

Transverse Charge and Spin Structures of the Nucleon

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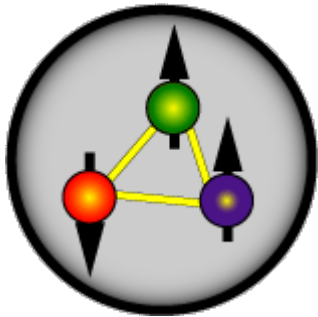
Nucleon

What we know about the Proton

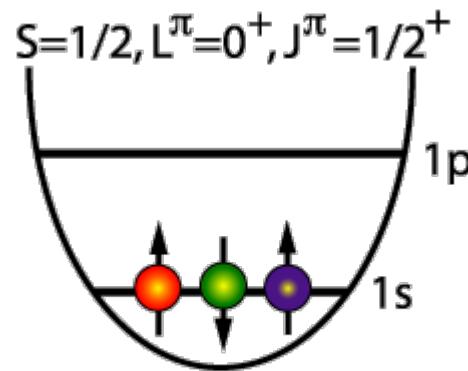


Experimentally, we know about

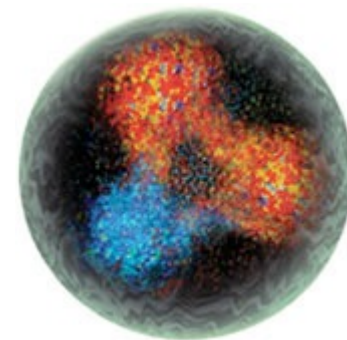
- Mass = 938.272 MeV
- Spin: $s = \frac{1}{2}\hbar$
 - Magnetic moment $\mu_p = 2.79\mu_N$
 - Anomalous magnetic moment $\mu_a = 1.79\mu_N$



Naive Quark model



Quark potential model



Picture of QCD

Nucleon, one of the most messy objects in the Universe!

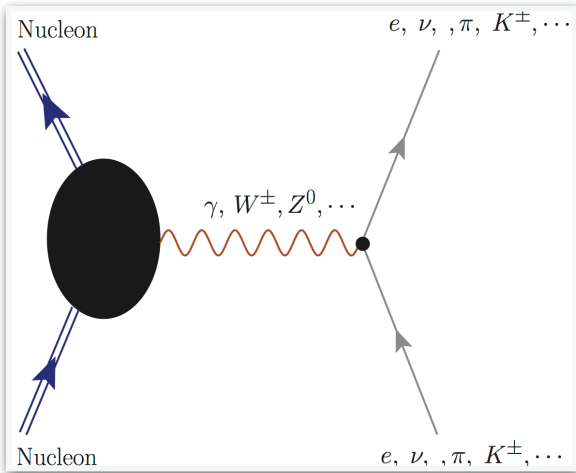
How to study the Nucleon



How to study the Nucleon



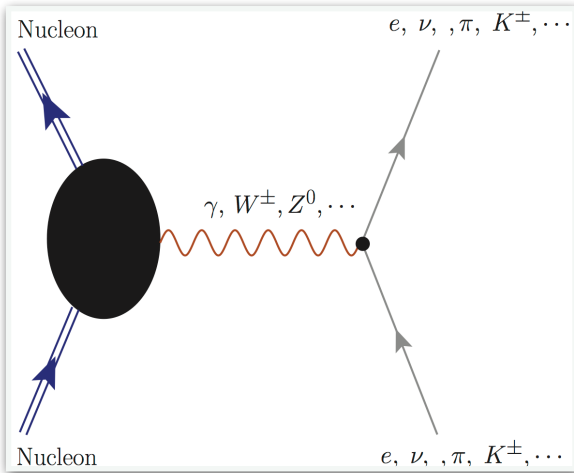
Elastic Scattering



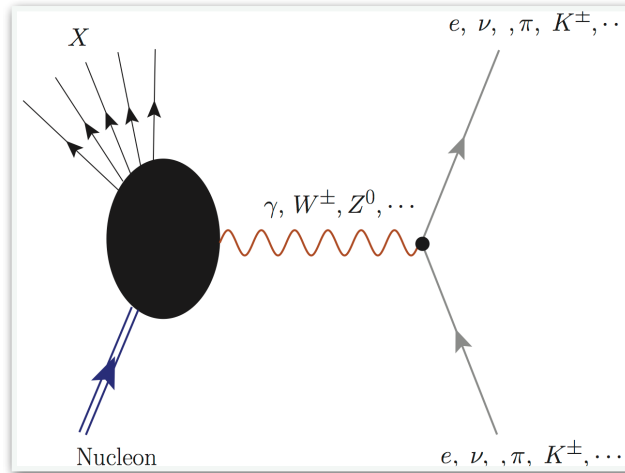
Radii,
Form factors,
densities

How to study the Nucleon

Elastic Scattering



Inelastic Scattering



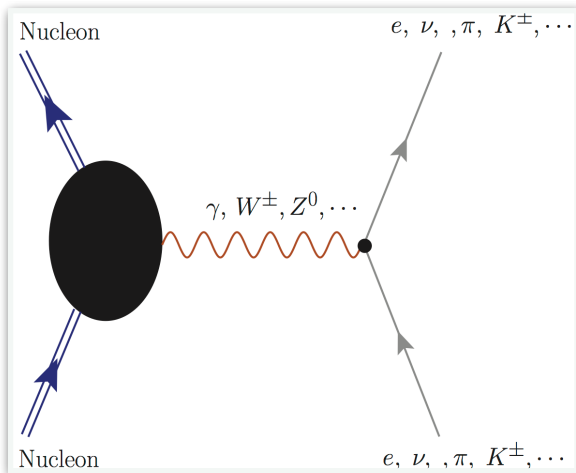
Radii,
Form factors,
densities

Parton distributions,
Structure functions

How to study the Nucleon

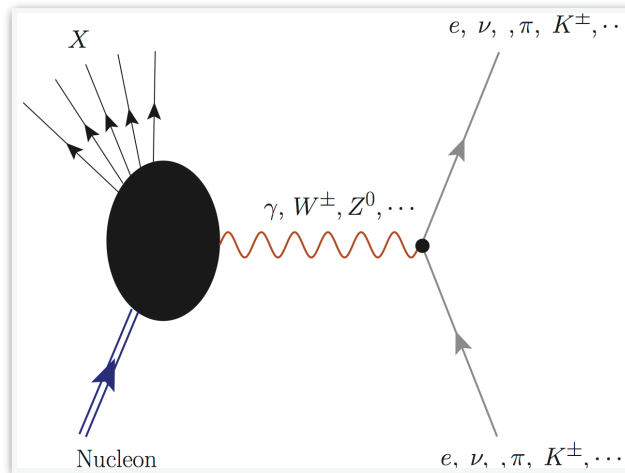


Elastic Scattering



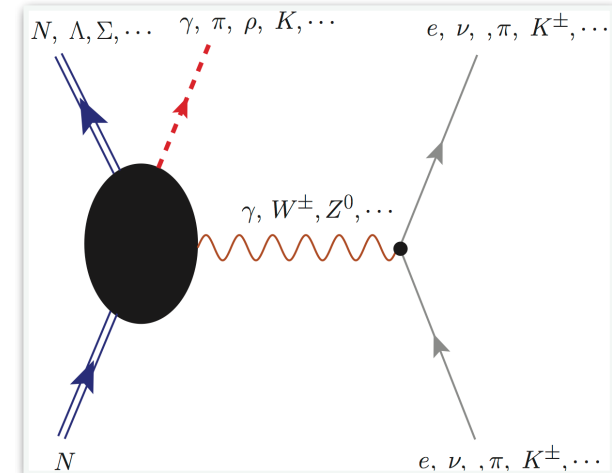
Radii,
Form factors,
densities

Inelastic Scattering



Parton distributions,
Structure functions

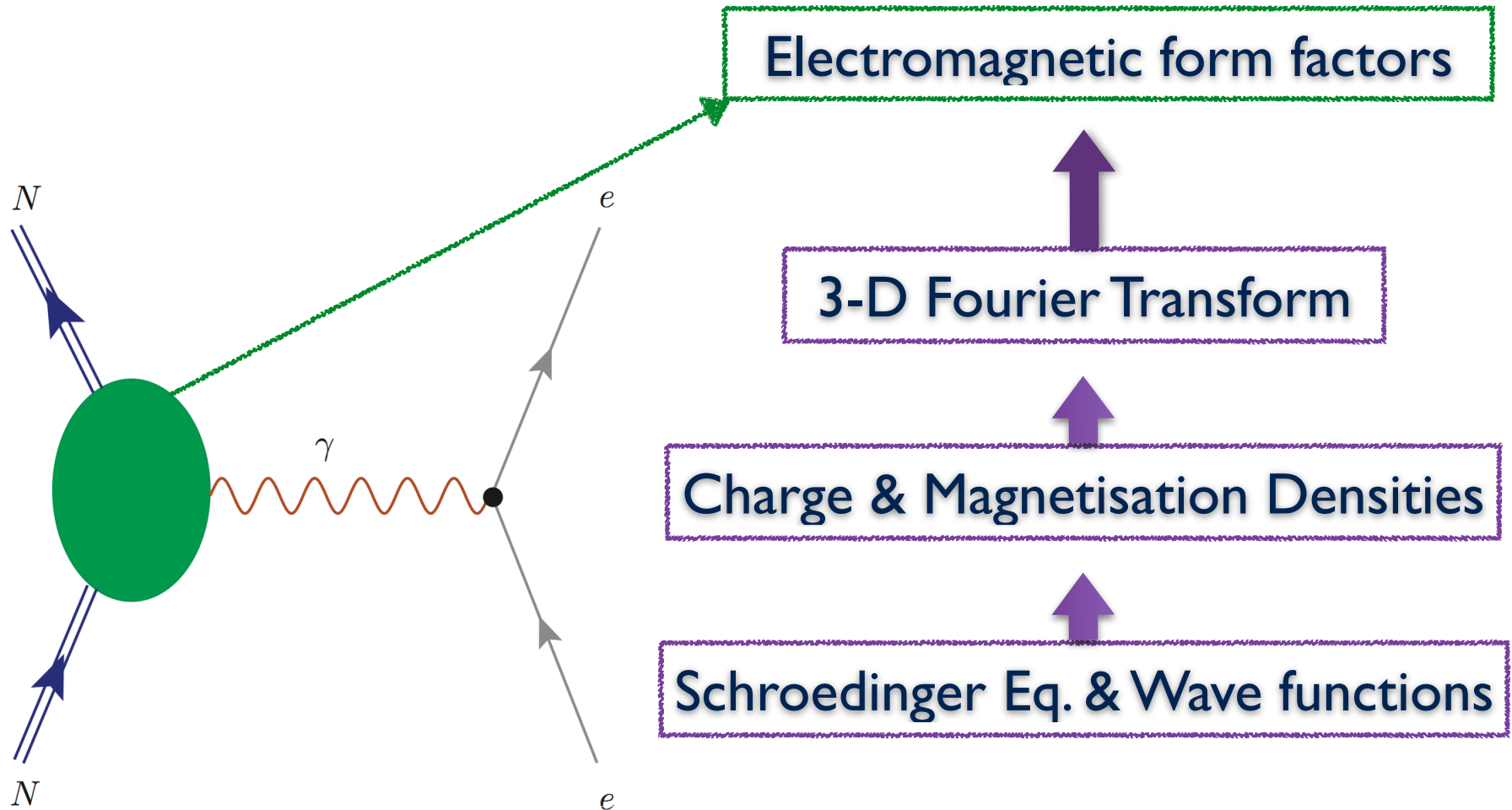
Exclusive Scattering



Generalised
Parton Distributions,
Generalised
Form factors

Interpretation of the Form factors

Non-Relativistic picture of the EM form factors



Interpretation of the EMFFs

Traditional interpretation of the nucleon form factors

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \rho(\mathbf{r}) \rightarrow \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

However, the initial and final momenta are different in a relativistic case. Thus, the initial and final wave functions are different.



Probability interpretation is wrong in a relativistic case!



We need a correct interpretation of the form factors

Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

G.A. Miller, PRL **99**, 112001 (2007)

Interpretation of the EMFFs

R: Size of the system
M: Mass of the system

Non-Relativistic description

$$R \gg 1/M \text{ (Compton length)}$$

$$M_{\text{atom}} R_{\text{atom}} = M_{\text{atom}} / (m_e \alpha) \sim 10^5$$

$$\|Q\| \ll M_{\text{atom}} \quad 1/\|Q\| \leq R$$

$$\rho(\mathbf{r}) = \sum \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r})$$

Particle number fixed.

Form factors can be measured and well interpreted (almost no recoil effect).

Relativistic description

$$M_N R_N \sim 1 \quad \|Q\| \geq M_N$$

Particle creation & annihilation

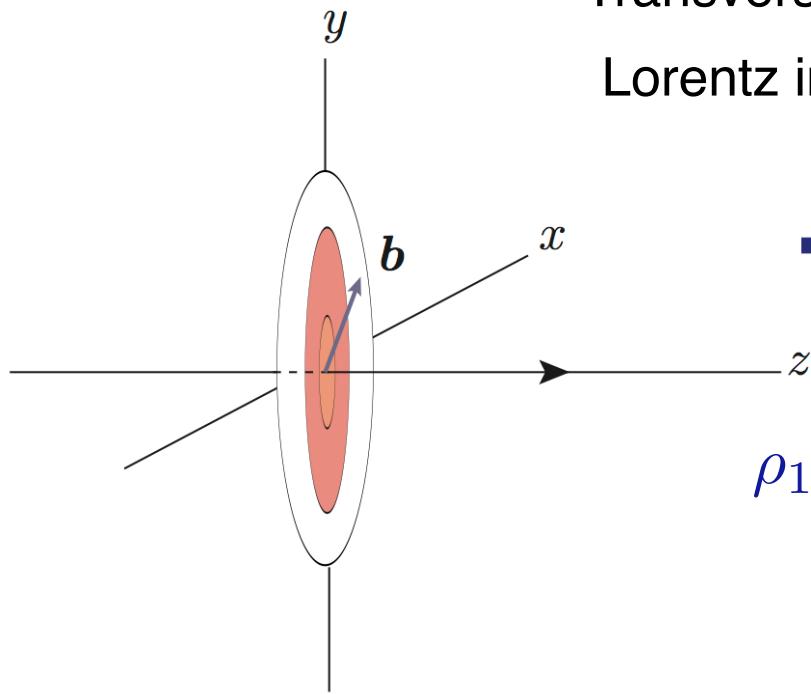
Initial and final momenta are different!



Nucleon cannot be treated non-relativistically!

Interpretation of the EMFFs

Modern understanding of the form factors



Transverse Charge densities $\rho_1(\mathbf{b})$

Lorentz invariant: independent of any observer.

\xrightarrow{p} Infinite momentum framework

$$\rho_1(b) = \sum_q e_q^2 \int dx \boxed{f_{q-\bar{q}}(x, \mathbf{b})}$$

GPDs

Dirac & Pauli form factors

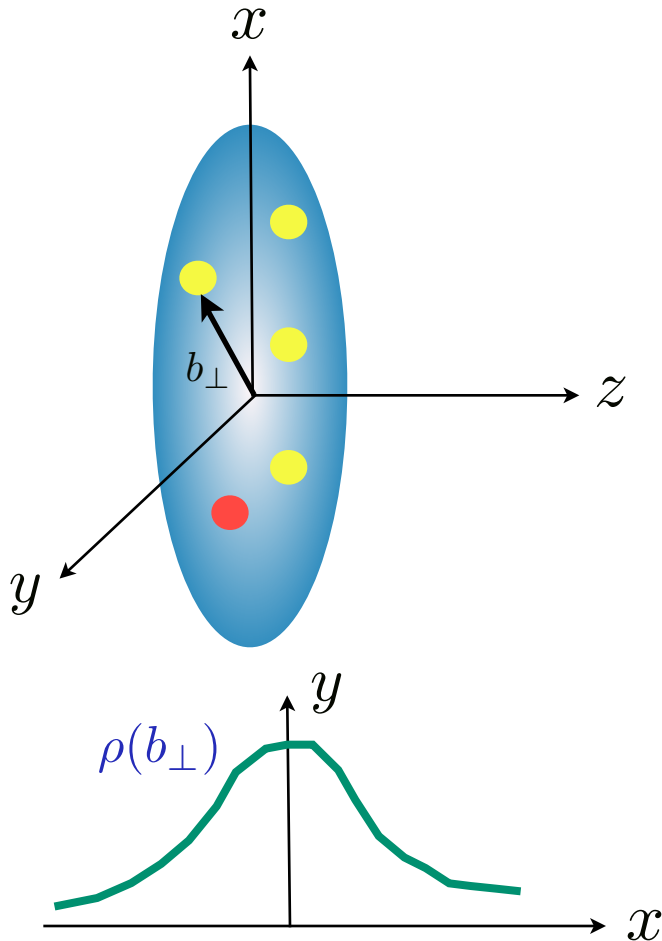
$$F_{1,2}(\Delta) = \int d^2b e^{i\Delta_\perp \cdot \mathbf{b}} \rho_{1,2}(\mathbf{r})$$

Nucleon Tomography



Nucleon Tomography

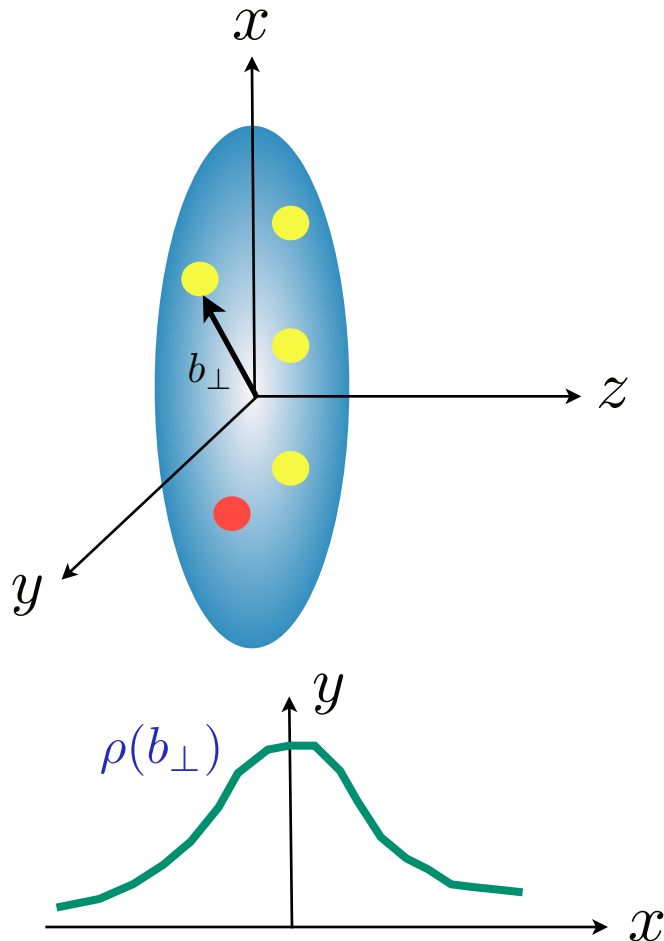
Axial & **Tensor** Form factors, Axial-vector charges, **Tensor** charges



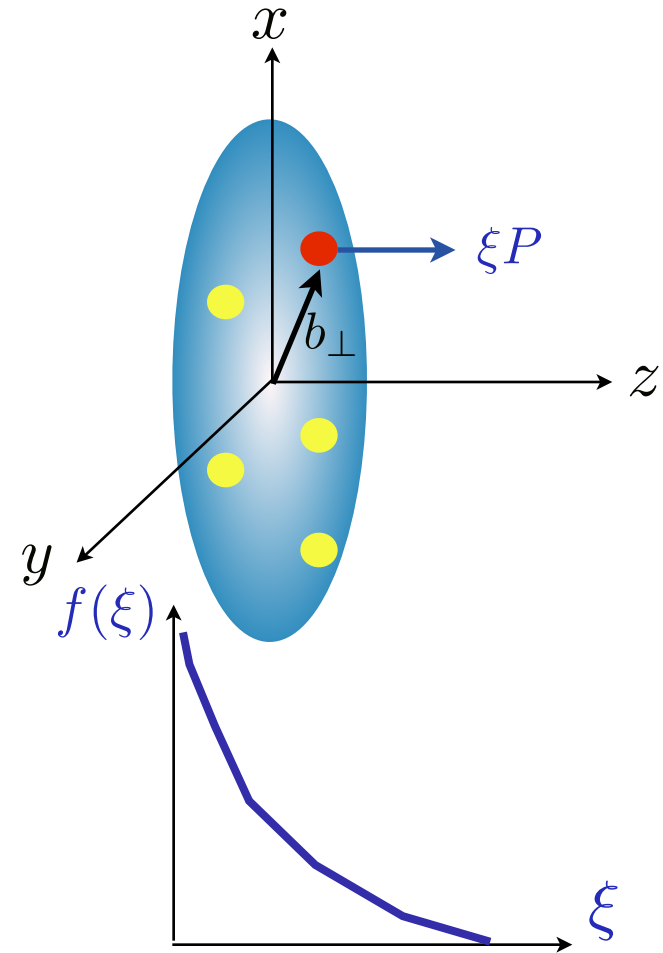
Nucleon Tomography



Axial & **Tensor** Form factors, Axial-vector charges, **Tensor** charges



Structure functions



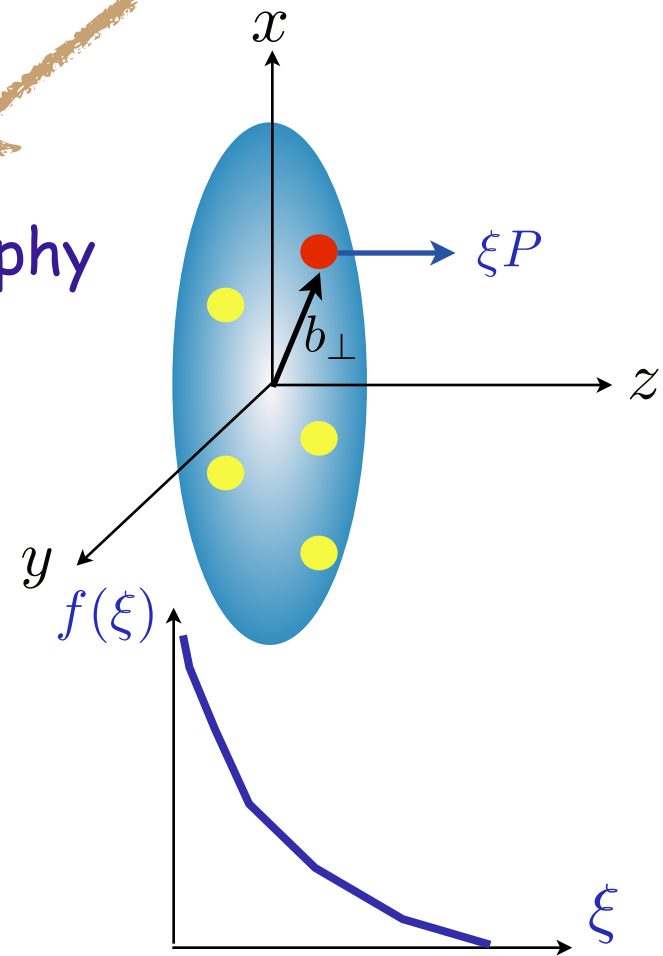
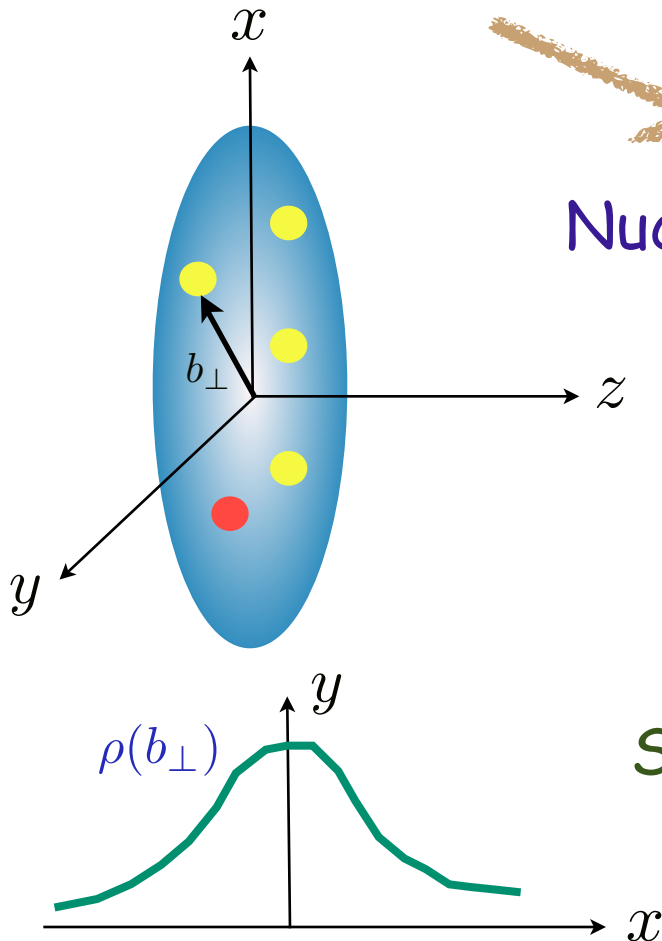
Nucleon Tomography

Axial & **Tensor** Form factors, Axial-vector charges, **Tensor** charges

Structure functions

Nucleon Tomography
(GPDs)

