \sqrt{\frac{\hbar}{2\,\omega_\lambda}}\,[\,a_\lambda\,\boldsymbol{F}_\lambda + a_\lambda^\dagger\,\boldsymbol{F}_\lambda^*\,]$$

$$\boldsymbol{E}_\perp = \sum_\lambda i\,\sqrt{\frac{\hbar\,\omega_\lambda}{2}}\,[\,a_\lambda\,\boldsymbol{F}_\lambda - a_\lambda^\dagger\,\boldsymbol{F}_\lambda^*\,]$$

$$\boldsymbol{B}_\perp = \sum_\lambda i\,\sqrt{\frac{\hbar}{2\,\omega_\lambda}}\,[\,a_\lambda\,\nabla\times\boldsymbol{F}_\lambda + a_\lambda^\dagger\,\nabla\times\boldsymbol{F}_\lambda^*\,]$$

$$[a_\lambda, a_{\lambda'}^\dagger] = \delta_{\lambda\lambda'}$$

This gives

$$\boldsymbol{S} \equiv \int d^3r\,\boldsymbol{E}_\perp\times\boldsymbol{A}_\perp = \frac{1}{2}\sum_{\lambda,\lambda'}[a_\lambda^\dagger\,a_{\lambda'} + a_{\lambda'}\,a_\lambda^\dagger]\,\langle\boldsymbol{F}_\lambda\,|\,\hat{\boldsymbol{S}}\,|\,\boldsymbol{F}_{\lambda'}\rangle$$

$$\boldsymbol{L} \equiv \int d^3r\,E_l^\perp\,(\boldsymbol{r}\times\nabla)\,A_l^\perp = \frac{1}{2}\sum_{\lambda,\lambda'}[a_\lambda^\dagger\,a_{\lambda'} + a_{\lambda'}\,a_\lambda^\dagger]\,\langle\boldsymbol{F}_\lambda\,|\,\hat{\boldsymbol{L}}\,|\,\boldsymbol{F}_{\lambda'}\rangle$$

with

$$(\hat{\boldsymbol{S}})_{ij} = -\,i\,\hbar\,\varepsilon_{ijk}, \quad \hat{\boldsymbol{L}} = -\,i\,\hbar\,(\boldsymbol{r}\times\nabla)$$

These $\hat{\boldsymbol{S}}$ and $\hat{\boldsymbol{L}}$ certainly satisfy the familiar SU(2) algebra :

$$[\,\hat{S}_i, \hat{S}_j\,] = i\,\hbar\,\hat{S}_k, \quad [\,\hat{L}_i, \hat{L}_j\,] = i\,\hbar\,\hat{L}_k$$

However, what correspond to **observables** are not $\hat{\boldsymbol{S}}$ and $\hat{\boldsymbol{L}}$ but $\boldsymbol{S}$ and $\boldsymbol{L}$, because the latters are operators acting on **physical Fock space**.

What are the C.R.'s of $\boldsymbol{S}$ and $\boldsymbol{L}$ like, then ?

Choose circularly polarized plane waves as field modes, again

$$\boldsymbol{F}_\lambda = \frac{1}{\sqrt{V}} \boldsymbol{\varepsilon}_{\boldsymbol{k},s} \, e^{i\,\boldsymbol{k}\cdot\boldsymbol{r}} \qquad (s = \pm 1)$$

In this case

$$\boldsymbol{S} = \sum_{\boldsymbol{k}} \frac{\hbar\,\boldsymbol{k}}{k} (a^\dagger_{\boldsymbol{k},1} a_{\boldsymbol{k},1} - a^\dagger_{\boldsymbol{k},-1} a_{\boldsymbol{k},-1})$$

so that

$$[S_i, S_j] = 0$$

Somewhat unexpectedly, $\boldsymbol{S}$ do not satisfy angular momentum C.R. !

Thus, $\boldsymbol{S}$ do not generate general rotations of photon polarization.

Only the components of the operator $\boldsymbol{S}$ along $\boldsymbol{k}$ is a true spin angular momentum operator, in the sense that this component generate spin rotation.

components of the operator $\boldsymbol{S}$ along $\boldsymbol{k}$ $\Longrightarrow$ **Helicity !**

What about C.R. of $\boldsymbol{L}$?

- First, total $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ must obey the standard C.R.

$$[J_i, J_j] \;=\; i\,\hbar\,\varepsilon_{ijk}\,J_k$$

- Second, $\boldsymbol{S}$ and $\boldsymbol{L}$ must transform as **vectors** under rotation, so that

$$[J_i, S_j] \;=\; i\,\hbar\,\varepsilon_{ijk}\,S_k$$
$$[J_i, L_j] \;=\; i\,\hbar\,\varepsilon_{ijk}\,L_k$$

- Combining these relations with $[S_i, S_j] \;=\; 0$, it follows that

$$[L_i, S_j] \;=\; i\,\hbar\,\varepsilon_{ijk}\,S_k$$
$$[L_i, L_j] \;=\; i\,\hbar\,\varepsilon_{ijk}\,(L_k \,-\, S_k)$$

Thus, $\boldsymbol{L}$ does not satisfy the standard angular momentum algebra either, even though it can definitely be measured !

(ex.) as orbital angular momentum of paraxial laser beam

All these **delicacies of photon spin decomposition** comes from the fact that there is **no rest frame** for **massless photon** !

