Structure functions at small x via the Pomeron exchange in AdS space

Akira Watanabe
(Academia Sinica)

This talk is based on:
AW, K. Suzuki, Phys. Rev. D86, 035011 (2012)
AW, K. Suzuki, arXiv:1312.7114 [hep-ph]

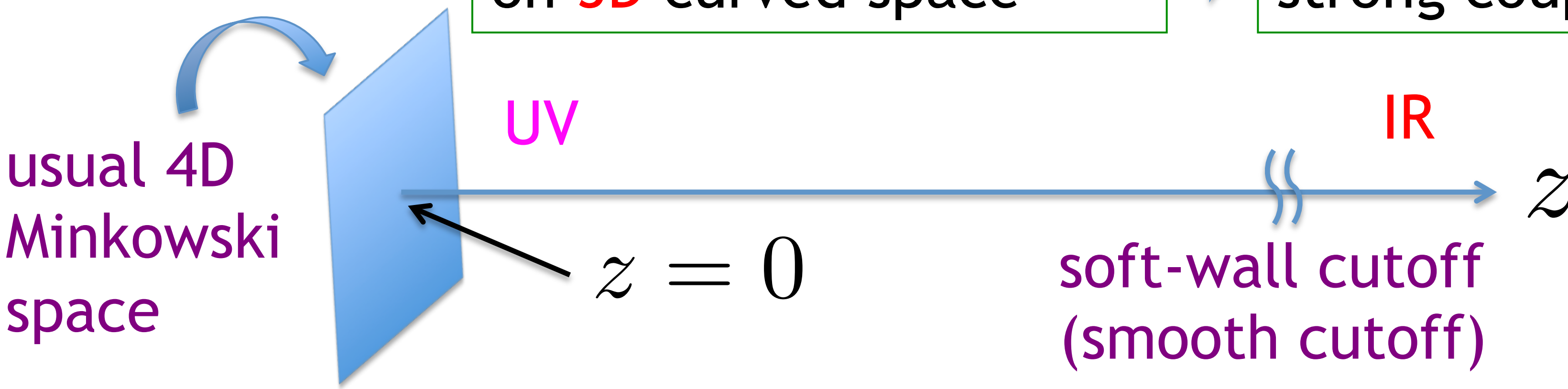
KEK-Tokai workshop on high-energy QCD and nucleon structure

March 7, 2014 @ KEK Tokai campus

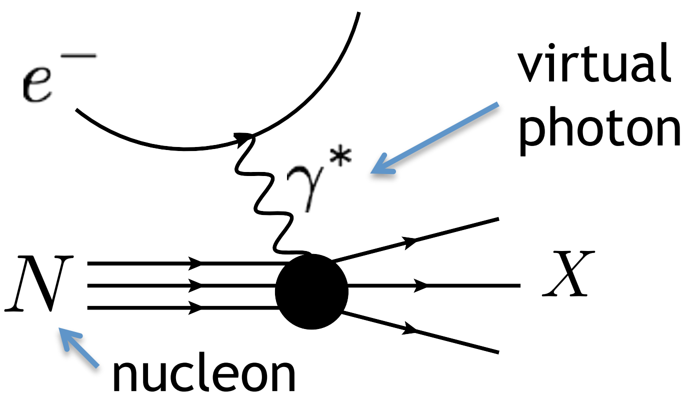
Outline of the study

Holographic QCD

Classical theory defined on 5D curved space → 4D QCD at strong coupling

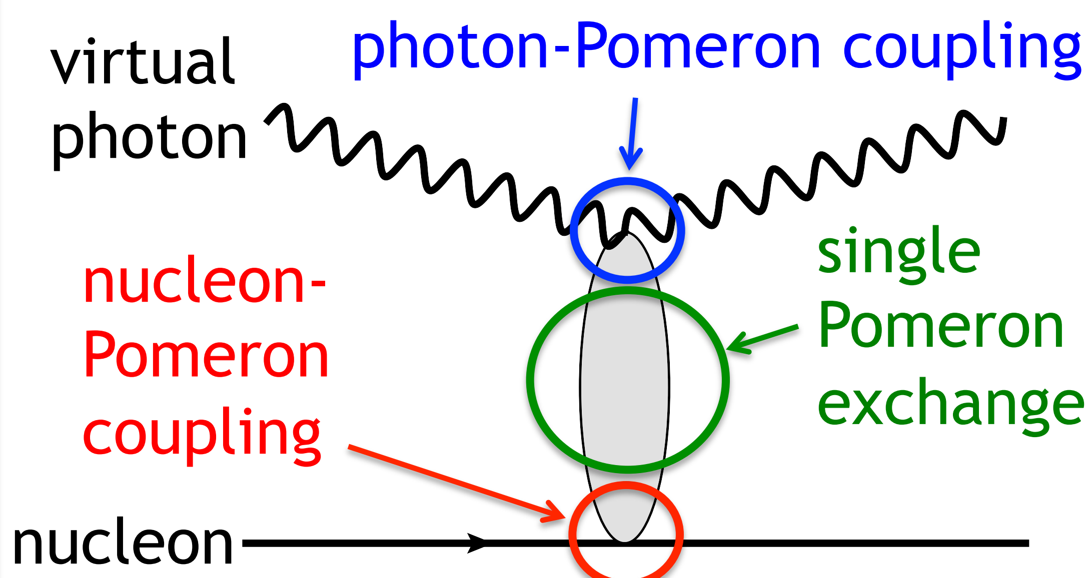


Deep inelastic scattering



structure function

$F_2 \propto \sigma^{\gamma^* N}_{\text{total}}$



Gauge/string correspondence (holography)

many correspondences…

AdS/CFT

Kerr/CFT

dS/CFT

holographic QCD

AdS/QCD

Lifshitz/CFT(?)

AdS/CMP

fluid/gravity

holographic superconductor

We are interested in hadron physics and QCD!

Holographic approaches to hadron physics

many studies…

mass spectra

QGP

coupling constants

decay constants

nuclear structure functions

form factors

heavy-light mesons

phase diagram

deep inelastic scattering (DIS)

exotic mesons

- Polchinski-Strassler (2003)
- Brower-Polchinski-Strassler-Tan (2007)
- Cornalba-Costa-Penedones (2007, 2010)
- Hatta-Iancu-Mueller (2008)
- Bayona - Boschi-Filho - Braga (2008)
- Brower-Strassler-Tan (2009)
- Brower-Djuric-Sarcevic-Tan (2010)
- Watanabe-Suzuki (2012, 2013) and so on…

Motivation (1)

- (2003-2009) Important studies on DIS from holography have been done almost completely…
  - e.g. In 2003, Polchinski and Strassler have demonstrated string calculations for structure functions and shown that the Callan-Gross relation is satisfied.
- (2010) Brower-Djuric-Sarcevic-Tan have studied nucleon structure functions at small *x* and well reproduced the HERA data for F_2^p.

Motivation (2)

- So far, holographic technique has been considered to be suitable only for qualitative studies, because the original gauge theory is the large-N theory. There is a fundamental uncertainty of 30%.
- However, if it works well in nonperturbative regions as an effective model and predict some unknown quantities, it will be a powerful theoretical tool.
- As our first trial, we try to study DIS at small x in the framework of holographic QCD by building a more realistic model.

Talk plan

1. Brief introduction of holographic QCD
2. Nucleon structure functions at small *x*
   - Model setup
   - Results for F_2^p and F_L^p
3. Future works and summary

Brief introduction of holographic QCD

What is holographic QCD?

- A generic name of QCD-like theories constructed based on the AdS/CFT correspondence.

AdS/CFT correspondence

- A conjectured correspondence between a weak coupling superstring theory in the higher dimensional anti-de Sitter (AdS) space and a strong coupling conformal field theory (CFT) in the usual 4D space proposed by J. M. Maldacena in 1997.

  (NB: superstring at low energy -> supergravity)
- Still not proven mathematically.
- So many applications have been studied.
- Breaking conformal symmetry and supersymmetry, then considering QCD instead of CFT, it is called holographic QCD or AdS/QCD.

Original AdS/CFT correspondence

type IIB supergravity theory on $S^5 \times AdS_5$ ⟺ strong coupling 4D N=4 supersymmetric Yang-Mills (SYM) theory

An example of holographic QCD

supergravity theory (classical theory) on AdS_5 ⟺ usual 4D QCD at strong coupling

Two approaches to QCD

top-down approach
(e.g. Sakai-Sugimoto model)

- from 10D superstring theory to 4D QCD
- considering appropriate configurations of D-branes to take out 4D QCD

bottom-up approach
(e.g. a model by Erlich-Katz-Son-Stephanov)

- from 4D QCD to 5D model in AdS space
- based on phenomenology of QCD, constructing 5D action by hand

- Both are according to the field/operator correspondence provided by the original AdS/CFT correspondence
- Both can (basically) only treat color singlet objects (there are no quarks, no gluons!)

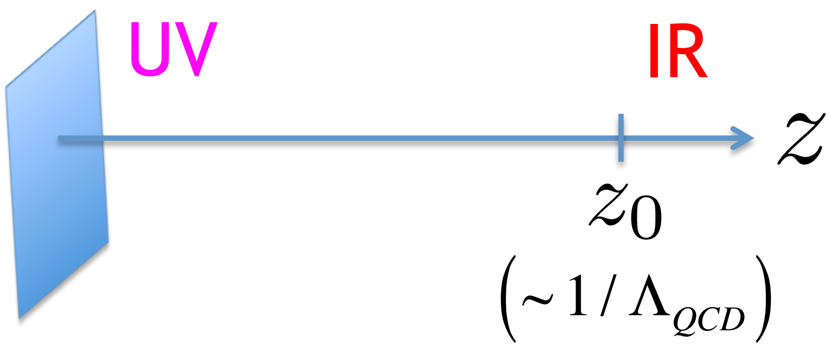
Holographic model of hadrons (bottom-up approach to QCD)

- In this study, we describe target hadrons in DIS by using the holographic model, which is constructed based on the phenomenology of QCD.
- In this model, chiral symmetry breaking and vector meson dominance are naturally included.
- Easier to handle than models constructed by the top-down approach.
- Just a model, but important physical quantities (mass spectrum, form factors, coupling constants…) can be well reproduced.