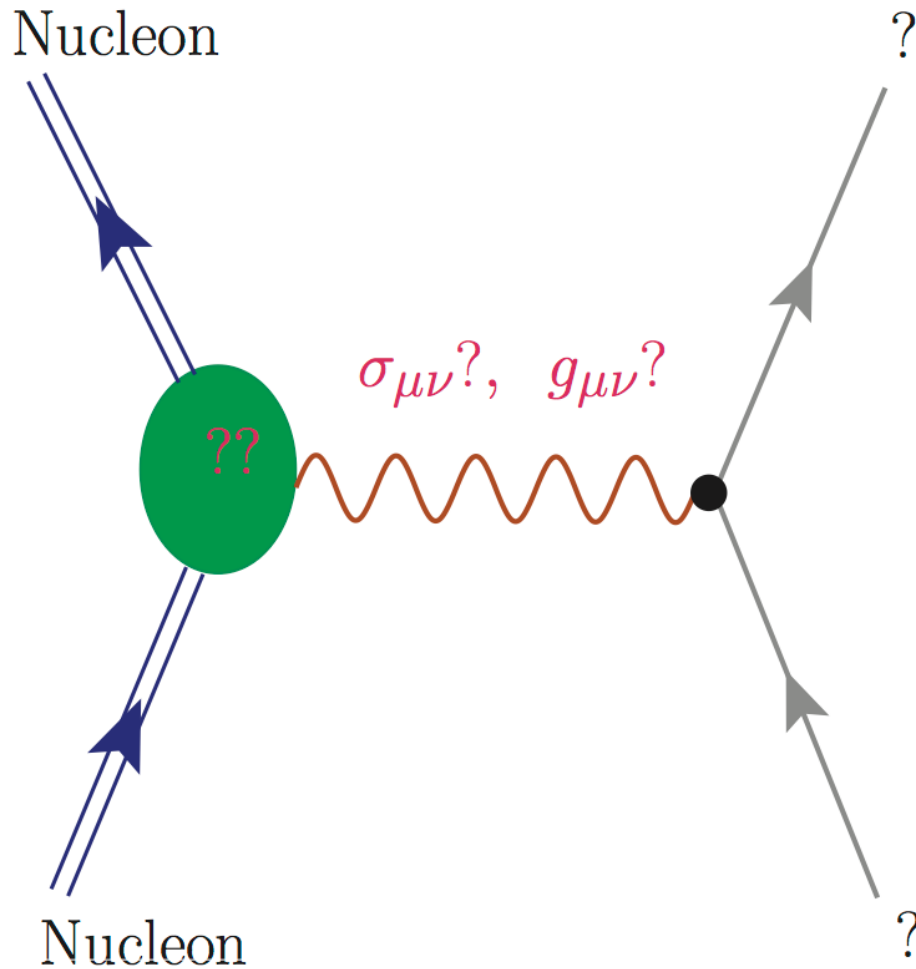
Spin Structure



Generalised Parton Distributions



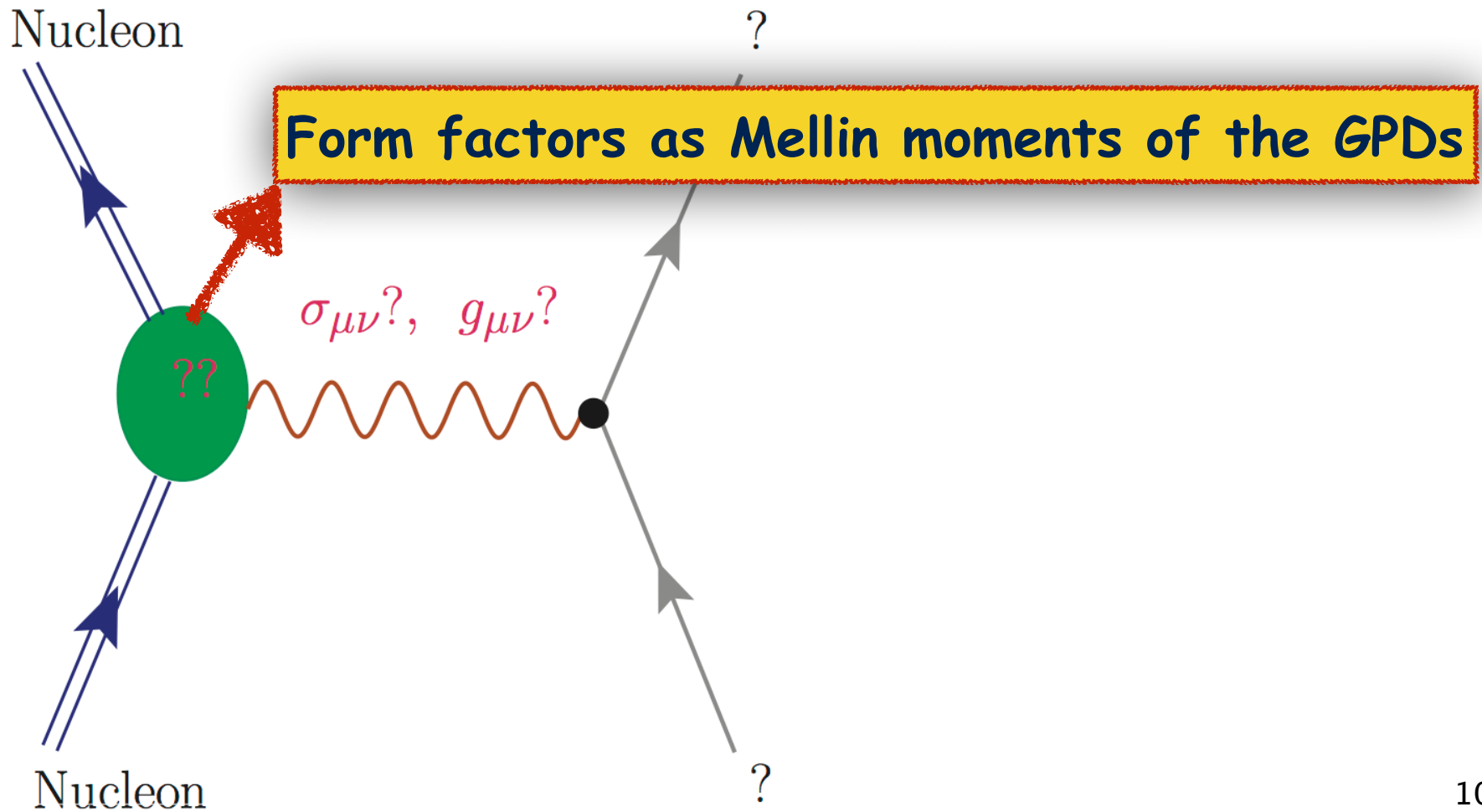
Probes are unknown for **Tensor** form factors
and the **Energy-Momentum Tensor** form factors!



Generalised Parton Distributions



Probes are unknown for **Tensor** form factors
and the **Energy-Momentum Tensor** form factors!



Model

Chiral quark–soliton model



Merits of the chiral quark–soliton model

- Fully relativistic field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalisation scale is naturally given.
 $1/\rho \approx 600 \text{ MeV}$
- All relevant parameters were fixed already.

$$Z_{\chi\text{QSM}} = \int \mathcal{D}U \exp(-S_{\text{eff}})$$

$$S_{\text{eff}} = -N_c \text{Tr} \ln D(U)$$

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$$H(U) = -i\gamma_4\gamma_i\partial_i + \gamma_4 M U \gamma_5$$
$$S_{\text{eff}} = -N_c \text{Tr} \ln D(U)$$
$$D(U) = \partial_4 + H(U) + \hat{m}$$

The diagram shows the relationship between the Dirac operator $D(U)$ and the Hamiltonian $H(U)$. A green circle highlights $D(U)$ in the effective action, with an arrow pointing to its definition. A pink circle highlights $H(U)$ in the definition of $D(U)$, with an arrow pointing to its definition in the box above.

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$$H(U) = -i\gamma_4\gamma_i\partial_i + \gamma_4 M U \gamma_5$$
$$D(U) = \partial_4 + H(U) + \hat{m}$$
$$\hat{m} = \text{diag}(m_u, m_d, m_s)\gamma_4$$

The diagram illustrates the components of the chiral quark-soliton model. The partition function $\mathcal{Z}_{\chi\text{QSM}}$ is defined by an integral over the gauge field U of the exponential of the negative effective action $-S_{\text{eff}}$. The effective action S_{eff} is given by $-N_c \text{Tr} \ln D(U)$, where $D(U)$ is the Dirac operator. The Dirac operator $D(U)$ is composed of the derivative ∂_4 , the term $H(U)$, and the mass term \hat{m} . The term $H(U)$ is defined as $-i\gamma_4\gamma_i\partial_i + \gamma_4 M U \gamma_5$. The mass term \hat{m} is defined as $\text{diag}(m_u, m_d, m_s)\gamma_4$. Arrows indicate the relationships: a green arrow from $D(U)$ to the definition box, a pink arrow from $H(U)$ to its definition box, and a red arrow from \hat{m} to its definition box.

Chiral quark-soliton model



Classical solitons

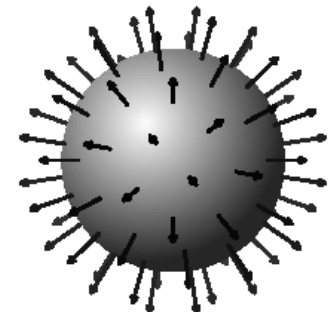
$$\langle J_N(\vec{x}, T) J_N^\dagger(\vec{y}, -T) \rangle_0 \sim \Pi_N(T) \sim e^{-[(N_c E_{\text{val}} + E_{\text{sea}})T]}$$



$$\frac{\delta}{\delta U}(N_c E_{\text{val}} + E_{\text{sea}}) = 0 \rightarrow M_{\text{cl}} = N_c E_{\text{val}}(U_c) + E_{\text{sea}}(U_c)$$

Hedgehog Ansatz:

$$U_{\text{SU}(2)} = \exp[i\gamma_5 \mathbf{n} \cdot \boldsymbol{\tau} P(r)]$$



hedgehog

Chiral quark–soliton model

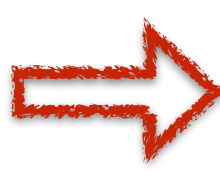


Collective (Zero-mode) quantisation

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

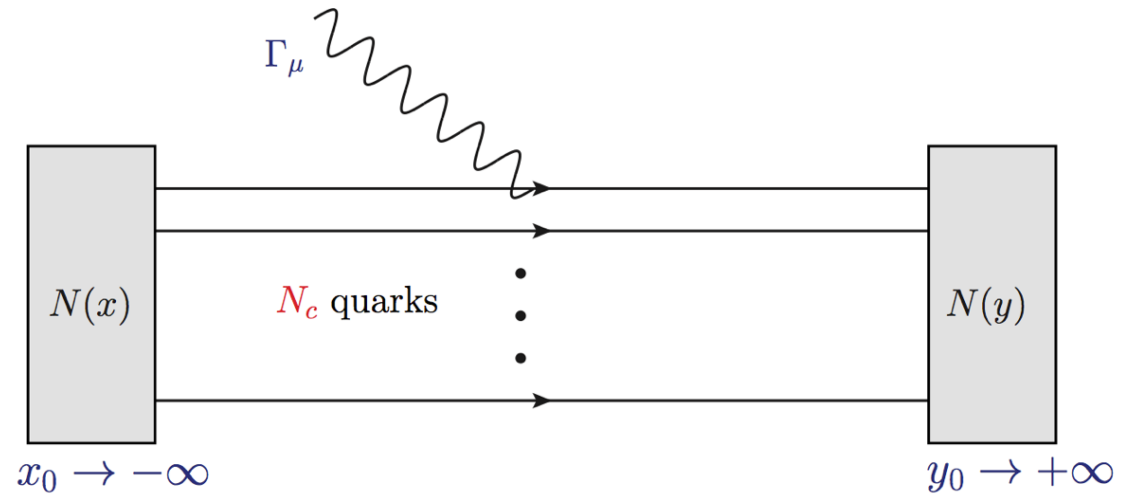
Zero-mode quantisation

$$U(\boldsymbol{x}, t) = R(t)U_c(\boldsymbol{x} - \boldsymbol{Z}(t))R^\dagger(t)$$
$$\int D\boldsymbol{U}[\cdots] \rightarrow \int D\boldsymbol{A}D\boldsymbol{Z}[\cdots]$$

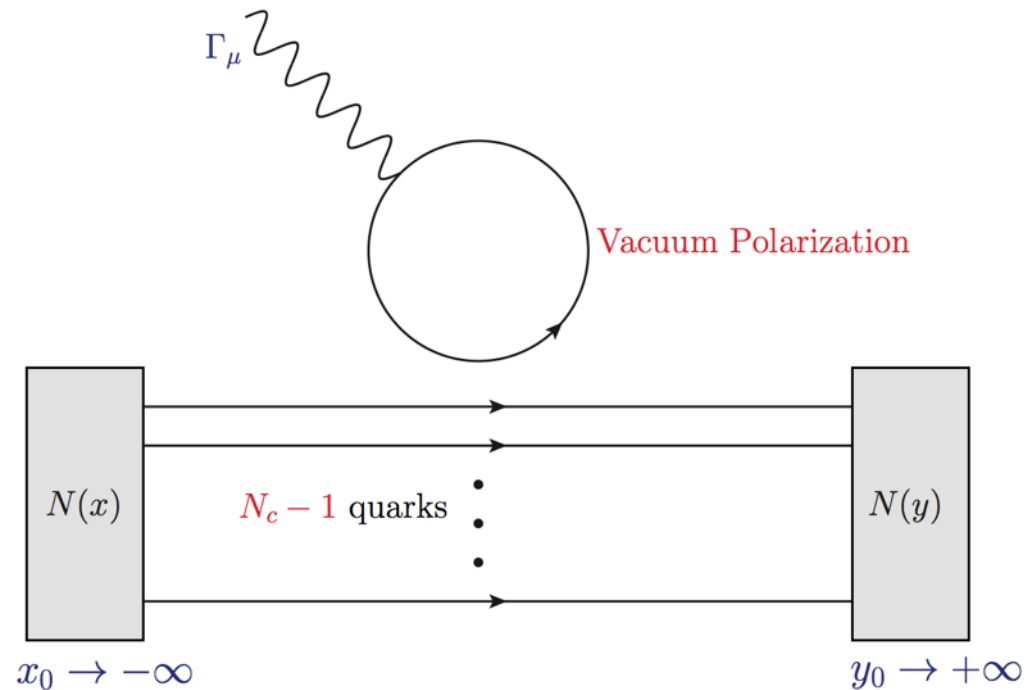

$$\mathcal{L} = -M_{sol} + \frac{I_1}{2} \sum_{i=1}^3 \Omega_i^2 + \frac{I_2}{2} \sum_{i=4}^7 \Omega_i^2 + \frac{N_c}{2\sqrt{3}} \Omega_8$$

Observables

Valence part



Sea part

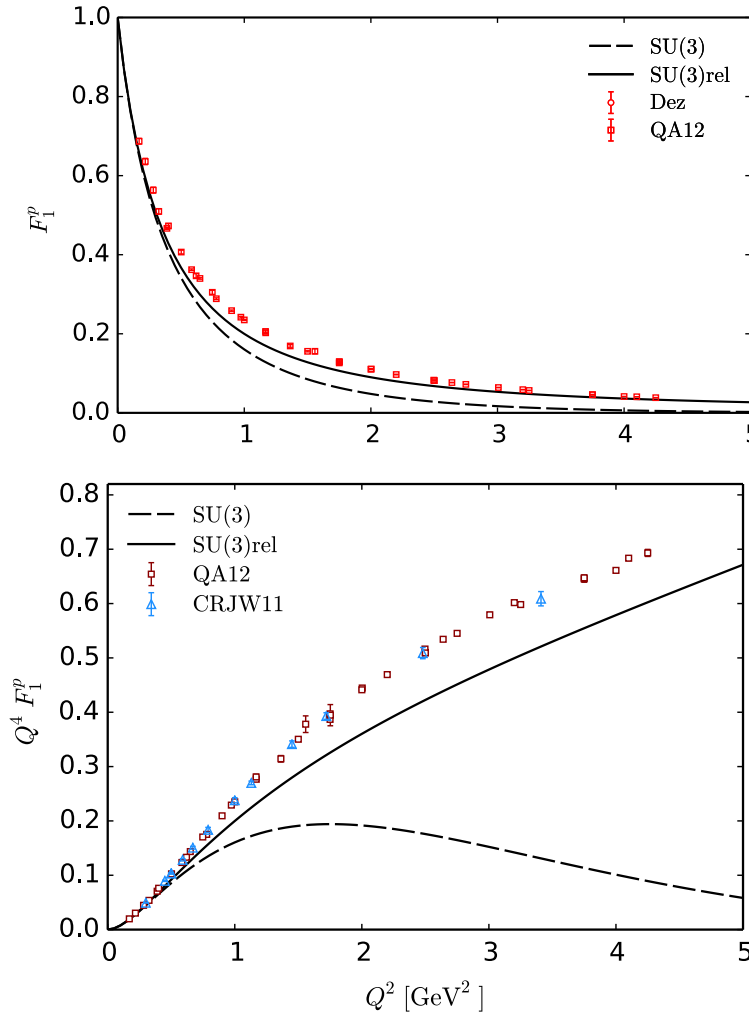


Transverse Charge Densities

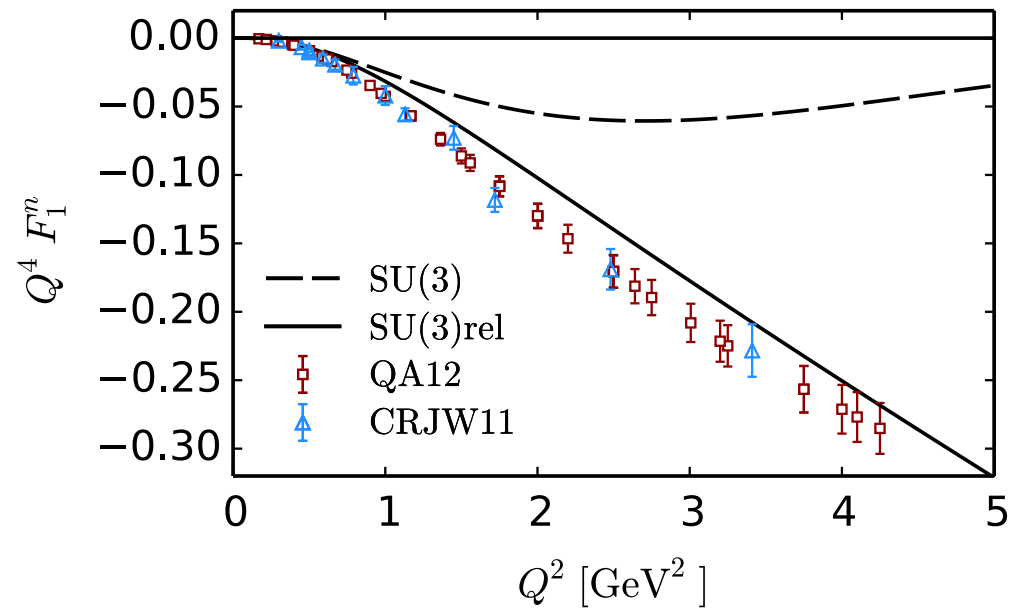
Dirac & Pauli Form factors



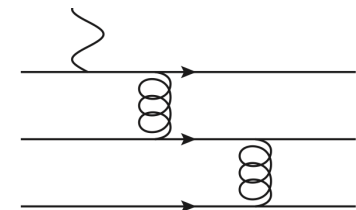
Proton



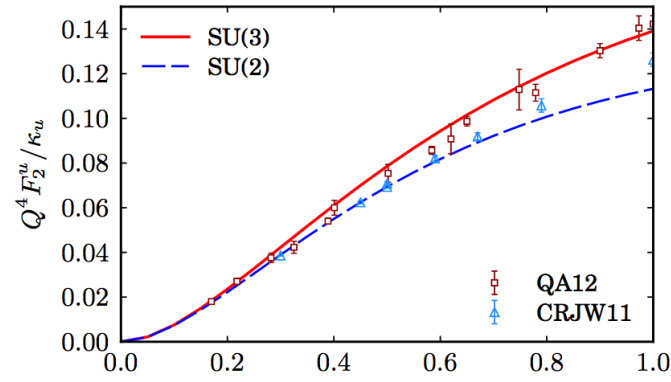
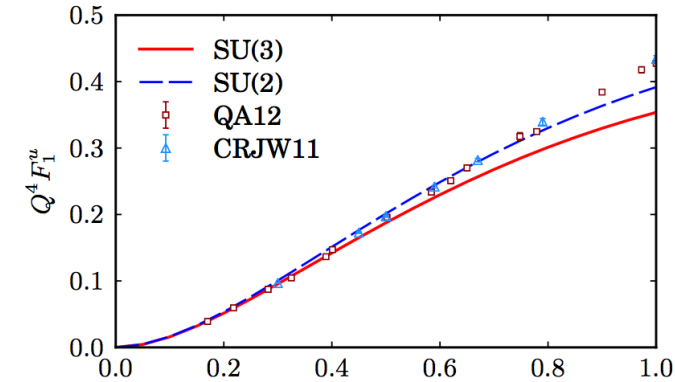
Neutron



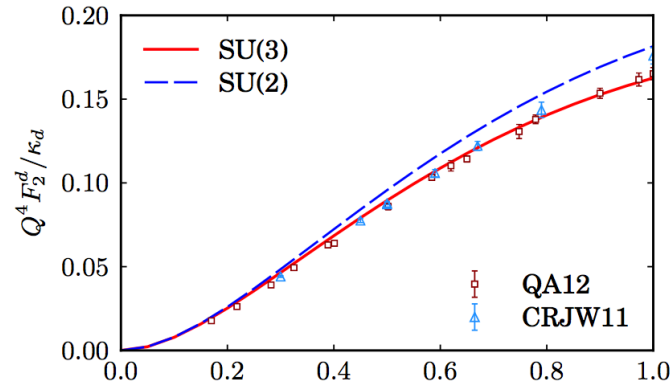
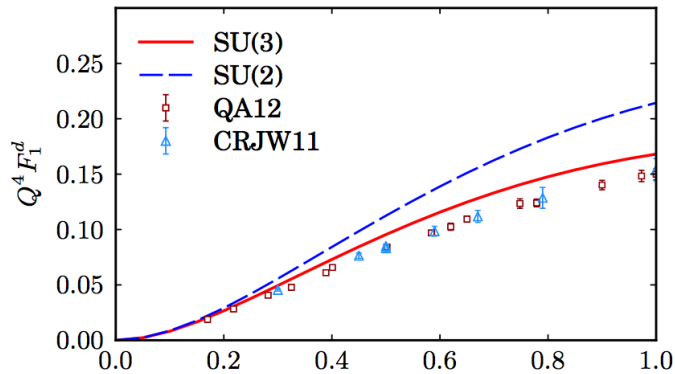
$$F_1(Q^2) \sim \frac{1}{Q^4}, \quad Q^2 \rightarrow \infty \quad F_2(Q^2) \sim \frac{1}{Q^6}, \quad Q^2 \rightarrow \infty$$



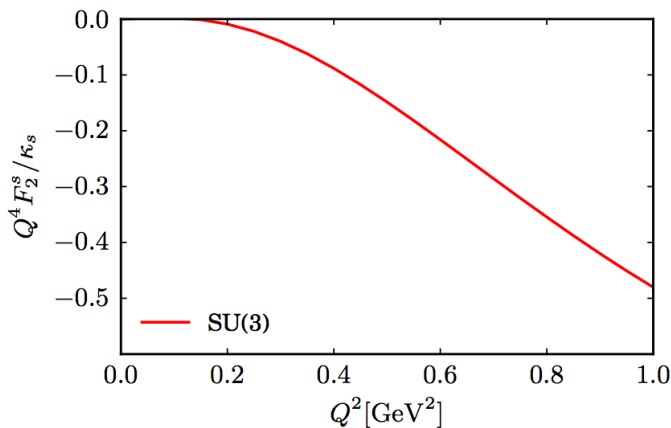
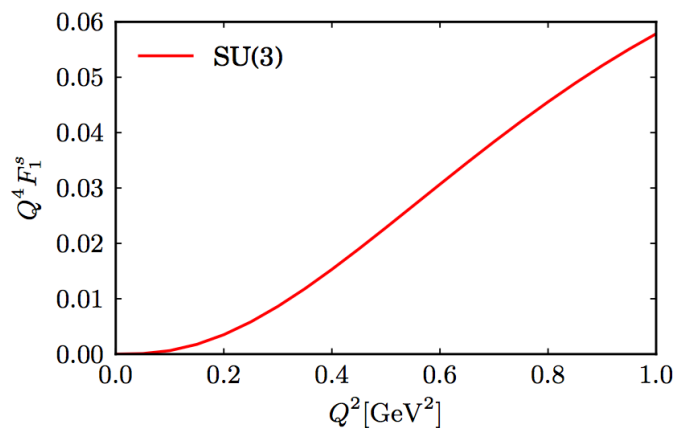
Dirac & Pauli Form factors



Up quark FFs



Down quark FFs



Strange quark FFs

Transverse charge densities



Why transverse charge densities?

$$\begin{aligned} & \langle P', S' | \bar{\psi}(\mathbf{0}) \gamma_\mu \hat{Q} \psi(\mathbf{0}) | P, S \rangle \\ &= \bar{u}(p', s') \left(\gamma_\mu F_1(t) + i \frac{\sigma^{\mu\nu} \Delta_n u}{2M_N} F_2(t) \right) u(p, s) \end{aligned}$$

Transverse charge densities



Why transverse charge densities?