We have little time to go into the detail, but, by introducing the interaction of the photon beam with atoms, the following conclusion can be drawn.

Both "spin" $\boldsymbol{S}$ and "orbital" angular momentum $\boldsymbol{L}$ of a photon are well defined quantities and can "in principle" be measured separately.

However, only the components along the propagation direction can be measured by detecting the change in internal and external angular momentum of atoms.

To sum up

> The observability of $\boldsymbol{S}^\gamma$ and $\boldsymbol{L}^\gamma$ have little to do their SU(2) C.R. !

[Note added]

The total angular momentum of a composite system must naturally satisfy the angular momentum algebra as a subgroup of Poincare algebra.

3. What is potential angular momentum ?

We ask a question "which is more physical from the observational viewpoint ?

"mechanical" OAM or generalized "canonical" OAM ?

our relation

$$\boldsymbol{L}_{can} = \boldsymbol{L}_{mech} + \boldsymbol{L}_{pot}$$

with

$$\boldsymbol{L}_{can} = \int \psi^\dagger \, \boldsymbol{r} \times \left(\frac{1}{i} \nabla - g \, \boldsymbol{A}_{pure} \right) \psi \, d^3 r$$

$$\boldsymbol{L}_{mech} = \int \psi^\dagger \, \boldsymbol{r} \times \left(\frac{1}{i} \nabla - g \, \boldsymbol{A} \right) \psi \, d^3 r$$

$$\boldsymbol{L}_{pot} = g \int \psi^\dagger \, \boldsymbol{r} \times \boldsymbol{A}_{phys} \, \psi \, d^3 r$$

This is different from the definition of Hatta and Yoshida.

$$\boldsymbol{L}_{mech} = \boldsymbol{L}_{can} + \boldsymbol{L}'_{pot},$$

with

$$\boldsymbol{L}'_{pot} = - g \int \psi^\dagger \, \boldsymbol{r} \times \boldsymbol{A}_{phys} \, \psi \, d^3 r.$$

Is it simply a matter of sign convention of $\boldsymbol{L}_{pot}$ term ?

The reason of HY definition can be easily imagined.

$$L'_{pot} \quad \Leftrightarrow \quad \texttt{quark-gluon interaction}$$
$$(\texttt{twist-3 quark-gluon correlation})$$

Then, the subtraction of L'_{pot} from L_{mech} is thought to work to eliminate the physical component A_{phys} of the gluon field, thereby leading to the generalized (gauge-invariant) canonical OAM as $L_{can} = L_{mech} - L'_{pot}$.

$$L_{can} \quad \Leftrightarrow \quad \texttt{consistent with free partonic picture of}$$
$$\texttt{quark and gluon orbital motion}$$

This picture is not necessarily justified, as we shall discuss below.

The underlying reason is that what appear in the equation of motion are the mechanical momentum and mechanical OAM, not the canonical momentum and mechanical OAM.

$$\frac{d}{dt}\, \boldsymbol{p}_{mech} = q\,[\,\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\,] \quad : \quad \texttt{Lorentz force}$$
$$\frac{d}{dt}\, \boldsymbol{L}_{mech} = q\,\boldsymbol{r} \times [\,\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\,] \quad : \quad \texttt{torque by Lorentz force}$$

Here, we shall clarify the physical meaning of the potential angular momentum, which characterizes the difference between "canonical" and "mechanical" OAM, with use of easier QED system of photons and charged particles described by

$$H = \sum_i \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2 + \frac{1}{2} \int d^3 r \, [\, \boldsymbol{E}^2 + \boldsymbol{B}^2 \,]$$

Let us start with the expression for the total angular momentum of this system.

$$\boldsymbol{J} = \sum_i \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i + \int d^3 r \, \boldsymbol{r} \times [\, \boldsymbol{E} \times \boldsymbol{B} \,]$$

mechanical OAM $\qquad\qquad \boldsymbol{J}^\gamma$

There is no doubts that the two terms of the r.h.s are both gauge-invariant.

As mentioned, the vector potential $\boldsymbol{A}$ of the photon field can be decomposed into transverse and longitudinal components as

$$\boldsymbol{A} = \boldsymbol{A}_\perp + \boldsymbol{A}_{||}$$

with the divergence-free and irrotational conditions :

$$\nabla \cdot \boldsymbol{A}_\perp = 0, \qquad \nabla \times \boldsymbol{A}_{||} = 0$$

This transverse-longitudinal decomposition is **unique**, once the Lorentz frame of reference is specified. Under a general gauge-transformation given by

$$\boldsymbol{A} \;\rightarrow\; \boldsymbol{A}' \;=\; \boldsymbol{A} \;+\; \nabla\Lambda(x), \quad A^0 \;\rightarrow\; A'^0 \;=\; A^0 \;-\; \frac{\partial}{\partial t}\Lambda(x)$$

the transverse and longitudinal components transform as

$$\boldsymbol{A}_\perp \;\rightarrow\; \boldsymbol{A}'_\perp \;=\; \boldsymbol{A}_\perp, \quad \boldsymbol{A}_{||} \;\rightarrow\; \boldsymbol{A}'_{||} \;+\; \nabla\Lambda(x)$$

indicating that $\boldsymbol{A}_{||}$ carries **unphysical gauge degrees of freedom** !