CFT -> QCD

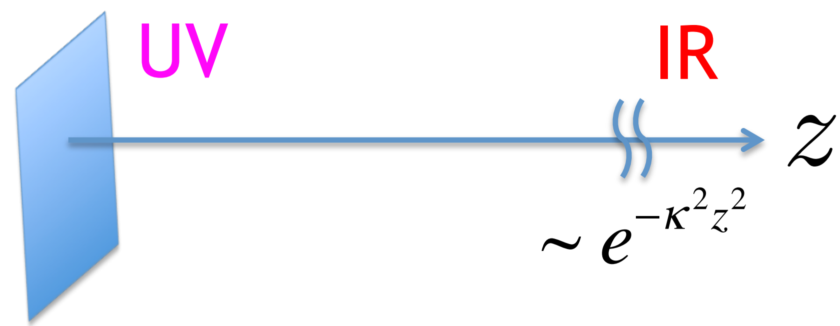
To break the conformal symmetry and consider QCD, we cut off the AdS geometry at IR and introduce the QCD scale.

hard-wall model



- sharp cutoff
- advantages in analytical calculations
- $m_n^2 \propto n^2$

soft-wall model



- smooth cutoff
- frequently numerical evaluations are needed
- $m_n^2 \propto n$ (correct Regge behavior)

An example calculation with a holographic model of vector mesons (1)

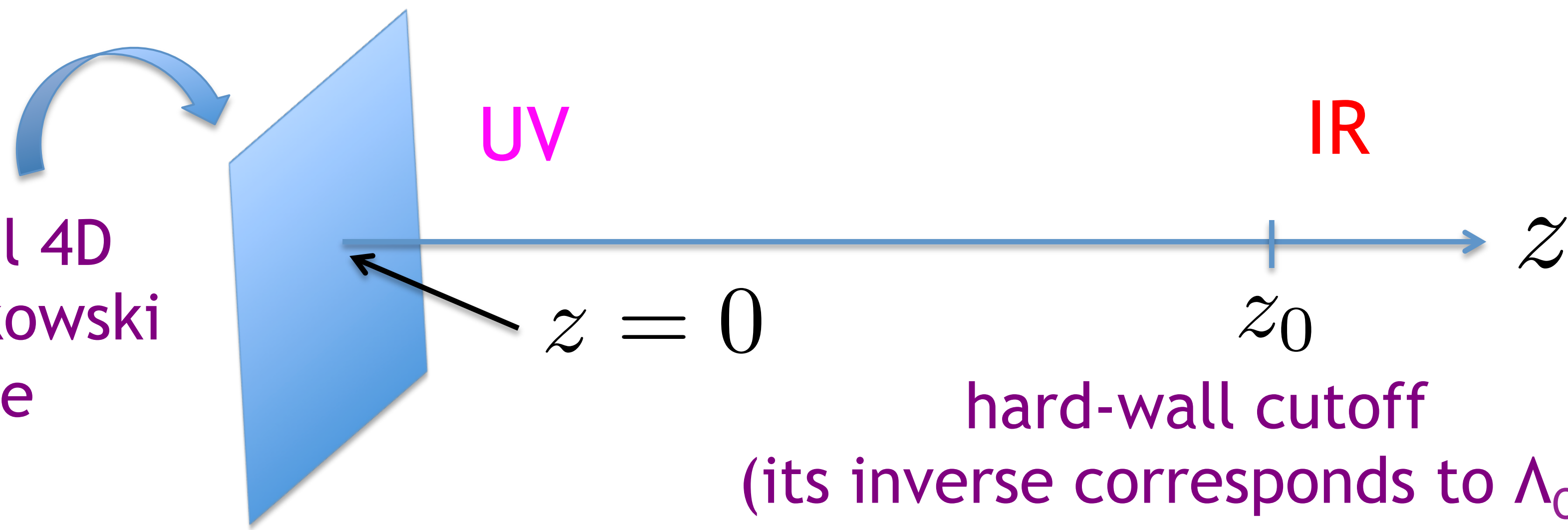
- An example: calculation for the mass spectrum of vector mesons

$\boxed{\text{background space-time (AdS}_5\text{)}}$

$$\boxed{ds^2 = \frac{1}{z^2}(dt^2 - d\vec{x}^2 - dz^2)}$$

z : 5th coordinate

$$\boxed{0 \le z \le z_0}$$



usual 4D Minkowski space

hard-wall cutoff
(its inverse corresponds to Λ_{QCD})

An example calculation with a holographic model of vector mesons (2)

$$ds^2 = \frac{1}{z^2}(dt^2 - d\vec{x}^2 - dz^2) \equiv g_{MN} dx^M dx^N$$

$M, N = 0 \sim 4$

5D action describing vector mesons:

$$S_{AdS_5}^{V} = \int d^5x \sqrt{\det(g_{MN})} \left(-\frac{1}{2g_5^2} F_V^{MN} F_{VMN} \right)$$

information of the space-time

5D field strength:

$$F_V^{MN} = \partial^M V^N - \partial^N V^M - i\left[V^M, V^N\right]$$

5D gauge field

fixing the gauge ($V_z = 0$), and we pick up the quadratic terms from the action

$$S_{AdS_5}^{V(2)} = \int d^5x \left\{ -\frac{1}{g_5^2 z^5} \left(\partial^\mu V^\nu \partial_\mu V_\nu - \partial^\mu V^\nu \partial_\nu V_\mu + \partial^z V^\nu \partial_z V_\nu \right) \right\}$$

An example calculation with a holographic model of vector mesons (3)

$$\boxed{S^{V(2)}_{AdS_5} = \int d^5x \left\{ -\frac{1}{g_5^2 z^5}\left(\partial^\mu V^\nu \partial_\mu V_\nu - \partial^\mu V^\nu \partial_\nu V_\mu + \partial^z V^\nu \partial_z V_\nu\right)\right\}}$$

to minimize the action in respect to the field, we calculate the Euler-Lagrange equation

$$\boxed{\partial_z\left(\frac{1}{z}\partial_z V(p,z)\right) + \frac{p^2}{z} V(p,z) = 0}$$

solving this EoM with $p^2 = m^2$ (m : mass of vector meson), then we can obtain the mass spectrum

⋮

2nd excited state

1st excited state

ground state

determining a parameter z_0 with the experimental value of rho meson mass

More on holographic model of mesons (1) (a model by Erlich-Katz-Son-Stephanov)

$$S_{\mathrm{AdS}} = \mathrm{Tr}\int d^4x dz \left[\frac{1}{z^3}|DX|^2 + \frac{3}{z^5}|X|^2 - \underbrace{\frac{1}{2g_5^2 z}}_{\text{5D coupling}}\left(F_V^2 + F_A^2\right)\right]$$

vector

axial vector

$$\left\{\begin{aligned} X &= X_0 \exp\left(2i\pi^a t^a\right) \\ D^M X &= \partial^M X - i\left[V^M, X\right] - i\left\{A^M, X\right\} \\ F_V^{MN} &\equiv \partial^M V^N - \partial^N V^M - i\left(\left[V^M, V^N\right] + \left[A^M, A^N\right]\right) \\ F_A^{MN} &\equiv \partial^M A^N - \partial^N A^M - i\left(\left[V^M, A^N\right] + \left[A^M, V^N\right]\right) \\ X_0(z) &= \underline{m_{\mathrm{q}} z/2 + \sigma z^3/2} \end{aligned}\right.$$

chiral symmetry breaking
(explicit and spontaneous)

$$\boxed{M, N = 0 \sim 3, z}$$

$$\boxed{V_z = A_z = 0}$$

More on holographic model of mesons (2) (a model by Erlich-Katz-Son-Stephanov)

The results of their study:

Erlich-Katz-Son-Stephanov (2005)

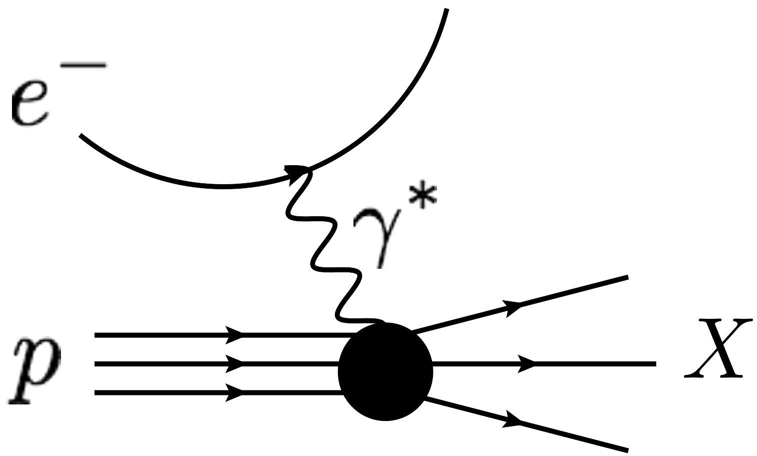
Observable	Measured (MeV)	Model A (MeV)
m_π	139.6 ± 0.0004 [8]	139.6^*
m_ρ	775.8 ± 0.5 [8]	775.8^*
m_{a_1}	1230 ± 40 [8]	1363
f_π	92.4 ± 0.35 [8]	92.4^*
$F_\rho^{1/2}$	345 ± 8 [15]	329
$F_{a_1}^{1/2}$	433 ± 13 [6]	486
$g_{\rho\pi\pi}$	6.03 ± 0.07 [8]	4.48

NB: inputs are indicated by $*$

Nucleon structure functions at small x

DIS structure functions

$$\frac{d^2\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[\left\{1+(1-y)^2\right\}F_2\left(x,Q^2\right) - y^2 F_L\left(x,Q^2\right)\right]$$



$$F_2\left(x,Q^2\right) = \frac{Q^2}{4\pi^2\alpha}\sigma_{tot}\left(x,Q^2\right)$$

$$F_L\left(x,Q^2\right) = \frac{Q^2}{4\pi^2\alpha}\sigma_L\left(x,Q^2\right)$$

- Structure functions are physical quantities which have information on the internal structure of hadrons.
- They depend on two kinematic variables, Bjorken-*x* and photon 4-momentum *Q*.

Longitudinal structure function F_L

- In the quark-parton model, F_2 can be written as:

$$F_2 = x\sum_q e_q^2 q_i(x)$$

$$F_L = 0$$

- F_L is expressed, for example by Altarelli-Martinelli equation, as:

Altarelli-Martinelli (1978)

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3}\left[\frac{8}{3}F_2(y,Q^2) + 4\sum_q e_q^2\left(1-\frac{x}{y}\right) yg(y,Q^2)\right]$$

At small x, the second term becomes dominant and F_L is approximately expressed by

Cooper-Sarkar et al. (1988)

$$F_L \approx 0.3\frac{4\alpha_s}{3\pi} xg(2.5x,Q^2)$$

Hence, studying F_L is related to the investigations on the gluon distribution and the higher order contribution of QCD.

Remarks

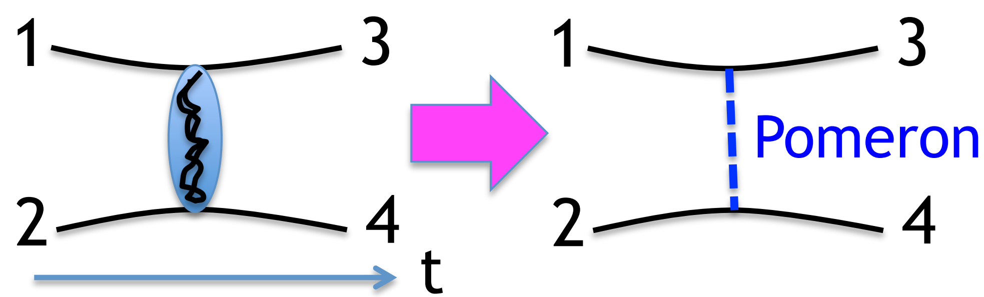
- In the region where pQCD works well (1 $GeV^2 \ll Q^2$), demonstrating rigorous calculations is a natural way.
- In this study, we focus on the kinematic region where $10^{-6} < x < 10^{-2}$ and $0.1 < Q^2 < 10$ [GeV^2].
- Physics at mall x = Pomeron physics!