Electromagnetic form factors:

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GPDs

$$\begin{aligned} & \int \frac{dx^-}{4\pi} \langle P', S' | \bar{q}(-\frac{x^-}{2}, \mathbf{0}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{0}_\perp) | P, S \rangle \\ &= \frac{1}{2\bar{p}^+} \bar{u}(p', s') \left(\gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M_N} E_q(x, \xi, t) \right) u(p, s) \end{aligned}$$

Transverse charge densities



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GPDs

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$$F_1(t) = \sum_q e_q \int dx H_q(x, 0, t),$$

$$F_2(t) = \sum_q e_q \int dx E_q(x, 0, t)$$

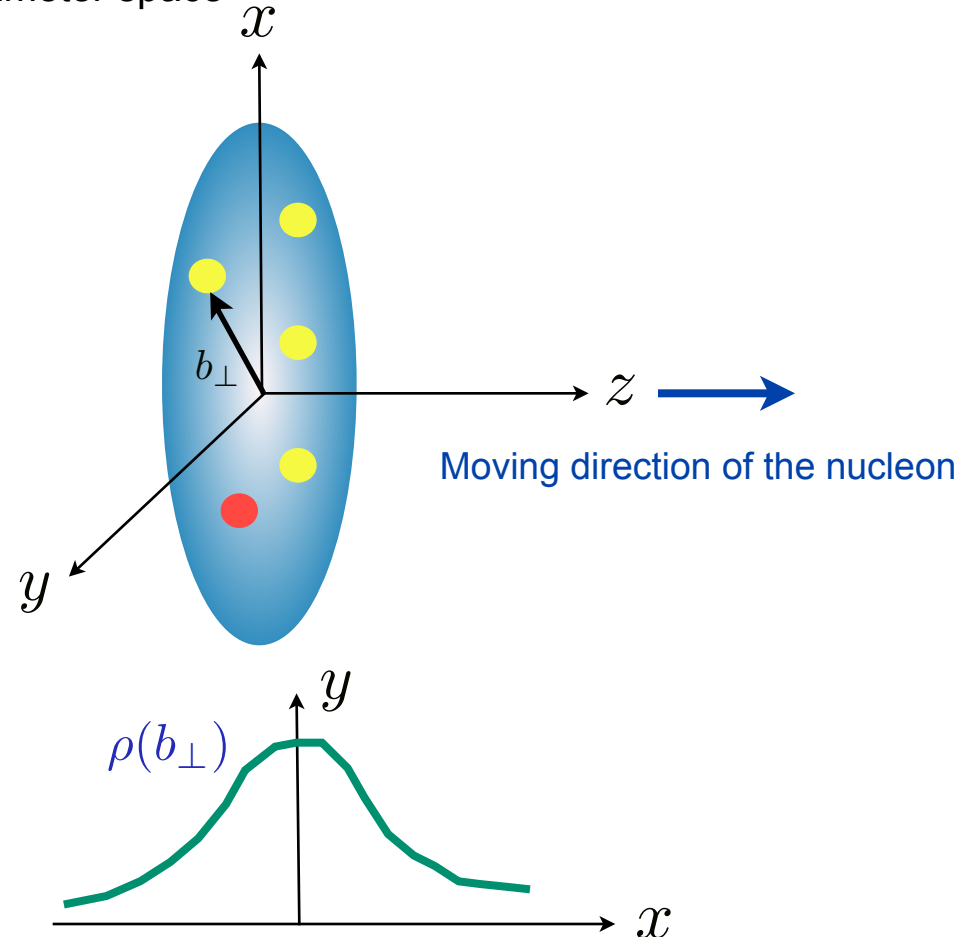
Transverse charge densities



Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

$$q(x, \mathbf{b}) = \int \frac{d^2 q}{(2\pi)^2} e^{i\mathbf{q} \cdot \mathbf{b}} H_q(x, -\mathbf{q}^2)$$



Transverse charge densities



Why transverse charge densities?

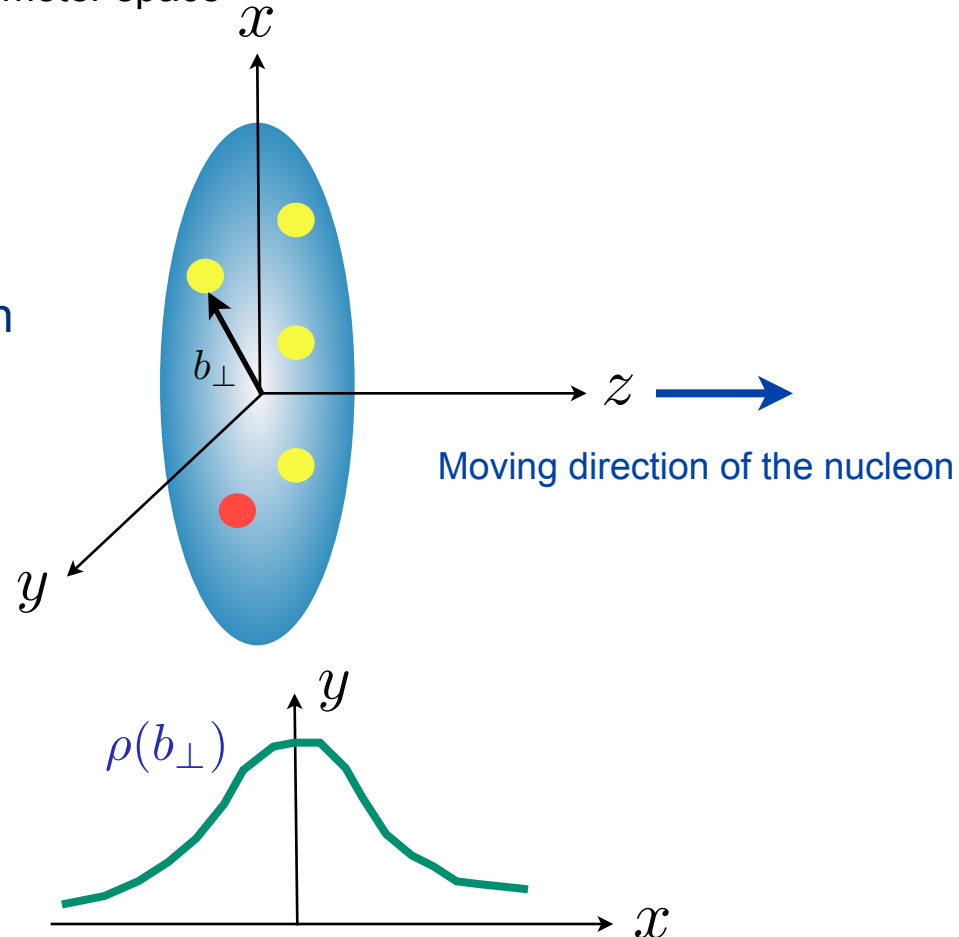
2-D Fourier transform of the GPDs in impact-parameter space

$$q(x, \mathbf{b}) = \int \frac{d^2 q}{(2\pi)^2} e^{i\mathbf{q} \cdot \mathbf{b}} H_q(x, -\mathbf{q}^2)$$

➡ It can be interpreted as the probability distribution of a quark in the transverse plane.

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

$$\begin{aligned} \rho(\mathbf{b}) &:= \sum_q e_q \int dx q(x, \mathbf{b}) \\ &= \int \frac{d^2 q}{(2\pi)^2} F_1(Q^2) e^{i\mathbf{q} \cdot \mathbf{b}} \end{aligned}$$



Transverse charge densities



Inside an unpolarized nucleon

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).

G.A. Miller, PRL **99**, 112001 (2007)

$$\rho_{\text{ch}}^{\chi}(b) = \int_0^{\infty} \frac{dQ}{2\pi} Q J_0(Qb) F_1^{\chi}(Q^2)$$

Inside a polarized nucleon

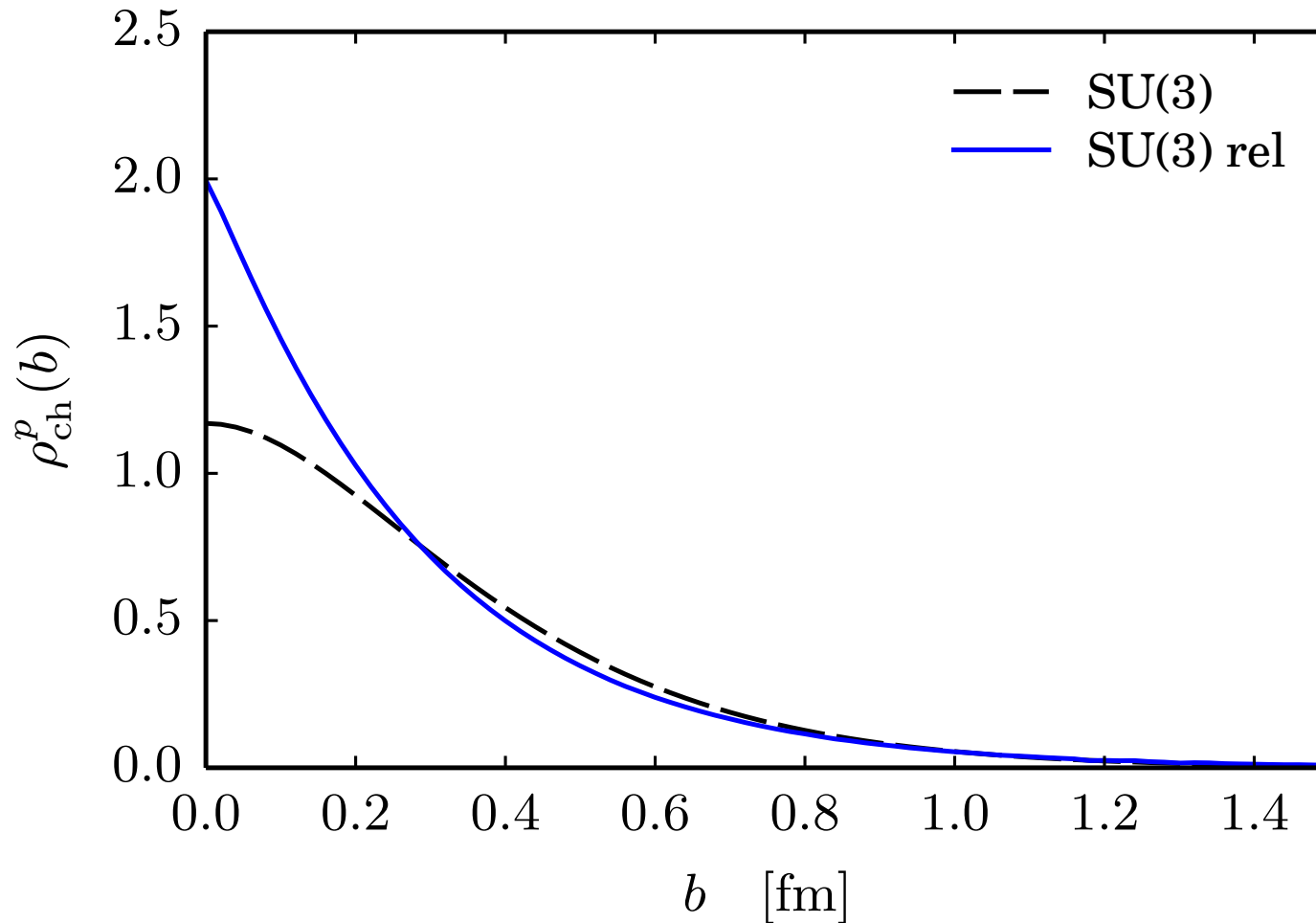
Carlson and Vanderhaeghen, PRL **100**, 032004

$$\rho_T^{\chi}(b) = \rho_{\text{ch}}^{\chi}(b) - \sin(\phi_b - \phi_S) \frac{1}{2M_N} \int_0^{\infty} \frac{dQ}{2\pi} Q^2 J_1(Qb) F_2^{\chi}(Q^2)$$

Results



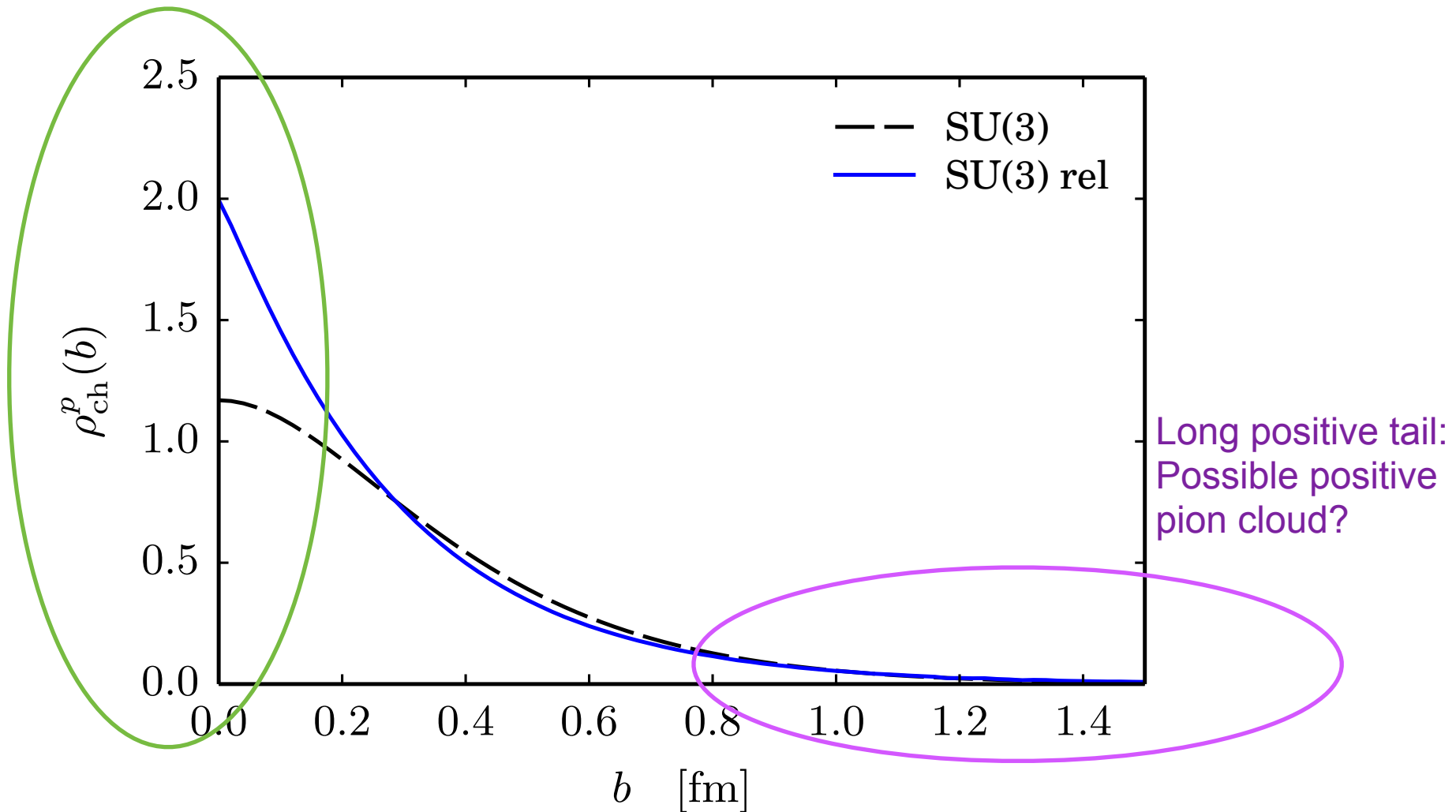
Transverse charge densities inside an **unpolarized** proton



Results



Transverse charge densities inside an **unpolarized** proton

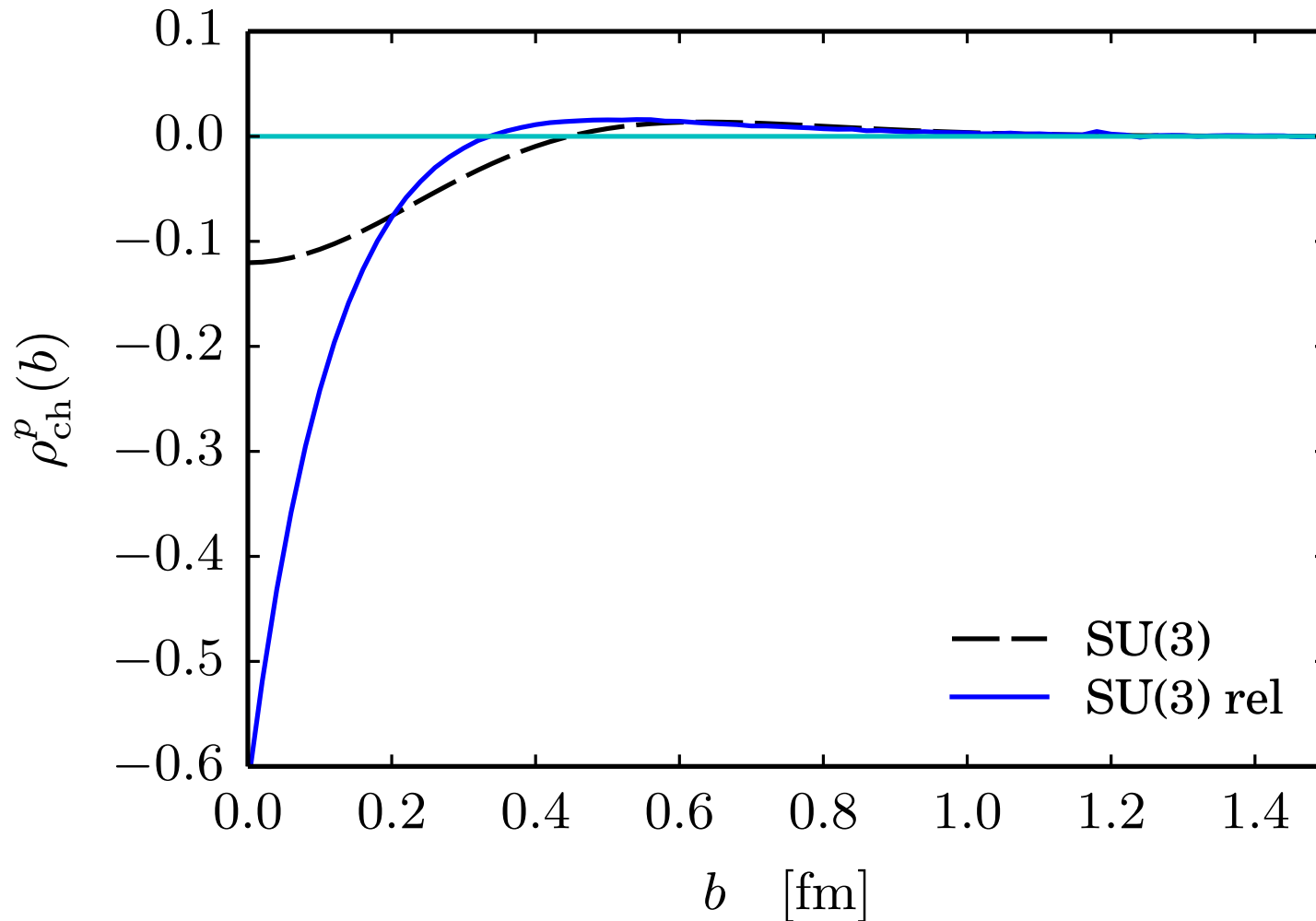


Centered positive charge distribution

Results



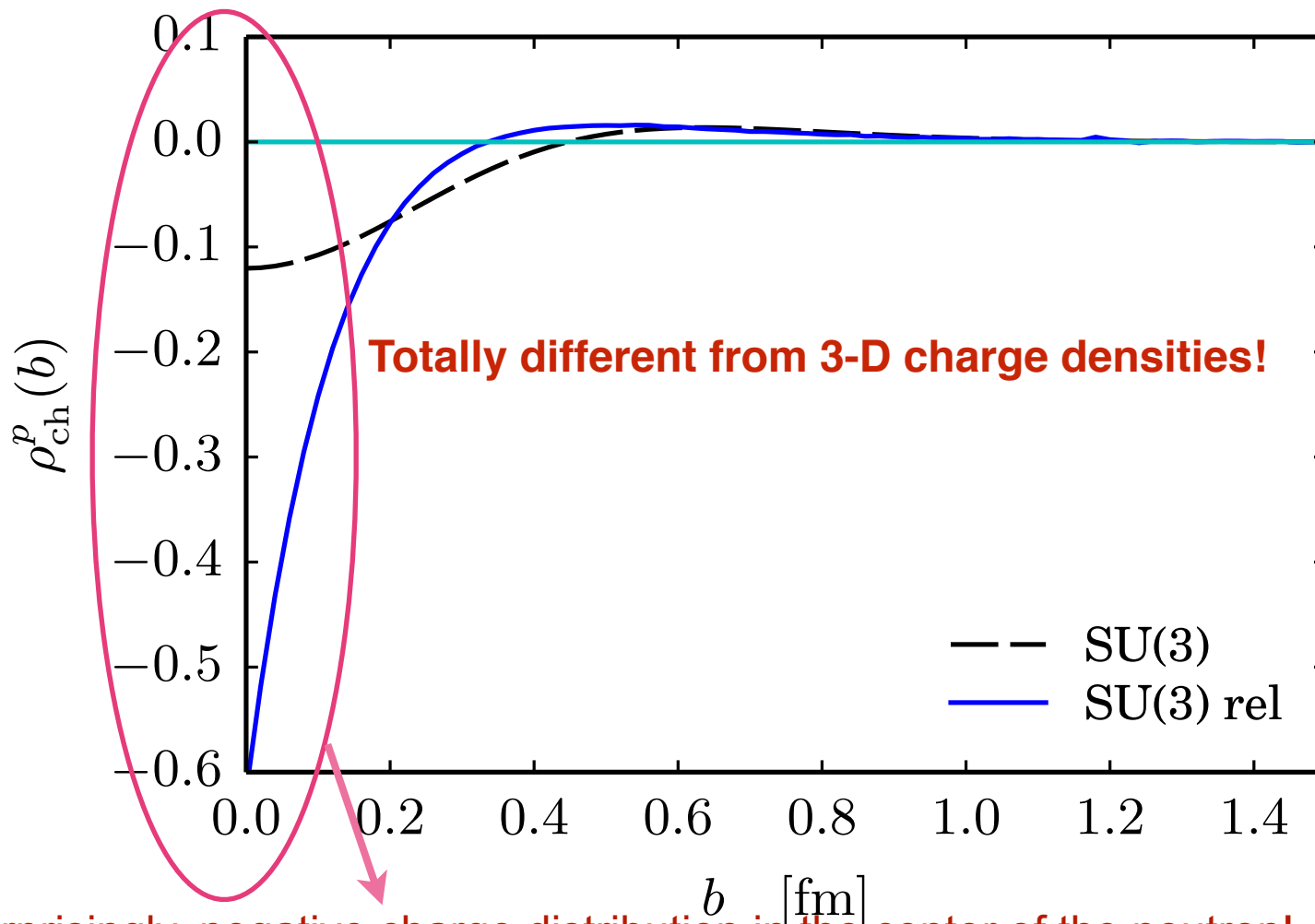
Transverse charge densities inside an **unpolarized** neutron



Results



Transverse charge densities inside an **unpolarized** neutron

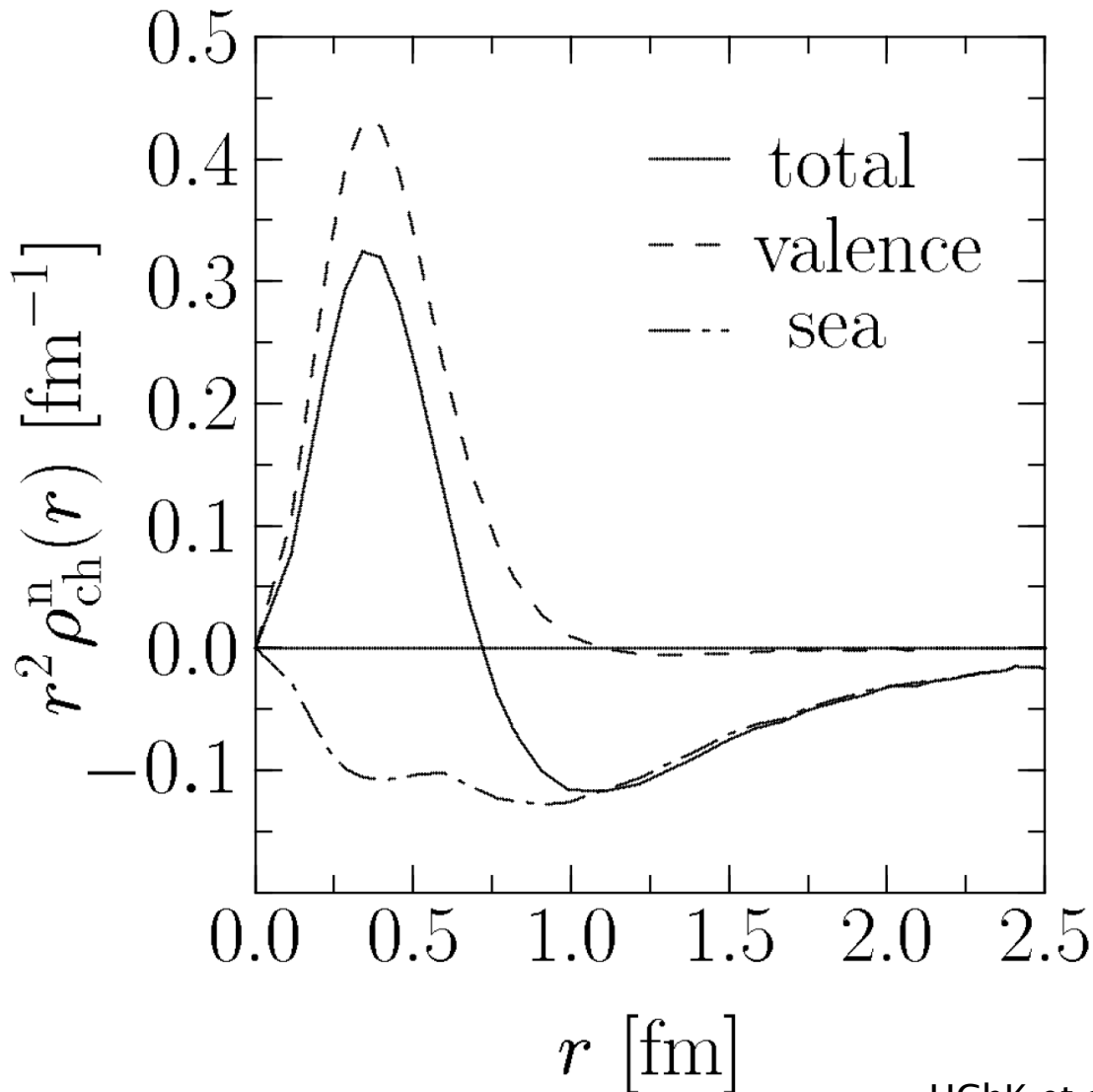


Surprisingly, negative charge distribution in the center of the neutron!