To avoid misunderstanding, we emphasize that the above transverse-longitudinal decomposition should not be confused with the Coulomb gauge fixing.

The Coulomb gauge fixing is to require $\nabla \cdot \boldsymbol{A} = 0$.

Because $\nabla \cdot \boldsymbol{A}_\perp \;=\; 0$ by definition, this is equivalent to requiring that

$$\nabla \cdot \boldsymbol{A}_{||} \;=\; 0 \quad : \quad \texttt{Coulomb gauge condition}$$

Now that $\boldsymbol{A}_{||}$ is divergence-free as well as irrotational by definition, one can set

$$\boldsymbol{A}_{||} \;=\; 0 \quad \texttt{in Coulomb gauge}$$

Without gauge-fixing, the decomposition can be made as follows :

$$\begin{aligned} \boldsymbol{J}^\gamma &= \int d^3r\ \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B}) \\ &= \int d^3r\ \boldsymbol{r} \times (\boldsymbol{E}_\perp \times \boldsymbol{B}) \ + \ \int d^3r\ \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}) \\ &\equiv \qquad \boldsymbol{J}_{trans} \qquad + \qquad \boldsymbol{J}_{long} \end{aligned}$$

Using the Gauss law $\nabla \cdot \boldsymbol{E}_{||} = \rho$, the 2nd part can also be written in the form :

$$\boldsymbol{J}_{long} = \sum_i q_i\ \boldsymbol{r}_i \times \boldsymbol{A}_\perp(\boldsymbol{r}_i)$$

I called it the "**potential angular momentum**" term. It is solely gauge-invariant. It is also important to recognize that **this term vanishes for free photon**, i.e. if there is no charged particle sources for photon.

The 1st part can further be split into 2 pieces in a gauge-invariant way :

$$\boldsymbol{J}_{trans} = \int d^3r\ E^l_\perp\ (\boldsymbol{r} \times \nabla)\ A^l_\perp \ + \ \int d^3r\ \boldsymbol{E}_\perp \times \boldsymbol{A}_\perp$$

photon canonical OAM **intrinsic photon spin**

definitely gauge-invariant !

To sum up, the total angular momentum of the photon can be split into 3 pieces as

$$\boldsymbol{J}^{\gamma} = \boldsymbol{J}_{long} + \boldsymbol{J}_{trans}$$

$$\boldsymbol{J}_{long} = \sum_i q_i \, \boldsymbol{r}_i \times \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) \Rightarrow \text{potential angular momentum}$$

$$\boldsymbol{J}_{trans} = \int d^3 r \, E^l_{\perp} (\boldsymbol{r} \times \nabla) A^l_{\perp} + \int d^3 r \, \boldsymbol{E}_{\perp} \times \boldsymbol{A}_{\perp}$$

What happens if we combine the potential angular momentum term with the "mechanical OAM" of charged particles ? We get

$$\sum_i \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i + \sum_i \boldsymbol{r}_i \times q_i \boxed{\boldsymbol{A}_{\perp}(\boldsymbol{r}_i)} = \sum_i \boldsymbol{r}_i \times \left(\boldsymbol{p}_i - q_i \boxed{\boldsymbol{A}_{\parallel}(\boldsymbol{r}_i)} \right)$$

Here, we have used the usual definition of the canonical momentum $\boldsymbol{p}_i$

$$\begin{aligned} \boldsymbol{p}_i &\equiv m_i \dot{\boldsymbol{r}}_i - q_i \boldsymbol{A}(\boldsymbol{r}_i) \\ &= m_i \dot{\boldsymbol{r}}_i - q_i \left(\boxed{\boldsymbol{A}_{\parallel}(\boldsymbol{r}_i)} + \boxed{\boldsymbol{A}_{\perp}(\boldsymbol{r}_i)} \right) \end{aligned}$$

This leads to the gauge-invariant “canonical” decomposition *a la* Chen et al.

$$\boldsymbol{J} \;=\; \boldsymbol{L}'_p \,+\, \boldsymbol{S}'_\gamma \,+\, \boldsymbol{L}'_\gamma$$

where

$$\begin{aligned} \boldsymbol{L}'_p &= \sum_i \boldsymbol{r}_i \times (\boldsymbol{p}_i - q_i \boxed{\boldsymbol{A}_{||}(\boldsymbol{r}_i)}) \;\Rightarrow\; \sum_i \boldsymbol{r}_i \times \frac{1}{i} \boldsymbol{D}_{i,pure} \\ \boldsymbol{S}'_\gamma &= \int d^3 r \, \boldsymbol{E}_\perp \times \boldsymbol{A}_\perp \\ \boldsymbol{L}'_\gamma &= \int d^3 r \, E^k_\perp \, (\boldsymbol{r} \times \nabla) \, A^k_\perp \end{aligned}$$

The gauge-invariance of the 1st term can easily be convinced from the gauge transformation property of the longitudinal component

$$\boldsymbol{A}_{||}(\boldsymbol{r}_i) \;\rightarrow\; \boldsymbol{A}_{||}(\boldsymbol{r}_i) \;+\; \nabla \Lambda(\boldsymbol{r}_i)$$

and the gauge transformation property of quantum mechanical w.f. of charged particle system :

$$\Psi(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_N) \;\rightarrow\; \left(\prod_i^N e^{i\, q_i \, \Lambda(\boldsymbol{r}_i)} \right) \Psi(\boldsymbol{r}_1, \cdots \boldsymbol{r}_N)$$

I emphasize once again that the pure gauge covariant derivative in the Chen formalism appears quite naturally or automatically.