Pomeron exchange picture

- Description of the high energy scattering before QCD
- Pomeron: many gluons interacted each other intricately
- Total cross section for the high energy two-to-two scattering can be well expressed by the single Pomeron exchange

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$



can be expressed by a single parameter (Pomeron intercept)

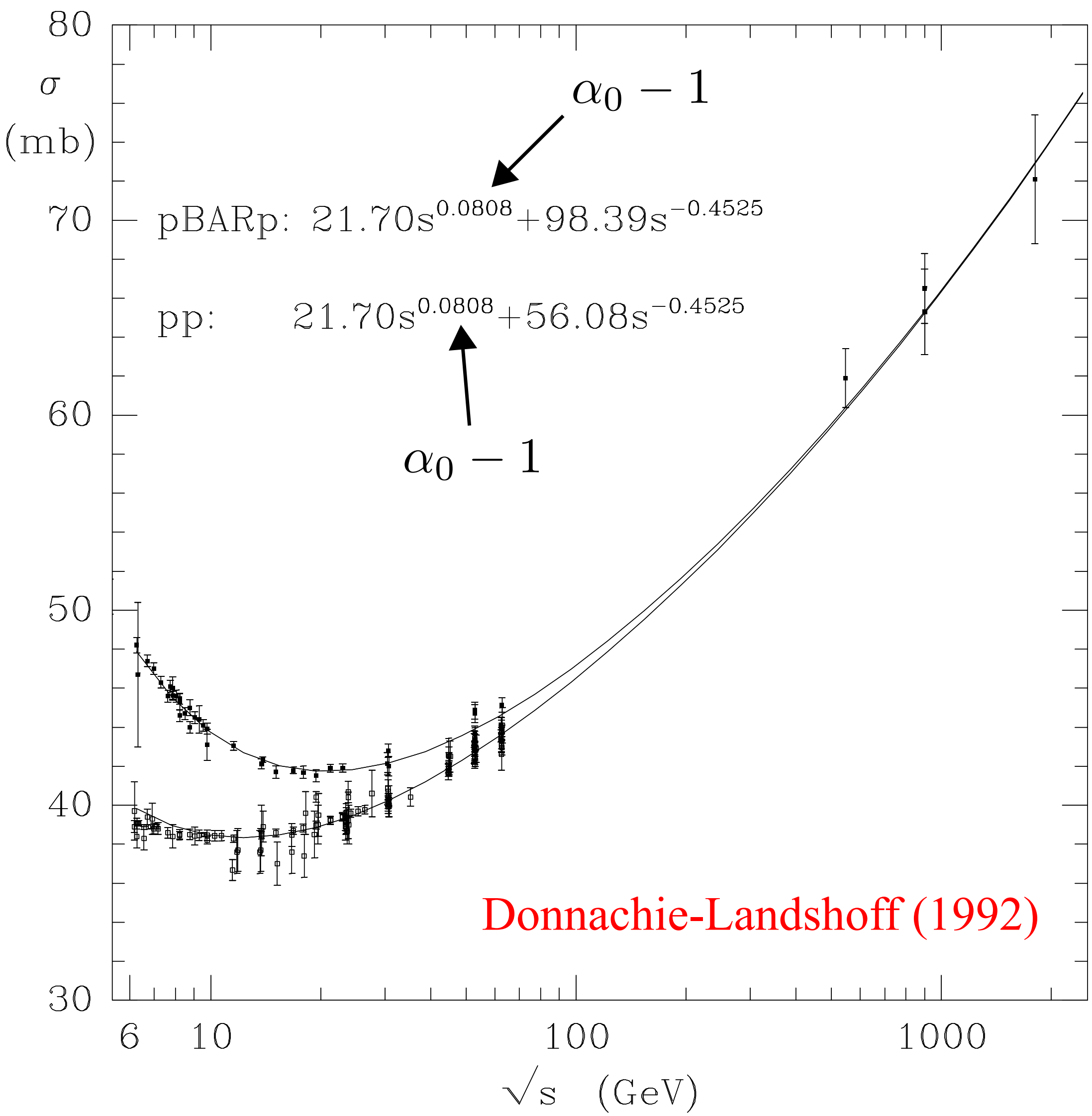
Therefore, F_2 structure function can be written as

$$F_2(x, Q^2) \sim x^{1-\alpha_0}$$

(this is effective only in the small *x* region)

Can it really describe the experimental data?

proton-(anti)proton total cross section



$$\sigma_{tot}(s) \sim s^{\alpha_0 - 1}$$

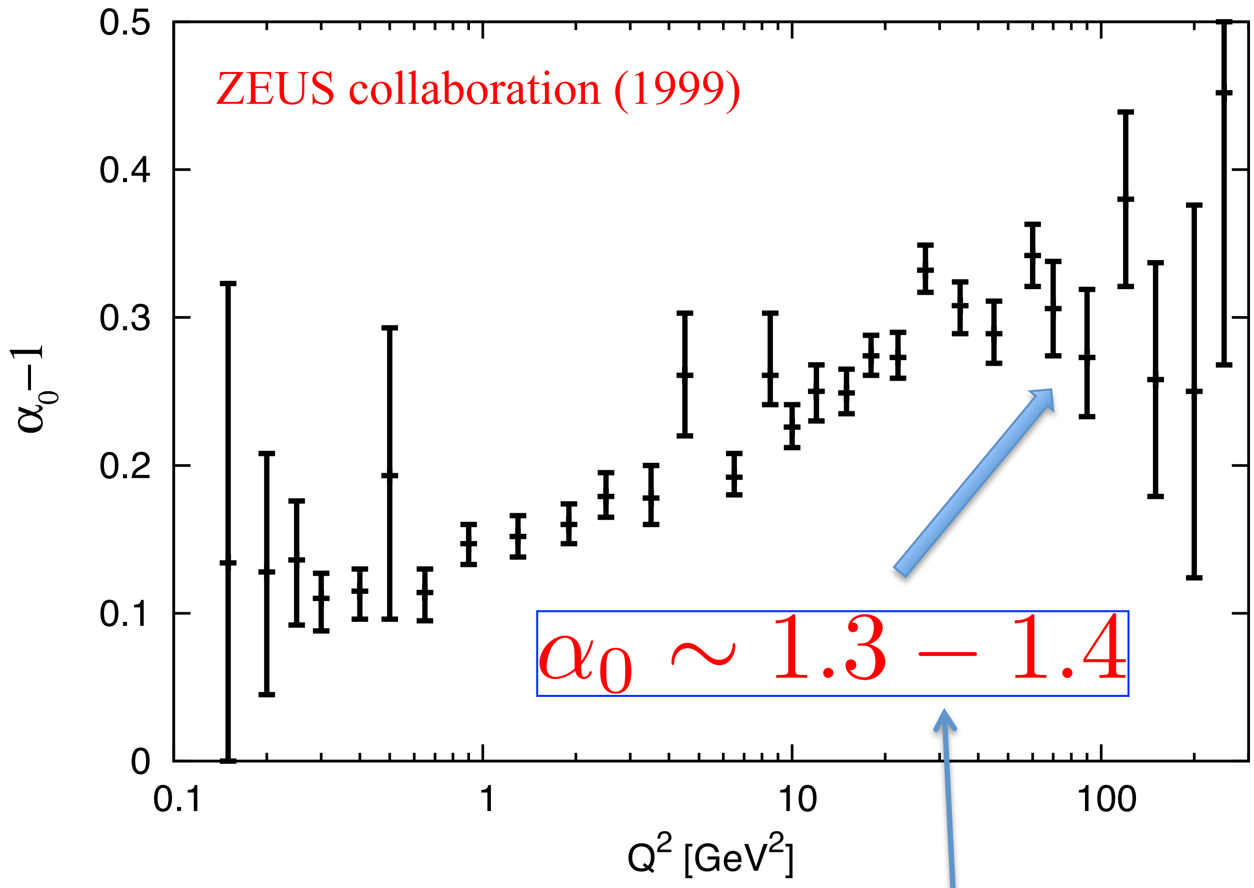
$$\alpha_0 = 1.0808$$

(soft Pomeron intercept)

well describing the experimental data

Is the Pomeron intercept a constant?

Pomeron intercept calculated from experimental data of F_2^p



$F_2(x, Q^2) \sim x^{1-\alpha_0}$

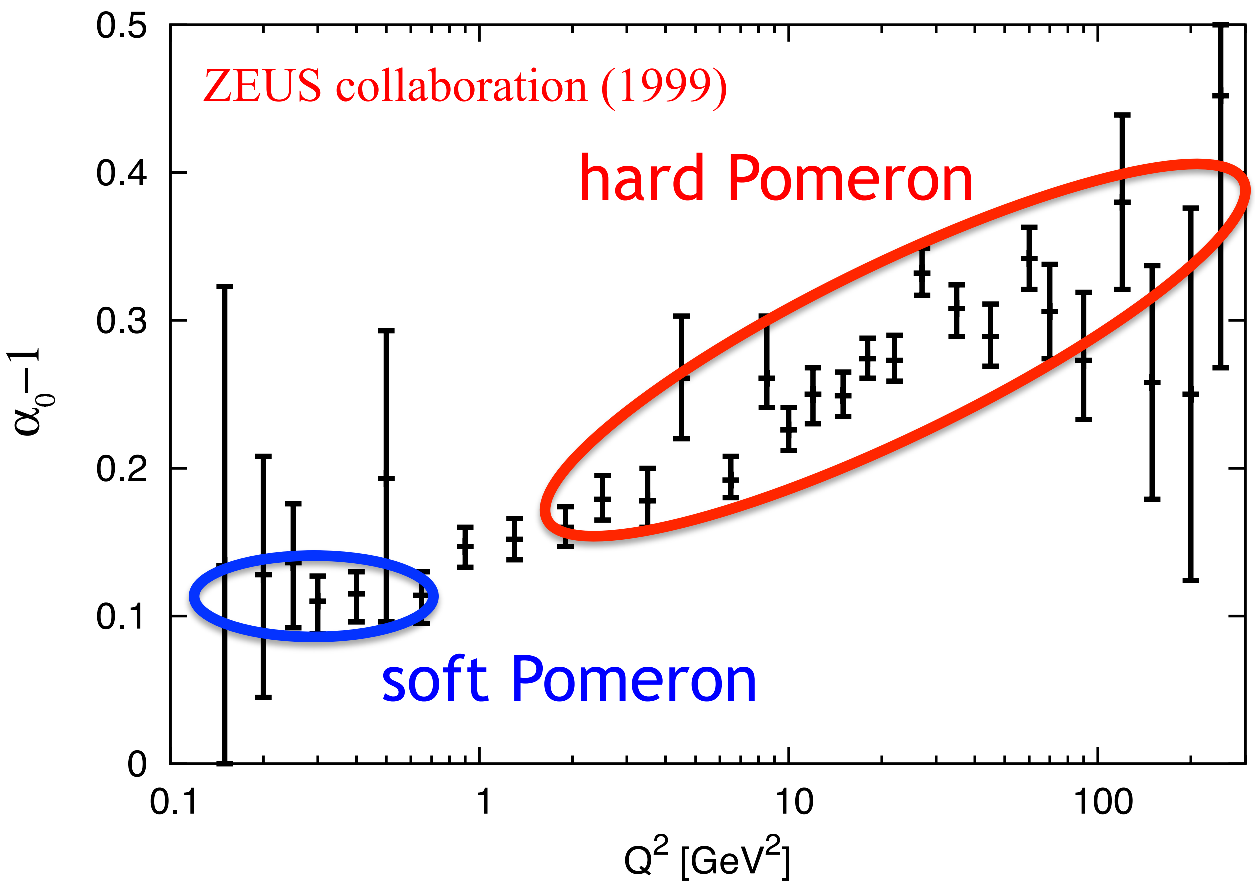
Once a hard scale (e.g. Q^2, quark mass, etc.) enters into the process, the Pomeron intercept has scale dependence.