Results



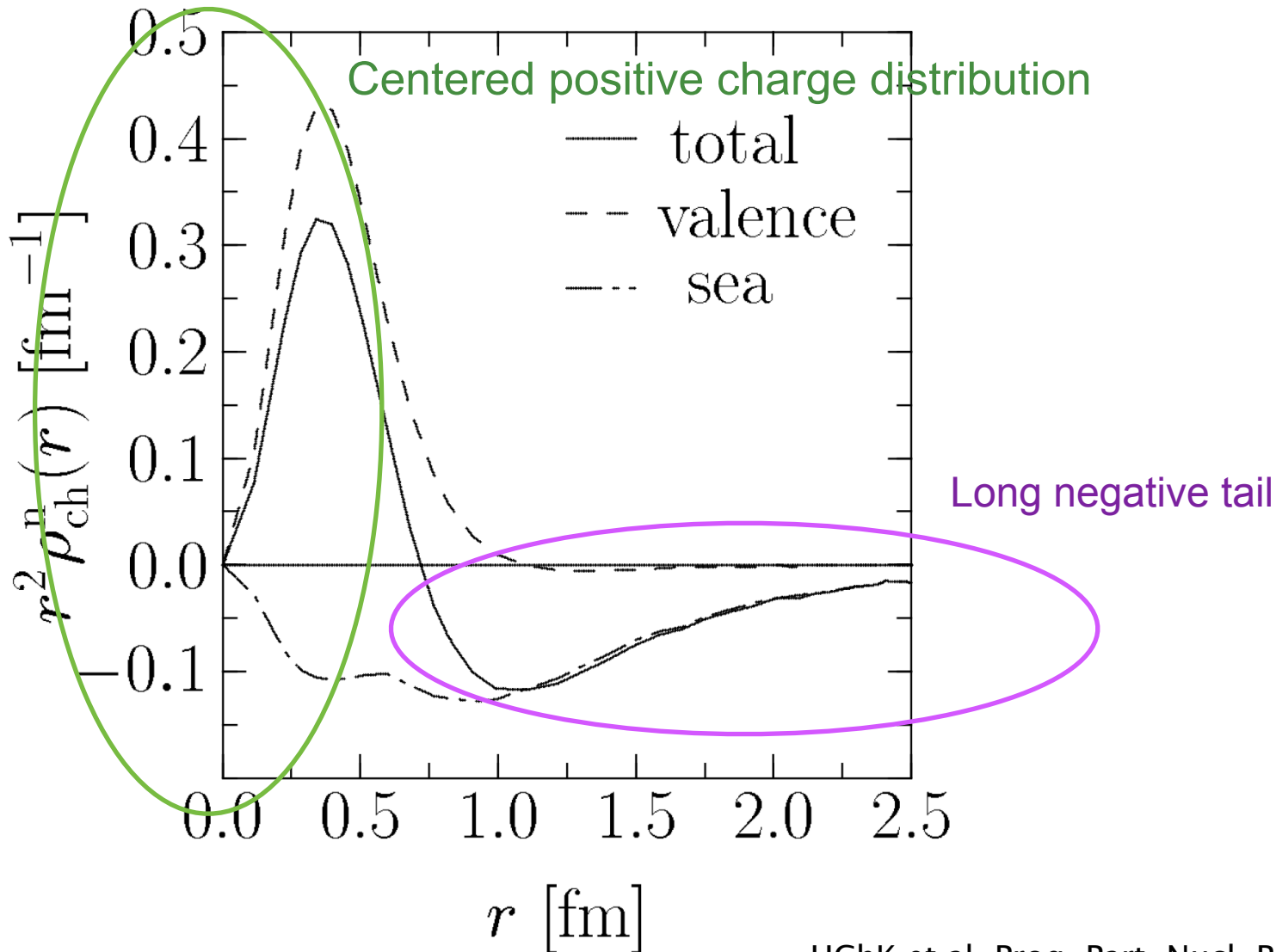
Old **3-D charge** densities inside an **unpolarized** neutron



Results

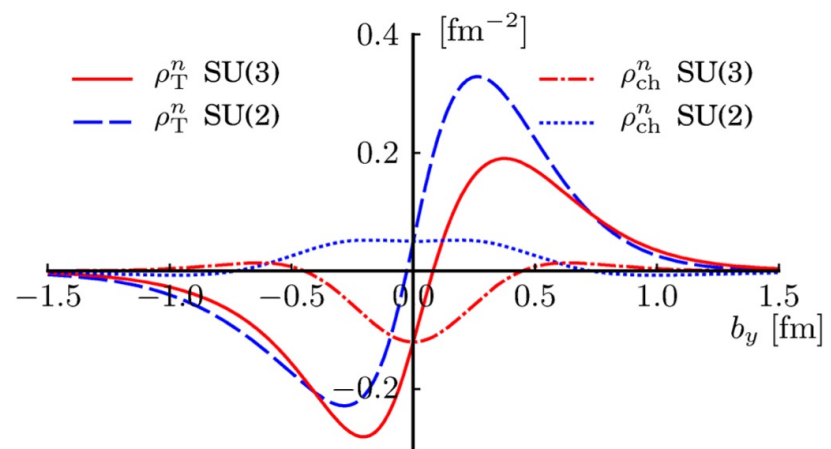
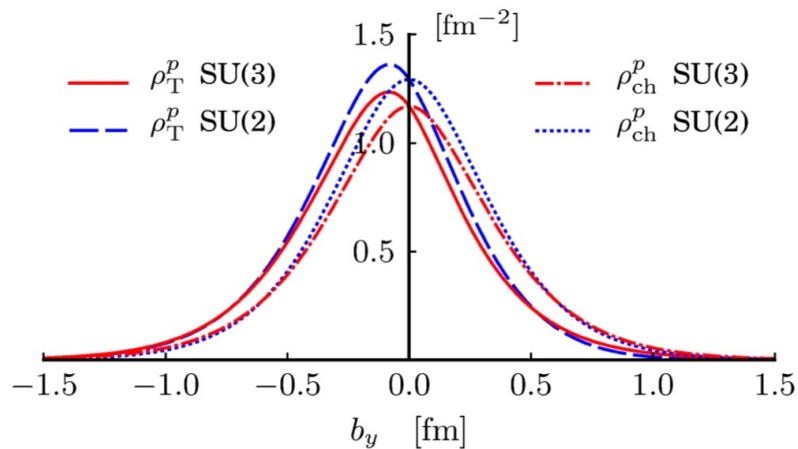
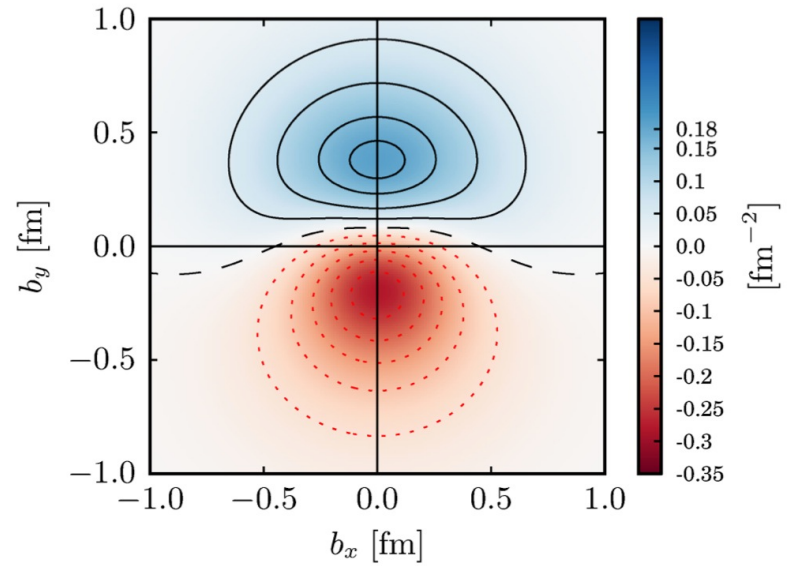
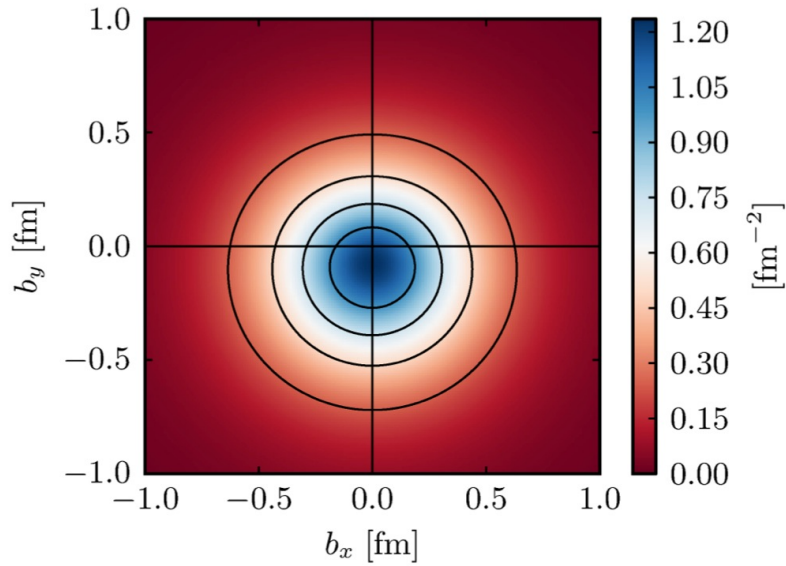


Old **3-D charge** densities inside an **unpolarized** neutron



Results

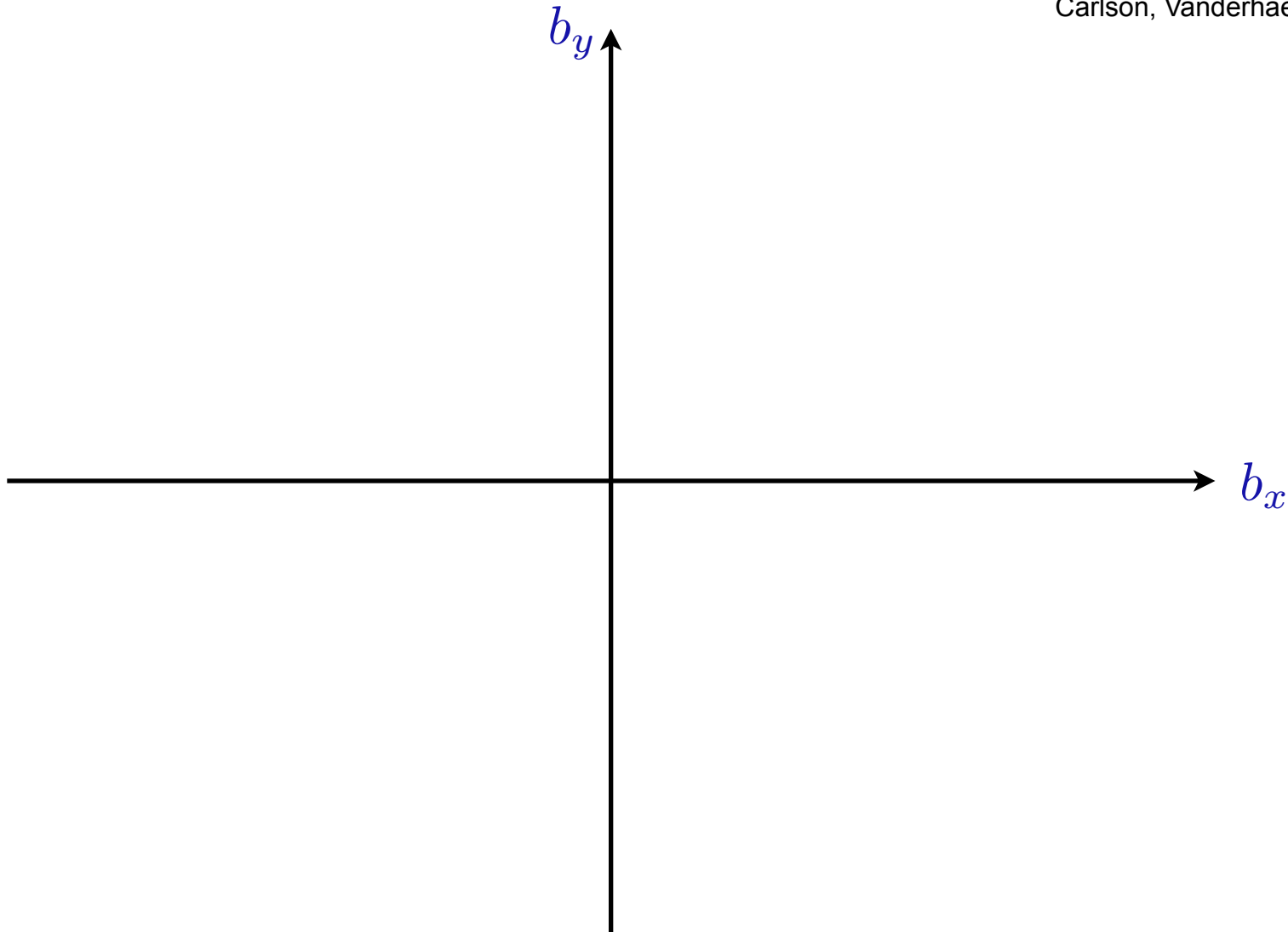
Transverse charge densities inside an **polarized** nucleon



Discussion



Carlson, Vanderhaeghen, PRL **100**, 032004

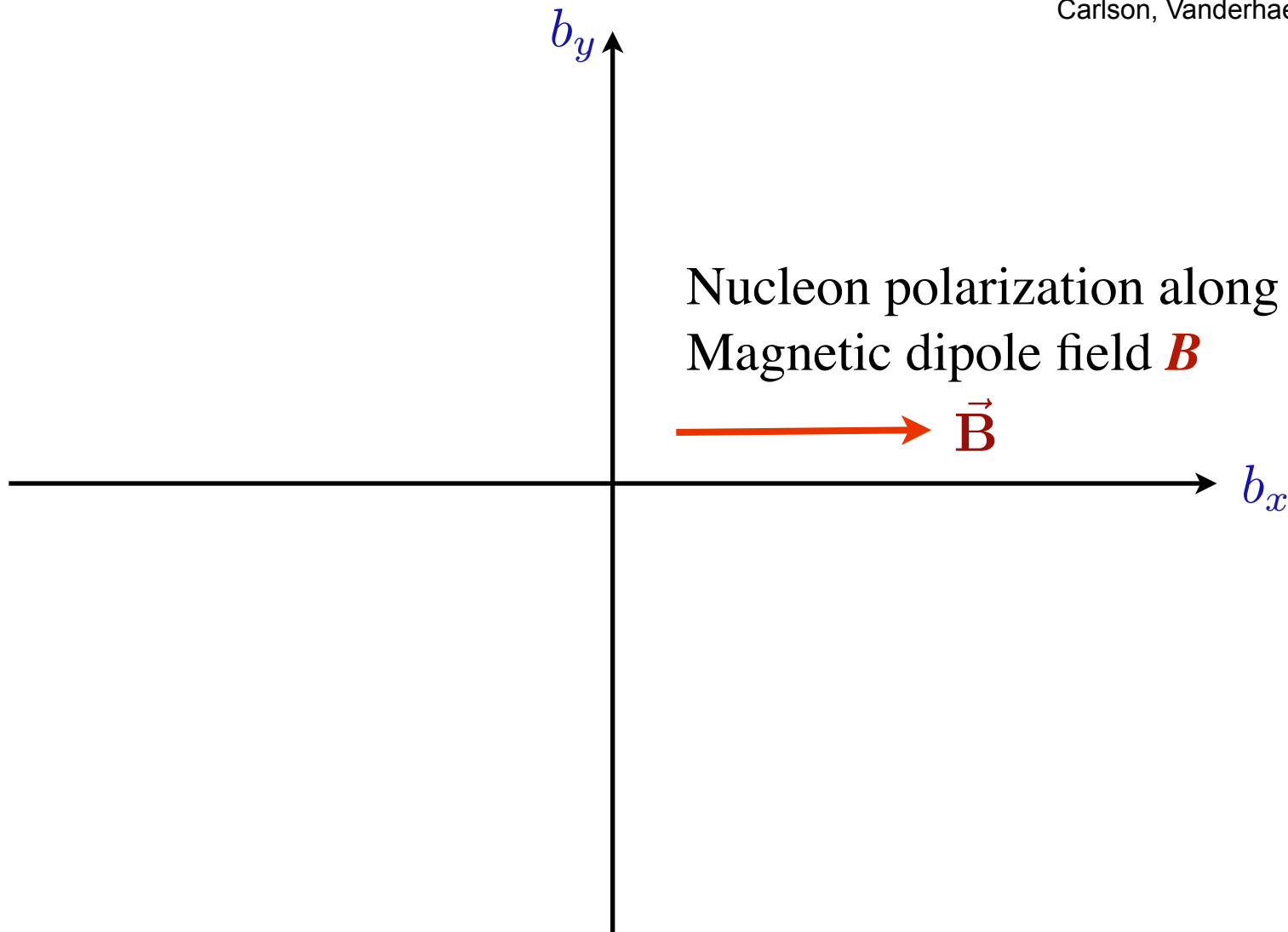


Silva, Urbano, HChK, hep-ph/1305.6373

Discussion



Carlson, Vanderhaeghen, PRL **100**, 032004

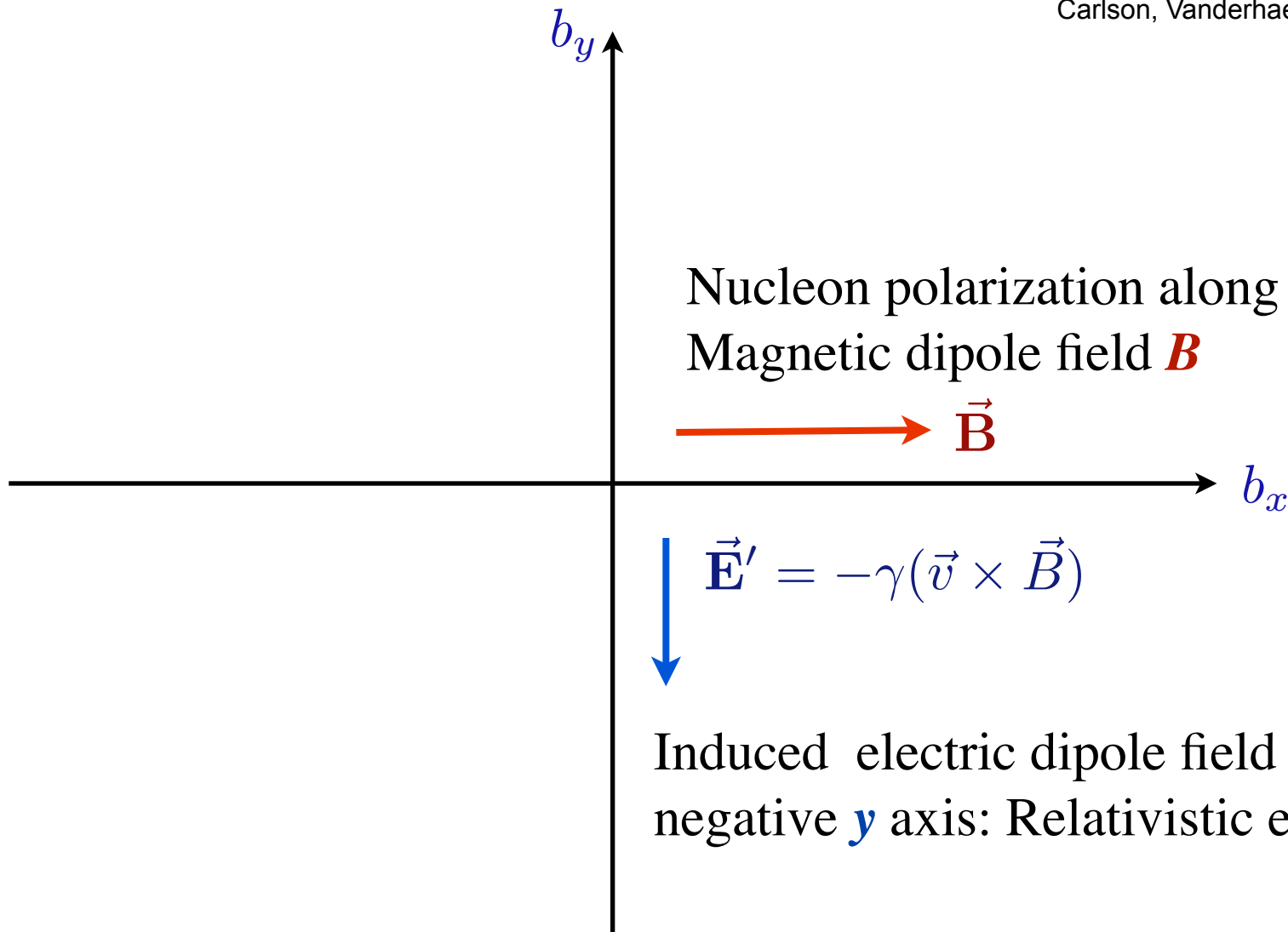


Silva, Urbano, HChK, hep-ph/1305.6373

Discussion



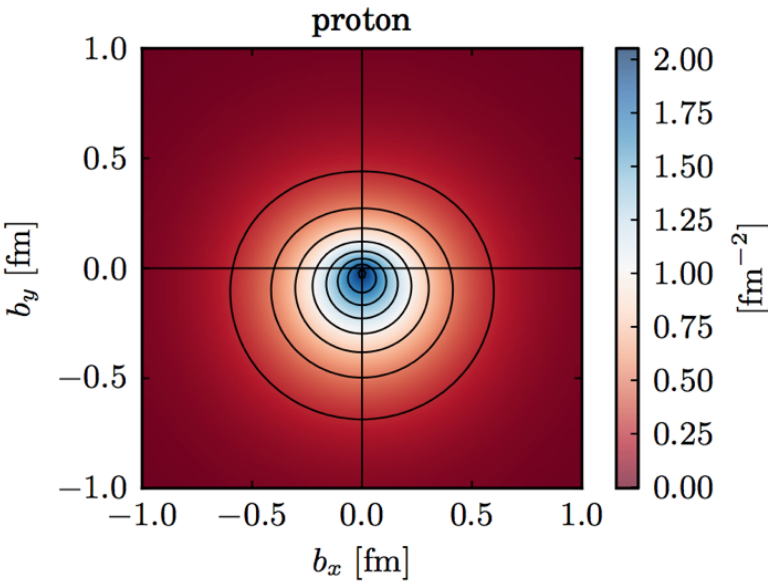
Carlson, Vanderhaeghen, PRL **100**, 032004



Discussion




Carlson, Vanderhaeghen, PRL **100**, 032004



Nucleon polarization along the **x** axis:
Magnetic dipole field ***B***



\vec{B}

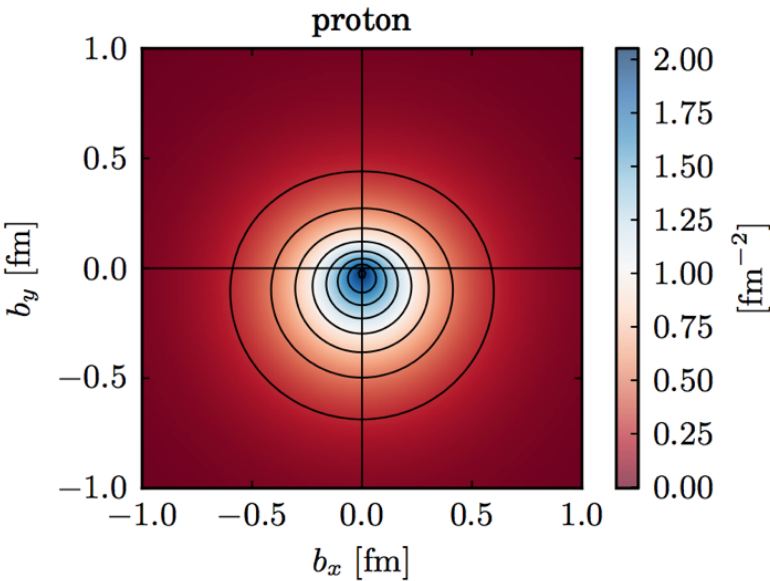

$$\vec{E}' = -\gamma(\vec{v} \times \vec{B})$$

Induced electric dipole field along the
negative **y** axis: Relativistic effects

Discussion



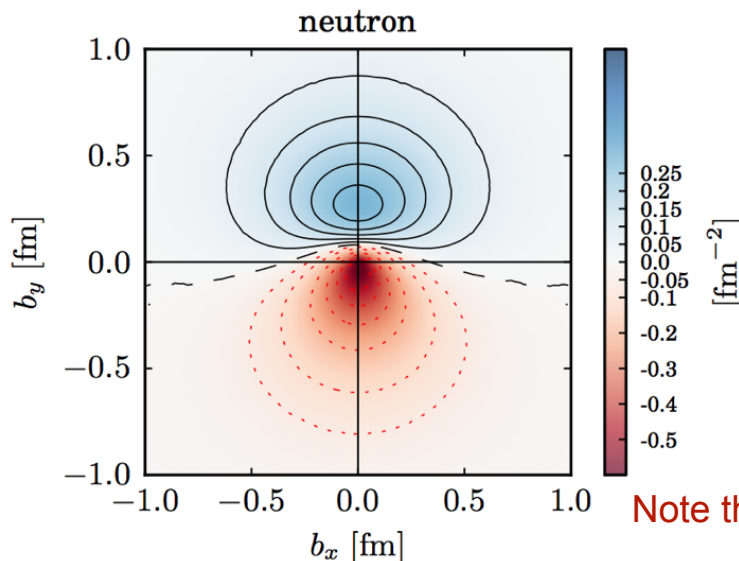
Carlson, Vanderhaeghen, PRL **100**, 032004