The gauge degrees of freedom, carried by the longitudinal component

$$\boldsymbol{A}_{pure} \equiv \boldsymbol{A}_{||}$$

is not introduced by hand. **It exists from the beginning in the original theory !**

The Chen decomposition is not a GIE by the Stückelberg trick !

Note however that the Chen decomposition is not only one GI decomposition !

Because the potential angular momentum

$$\boldsymbol{J}_{long} = \sum_i q_i \, \boldsymbol{r}_i \times \boldsymbol{A}_\perp(\boldsymbol{r}_i) = \int d^3r \, \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}_\perp)$$

is solely gauge-invariant, we can **leave** it in the **photon part**, which leads to another GI decomposition.

"mechanical" decomposition in our terminology “mechanical” decomposition

$$\boldsymbol{J} \;=\; \boldsymbol{L}_p \;+\; \boldsymbol{S}_\gamma \;+\; \boldsymbol{L}_\gamma$$

where

$$\boldsymbol{L}_p \;=\; \sum_i m_i\, \boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i = \sum_i m_i\, \boldsymbol{r}_i \times (\,\boldsymbol{p}_i - q_i\, \boldsymbol{A}(\boldsymbol{r}_i)\,) \Rightarrow \sum_i \boldsymbol{r}_i \times \frac{1}{i}\, \boldsymbol{D}_i$$

$$\boldsymbol{S}_\gamma \;=\; \boldsymbol{S}'_\gamma \;=\; \int d^3 r\, \boldsymbol{E}_\perp \times \boldsymbol{A}_\perp$$

$$\boldsymbol{L}_\gamma \;=\; \int d^3 r\; E^k_\perp\, (\boldsymbol{r} \times \nabla)\, A^k_\perp \;+\; \int d^3 r\; \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}_\perp)$$

canonical OAM term **potential OAM term**

characteristic features

- The difference between the two decompositions exist only in orbital parts.
- The intrinsic photon spin part is just common in the two decompositions.

another important remark

interaction term ?

It is a wide-spread belief that, among the following two quantities :

$$\boldsymbol{L}_{can} = \boldsymbol{r} \times \boldsymbol{p} \quad \Longleftrightarrow \quad \boldsymbol{L}_{mech} = \boldsymbol{r} \times (\boldsymbol{p} - e\,\boldsymbol{A}_{\perp})$$

what is closer to a physical image of orbital motion is the former, because the latter appears to contain an extra **interaction term** with the gauge field !

The fact is just opposite !

$$\begin{aligned} \boldsymbol{L}_{\text{“}can\text{”}} &= \boldsymbol{L}_{mech} + \sum_i \boldsymbol{r}_i \times q_i\,\boldsymbol{A}_{\perp}(\boldsymbol{r}_i) \\ &= \sum_i m_i\,\boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i + \int d^3r\,\boldsymbol{r} \times (\boldsymbol{E}_{\parallel} \times \boldsymbol{B}_{\perp}) \end{aligned}$$

orbital motion !

- What has a natural physical interpretation as orbital motion of particles is the “mechanical” OAM $\boldsymbol{L}_{mech}$ not the “canonical” OAM $\boldsymbol{L}_{\text{“}can\text{”}}$!
- It may sound **paradoxical**, but what contains an extra interaction term is rather the “canonical” angular momentum than the “mechanical” angular momentum !

One might suspect that the argument above is just a matter of philosophy.

Naturally, what discriminates physics from philosophy is observation !

In what follows, I will show that the above-mentioned difference between the "canonical" OAM and the "mechanical" OAM has an important influence on their observability by means of high-energy DIS measurements.

If we say to Shakespeare's style

Which does the nature favor ?

"canonical" decomposition or "mechanical" decomposition ?

That is the question !

4. On the relation with deep-inelastic-scattering observables ?

Historically, it was a common belief that the canonical OAM appearing in the Jaffe-Manohar decomposition would not correspond to observables, because they are not gauge-invariant quantities.

This nebulous impression did not change even after a gauge-invariant version of the Jaffe-Manohar decomposition by Bashinsky and Jaffe or that by Chen et al. appeared.

However, the situation has changed drastically after Lorcé and Pasquini showed that the canonical quark OAM can be related to a certain moment of a quark distribution function in a phase space, called the Wigner distribution.

$$\rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp; \mathcal{W}) = \int \frac{d^2 \boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i \boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(x \bar{P}^+ z^- - \boldsymbol{k}_\perp \cdot \boldsymbol{z}_\perp)} \times \langle P'^+, \frac{\boldsymbol{\Delta}_\perp}{2}, S \,|\, \bar{\psi}\left(-\frac{z}{2}\right) \gamma^+ \mathcal{W} \psi\left(\frac{z}{2}\right) |\, P^+, -\frac{\boldsymbol{\Delta}_\perp}{2}, S \rangle \,|_{z^+=0}$$

$x = k^+ / \bar{P}^+$, $\quad \boldsymbol{k}_\perp$: transverse momentum

$\mathcal{W}$: gauge-link, $\quad \boldsymbol{b}_\perp$: impact parameter

A natural definition of quark OAM density in the phase-space by Lorcé-Pasquini

$$L_z^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W}) \;=\; (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z \; \rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W})$$