$\alpha_0 \to \alpha_0(Q)$

(hard Pomeron intercept)

Transition from soft Pomeron to hard Pomeron

Pomeron intercept calculated from experimental data of F_2^p



$$F_2(x,Q^2) \sim x^{1-\alpha_0}$$

problems:

- Where is the boundary located?
- Can this transition be described in holographic QCD?

Holographic description of structure functions

- **Polchinski-Strassler (2003)**
- **Brower-Polchinski-Strassler-Tan (2007)** → derived Pomeron exchange kernel
- **Brower-Djuric-Sarcevic-Tan (2010)** → studied nucleon structure functions

$$F_2\left(x,Q^2\right)=\frac{Q^2}{2\pi^2}\int dz dz' P_{13}\left(z,Q^2\right)P_{24}\left(z'\right)\left(zz'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$

z and z' : 5th coordinate

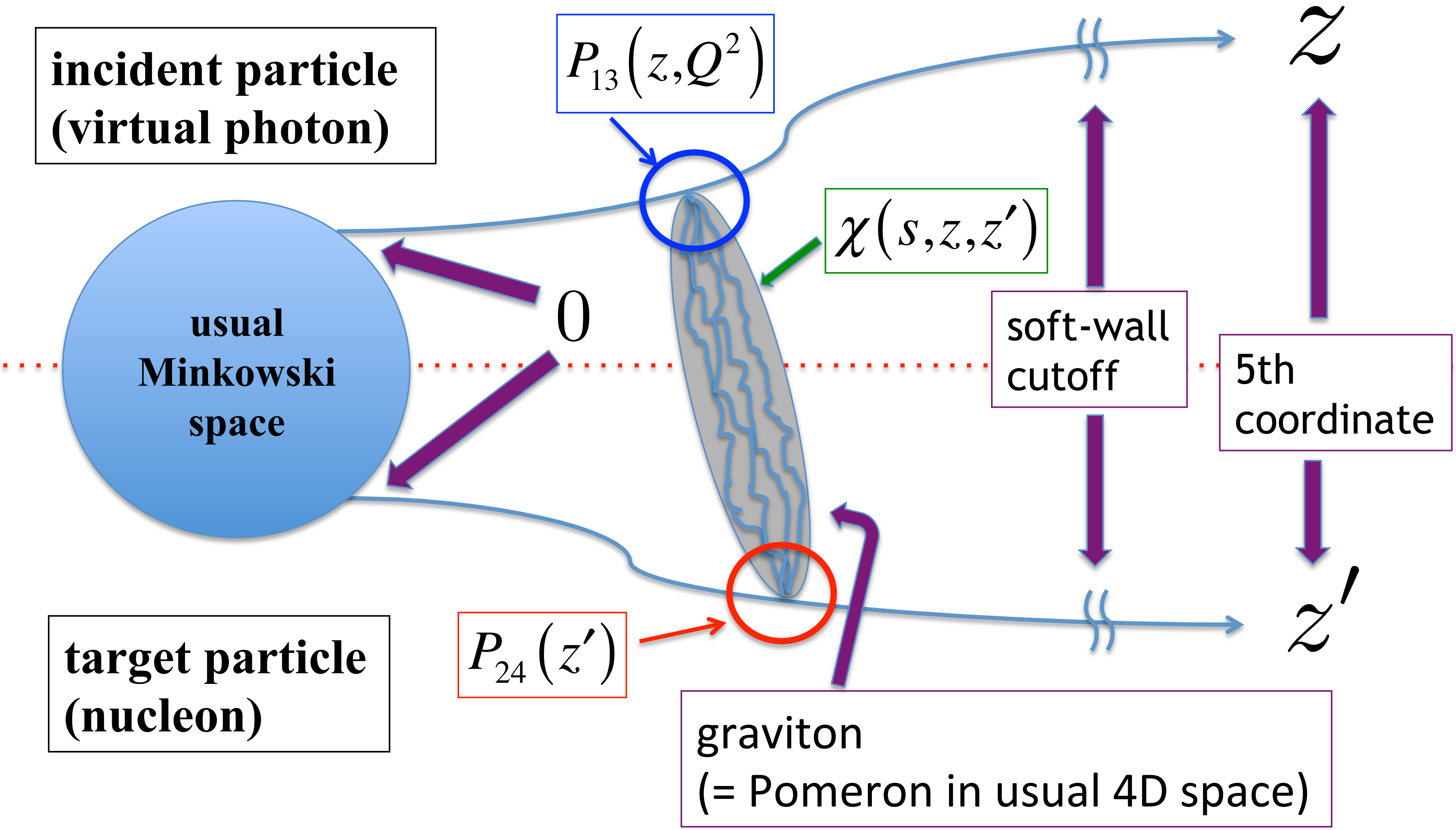
χ : Pomeron exchange kernel in the AdS space

$P_{13}(z,Q^2)$: incident particle (virtual photon, 4-momentum Q)

$P_{24}(z')$: target particle

overlap functions (density distributions in the AdS space)

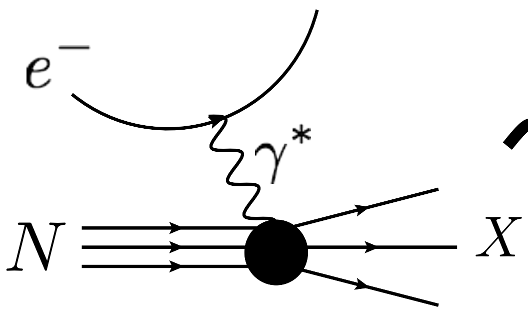
5D background space-time (AdS_5)



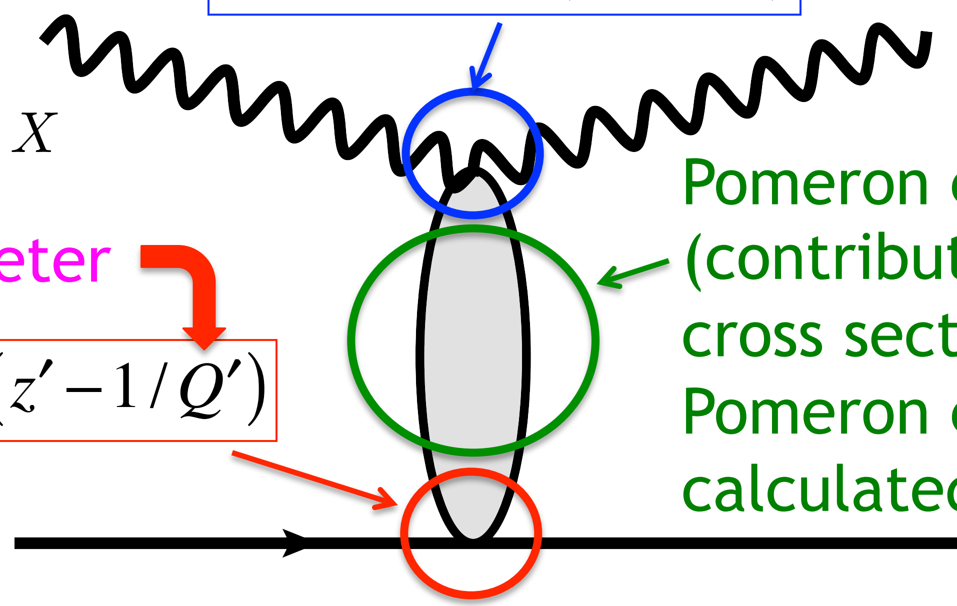
A preceding study

Brower-Djuric-Sarcevic-Tan (2010)

$$F_2\left(x,Q^2\right)=\frac{Q^2}{2\pi^2}\int dz dz' P_{13}\left(z,Q^2\right)P_{24}\left(z'\right)\left(zz'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]$$



$$P_{13}(z,Q^2)\approx\delta\left(z-1/Q\right)$$



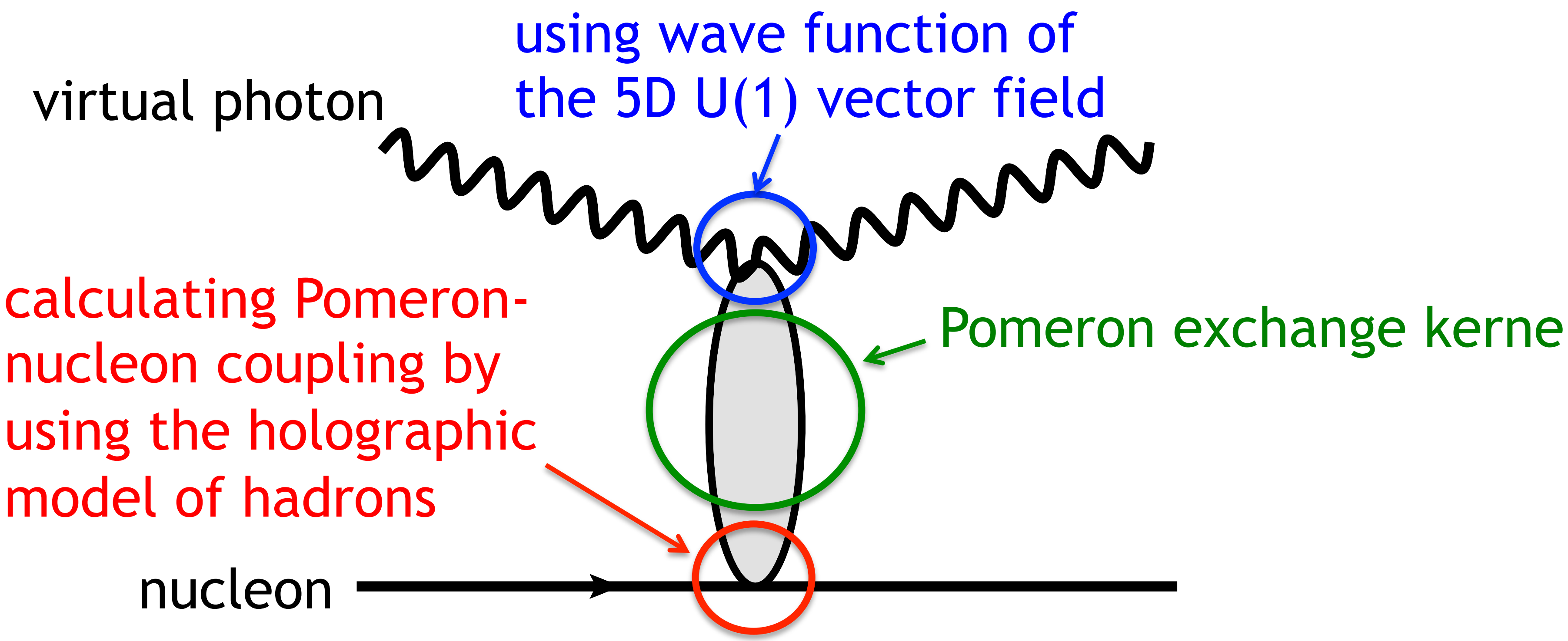
parameter

$$P_{24}(z')\approx\delta\left(z'-1/Q'\right)$$

Pomeron exchange kernel (contribution to the total cross section for the Pomeron exchange, calculated by AdS/CFT)

- Incident and target particle are simply replaced with delta functions.

Our model setup



- This is a more consistent description of structure functions.
- In this model, we can consider structure functions of various hadrons and longitudinal structure functions, which can not be considered in the preceding model.