Nucleon polarization along the **x** axis:
Magnetic dipole field **B**



$$\vec{E}' = -\gamma(\vec{v} \times \vec{B})$$



Induced electric dipole field along the
negative **y** axis: Relativistic effects

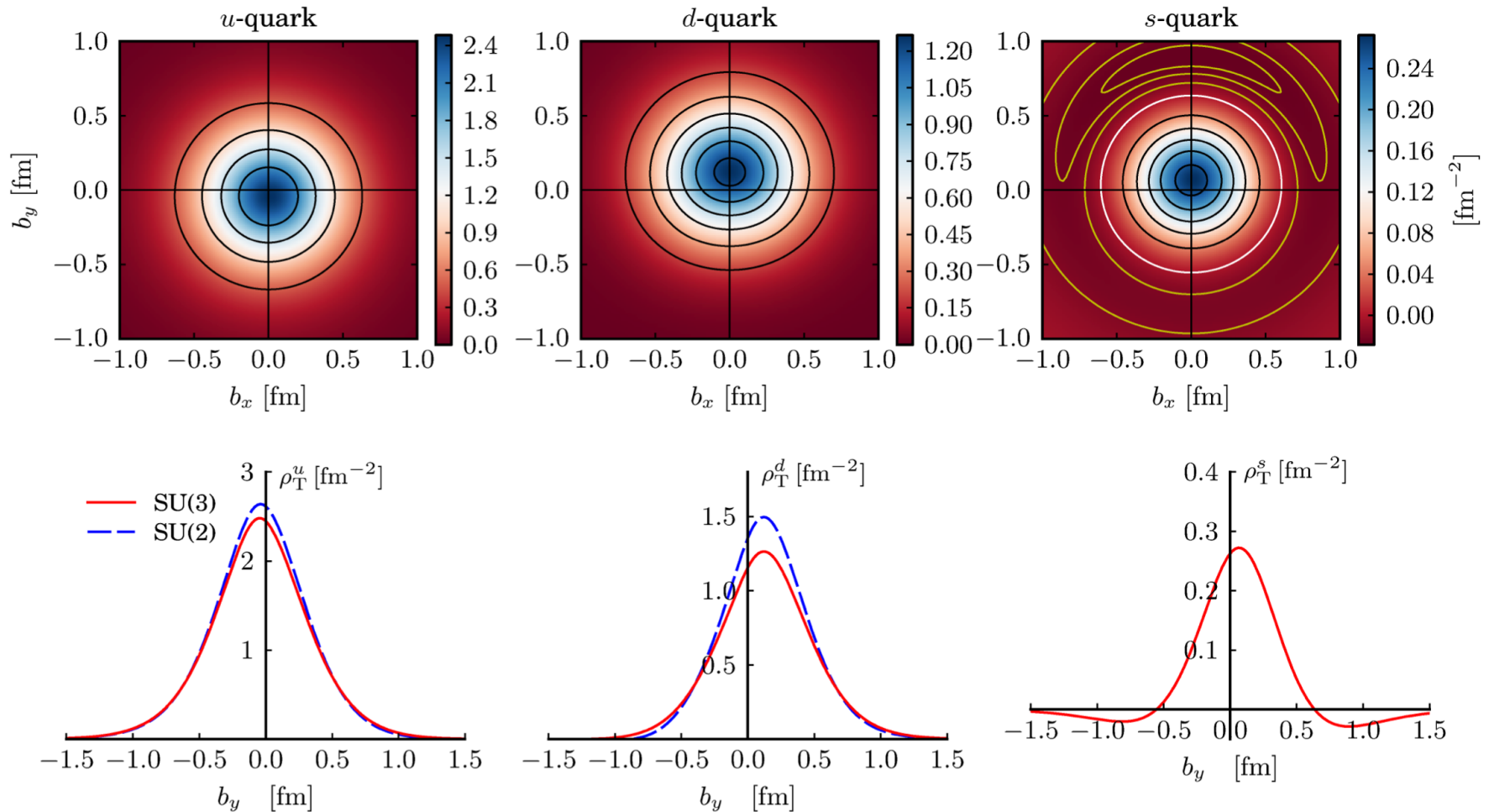
Note that the neutron anomalous magnetic moment is negative!

Silva, Urbano, HChK, hep-ph/1305.6373

Results



Flavor-decomposed Transverse charge densities inside a **polarized** nucleon

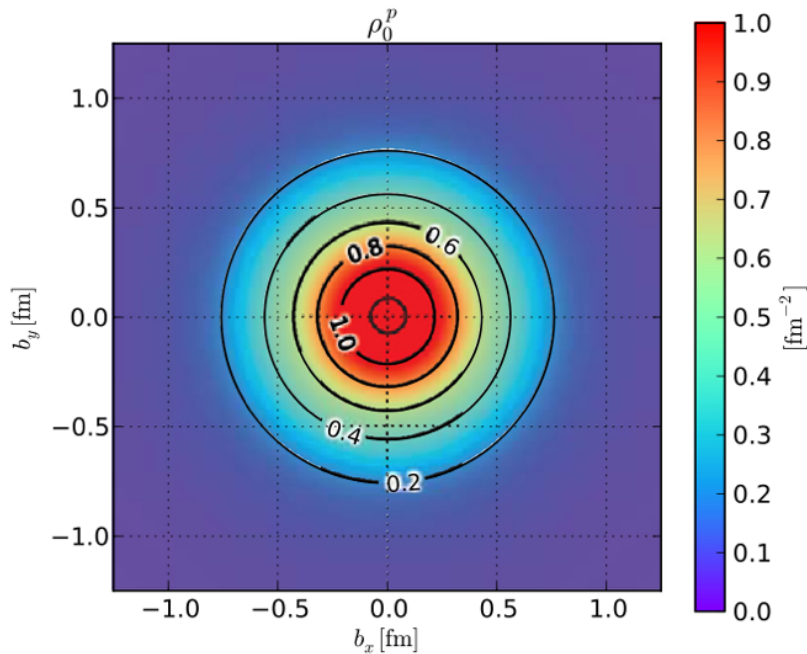


Results

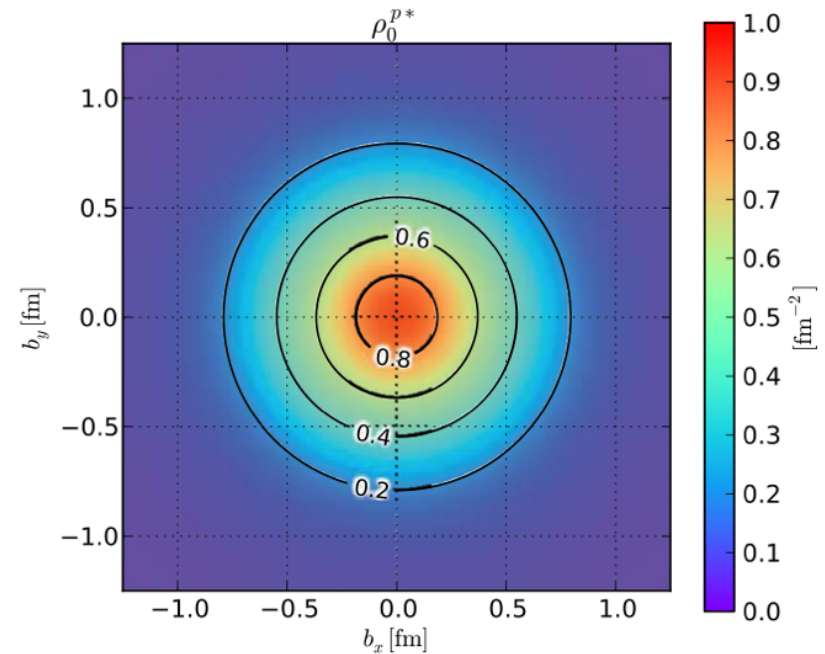


Transverse charge densities inside an **unpolarized proton in nuclear matter**

Free-space Skyrme Model



Medium-Modified Skyrme Model



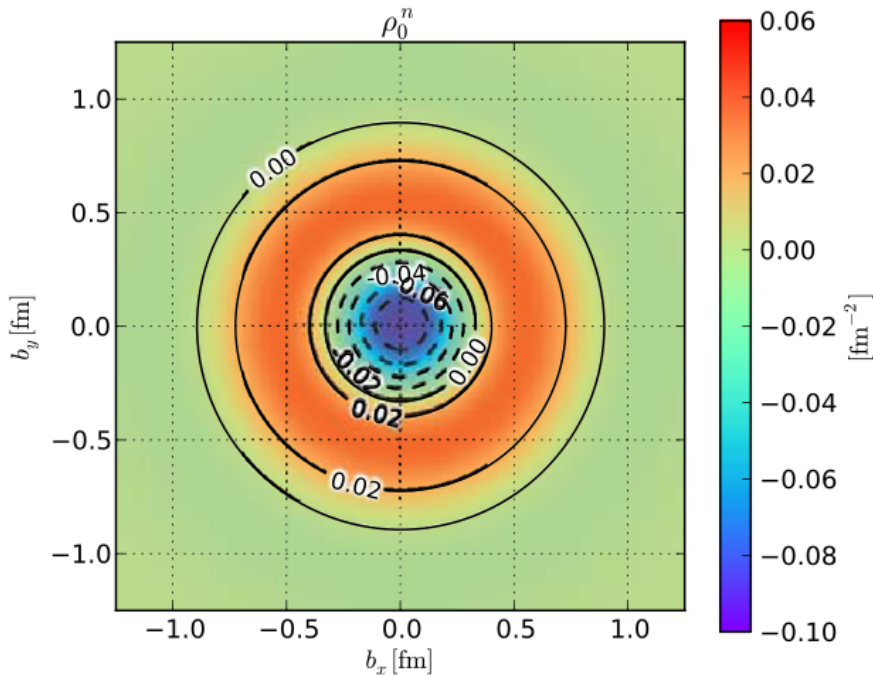
The proton bulges out in nuclear matter!

Results

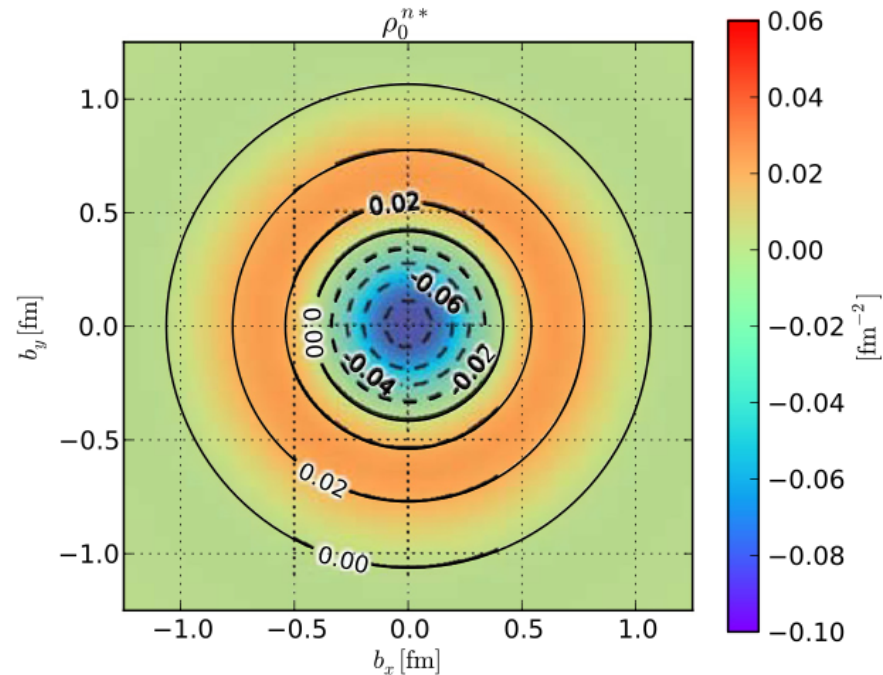


Transverse charge densities inside an **unpolarized neutron** in nuclear matter

Free-space Skyrme Model



Medium-Modified Skyrme Model



The neutron also bulges out in nuclear matter!

Transverse Spin Densities

Tensor form factors



$$\langle N_{s'}(p') | \bar{\psi}(0) i \sigma^{\mu\nu} \lambda^x \psi(0) | N_s(p) \rangle = \bar{u}_{s'}(p') \left[H_T^\chi(Q^2) i \sigma^{\mu\nu} + E_T^\chi(Q^2) \frac{\gamma^\mu q^\nu - q^\mu \gamma^\nu}{2M} + \tilde{H}_T^\chi(Q^2) \frac{(n^\mu q^\nu - q^\mu n^\nu)}{2M^2} \right] u_s(p)$$

$$\int_{-1}^1 dx H_T^\chi(x, \xi = 0, t) = H_T^\chi(q^2),$$

$$\int_{-1}^1 dx E_T^\chi(x, \xi = 0, t) = E_T^\chi(q^2),$$

$$\int_{-1}^1 dx \tilde{H}_T^\chi(x, \xi = 0, t) = \tilde{H}_T^\chi(q^2)$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

Tensor form factors



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$$H_T^{*\chi}(Q^2) = \frac{2M}{\mathbf{q}^2} \int \frac{d\Omega}{4\pi} \langle N_{\frac{1}{2}}(p') | \psi^\dagger \gamma^k q^k \lambda^x \psi | N_{\frac{1}{2}}(p) \rangle$$

$$\kappa_T^\chi = -H_T^\chi(0) - H_T^{*\chi}(0)$$

Together with the anomalous magnetic moment, this will allow us to describe the **transverse spin quark densities** inside the nucleon.

Tensor form factors



Tensor charges and anomalous tensor magnetic moments are **scale-dependent**.

$$\delta q(\mu^2) = \left(\frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)} \right)^{4/27} \left[1 - \frac{337}{486\pi} (\alpha_S(\mu_i^2) - \alpha_S(\mu^2)) \right] \delta q(\mu_i^2),$$

$$\alpha_S^{NLO}(\mu^2) = \frac{4\pi}{9 \ln(\mu^2/\Lambda_{\text{QCD}}^2)} \left[1 - \frac{64}{81} \frac{\ln \ln(\mu^2/\Lambda_{\text{QCD}}^2)}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)} \right]$$

$$\Lambda_{\text{QCD}} = 0.248 \text{ GeV}$$

M. Gluck, E. Reya, and A. Vogt, Z.Phys. C 67, 433(1995).

Results



Proton	This work	SU(2)	Lattice	SIDIS	NR
$ \delta d/\delta u $	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

Results



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SIDIS [16] (0.80 GeV ²):	$\delta u = 0.54^{+0.09}_{-0.22}$,	$\delta d = -0.231^{+0.09}_{-0.16}$,
SIDIS [16] (0.36 GeV ²):	$\delta u = 0.60^{+0.10}_{-0.24}$,	$\delta d = -0.26^{+0.1}_{-0.18}$,
Lattice [21] (4.00 GeV ²):	$\delta u = 0.86 \pm 0.13$,	$\delta d = -0.21 \pm 0.005$,
Lattice [21] (0.36 GeV ²):	$\delta u = 1.05 \pm 0.16$,	$\delta d = -0.26 \pm 0.01$,
χ QSM (0.36 GeV ²):	$\delta u = 1.08$,	$\delta d = -0.32$,

[16] M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

[21] M. Goeckeler et al., PLB 627, 113 (2005)

Results



$$\mu^2 = 0.36 \text{ GeV}^2$$

	Present work SU(3)	Present work SU(2)	Lattice
κ_T^u	3.56	3.72	3.00 (3.70)
κ_T^d	1.83	1.83	1.90 (2.35)
κ_T^s	$0.2 \sim -0.2$		
κ_T^u / κ_T^d	1.95	2.02	1.58

Results



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The present results are comparable with the lattice data!

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Transverse spin density



$$\begin{aligned} \rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = & \frac{1}{2} \left[H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right. \\ & + s^i S^i \left\{ H_T(b^2) - \frac{1}{4M_N^2} \nabla^2 \tilde{H}_T(b^2) \right\} \\ & \left. + s^i (2b^i b^j - b^2 \delta^{ij}) S^j \frac{1}{M_N^2} \left(\frac{\partial}{\partial b^2} \right)^2 \tilde{H}_T(b^2) \right] , \end{aligned}$$

Transverse spin density



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$$[\mathbf{S}, \mathbf{s}] = [(1, 0), (0, 0)], \quad [\mathbf{S}, \mathbf{s}] = [(0, 0), (1, 0)]$$

Transverse spin density



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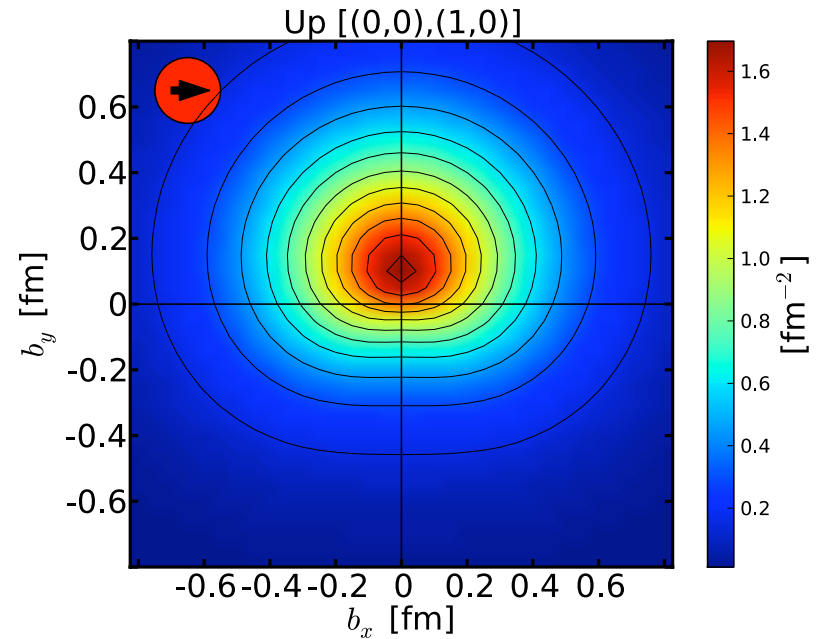
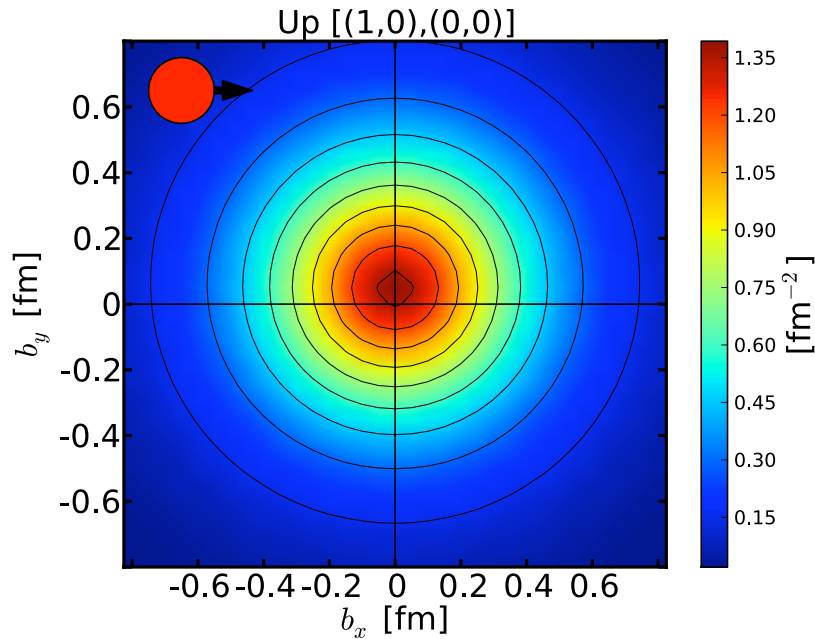
$$[\mathbf{S}, \mathbf{s}] = [(1, 0), (0, 0)], \quad [\mathbf{S}, \mathbf{s}] = [(0, 0), (1, 0)]$$

$$\mathcal{F}^x(b^2) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F^x(Q^2)$$

$$H(b^2) = F_1(b^2), \quad E(b^2) = F_2(b^2)$$

Results

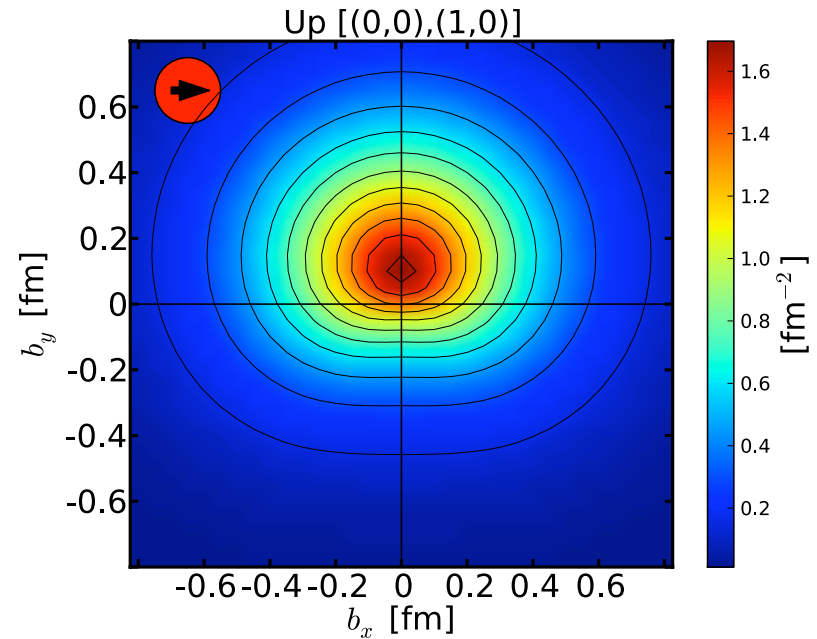
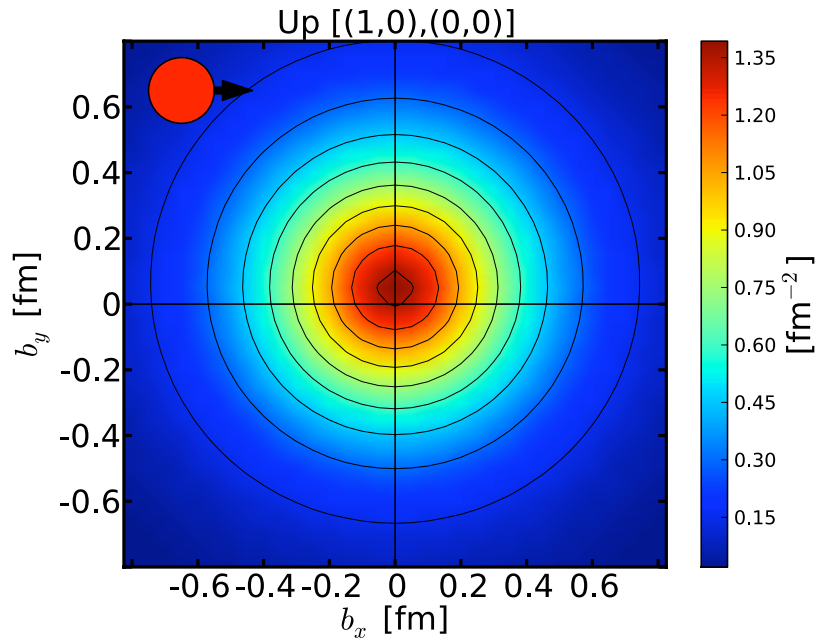
Up quark transverse spin density inside a nucleon



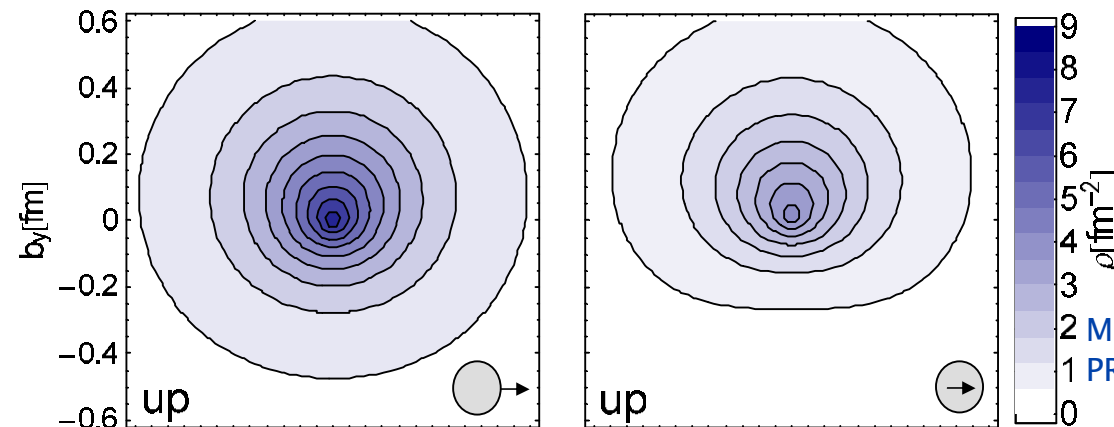
T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