After integrating over x, $\boldsymbol{k}_\perp$, and $\boldsymbol{b}_\perp$, they found a remarkable relation

$$\langle L_z^q \rangle^{\mathcal{W}} \;=\; \int dx\, d^2k_\perp\, d^2b_\perp\, L_z^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W}) \;=\; - \int dx\, d^2k_\perp\, \frac{\boldsymbol{k}_\perp^2}{M^2}\, F_{1,4}^q(x, 0, \boldsymbol{k}_\perp^2, 0, 0, \mathcal{W})$$

where

$$\begin{aligned} \rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp ; \mathcal{W}) \;&=\; F_{1,1}^q(x, \boldsymbol{k}_\perp^2, \boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp, \boldsymbol{b}_\perp^2 ; \mathcal{W}) \\ &-\; \frac{1}{M^2} (\boldsymbol{k}_\perp \times \nabla_{\boldsymbol{b}_\perp})_z\, F_{1,4}^q(x, \boldsymbol{k}_\perp^2, \boldsymbol{k}_\perp \cdot \boldsymbol{b}_\perp, \boldsymbol{b}_\perp^2 ; \mathcal{W}) \end{aligned}$$

A delicacy here is that the Wigner distribution ρ^q generally depends on the chosen path of the gauge-link $\mathcal{W}$ connecting the points $z/2$ and $-z/2$.

As shown by a careful study by Hatta, with the choice of a staple-like gauge-link in the light-front direction, corresponding to the kinematics of the semi-inclusive reactions or the Drell-Yan processes, the above quark OAM turns out to coincide with the (gauge-invariant) canonical quark OAM not the dynamical OAM :

$$L_{can}^q \;=\; \langle L_z^q \rangle^{\mathcal{W} = \mathcal{W}^{LC}}$$

This observation holds out a hope that the canonical quark OAM in the nucleon would also be a measurable quantity, at least in principle.

In a recent paper (arXiv:1310.5157), however, Courtoy et al. throws a serious doubt on the practical observability of the Wigner function F_{14}^q appearing in the above intriguing sum rule.

According to them, even though F_{14}^q may be nonzero in particular models and also in real QCD, its observability would contradict the following observations :

- it drops out in both the formulation of GPDs and TMDs ;
- it is parity-odd (this statement may be wrong !)
- it is nonzero only for imaginary values of the quark-proton helicity amplitudes.

Anyhow, their observations indicate that F_{14}^q would not appear in the cross section formulas of any DIS processes at least at the leading order approximation.

What is indicated by their arguments is the fact that the existence of a simple partonic picture of the canonical quark OAM in the Fock space and its observability are different things.

It appears to us that this takes a discussion on the observability of the canonical OAM back to its starting point ?

What about observability of another OAMs, i.e. the mechanical OAMs, then ?

already known (indirect) relation

$$L^q_{mech} = \frac{1}{2}\int x\,[\,H^q(x,0,0) + E^q(x,0,0)\,]\,dx - \frac{1}{2}\int \tilde{H}^q(x,0,0)\,dx, \quad \text{(X. Ji)}$$

$$L^G_{mech} = \frac{1}{2}\int x\left[H^G(x,0,0) + E^G(x,0,0)\right]dx - \int \tilde{H}^G(x,0,0)\,dx \quad \text{(M. W.)}$$

more direct relation with GPD

due to Penttinen et al. (2000), Kiptily and Polyakov (2004), Hatta and Yoshida (2012)

$$L^q_{mech} = -\int x\, G^q_2(x,0,0)\,dx$$

where

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{i\,x\,\bar{P}^+ z^-}\,\langle P', S' \,|\, \bar{\psi}\left(-\frac{z^-}{2}\right)\gamma^j\,\mathcal{W}\,\psi\left(\frac{z^-}{2}\right) |\, P, S\rangle$$

$$= \frac{1}{2\,\bar{P}^+}\,\bar{u}(P',S')\left[\frac{\Delta^j_\perp}{2\,M}\,G^q_1 + \gamma^j\,(\,H^q + E^q + G^q_2\,) + \frac{\Delta^j_\perp\,\gamma^+}{P^+}\,G^q_3 + \frac{i\,\epsilon^{jk}_T\,\Delta^k_\perp\,\gamma^+\gamma_5}{P^+}\,G^q_4\right]u(P,S)$$

An interesting observation by Kiptily and Polyakov

$$G_2^q(x,0,0) \;=\; G_2^{q,WW}(x,0,0) \;+\; \bar{G}_2^q(x,0,0)$$

WW part　　　　genuine twist-3

The WW part is represented by the forward limits of the 3 twist-2 PDFs as

$$G_2^{q,WW}(x,0,0) \;=\; -\,(H^q(x,0,0) + E^q(x,0,0)) \;+\; \frac{1}{x}\,\tilde{H}^q(x,0,0)$$
$$+\; \int_x^{\epsilon(x)} \frac{dy}{y}\,(H^q(y,0,0) \;+\; E^q(y,0,0)) \;-\; \int_x^{\epsilon(x)} \frac{dy}{y^2}\,\tilde{H}^q(y,0,0)$$

whereas the 2nd moment of the genuine twist-3 part of G_2^q vanishes.

$$\int_{-1}^{1} x\,\bar{G}_2^q(x,0,0)\,dx \;=\; 0 \qquad \left(\text{remember } L^q_{mech} \;=\; -\int x\,G_2^q(x,0,0)\,dx\right)$$

This means that the genuine twist-3 part of G_2^q does not contribute at all to the net (or integrated) mechanical quark OAM L^q_{mech} .