Pomeron exchange kernel

Brower-Polchinski-Strassler-Tan (2007)
Brower-Djuric-Sarcevic-Tan (2010)

$$\boxed{F_i\left(x,Q^2\right)=\frac{g_0^2\rho^{3/2}Q^2}{32\pi^{5/2}}\int dz\,dz' P_{13}^{(i)}\left(z,Q^2\right)P_{24}\left(z'\right)\left(zz'\right)\mathrm{Im}\left[\chi\left(s,z,z'\right)\right]}$$

$\boxed{i=2 \text{ or } L}$

$$\mathrm{Im}\left[\chi_c\left(s,z,z'\right)\right]\equiv e^{(1-\rho)\tau}e^{-\frac{\log^2 z/z'}{\rho\tau}}/\tau^{1/2}$$

$$\tau=\log\left(\rho zz's/2\right)$$

$$\mathrm{Im}\left[\chi_{\mathrm{mod}}\left(s,z,z'\right)\right]\equiv\mathrm{Im}\left[\chi_c\left(s,z,z'\right)\right]+\boxed{\mathcal{F}\left(z,z',\tau\right)\mathrm{Im}\left[\chi_c\left(s,z,z_0^2/z'\right)\right]}$$

$$\mathcal{F}\left(z,z',\tau\right)=1-2\sqrt{\rho\pi\tau}\,e^{\eta^2}\mathrm{erfc}\left(\eta\right)$$

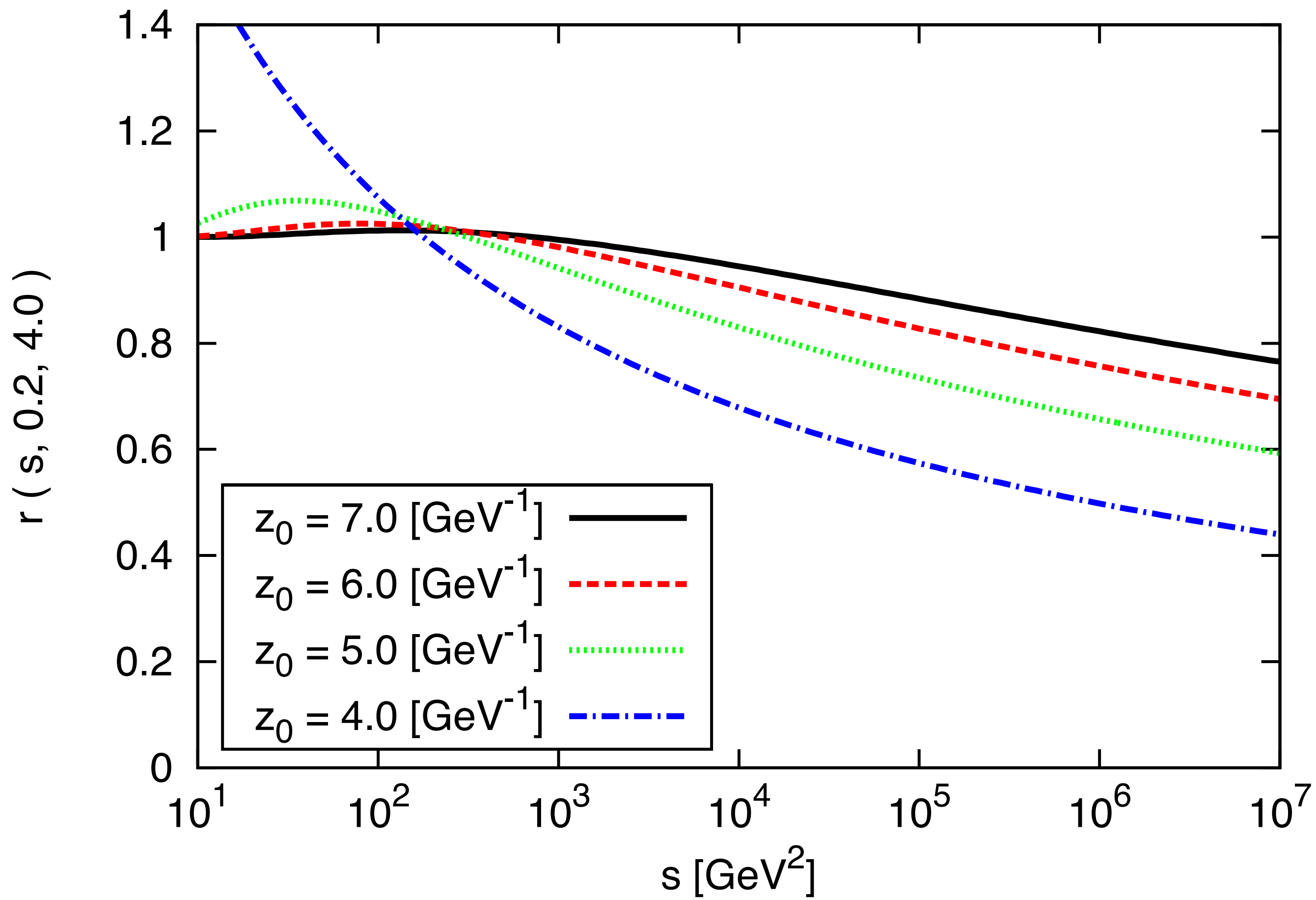
$$\eta=\left(-\log\frac{zz'}{z_0^2}+\rho\tau\right)/\sqrt{\rho\tau}$$

confinement effect

3 adjustable parameters : $\boxed{\rho,\ g_0^2,\ z_0}$

Kernel ratio

$$r(s,z,z') \equiv \mathrm{Im}[\chi_{mod}]/\mathrm{Im}[\chi_c]$$

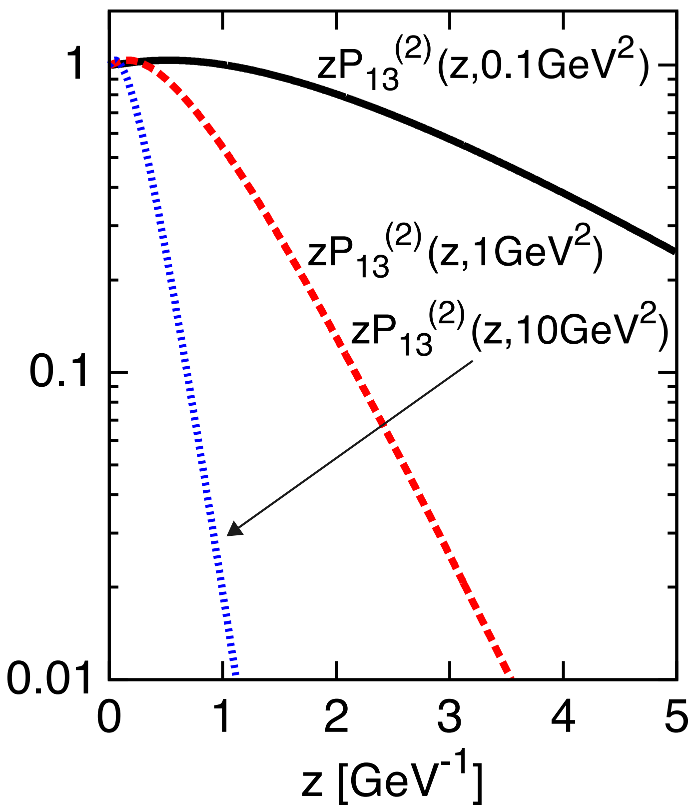


The effect of the added term ～30%

Density distribution for the incident particle

Polchinski-Strassler (2003)

- As a density distribution of the incident particle (virtual photon), we use wave function of the 5D U(1) vector field



$$\boxed{P_{13}^{(2)}(z,Q^2) = Q^2 z\left(K_0^2(Qz) + K_1^2(Qz)\right)}$$

(to calculate F_2)

$$\boxed{P_{13}^{(L)}(z,Q^2) = Q^2 z K_0^2(Qz)}$$

(to calculate F_L)

$P_{13}^{(2)}$ are localized at the origin with Q^2 increasing
(the behavior of $P_{13}^{(L)}$ is similar)

Density distribution for the target particle (1)

Abidin-Carlson (2009)

The matrix element of the energy momentum tensor in respect to spin 1/2 particle:

$$\left\langle p_2, s_2 \middle| T^{\mu\nu}(0) \middle| p_1, s_1 \right\rangle = u(p_2, s_2)\left(A(t)\gamma^{(\mu} p^{\nu)} + B(t)\frac{ip^{(\mu}\sigma^{\nu)\alpha} q_\alpha}{2m} + C(t)\frac{q^\mu q^\nu - q^2 \eta^{\mu\nu}}{m} \right) u(p_1, s_1)$$

To calculate A(t) etc. with the holographic model of nucleons,

$$S_F = \int d^5x \sqrt{g} e^{-\kappa^2 z^2} \left(\frac{i}{2}\bar{\Psi} e_A^N \Gamma^A D_N \Psi - \frac{i}{2}(D_N \Psi)^\dagger \Gamma^0 e_A^N \Gamma^A \Psi - (M + \kappa^2 z^2)\bar{\Psi}\Psi \right)$$

we introduce the metric perturbation, $\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$, in the 5D classical action, and pick up the hψψ terms.
By comparing the Lorentz structure of them, we can obtain the form factors. (in this case, only A(t) remains)

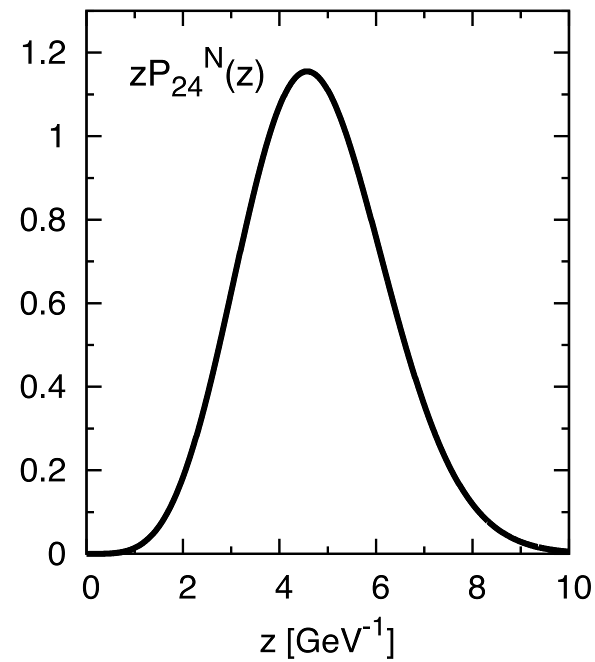
Density distribution for the target particle (2)

Finally, one can obtain the density distribution for the target nucleon:

$$\boxed{P_{24}(z') = \frac{e^{-\kappa^2 z^2}}{2z'^3}\left(\psi_L^2(z') + \psi_R^2(z')\right)} \qquad \boxed{\int dz' P_{24}(z') = 1}$$

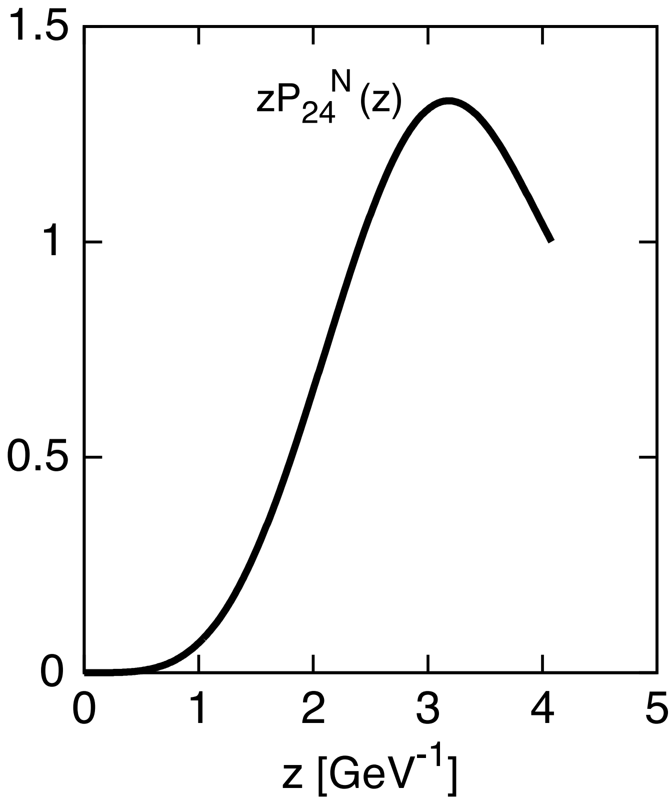
where $\psi_{L,R}$ are 5D wave functions describing a nucleon as a 5D Dirac fermion with chiral symmetry breaking.