Results

Up quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

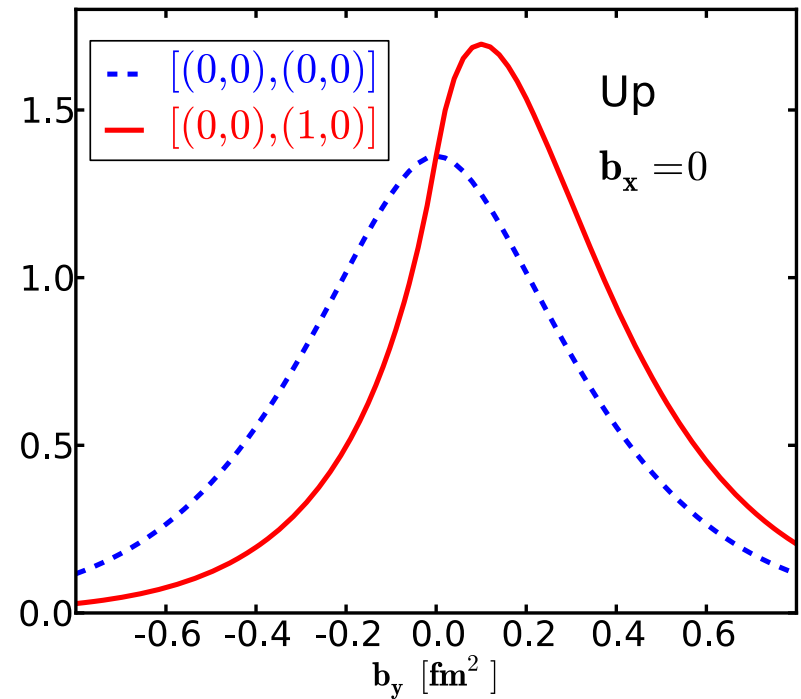
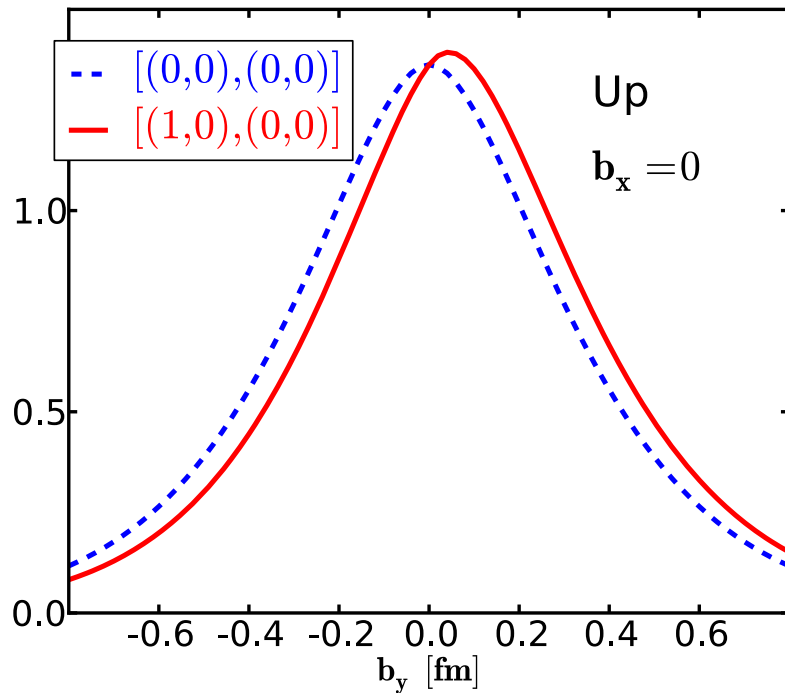


Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

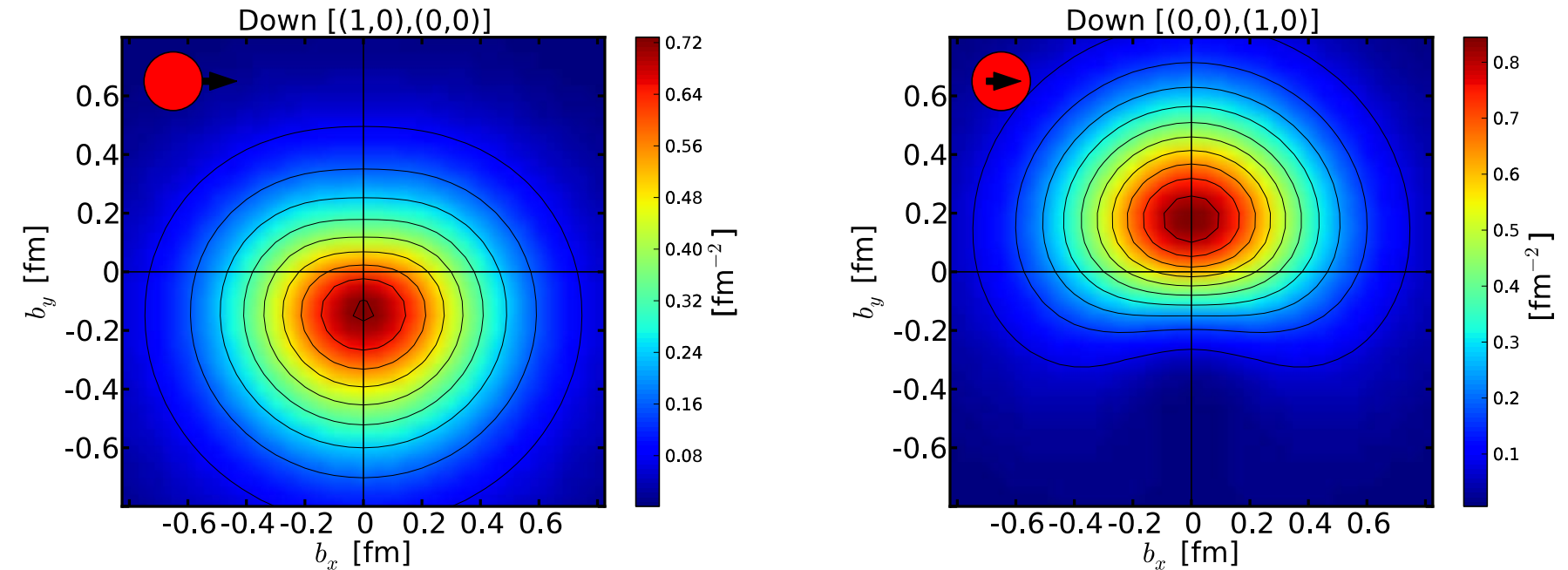
Results

Up quark transverse spin density inside a nucleon



Results

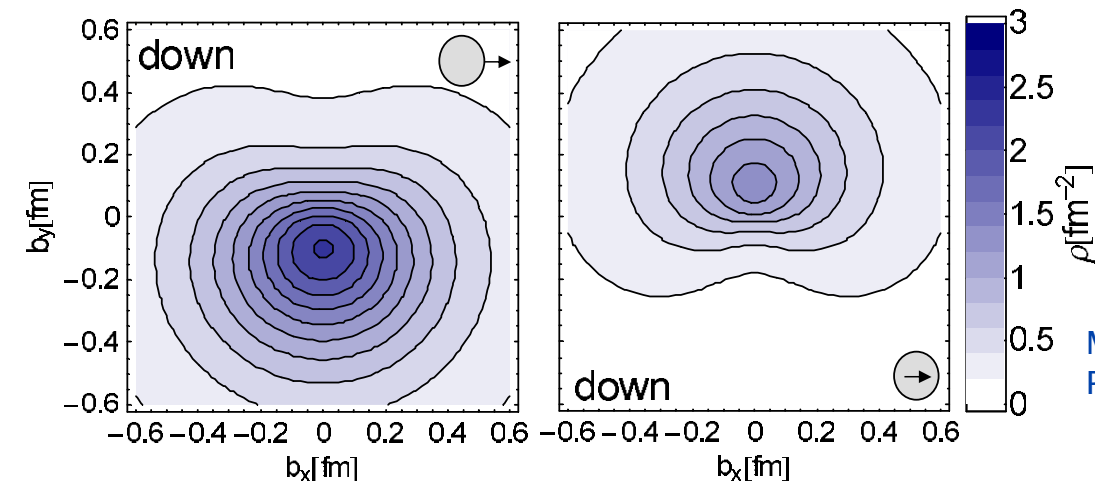
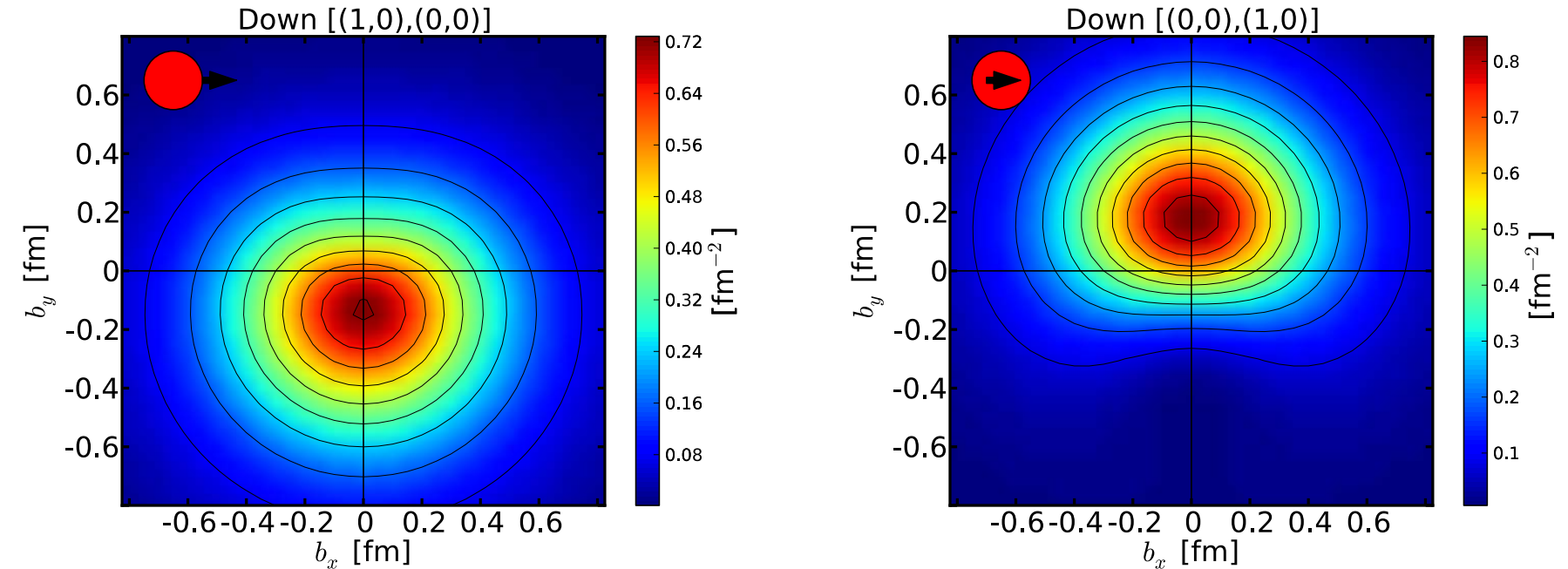
Down quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

Results

Down quark transverse spin density inside a nucleon



T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

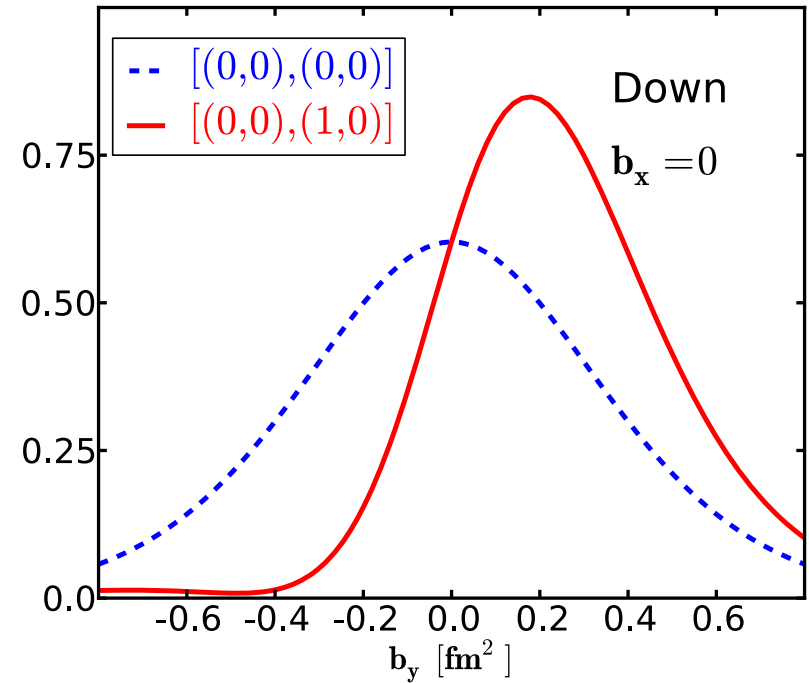
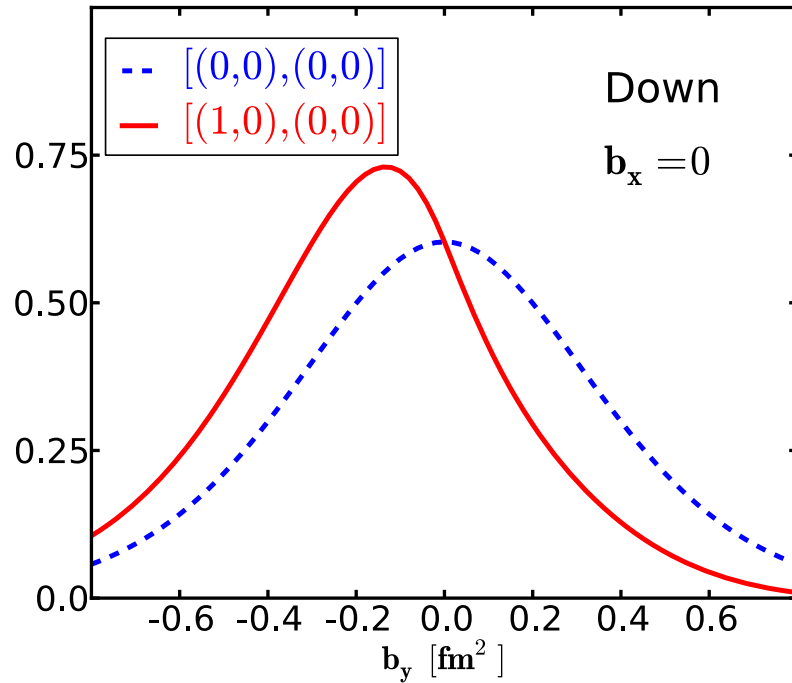
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M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Results

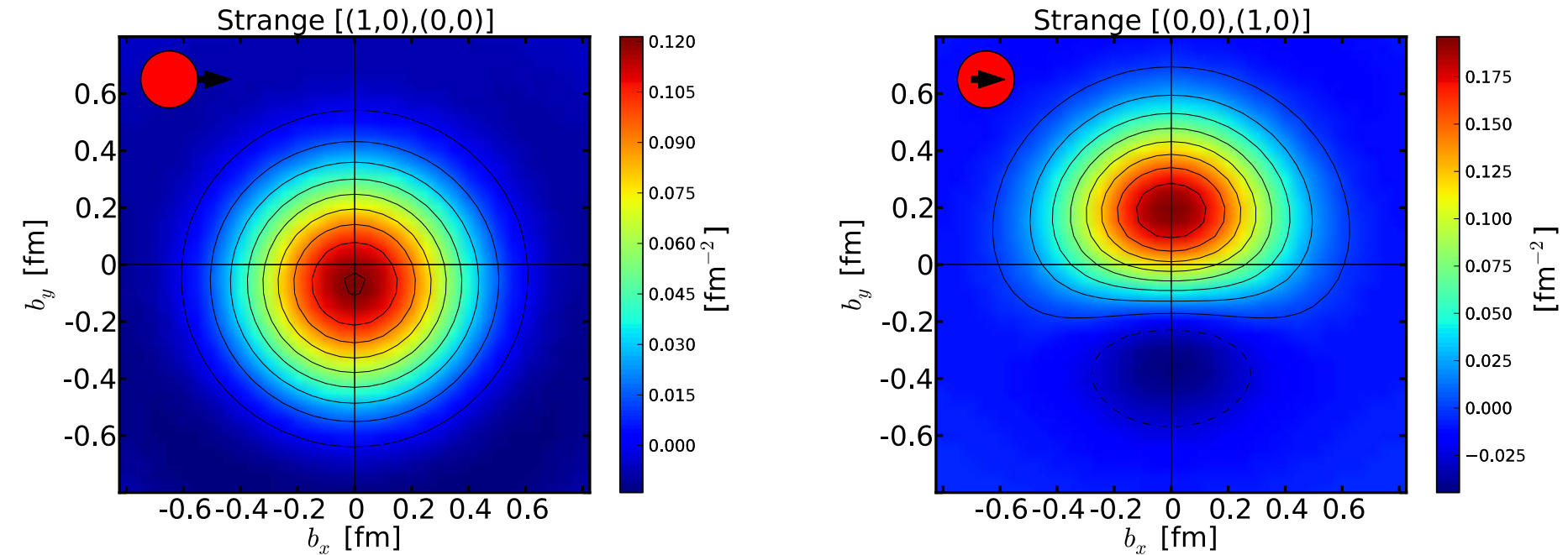


Down quark transverse spin density inside a nucleon



Results

Strange quark transverse spin density inside a nucleon



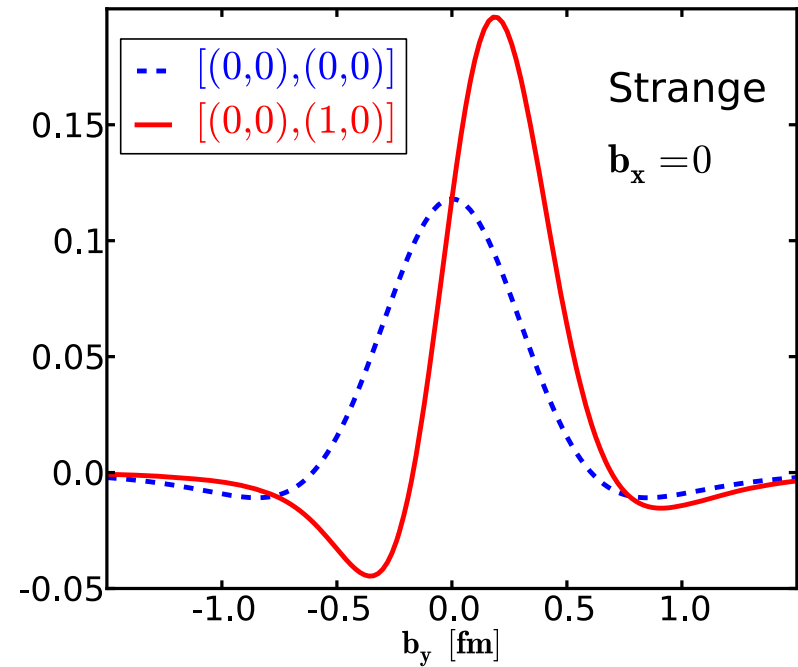
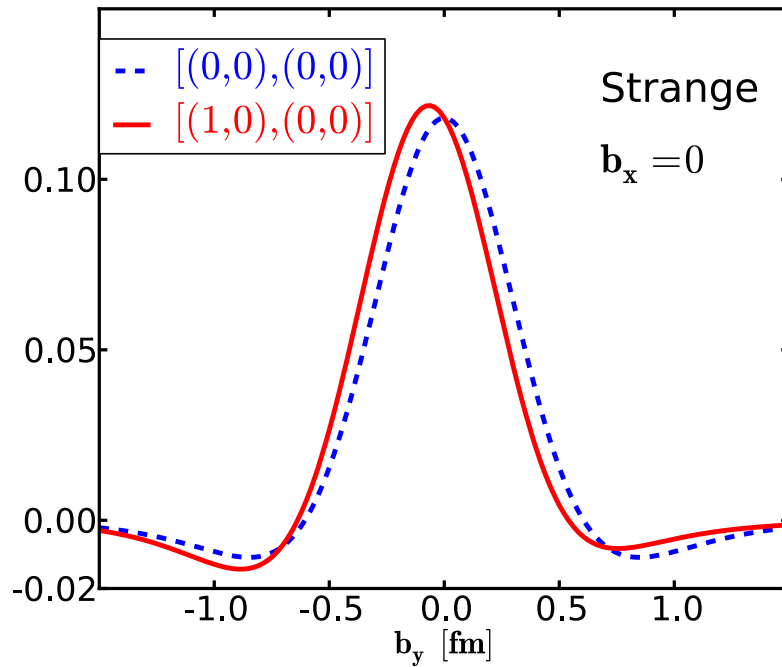
T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

This is the **first** result of the strange quark transverse spin density inside a nucleon

Results

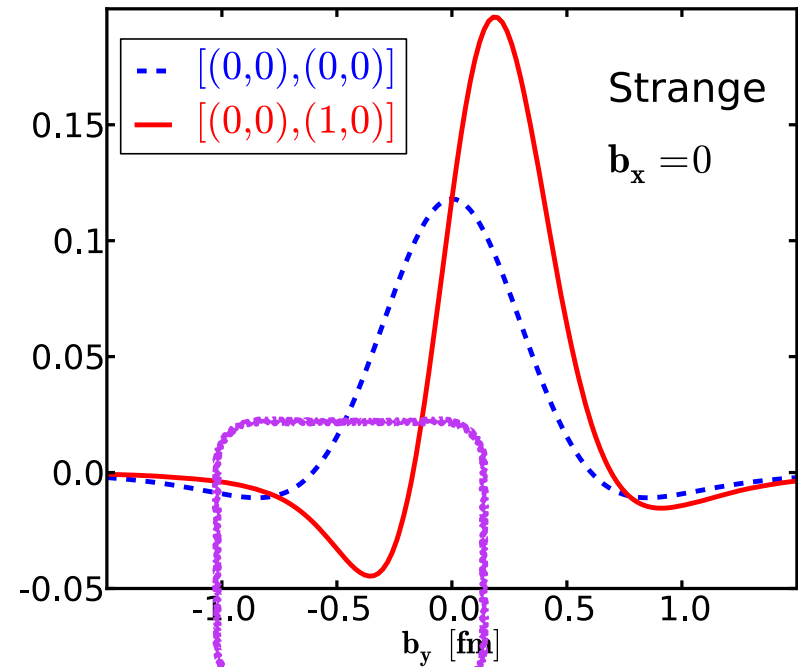
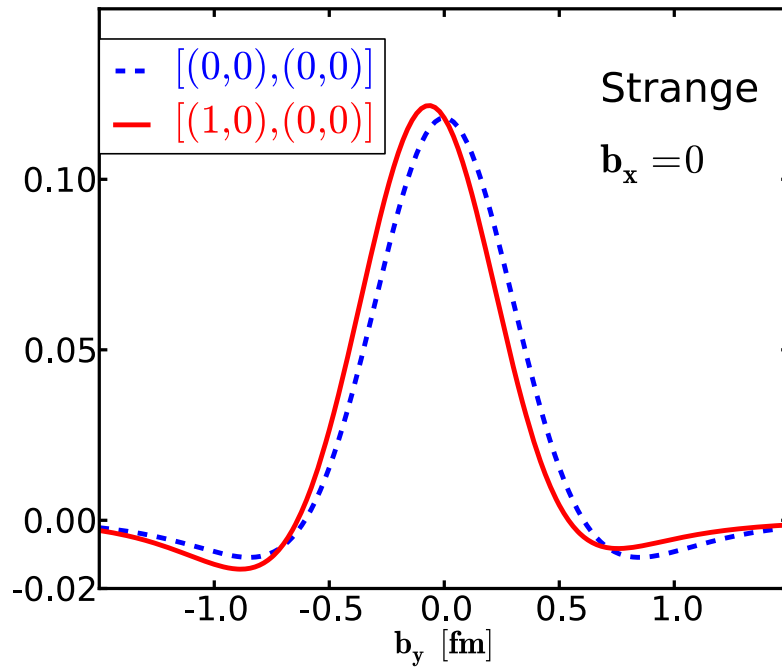


Strange quark transverse spin density inside a nucleon



Results

Strange quark transverse spin density inside a nucleon



Polarized to the negative direction in the b plane.

Summary & Conclusion

Summary



- We have reviewed recent investigations on the charge and spin structures of the nucleon, based on the chiral quark-soliton model.
- We have derived the EM and tensor form factors of the nucleon, from which we have obtained its transverse charge & spin densities. The results are compared with the lattice and "experimental" data.
- The first strange anomalous tensor magnetic moment was obtained, though it is compatible with zero.
- The strange quark transverse spin density was first announced in this work.
- We also extended the investigation to nuclear matter case.

- The transverse charge and spin densities for the transition processes can be studied (**K-pi transition is under way**).
- The excited states for the nucleon and the hyperon can be investigated (Generalisation of the XQSM is under way).
- **Internal structure of Heavy-light quark systems**
(Derivation of the Partition function is under way.)
- **New perspective on hadron tomography**

*Though this be madness,
yet there is method in it.*

Hamlet Act 2, Scene 2

Thank you very much!