Putting it in another way, the net mechanical quark OAM is determined solely by 3 twist-2 PDFs $H^q(x,0,0)$, $E^q(x,0,0)$, and $\tilde{H}^q(x,0,0)$.

Remember argument on the relations between the canonical and mechanical OAMs

Hatta-Yoshida definition

$$L^q_{mech} = L^q_{can} + L'_{pot} \quad \left(L'_{pot} = \int dx_1 \, dx_2 \, \mathcal{P} \frac{1}{x_1 - x_2} \Phi_F(x_1, x_2) \right)$$

no genuine twist-3 genuine twist-3 twist-3 quark-gluon correlator

The genuine twist-3 contribution in L^q_{can} and L'_{pot} must cancel each other !

Is this cancellation accidental ?

Natural interpretation based on our relation

$$L^q_{can} = L^q_{mech} + L_{pot} \quad (L_{pot} = -L'_{pot})$$

Now it is no surprise that the canonical OAM contains the genuine twist-3 part, since it is given as a sum of the mechanical OAM (given by the twist-2 GPDs alone), and the genuine twist-3 potential angular momentum.

We emphasize that this interpretation is in perfect harmony with the statement in sect.3, which tells that what contains the potential angular momentum is the canonical OAM rather than the mechanical OAM.

Burkardt's physical interpretation of the relation between the two OAMs

average transverse momentum and OAM of quarks

$$\langle \boldsymbol{k}^q_\perp \rangle^{\mathcal{W}} = \int dx\, d^2\boldsymbol{b}_\perp\, d^2\boldsymbol{k}_\perp\; \boldsymbol{k}_\perp\; \rho^q(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp; \mathcal{W})$$

$$\langle L^q_z \rangle^{\mathcal{W}} = \int dx\, d^2 b_\perp\, d^2 k_\perp\, (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z\, \rho^q(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp; \mathcal{W})$$

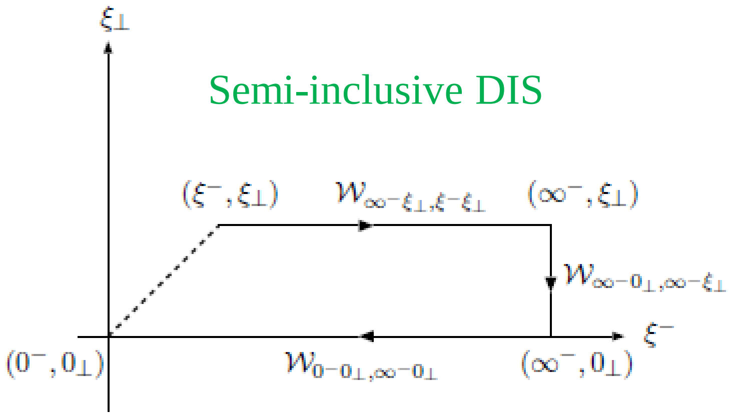
with

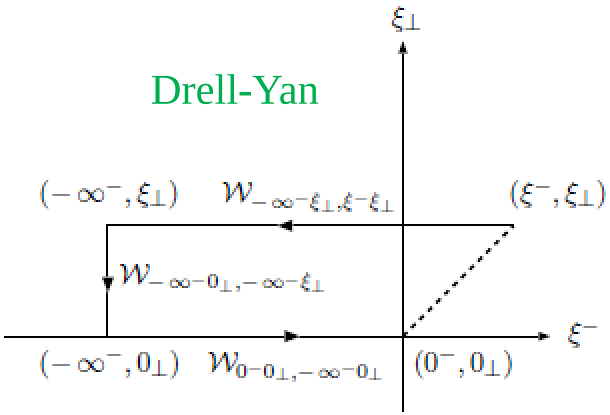
$$\rho^q(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp; \mathcal{W}) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\,\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp}\, \frac{1}{2} \int \frac{d\xi^-\, d^2\xi_\perp}{(2\pi)^3}\, e^{i\,(x\,\bar{P}^+ \xi^- - \boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp)}$$

$$\times\, \langle P'^+, \frac{\boldsymbol{\Delta}_\perp}{2}, S\,|\, \bar{\psi}(0)\, \gamma^+\, \mathcal{W}\, \psi(\xi)\,|\, P^+, -\frac{\boldsymbol{\Delta}_\perp}{2}, S \rangle\, |_{\xi^+=0}$$

generally path-dep.