- The peak position of P_{24} is in the large z region, which is obviously different from P_{13}.
- Also, this behavior is different from the hard-wall version.



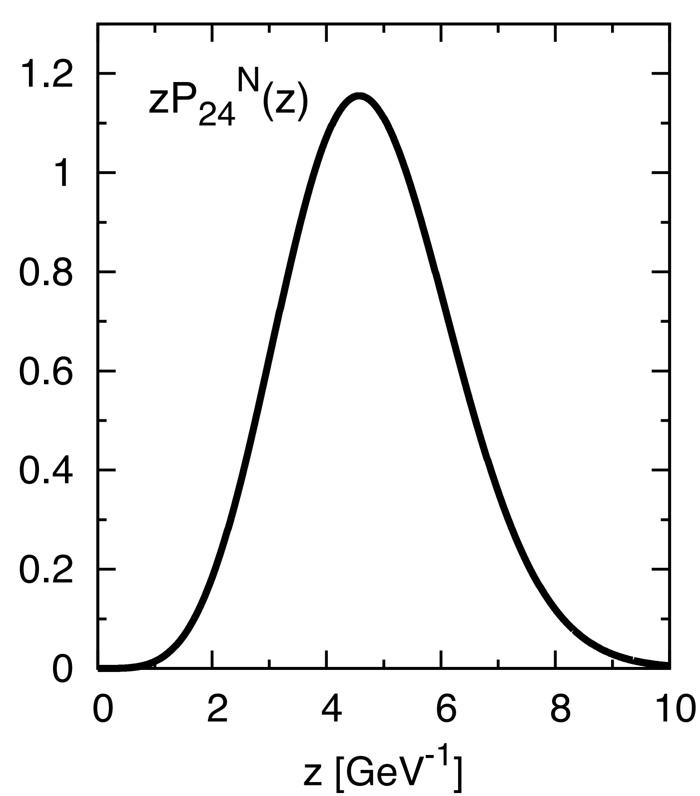
Density distribution for the target particle (3)

<u>hard-wall model</u>



$$P_{24}(z') = \frac{1}{2z'^3}\left(\psi_L^2(z') + \psi_R^2(z')\right)$$

<u>soft-wall model</u>



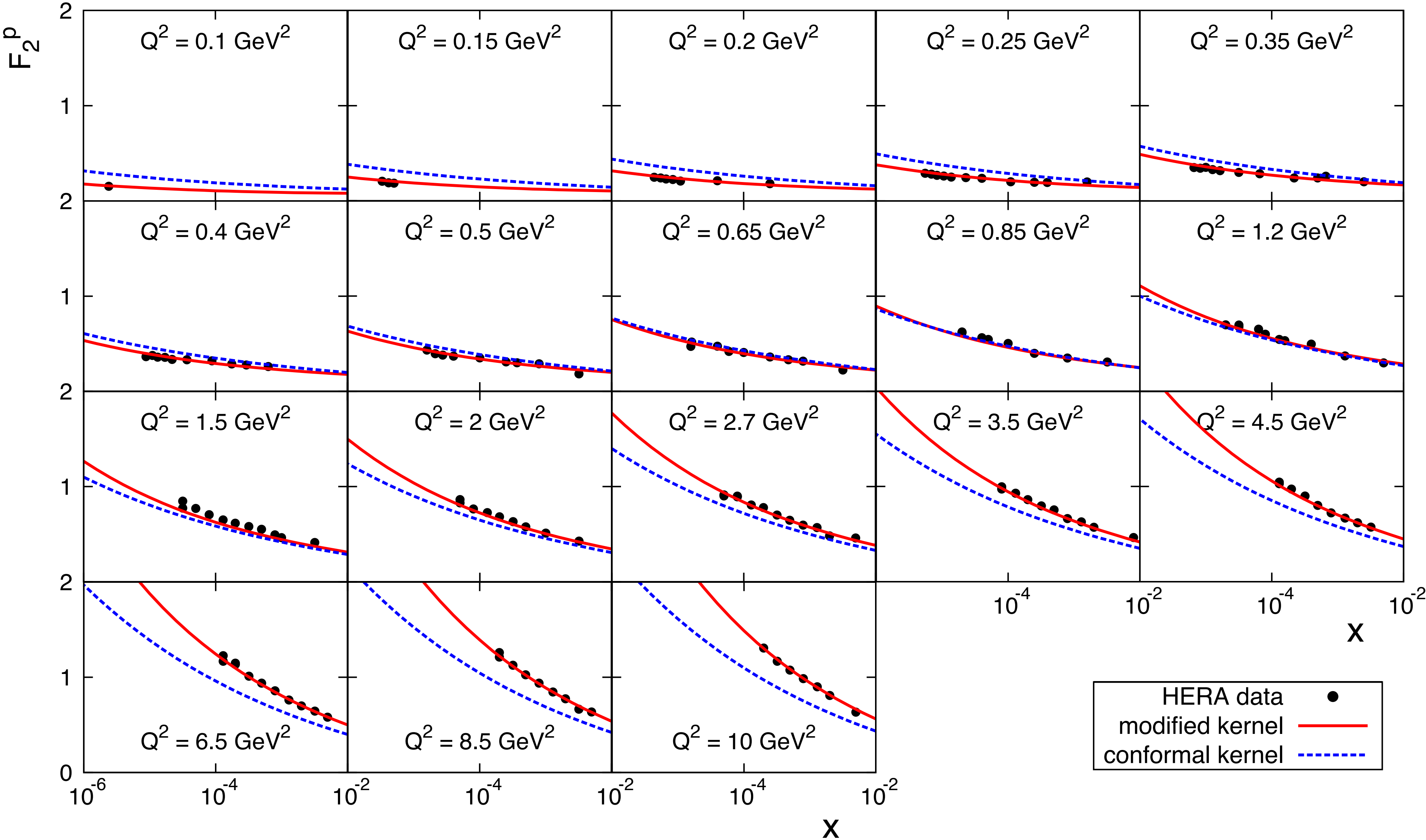
$$P_{24}(z') = \frac{e^{-\kappa^2 z^2}}{2z'^3}\left(\psi_L^2(z') + \psi_R^2(z')\right)$$

Fixing parameters

- There are 3 adjustable parameters (ρ, g_0, and z_0)
- They are fixed with the experimental data for F_2^p measured at HERA

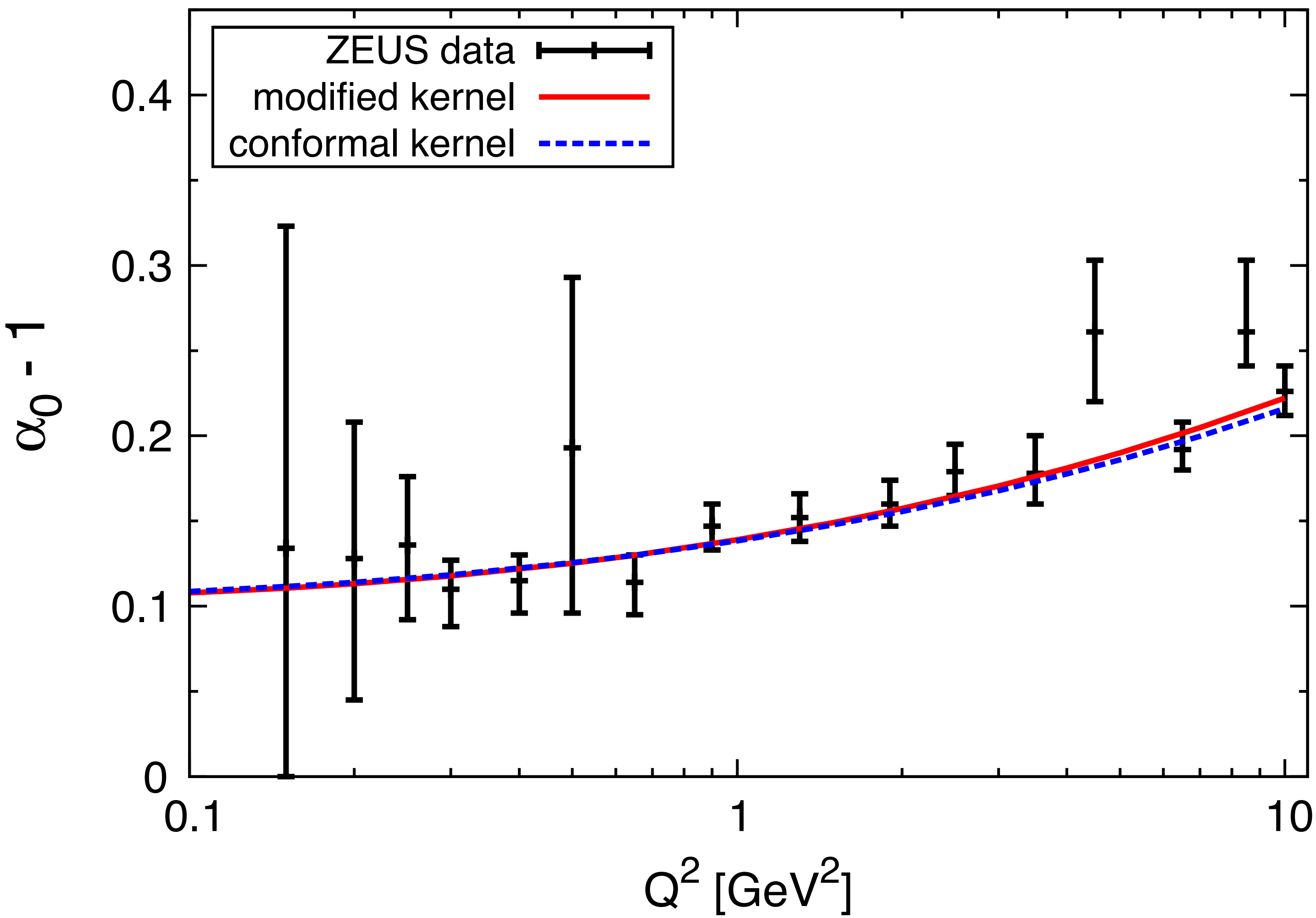
ZEUS collaboration (1999)
H1 and ZEUS collaborations (2010)