Back-up slides

Chiral quark–soliton model

$$S_{\text{eff}} = -N_c \text{Tr} \ln(i\cancel{D} + iMU\gamma^5 + i\hat{m})$$

Nucleon consisting of N_c quarks

$$\Pi_N = \langle 0 | J_N(0, T/2) J_N^\dagger(0, -T/2) | 0 \rangle$$

$$J_N(\vec{x}, t) = \frac{1}{N_c!} \varepsilon^{\beta_1 \cdots \beta_{N_c}} \Gamma_{JJ_3 Y' T T_3 Y}^{\{f\}} \psi_{\beta_1 f_1}(\vec{x}, t) \cdots \psi_{\beta_{N_c} f_{N_c}}(\vec{x}, t)$$

$$\lim_{T \rightarrow \infty} \Pi_N(T) \simeq e^{-M_N T}$$

$$\Pi_N(\vec{x}, t) = \Gamma_N^{\{f\}} \Gamma_N^{\{g\}*} \frac{1}{Z} \int dU \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle_{f,g} e^{-S_{\text{eff}}}$$

$$\lim_{T \rightarrow \infty} \frac{1}{Z} \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle \sim e^{-(N_c E_{\text{val}}(U) + E_{\text{sea}}(U))T}$$

Baryonic correlation functions

Baryonic observables

$$\lim_{x_0 \rightarrow -\infty} \langle 0 | J_N(x) \Gamma_\mu(z) J_N^\dagger(y) | 0 \rangle = \lim_{\substack{x_0 \rightarrow -\infty \\ y_0 \rightarrow \infty}} \mathcal{K}_\mu$$

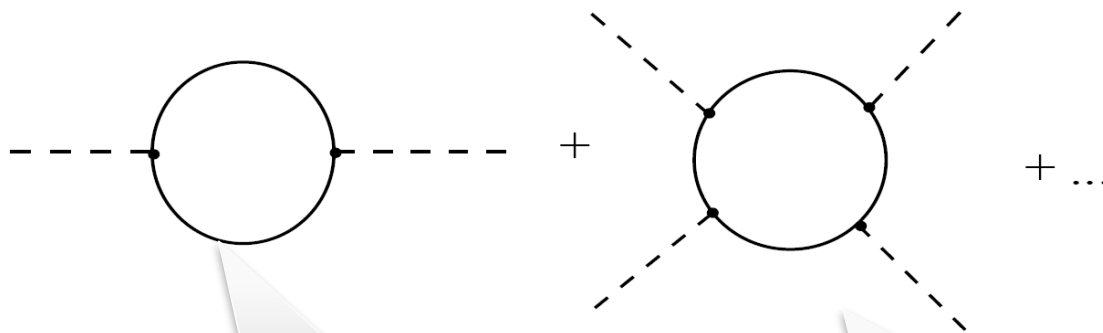
$$\mathcal{K}_\mu = \frac{1}{\mathcal{Z}} \int D\psi D\psi^\dagger DU J_N \Gamma_\mu J_N^\dagger \\ \times \exp \left[\int d^4x \psi^\dagger (i\not{\partial} + iMU\gamma^5 + i\hat{m}) \psi \right]$$

Skyrme model as a limit of the XQSM

Effective Chiral Lagrangian and LECs

$$S_{\text{eff}} = -N_c \text{Tr} \ln(i\not{\partial} + i\sqrt{M(i\not{\partial})} U^{\gamma_5} \sqrt{M(i\not{\partial})})$$

Derivative expansions: pion momentum as an expansion parameter



Weinberg term

Gasser-Leutwyler terms

Effective chiral Lagrangian

Weinberg Lagrangian

$$\mathcal{O}(p^2)$$

$$\text{Re}S_{\text{eff}}^{(2)}[\pi^a] - \text{Re}S_{\text{eff}}^{(2)}[0] = \int d^4x \mathcal{L}^{(2)}$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4} \langle D^\mu U^\dagger D_\mu U \rangle + \frac{F_\pi^2}{4} \langle \chi^\dagger U + \chi U^\dagger \rangle$$

Gasser-Leutwyler Lagrangian

$$\mathcal{O}(p^4)$$

$$\mathcal{L}^{(4)} = L_1 \langle L_\mu L_\mu \rangle^2 + L_2 \langle L_\mu L_\nu \rangle^2 + L_3 \langle L_\mu L_\mu L_\nu L_\nu \rangle$$

Low-energy constants

Gasser-Leutwyler Lagrangian

	$M_0(\text{MeV})$	$\Lambda(\text{MeV})$	$L_1(\times 10^{-3})$	$L_2(\times 10^{-3})$	$L_3(\times 10^{-3})$
local χQM	350	1905.5	0.79	1.58	-3.17
DP	350	611.7	0.82	1.63	-3.09
Dipole	350	611.2	0.82	1.63	-2.97
Gaussian	350	627.4	0.81	1.62	-2.88
GL			0.9 ± 0.3	1.7 ± 0.7	-4.4 ± 2.5
Bijnens			0.6 ± 0.2	1.2 ± 0.4	-3.6 ± 1.3
Arriola			0.96	1.95	-5.21
VMD			1.1	2.2	-5.5
Holdom(1)			0.97	1.95	-4.20
Holdom(2)			0.90	1.80	-3.90
Bolokhov et al.			0.63	1.25	2.50
Alfaro et al.			0.45	0.9	-1.8

Low-energy constants

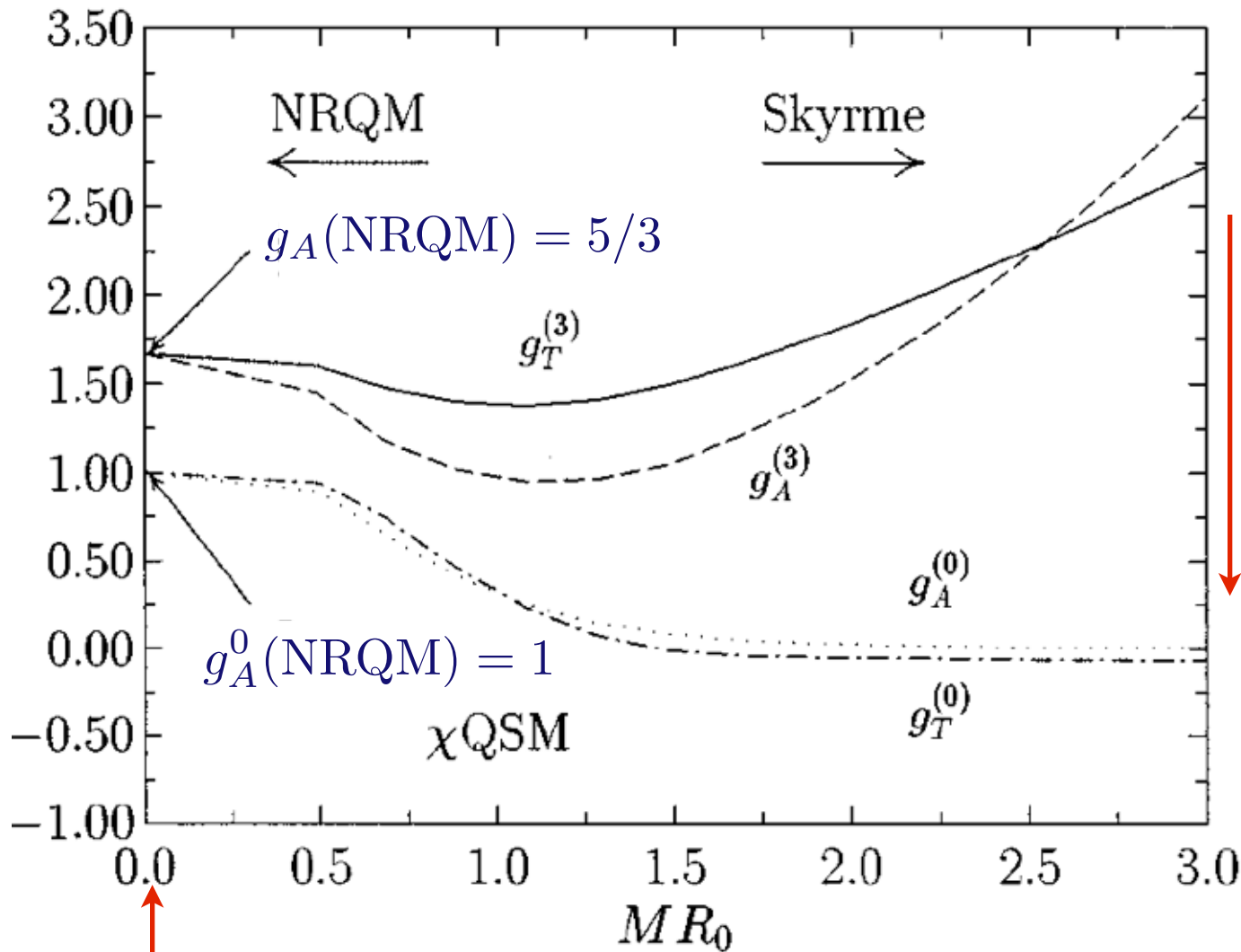
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Limit to the Skyrme model

Example:

Axial and Tensor Charges



Large soliton size:
Valence quarks dive
into the Dirac sea—
No quark and
topological winding
number=1.

$$g_A^0(\text{Skyrme}) \approx 0$$

zero soliton size: NRQM

Medium-modified Skyrme model

Medium-modified effective chiral Lagrangian

$$\begin{aligned}\mathcal{L}^* = & \frac{F_\pi^2}{4} \text{Tr} \left(\frac{\partial U}{\partial t} \right) \left(\frac{\partial U^\dagger}{\partial t} \right) - \frac{F_\pi^2}{16} \alpha_p(\mathbf{r}) \text{Tr} (\nabla U) \cdot (\nabla U^\dagger) \\ & + \frac{1}{32e^2\gamma(\mathbf{r})} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \\ & + \frac{F_\pi^2 m_\pi^2}{16} \alpha_s(\mathbf{r}) \text{Tr}(U + U^\dagger - 2)\end{aligned}$$

Medium-modified Skyrme model

Medium-modified effective chiral Lagrangian

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$$\alpha_p(\mathbf{r}) = 1 - \chi_p(\mathbf{r})$$

$\chi_{p,s}$: pion dipole susceptibility in medium

$$\alpha_s(\mathbf{r}) = 1 + \chi_s(\mathbf{r})/m_\pi^2$$

The parameters are fixed by pion-nucleus scattering data.

(See Ericson and Weise, “Pions in Nuclei”.)

Medium-modified Skyrme model

Medium-modified effective chiral Lagrangian

$$\begin{aligned}\mathcal{L}^* = & \frac{F_\pi^2}{4} \text{Tr} \left(\frac{\partial U}{\partial t} \right) \left(\frac{\partial U^\dagger}{\partial t} \right) - \frac{F_\pi^2}{16} \alpha_p(\mathbf{r}) \text{Tr} (\nabla U) \cdot (\nabla U^\dagger) \\ & + \frac{1}{32e^2 \gamma(\mathbf{r})} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 \\ & + \frac{F_\pi^2 m_\pi^2}{16} \alpha_s(\mathbf{r}) \text{Tr}(U + U^\dagger - 2)\end{aligned}$$

$$\alpha_p(\mathbf{r}) = 1 - \chi_p(\mathbf{r})$$

$\chi_{p,s}$: pion dipole susceptibility in medium

$$\alpha_s(\mathbf{r}) = 1 + \chi_s(\mathbf{r})/m_\pi^2$$

The parameters are fixed by pion-nucleus scattering data.

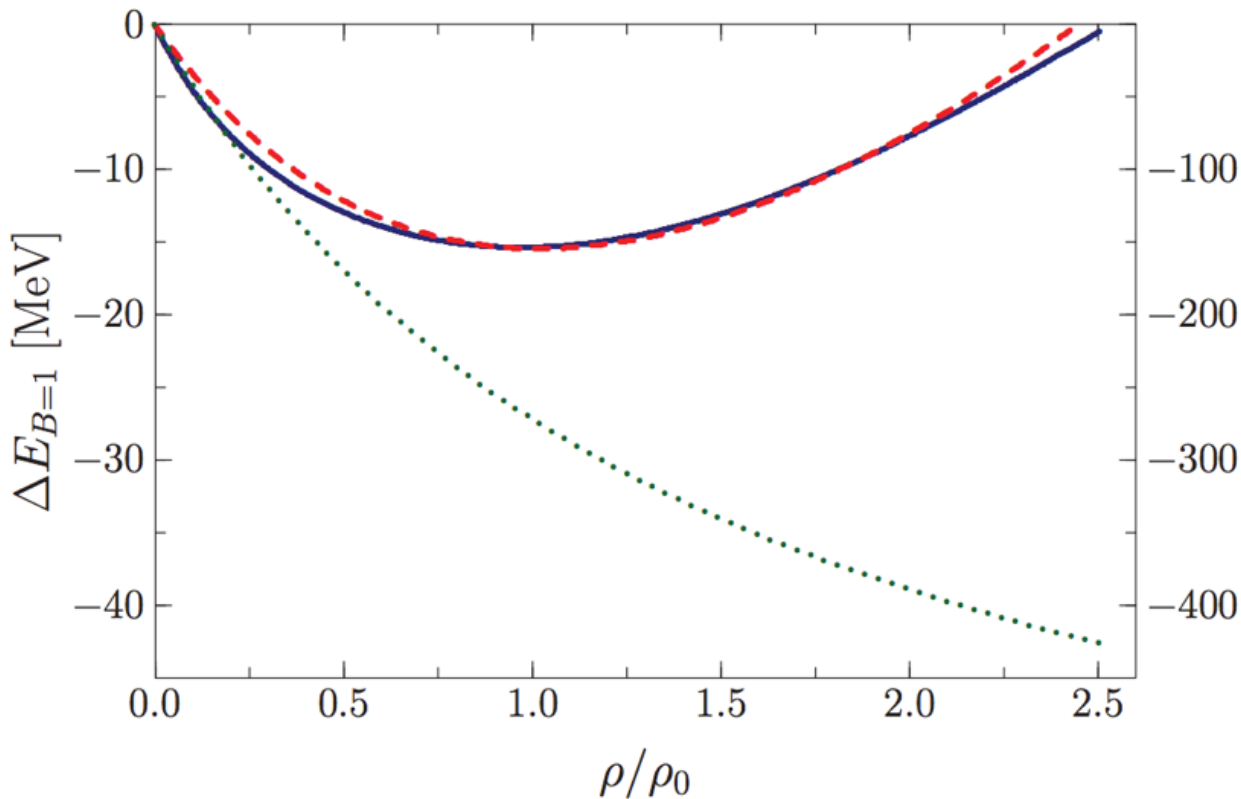
(See Ericson and Weise, “Pions in Nuclei”.)

$$\gamma(\mathbf{r}) = \exp \left(- \frac{\gamma_{\text{num}} \rho(\mathbf{r})}{1 + \gamma_{\text{den}} \rho(\mathbf{r})} \right)$$

Fitted to the volume term of the semi-empirical mass formula.

Medium-modified Skyrme model

Binding Energy per nucleon

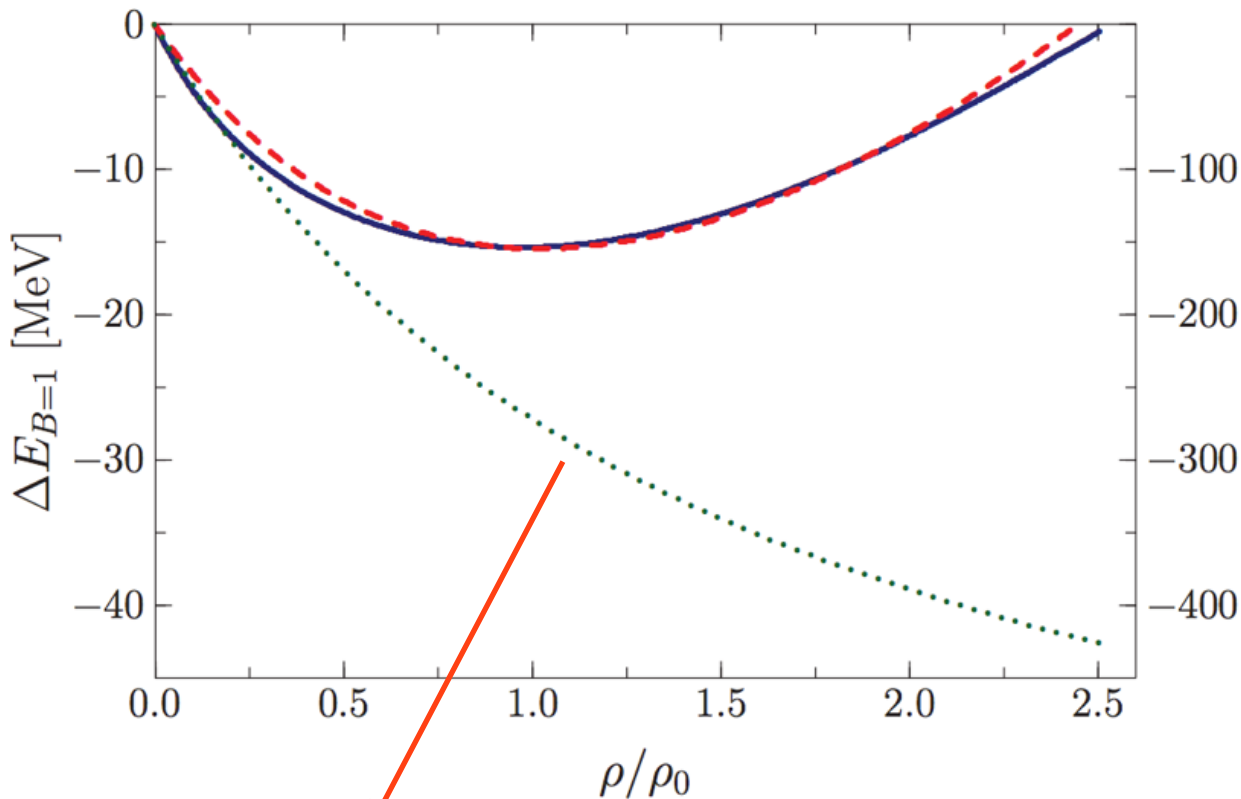


$$\gamma_{\text{num}} = 0.47 m_{\pi}^{-3}$$

$$\gamma_{\text{den}} = 0.17 m_{\pi}^{-3}$$

Medium-modified Skyrme model

Binding Energy per nucleon



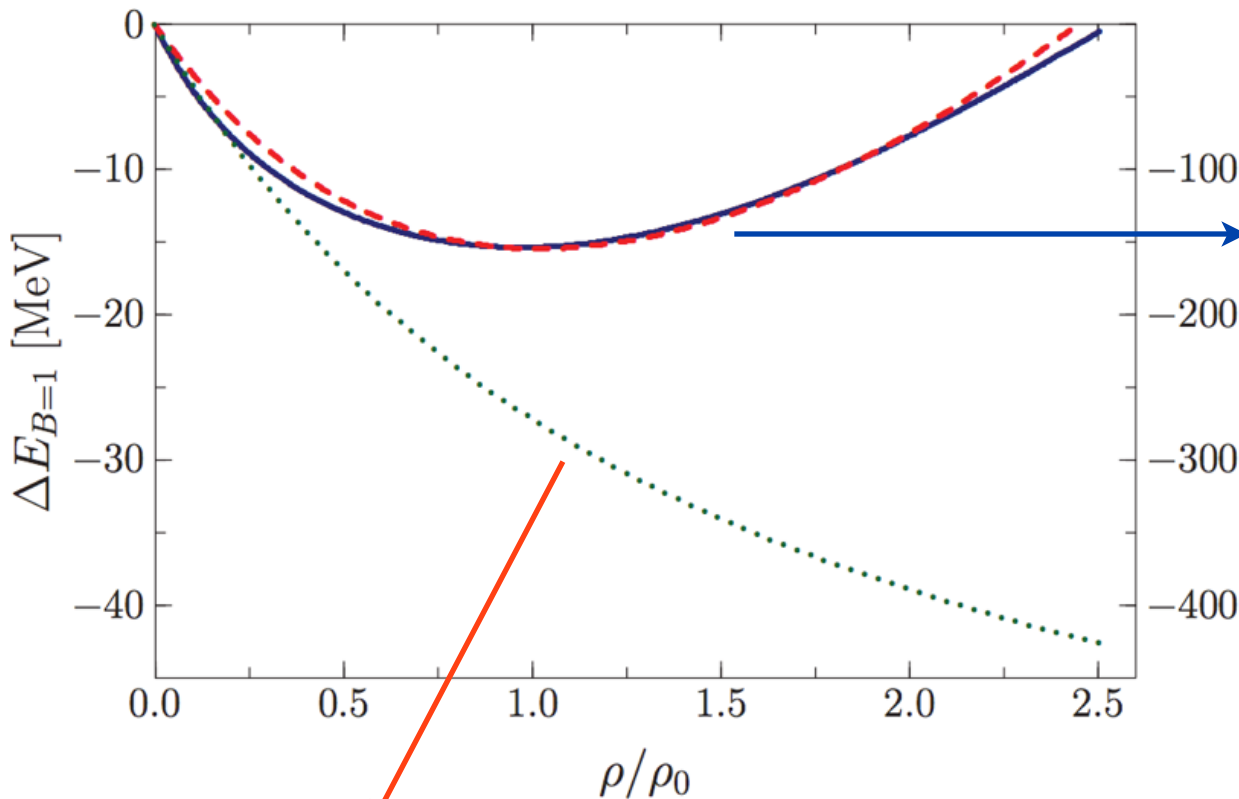
No modification of the Skyrme term

$$\gamma_{\text{num}} = 0.47 m_{\pi}^{-3}$$

$$\gamma_{\text{den}} = 0.17 m_{\pi}^{-3}$$

Medium-modified Skyrme model

Binding Energy per nucleon



With the Skyrme term modified

It is required to protect the Skyrmion from the collapse!

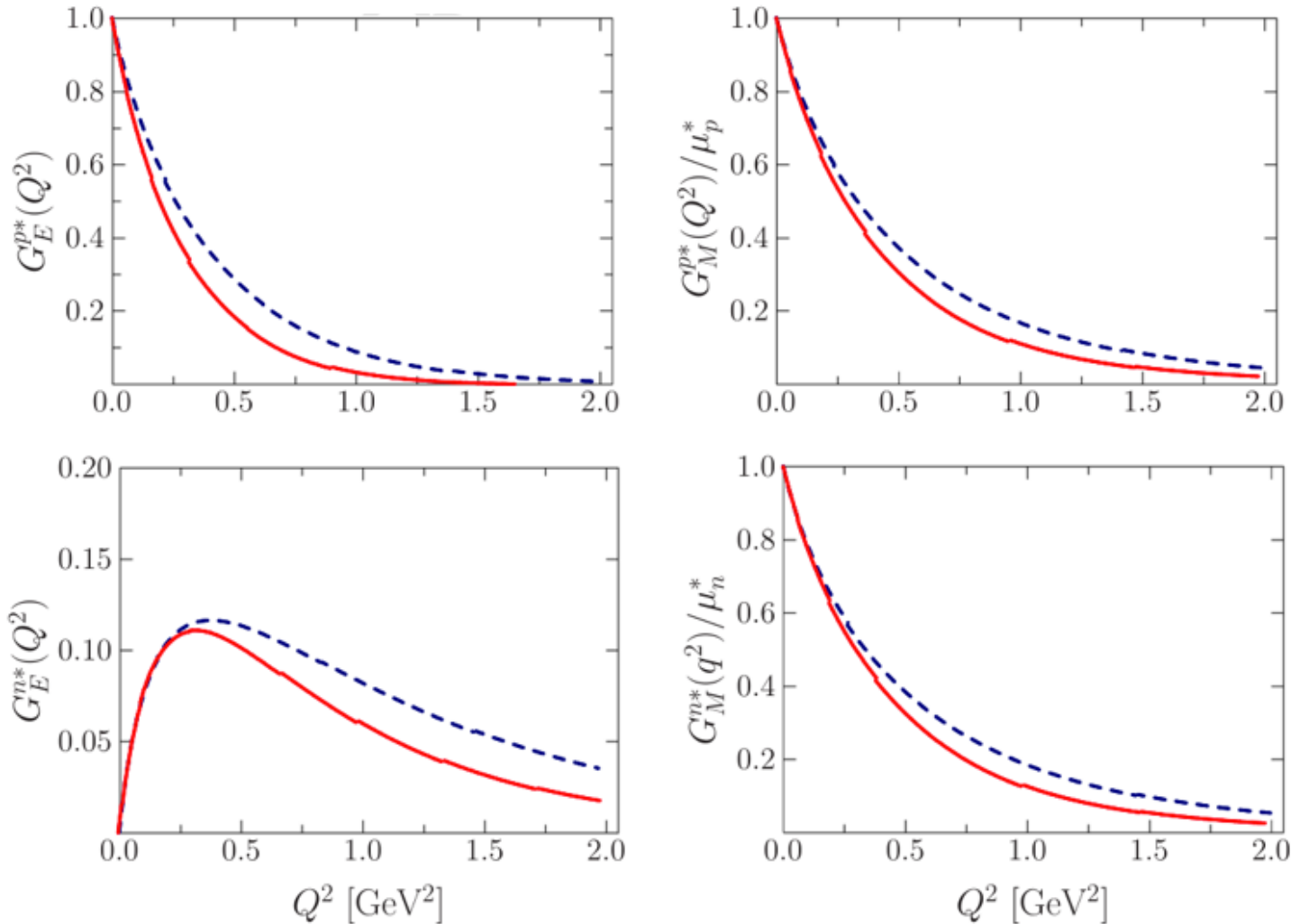
No modification of the Skyrme term

$$\gamma_{\text{num}} = 0.47 m_{\pi}^{-3}$$

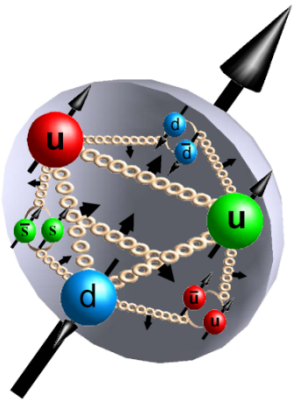
$$\gamma_{\text{den}} = 0.17 m_{\pi}^{-3}$$

Results

Electromagnetic form factors of the nucleon in nuclear matter



Transversity: Tensor Charges



$$\delta \mathbf{q}(\mathbf{x}) = \text{Diagram 1} - \text{Diagram 2}$$

The diagram shows the difference between two nucleon states. The first state (left) is a red sphere with a white core and a green arrow pointing up. The second state (right) is a red sphere with a white core and a green arrow pointing down. A minus sign is between them.

$$\langle N | \bar{\psi} \sigma_{\mu\nu} \lambda^x \psi | N \rangle \sim \text{Tensor charges}$$

- **No explicit probe** for the tensor charge! Difficult to be measured.
- Chiral-odd Parton Distribution Function can get accessed via the SSA of SIDIS (HERMES and COMPASS).