3 paths with physical interest





- future-pointing staple LC path $\mathcal{W}^{+LC}$
- past-pointing staple LC path $\mathcal{W}^{-LC}$
- straight-line path connecting ξ and 0

One can show the relation

$$\langle k_\perp^i \rangle^{+LC} - \langle k_\perp^i \rangle^{straight} = -\mathcal{N} \int d^3 r \, \langle PS \,|\, \bar{\psi}(\boldsymbol{r}) \, \gamma^+ \times \int_{r^-}^{\infty} \mathcal{W}_{r^- \boldsymbol{r}_\perp, z^- \boldsymbol{r}_\perp} \, F^{+i}(z^-, \boldsymbol{r}_\perp) \, \mathcal{W}_{z^- \boldsymbol{r}_\perp, r^- \boldsymbol{r}_\perp} \, \psi(\boldsymbol{r}) \,|\, PS \rangle$$

In the LC gauge, $\mathcal{W} \rightarrow 1$, and

$$-\sqrt{2} \, g \, F^{+y} = g \, (E^y - B^x) = g \, [\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})]^y$$

According to Burkardt, the r.h.s. can be interpreted as the change of transverse momentum for the struck quark by color Lorentz force as it leaves the target after being struck by the virtual photon in the semi-inclusive DIS processes.

Similarly

$$\langle L_z^q \rangle^{+LC} - \langle L_z^q \rangle^{straight} = -\mathcal{N} \int d^3 r \, \langle PS \,|\, \bar{\psi}(\boldsymbol{r}) \, \gamma^+ \int_{r^-}^{\infty} dz^- \, \mathcal{W}_{r^- \boldsymbol{r}_\perp, z^- \boldsymbol{r}_\perp} \times g \left(x \, F^{+y}(z^-, \boldsymbol{r}_\perp) - y \, F^{+x}(z^-, \boldsymbol{r}_\perp) \right) \mathcal{W}_{z^- \boldsymbol{r}_\perp, r^- \boldsymbol{r}_\perp} \, \psi(\boldsymbol{r}) \,|\, PS \rangle$$

Lorentz force $\Longrightarrow$ torque by Lorentz force

$$T^z(r^-, \boldsymbol{r}_\perp) \equiv -g \left(x \, F^{+y}(r^-, \boldsymbol{r}_\perp) - y \, F^{+x}(r^-, \boldsymbol{r}_\perp) \right)$$

A question of path- or process-dependence

$$\langle L_z^q \rangle^{\pm LC} = \mathcal{N} \int d^3 r \, \langle PS \,|\, \bar{\psi}(\boldsymbol{r}) \, \gamma^+ \left\{ \left[\boldsymbol{r} \times \left(\frac{1}{i} \nabla - g \, \boldsymbol{A} \right) \right]^z - \int_{r^-}^{\pm\infty} \mathcal{W}_{r^- \boldsymbol{r}_\perp, z^- \boldsymbol{r}_\perp} \; g \left(x \, F^{+y}(z^-, \boldsymbol{r}_\perp) - y \, F^{+x}(z^-, \boldsymbol{r}_\perp) \right) \mathcal{W}_{r^- \boldsymbol{r}_\perp, r^- \boldsymbol{r}_\perp} \right\} \psi(\boldsymbol{r}) \,|\, PS \rangle$$

The 1st term of the r.h.s. is nothing but the "mechanical" OAM L_{mech}.

Hatta showed that the 2nd term (FSI or ISI term) can be expressed as

$$\mathcal{N} \int d^3 r \, \langle PS \,|\, \bar{\psi}(\boldsymbol{r}) \, \gamma^+ \left(\boldsymbol{r} \times \boldsymbol{A}_{phys}(\boldsymbol{r}) \right)^z \psi(\boldsymbol{r}) \,|\, PS \rangle$$

with the definition of the physical component of the gluon field

$$A^\mu_{phys}(r^-, \boldsymbol{r}_\perp) = - \int dz^- \, \kappa(z^- - r^-) \, \mathcal{W}_{r^- \boldsymbol{r}_\perp, z^- \boldsymbol{r}_\perp} \, F^{+\mu}(z^-, \boldsymbol{r}_\perp) \, \mathcal{W}_{z^- \boldsymbol{r}_\perp, r^- \boldsymbol{r}_\perp}$$

$$\kappa(z^-) = \pm \, \theta(\pm \, z^-)$$

The 2nd (FIS or ISI) term therefore precisely coincides with the potential OAM.

$$L_{pot} \quad (\text{or} \; - \, L'_{pot})$$

As a consequence, we have

$$\langle L_z^q \rangle^{\pm LC} = L_{mech} + L_{pot}$$

Due to the parity and time-reversal (PT) symmetry

$$\langle L_z^q \rangle^{-LC} = \langle L_z^q \rangle^{+LC}$$

the above relation is consistent with

$$L_{can} = L_{mech} + L_{pot} \quad \texttt{with} \quad L_{can} = \langle L_z^q \rangle^{\pm LC}$$

What is crucial here is that the canonical OAM is basically process-independent !

$$\Downarrow$$

This is not the case for the average transverse momentum case !

Along the same line, one can show the relation

$$\langle k_\perp^i \rangle^{\pm LC} = \mathcal{N} \int d^3 r \, \langle PS \,|\, \bar{\psi}(\boldsymbol{r}) \left[\gamma^+ \left(\frac{1}{i} \nabla_\perp^i - g \, A^i(r^-, \boldsymbol{r}_\perp) \right) + g \, A^i_{phys}(r^-, \boldsymbol{r}_\perp) \right] \psi(\boldsymbol{r}) \,|\, PS \rangle$$

This formally gives

$$\langle k_\perp^i \rangle^{\pm LC} = \mathcal{N} \int d^3 r \, \langle PS \,|\, \bar{\psi}(\boldsymbol{r}) \, \gamma^+ \left(\frac{1}{i} \nabla_\perp^i - g \, A^i_{pure}(r^-, \boldsymbol{r}_\perp) \right) \psi(\boldsymbol{r}) \,|\, PS \rangle \stackrel{?}{=} \langle k_\perp^i \rangle_{can}$$

However, now the PT symmetry means that

$$\langle k_\perp^i \rangle^{-LC} = - \langle k_\perp^i \rangle^{+LC}$$

The definition of canonical transverse momentum is therefore not universal, but process-dependent.