Proton structure function



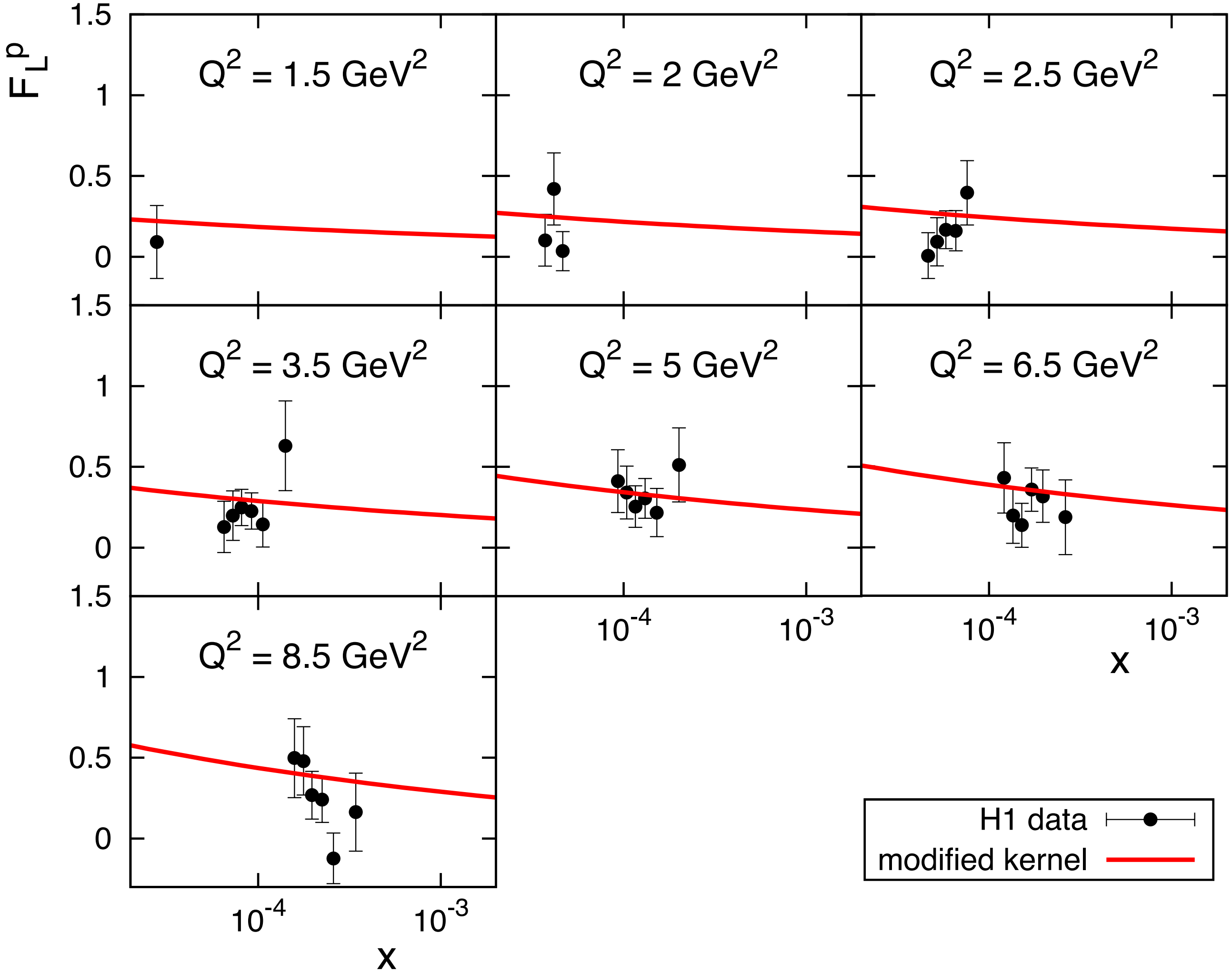
Pomeron intercept

$x < 0.01$



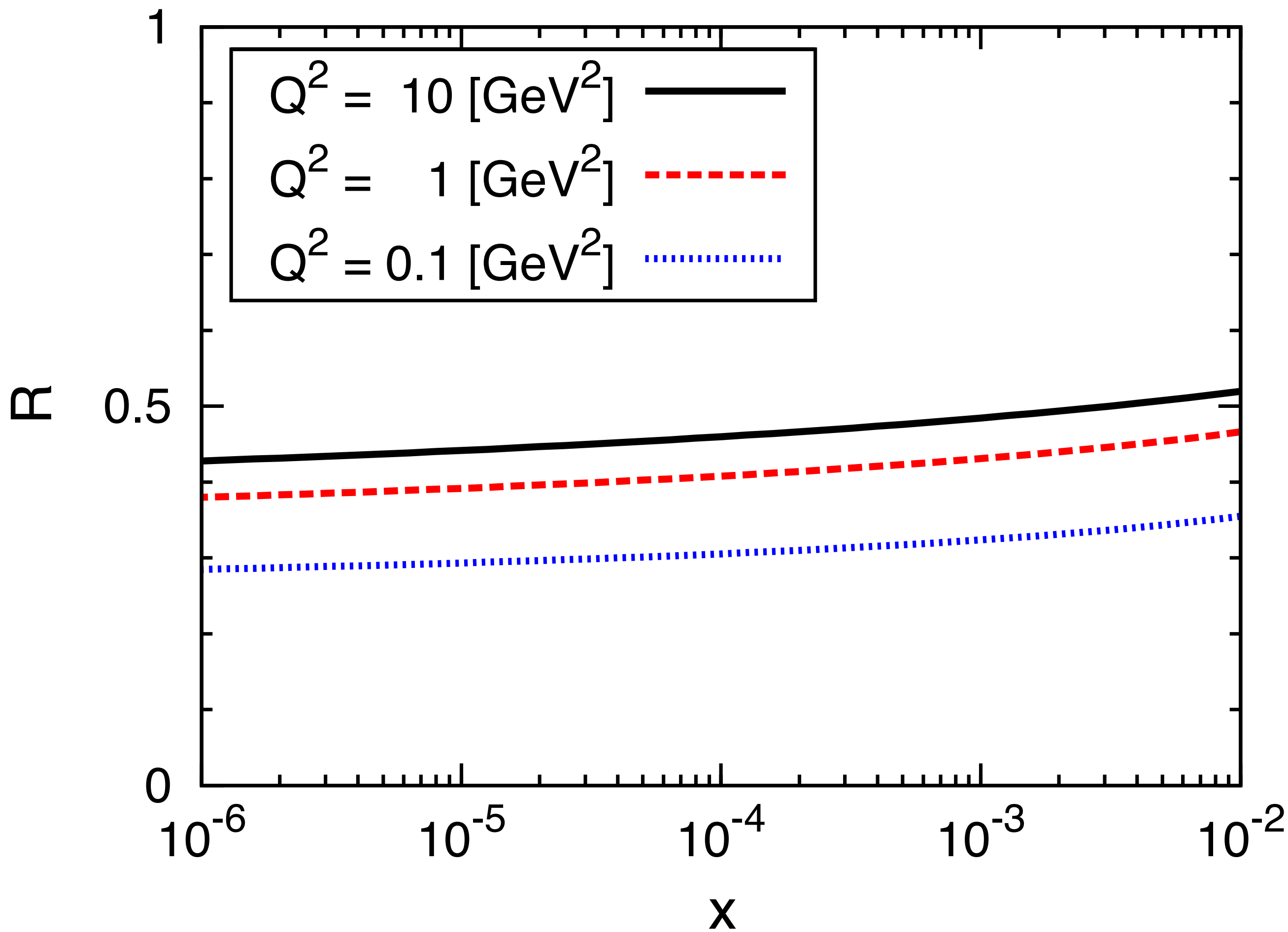
Proton longitudinal structure function

- Replacing the density distribution for incident particle with the longitudinal component



Scale dependence of R

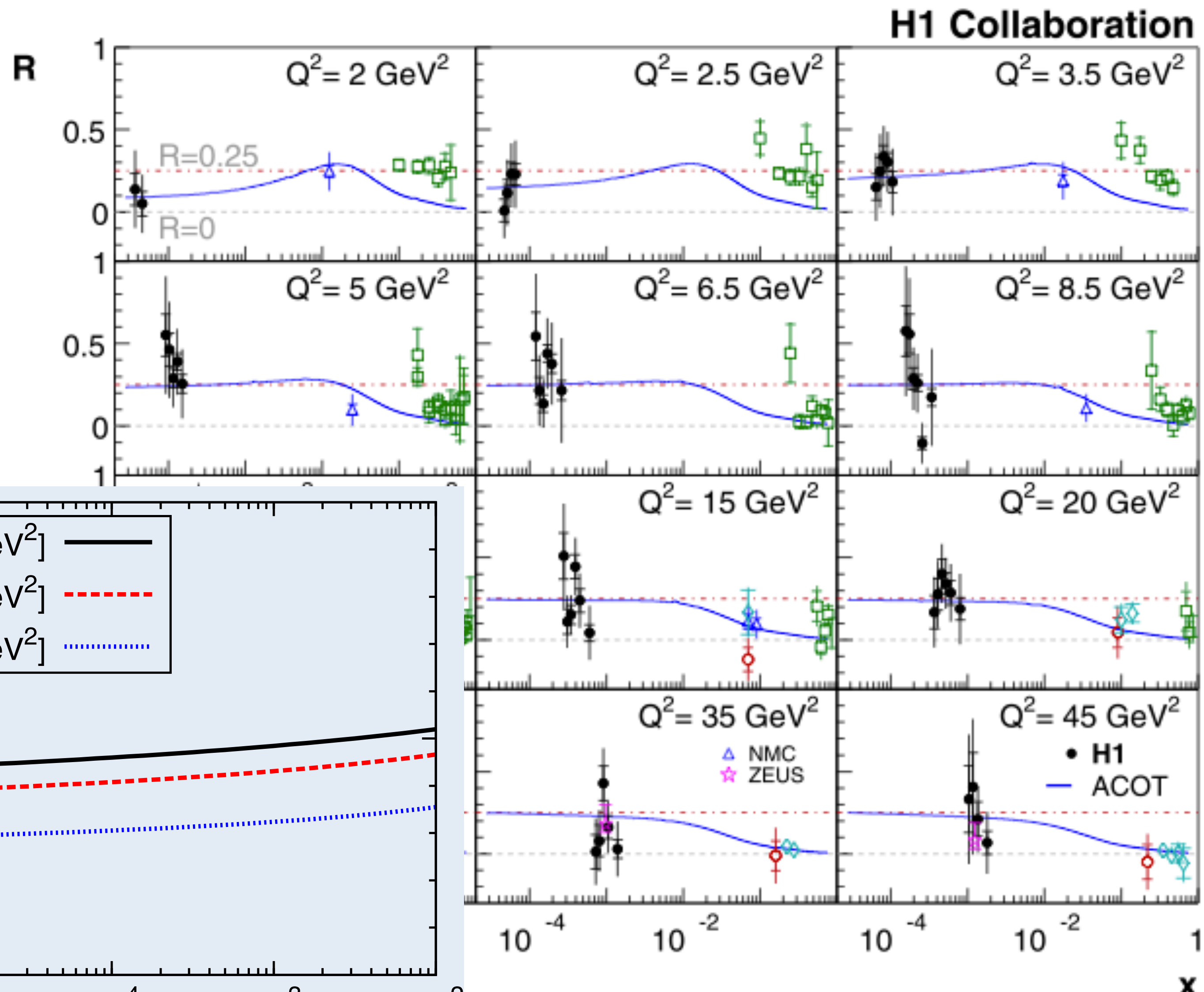
longitudinal-to-transverse ratio : $\boxed{R \equiv F_L^p / F_T^p \left(= F_2^p - F_L^p\right)}$