A. Airapetian et al. (HERMES Coll.), PRL 94, 012002 (2005).

E.S. Ageev et al. (COMPASS Coll.), NPB 765, 31 (2007).

CLAS & CLAS12 Coll.

ppbar Drell-Yan process (PAX Coll.): Technically too difficult for the moment (polarized antiproton: hep-ex/0505054).

Transversity: Tensor Charges

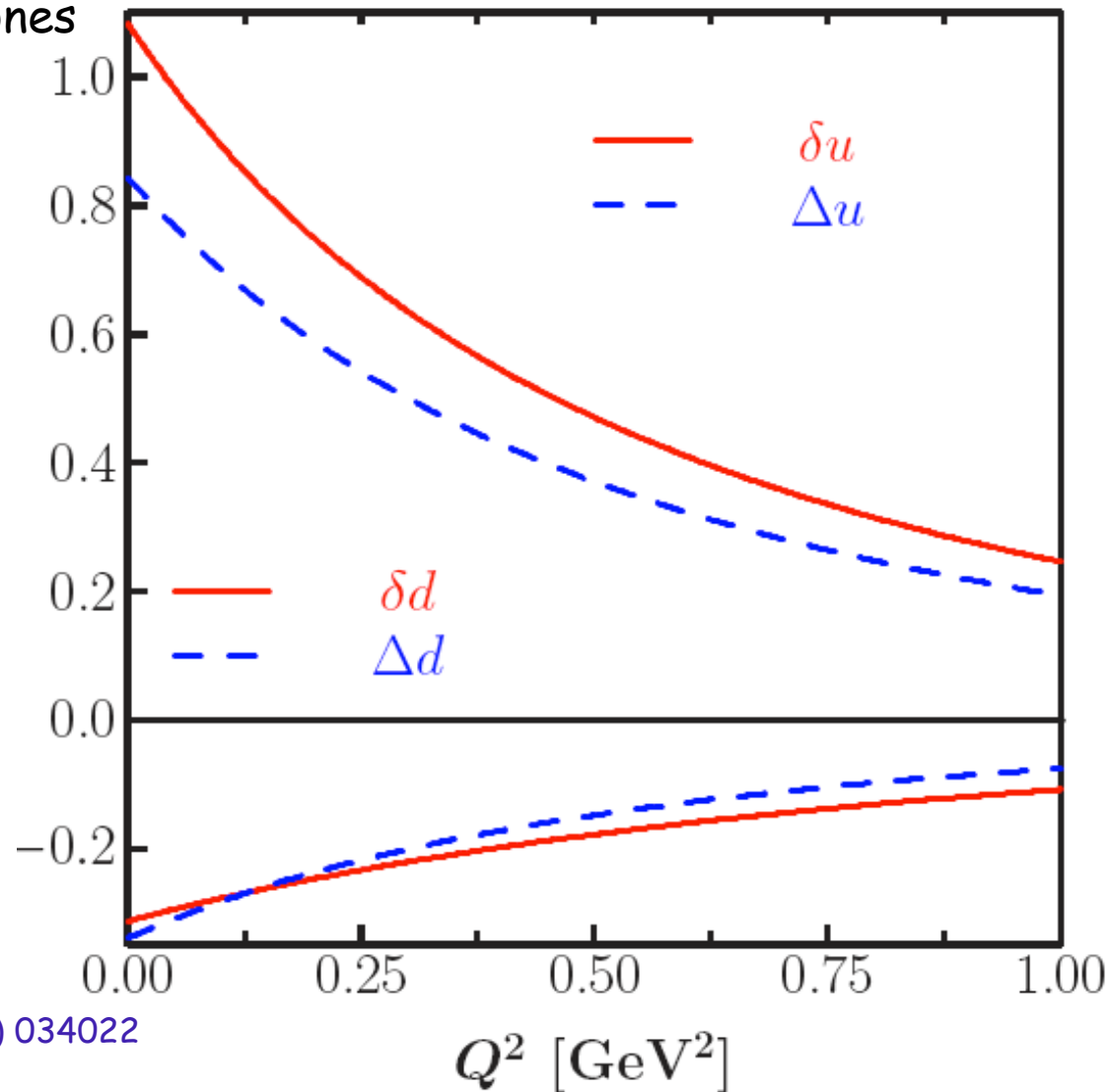
$$\delta u = 0.60^{+0.10}_{-0.24}, \quad \delta d = -0.26^{+0.1}_{-0.18} \text{ at } 0.36 \text{ GeV}^2$$

Based on SIDIS (HERMES) data:

M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

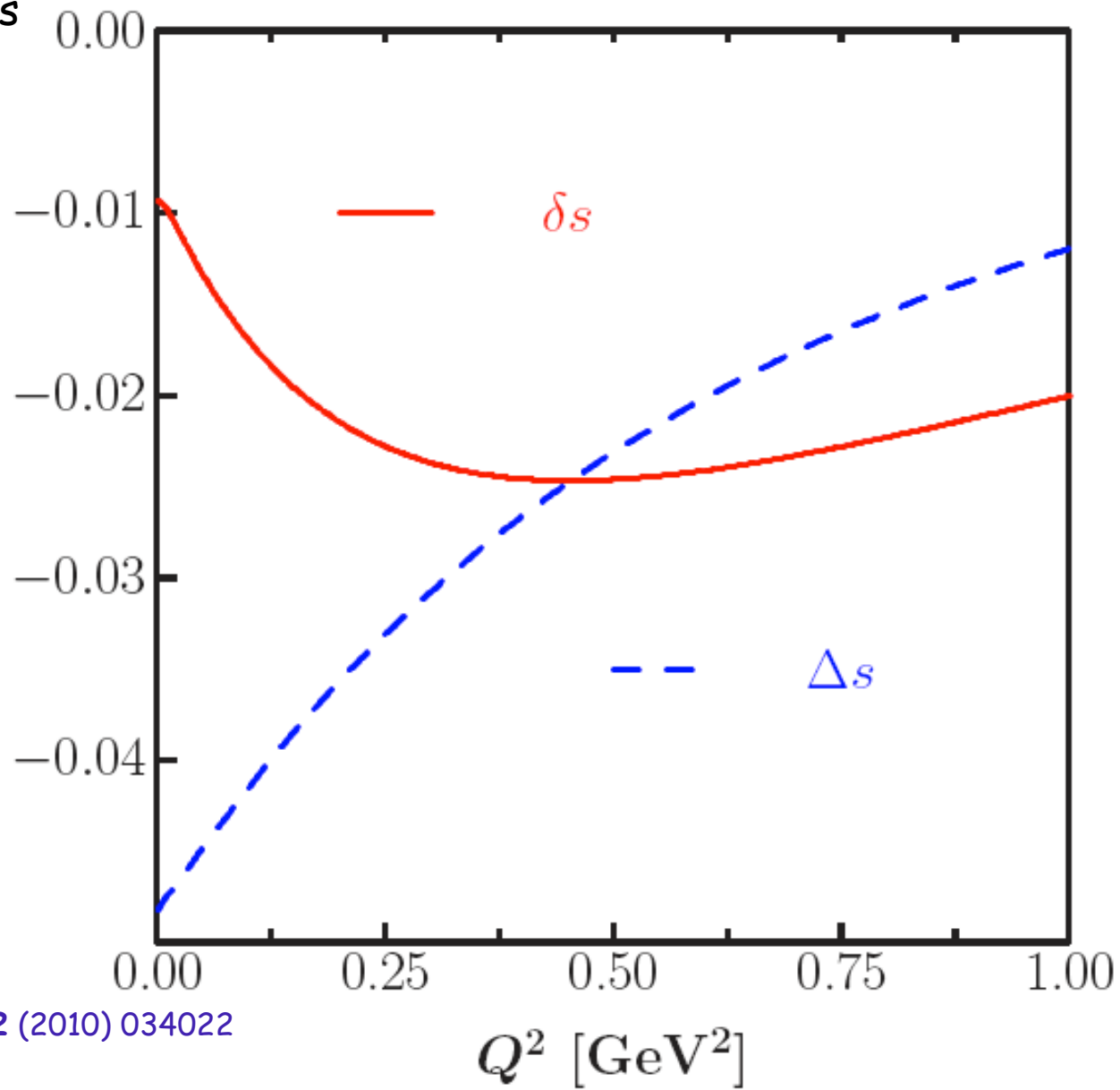
Results

Up and down tensor form factors
compared with the axial-vector ones



Results

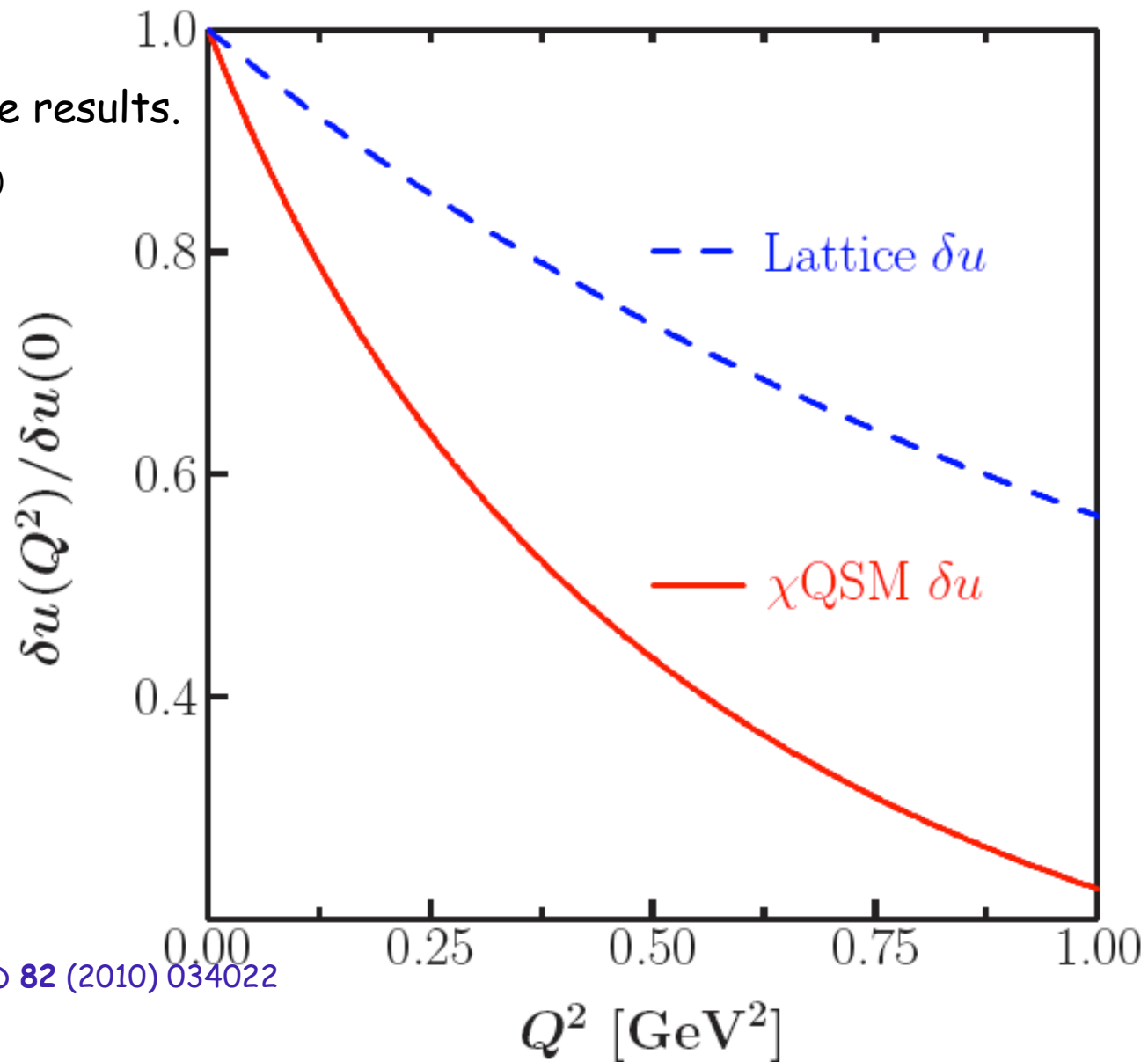
Strange tensor form factors
compared with
the axial-vector ones



Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)

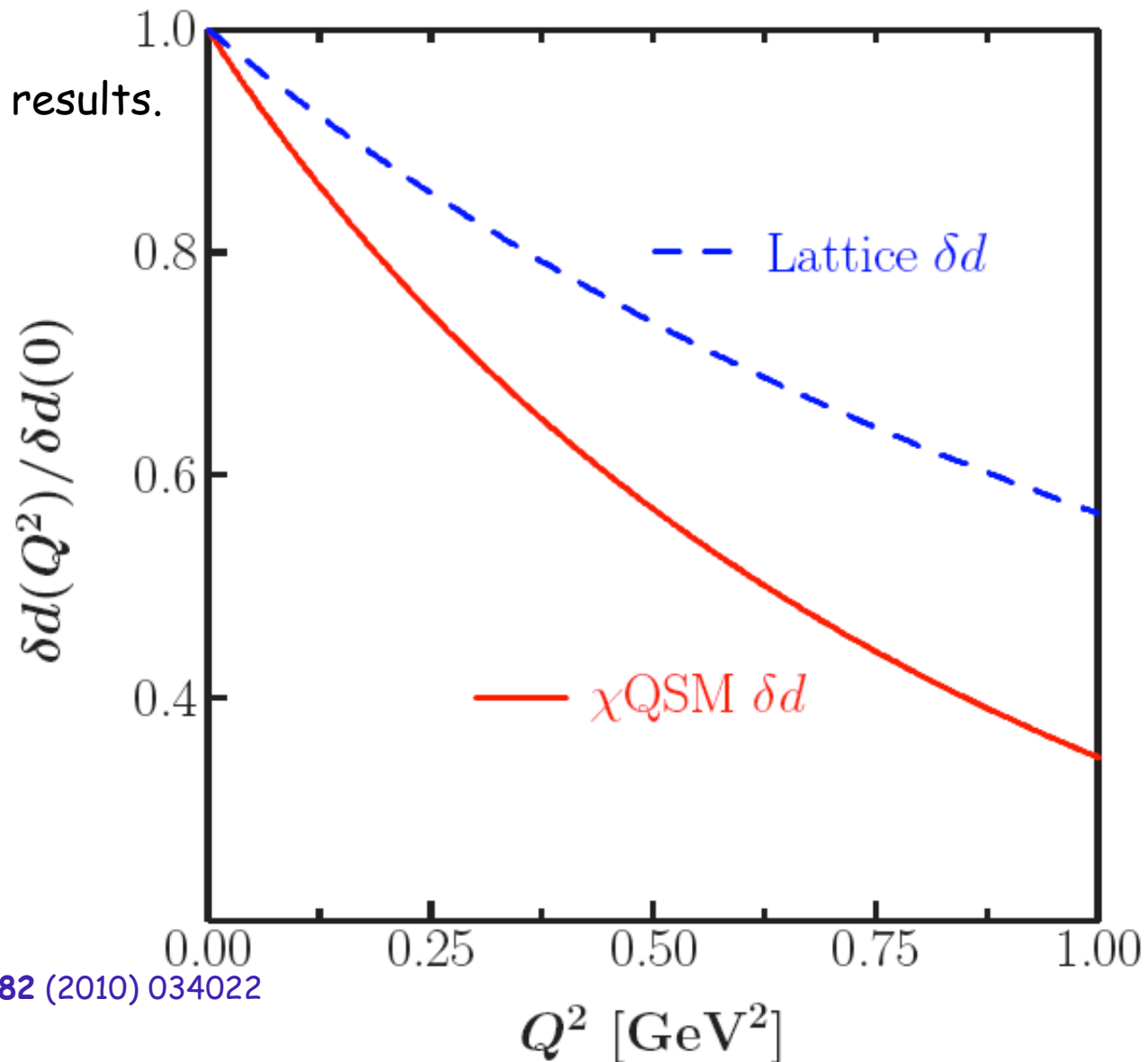


T. Ledwig, A. Silva, HChK, *Phys. Rev. D* **82** (2010) 034022

Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)



T. Ledwig, A. Silva, HChK, *Phys. Rev. D* **82** (2010) 034022

Results

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
δu	1.08	-0.32	-0.03	1.08	0.53	-0.02	-0.32	-0.02
δd	-0.32	1.08	-0.03	-0.02	0.53	1.08	-0.02	-0.32
δs	-0.01	-0.01	0.79	-0.29	-0.29	-0.29	1.06	1.06

Isospin relations

$$\begin{aligned}
 \delta u_p &= \delta d_n, & \delta u_n &= \delta d_p, & \delta u_\Lambda &= \delta d_\Lambda, & \delta u_{\Sigma^+} &= \delta d_{\Sigma^-}, \\
 \delta u_{\Sigma^0} &= \delta d_{\Sigma^0}, & \delta u_{\Sigma^-} &= \delta d_{\Sigma^+}, & \delta u_{\Xi^0} &= \delta d_{\Xi^-}, & \delta u_{\Xi^-} &= \delta d_{\Xi^0}, \\
 \delta s_p &= \delta s_n, & \delta s_{\Sigma^\pm} &= \delta s_{\Sigma^0}, & \delta s_{\Xi^0} &= \delta s_{\Xi^-},
 \end{aligned}$$

SU(3) relations

$$\begin{aligned}
 \delta u_p &= \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-}, \\
 \delta u_n &= \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}.
 \end{aligned}$$

Results

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
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$$\begin{aligned}
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 \delta u_{\Sigma^0} &= \delta d_{\Sigma^0}, & \delta u_{\Sigma^-} &= \delta d_{\Sigma^+}, & \delta u_{\Xi^0} &= \delta d_{\Xi^-}, & \delta u_{\Xi^-} &= \delta d_{\Xi^0}, \\
 \delta s_p &= \delta s_n, & \delta s_{\Sigma^\pm} &= \delta s_{\Sigma^0}, & \delta s_{\Xi^0} &= \delta s_{\Xi^-},
 \end{aligned}$$

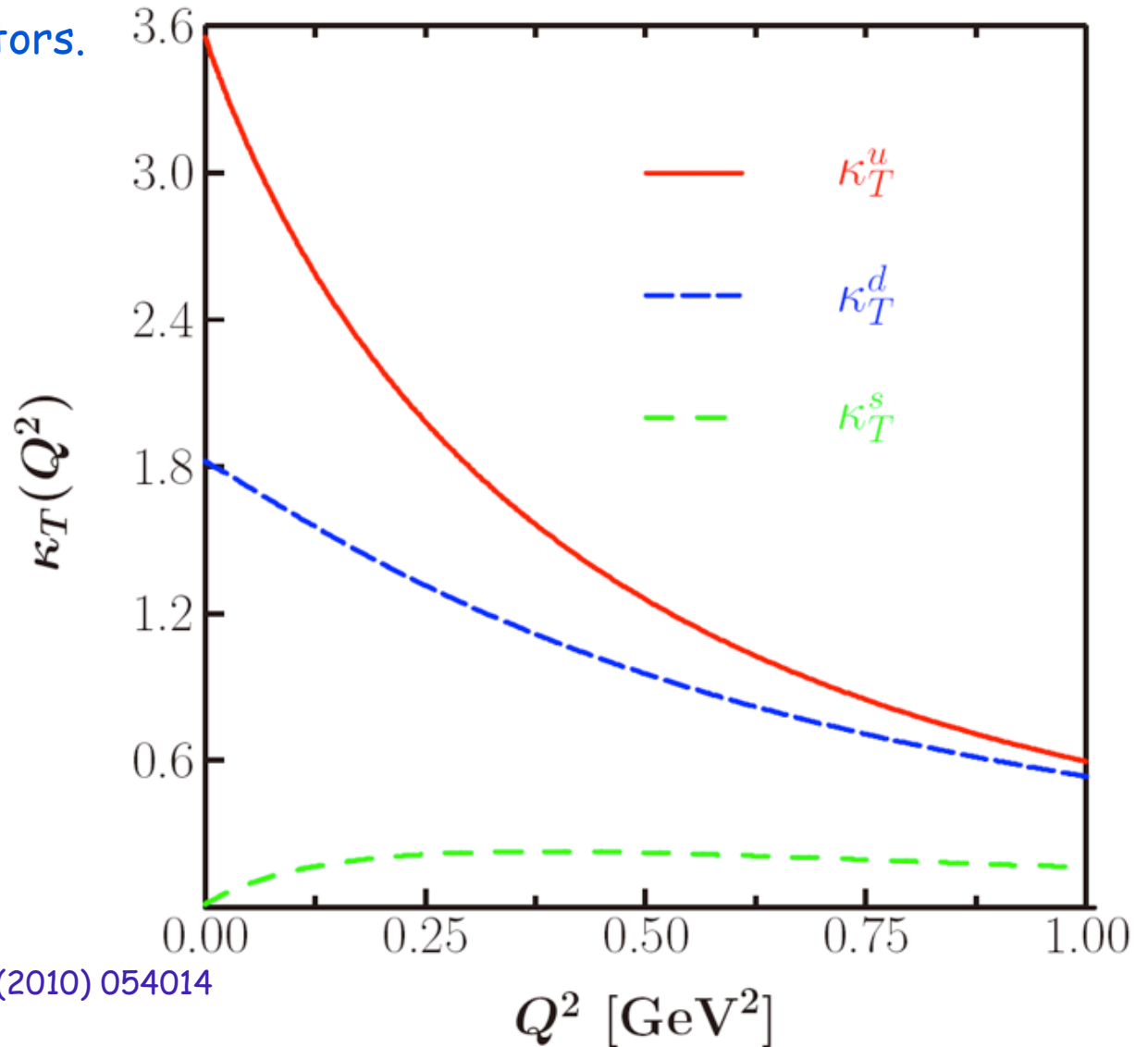
SU(3) relations

Effects of SU(3) symmetry breaking are almost negligible!

$$\begin{aligned}
 \delta u_p &= \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-}, \\
 \delta u_n &= \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}.
 \end{aligned}$$

Results

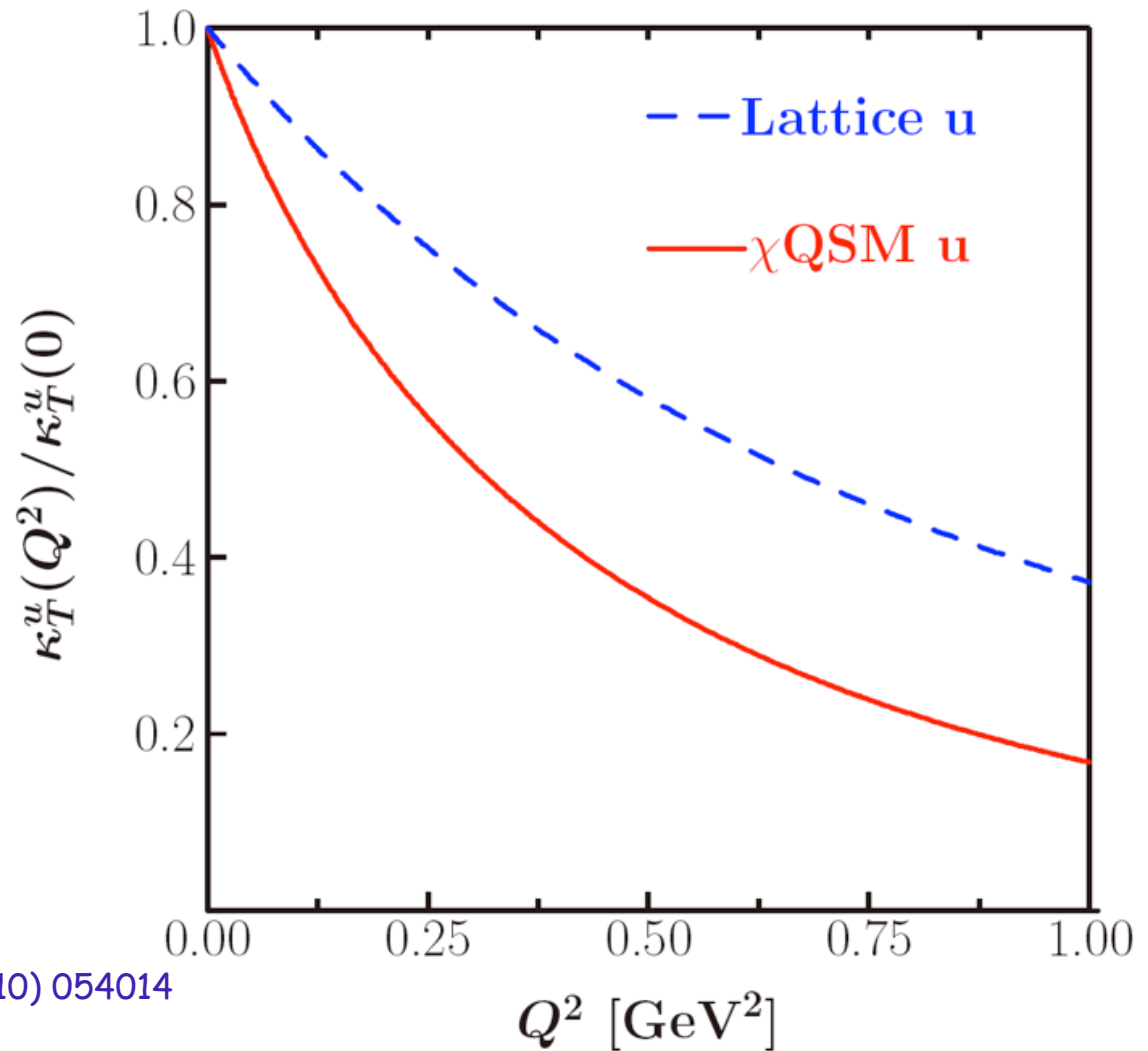
Flavor decomposition of the anomalous tensor magnetic form factors.



Results

Up anomalous tensor magnetic form factors compared with the lattice one.

M. Goekeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)



Results

Down anomalous tensor magnetic form factors compared with the lattice one.

M. Goekeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