This again supports our viewpoint that what contains the FSI or ISI (quark-gluon interaction) is the canonical momentum and canonical OAM not the mechanical momentum and mechanical OAM.

Don’t you think it wondering ?

5. Summary and conclusion

- We have carried out a comparative analysis of the two nucleon spin decompositions, which are characterized by two types of OAMs, i.e.

  (generalized) canonical OAMs & mechanical OAMs

- We have advocated a viewpoint which favors the mechanical OAMs rather than the canonical OAMs, since the former have closer connection with direct observables.

- However, one can also get some insight also into the canonical OAM although somewhat indirectly through twist-3 DIS mechanism.

- Anyhow, when one talks about the OAMs of quarks and gluons in the nucleon, one must at the least be clearly conscious of which OAMs one is thinking of.

[Backup Slides]

What can we learn form the recent controversies on the transverse spin sum rule ?

$$P^\mu = (P^0, 0, 0, P^3) \quad : \quad \texttt{nucleon momentum}$$

- X. Ji, X. Xiong, F. Yuan, P.L. B717 (2012)214.

$$J^\perp_{q/G} = \frac{1}{2}\left(A_{q/G}(0) + B_{q/g}(0)\right) \quad \Rightarrow \quad \texttt{incomplete}$$

- E. Leader, P.L. B720 (2013) 120.

$$J^\perp_{q/G} = \frac{1}{2}\left(A_{q/G}(0) + B_{q/g}(0)\right) + \frac{P^0 - M}{2\,P^0}\bar{C}_{q/G}(0)$$

- Y. Hatta, K. Tanaka, S. Yoshida, JHEP 02 (2013) 003.

$$J^\perp_{q/G} = \frac{1}{2}\left(A_{q/G}(0) + B_{q/g}(0)\right) + \frac{P^3}{2\,(P^0 + M)}\bar{C}_{q/G}(0)$$

- A. Harindranath, R. Kundu, A. Mukherjee, arXiv : 1308.519.

$$J^\perp_{q/G} = \frac{1}{2}\left(A_{q/G}(0) + B_{q/g}(0) + \bar{C}_{q/G}(0)\right)$$

The last three coincides only in the IMF limit ; $P^3, \, P^0 \rightarrow \infty$

How can we understand these differences ?

♣ The origin of differences

They all calculated the M.E. of the Pauli-Lubanski vector W^x between the transversely polarized nucleon state in the *x* direction :

$$\langle PS^x \,|\, W^x \,|\, PS^x \rangle \quad \texttt{with} \quad W_\mu = -\frac{1}{2}\epsilon_{\mu\alpha\beta\rho} J^{\alpha\beta} P^\rho$$

but with different $J^{\alpha\beta}$ and $|\, PS^x \rangle$

	$J^{\alpha\beta}$	$\|\, PS^x \rangle$
Leader	$\int d^3x \, M^{0\alpha\beta}$	Dirac spinors
HTY	$\int dx^- \, d^2x^\perp \, M^{+\alpha\beta}$	Dirac spinors
HKM	$\int dx^- \, d^2x^\perp \, M^{+\alpha\beta}$	Light-front spinors

$M^{0\alpha\beta}$: angular momentum tensor in ET formalism

$M^{+\alpha\beta}$: angular momentum tensor in LF formaliam

HKM claim that their result based on the LF (light-front) formalism is absolutely Lorentz-frame independent, but this statement is misleading.

It is known that the use of the LF spinors in the LF formalism is equivalent to working in the IMF (infinite-momentum-frame).

In the IMF, however, the dependence on the nucleon longitudinal momentum P^3 is naturally washed out.

What HKM have shown is actually the $P_\perp$-independence of their sum rule.

We thus conclude that the transverse spin sum rule is Lorentz-frame dependent due to the existence of the term $\bar{C}(0)$.

It is important to recognize the fact that the existence of plural forms of transverse spin decomposition has nothing to do with our gauge problem, because both of J_q and J_G are clearly gauge-invariant.

The truth is that there generally exist many definitions of relativistic spin, which would correspond to different observables. This originates from the fact that successive operations of Lorentz boost can generate spin rotation.

But !

We emphasize that this is not the case for the longitudinal spin sum rule.

In fact, one can easily verify that any of the afore-mentioned three choices leads to exactly the same sum rule for the longitudinal spin :

$$J^{||}_{q/G} = \frac{1}{2} \left(A_{q/G}(0) + B_{q/G}(0) \right)$$

which is nothing but the celebrated Ji sum rule.

Our discussion above thus indicates that further gauge- and frame-independent decomposition of J_q and J_G into their intrinsic spin and orbital parts can be made only for the longitudinal components.

longitudinal components $\Rightarrow$ helicity !

As emphasized by Zhang and Pak, the only frame-independent notion of spin for a massless particle is the **helicity**, which is described by a little group E(2) of the Lorentz group.