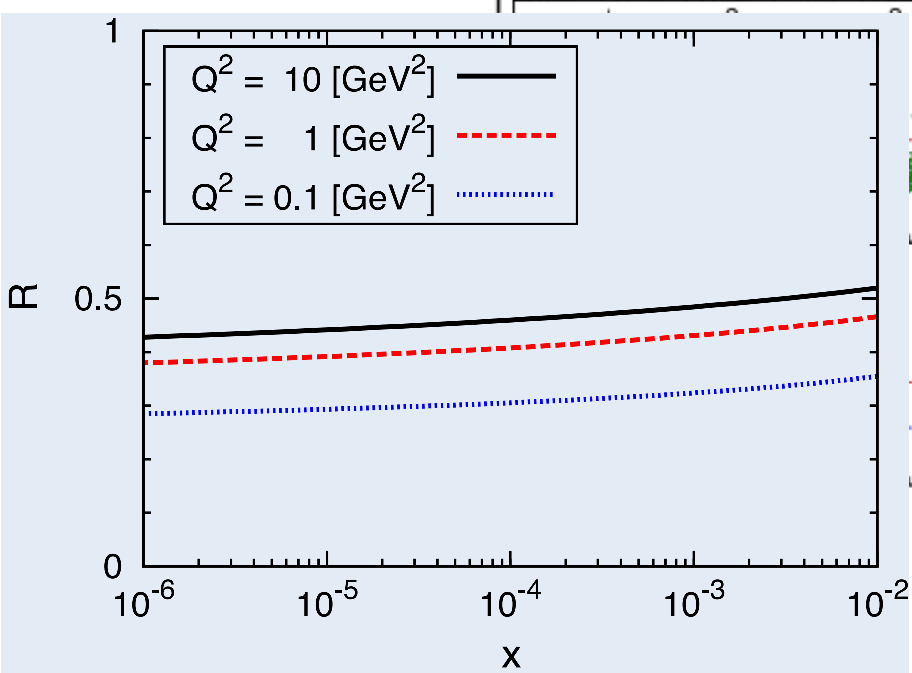


x dependence of R (experimental data)

H1 Collaboration (2011)

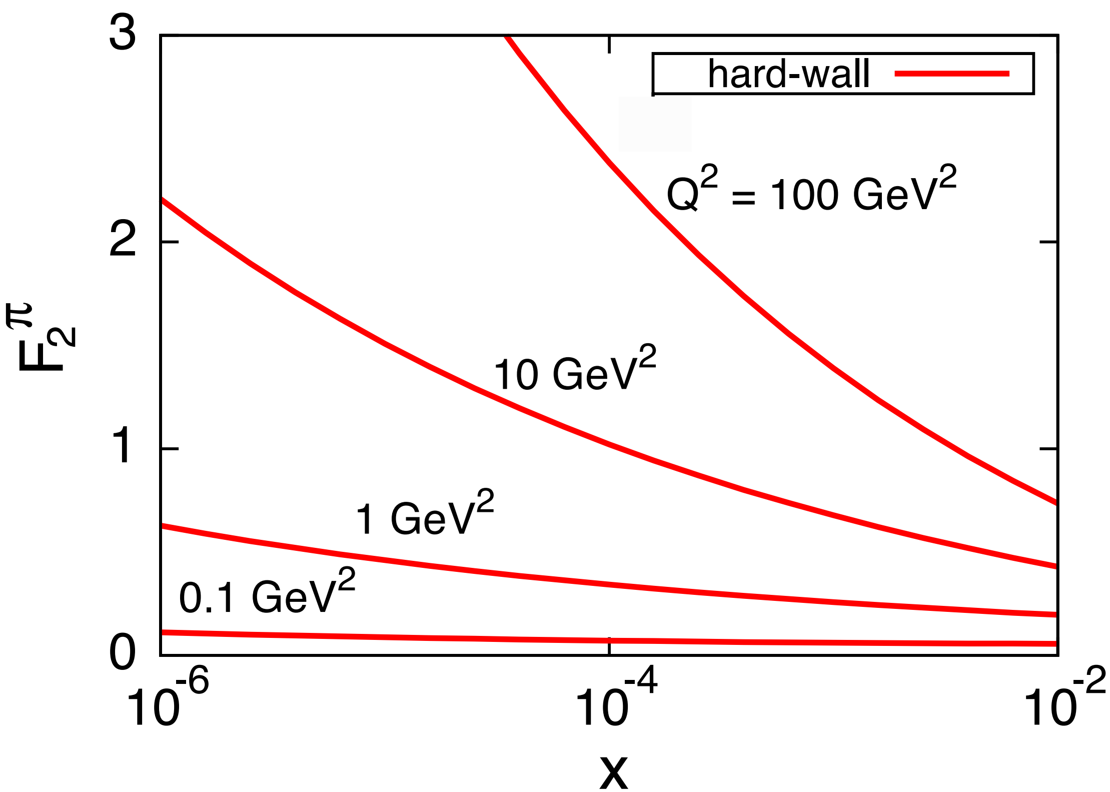
Fig. 14 The ratio R as a function of x in bins of Q^2. The inner error bars represent statistical error, the full error bars include the statistical and systematic uncertainties added in quadrature. The *solid curves* represent predictions of the DGLAP fit in the ACOT scheme. Results from other experiments are shown by the *open symbols*





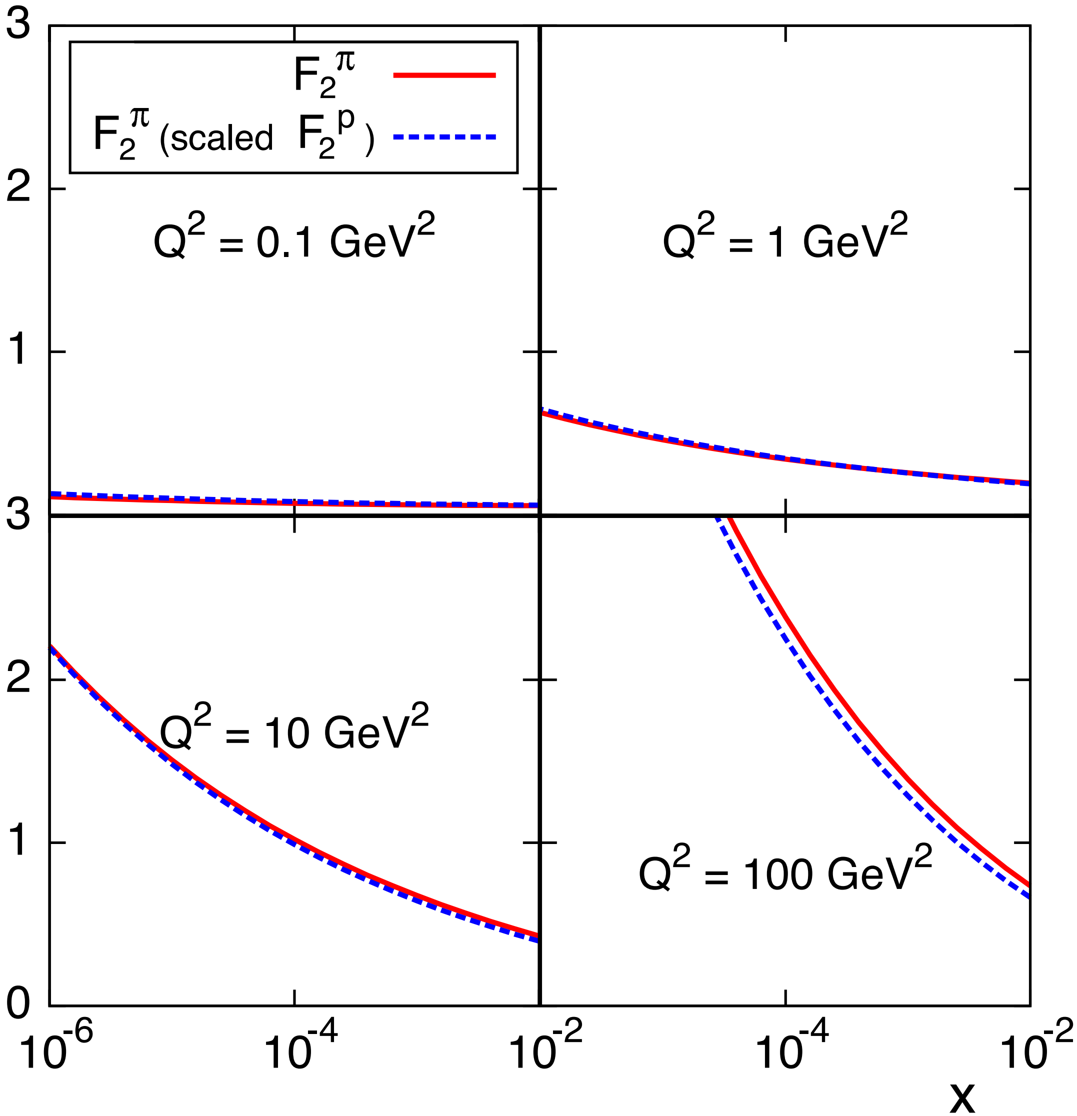
Pion structure function (from hard-wall model)

- Replacing the density distribution for target hadron with pion's, which can be calculated by using the holographic model of mesons Abidin-Carlson (2008)



- There is no experimental data

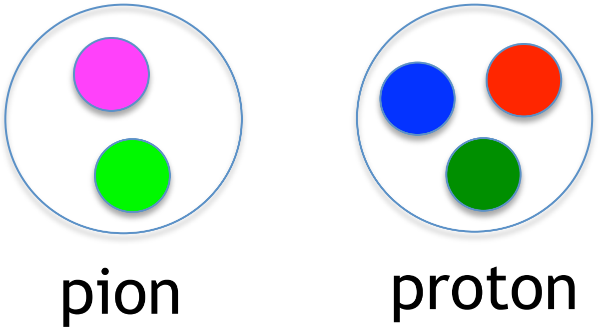
a relation between F_2^p and F_2^π (from hard-wall model)



Nikolaev-Speth-Zoller (2000)

$$F_2^\pi(x,Q^2) \simeq \frac{2}{3} F_2^p\left(\frac{2}{3}x, Q^2\right)$$

a relation based on the valence quark number



accidental coincidences?

Future works

- Improvement of the model
  - Saturation effect
- Applications to other high energy scattering processes
  - hadron-hadron scattering
  - Deeply virtual Compton scattering
  - Photoproduction of neutral vector mesons

Summary

- We have studied structure functions at the small *x* in the framework of holographic QCD.
- Results for F_2^p and F_L^p with the modified kernel are consistent with the experimental data, while those with the conformal kernel are not enough.
  -> QCD is significantly different from CFT at small *x*
  -> The holographic technique may be useful as a "building block" for building models to investigate the nonperturbative region in QCD
- Various applications can be considered, which are not only for the improvement of the model itself, but also for other high energy scattering processes.