Hadrons in nuclei and partial restoration of chiral symmetry

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I talk about something on hadrons in nuclei and partial restoration of chiral symmetry as a personal view.

especially reply to comment by Brodsky

for this purpose, first, I revisit spontaneous chiral symmetry breaking

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Motivation

Why do we study the properties of hadron in nucleus ?

- as **nuclear physics**
  - study many-body systems governed by strong force
  - discover new bound systems of strong interaction
  - exotic states are interesting

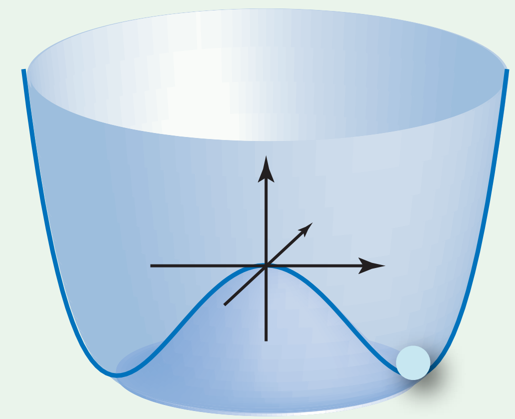
- as **hadron physics**
  - phenomenological proof of spontaneous **chiral symmetry** breaking
  - chiral symmetry breaking is a phase transition phenomenon
  - the order parameters change as environment changes

- for **other research areas**
  - provide basic informations of high density physics
  - important constraints

Chiral Symmetry

chiral symmetry (ChS)

- a fundamental symmetry in QCD
- spontaneously broken by physical states
- spontaneous ChS breaking, SChSB, determines **vacuum property**
  - hadrons are excitation modes upon vacuum
  - most of light hadron properties is determined by ChSB
  - vacuum condensates are more fundamental quantities
- SChSB is a **phase transition phenomenon**
  - broken symmetry can be restored
  - partial (incomplete) restoration takes in nuclear medium
  - complete and/or partial restoration of ChS can be proofs of SChSB



light pion mass, mass generation etc.

two important features of SChSB

determining vacuum property and phase transition phenomenon

Spontaneous Chiral Symmetry Breaking

correlator of axial vector current and pseudoscalar density

$$\Pi_5^{ab}(q) = \mathrm{F.T.}\ \partial^\mu \langle 0|\mathrm{T}[A_\mu^a(x)\phi_5^b(0)]|0\rangle,$$ **chiral limit**

$SU(2)_L \otimes SU(2)_R$ chiral symmetry

pseudoscalar field $\phi_5^a = \frac{1}{2}\bar{q}i\gamma_5\tau^a q$

scalar field $\phi = \frac{1}{2}\bar{q}q$

axial trans. $[Q_5^a, \phi_5^b] = -i\delta^{ab}\phi$

Ward-Takahashi identity (operator identity)

$$\partial^\mu \mathrm{T}[A_\mu^a(x)\phi_5^b(0)] = \mathrm{T}[\partial^\mu A_\mu^a(x)\phi_5^b(0)] + \delta(x_0)[A_0^a(x), \phi_5^b(0)]$$

take soft limit $q_\mu \to 0$

$$\lim_{p\to 0}\int d^4x\, e^{ip\cdot x}\delta(x_0)\langle 0|[A_0^a(x), \phi_5^b(0)]|0\rangle = \langle 0|[Q_5^a, \phi_5^b|0\rangle = -i\delta^{ab}\langle \bar{q}q\rangle$$

$$\boxed{\Pi_5^{ab}(0) = -i\delta^{ab}\langle \bar{q}q\rangle}$$

consequence of QCD

Spontaneous Chiral Symmetry Breaking

correlator of axial vector current and pseudoscalar density

$$\Pi_5^{ab}(q) = \mathrm{F.T.}\ \partial^\mu \langle 0|\mathrm{T}[A_\mu^a(x)\phi_5^b(0)]|0\rangle, \qquad \text{chiral limit}$$

hadronic modes in soft limit

$$i\delta^{ab} F G^{1/2} \frac{q^2}{q^2 - m^2 + i\epsilon} \to 0$$

except zero mode (m^2=0)

$$\boxed{\Pi_5^{ab}(0) = i\delta^{ab} F_\pi G_\pi^{1/2}}$$

matrix elements of pion

pion decay constant

$$\langle 0|A_\mu^a(x)|\pi^b(p)\rangle = \delta^{ab} i p_\mu F_\pi e^{-ip\cdot x}$$

wavefunction normalization

$$\langle 0|\phi_5^a(x)|\pi^b(p)\rangle = \delta^{ab} G_\pi^{1/2} e^{-ip\cdot x}$$

Glashow Weinberg relation

$$F_\pi G_\pi^{1/2} = -\langle \bar{q} q\rangle$$

Glashow, Weinberg, PRL 20 (1968) 224

if quark condensate is finite,
there must be a zero mode having finite decay constant

In-medium chiral condensate

DJ, Hatsuda, Kunihiro, PLB 670 (2008), 109.

correlator of axial vector current and pseudoscalar density in medium

$$\Pi_5^{ab}(q) = \mathrm{F.T.}\ \partial^\mu \langle \Omega | \mathrm{T}[A_\mu^a(x)\phi_5^b(0)] | \Omega \rangle$$

chiral limit

$|\Omega\rangle$ grand state of isospin symmetric nuclear matter

operator relation is independent of states

pion is not unique zero mode, p-h excitations can be ZMs

sum rule
in chiral limit

$$\sum_\alpha \mathrm{Re}\left[(N_\alpha^* + F_\alpha^*) G_\alpha^{*1/2}\right] = -\langle \bar{q} q \rangle^*$$

sum up all the zero modes

- valid for all densities
- particle-hole modes also account for in-medium quark condensate
- hadronic quantities define the in-medium quark condensate

Matrix elements

$$\begin{aligned}
\langle \Omega_\ell^b(k) | \phi_5^a(x) | \Omega \rangle &= \delta^{ab} G_\ell^{*1/2} e^{ik\cdot x}, \\
\langle \Omega | A_\mu^a(x) | \Omega_\ell^b(k) \rangle &= i\delta^{ab}[n_\mu (n\cdot k) N_\ell^* + k_\mu F_\ell^*] e^{-ik\cdot x}.
\end{aligned}$$

Gell-Mann Oakes Renner (GOR) relation

GW relation $$F_\pi G_\pi^{1/2} = -\langle \bar{q} q \rangle$$ in chiral limit

PCAC relation $$\partial^\mu A_\mu^a(x) = 2m_q \phi_5^a(x)$$ with quark mass

take matrix element

$$\langle 0 | \partial^\mu A_\mu(x) | \pi(p) \rangle = F_\pi p^2 = F_\pi m_\pi^2$$

$$2m_q \langle 0 | \phi_5(x) | \pi(p) \rangle = 2m_q G_\pi^{1/2}$$

pion decay constant

$$\langle 0 | A_\mu^a(x) | \pi^b(p) \rangle = \delta^{ab} i p_\mu F_\pi e^{-ip\cdot x}$$

wavefunction normalization

$$\langle 0 | \phi_5^a(x) | \pi^b(p) \rangle = \delta^{ab} G_\pi^{1/2} e^{-ip\cdot x}$$

$$F_\pi m_\pi^2 = 2m_q G_\pi^{1/2}$$

this relation is important
we will come back later

GOR relation $$F_\pi^2 m_\pi^2 = -2m_q \langle \bar{q} q \rangle$$

Gell-Mann, Oakes, Renner, PR175 (1968) 2195

a fundamental relation to connect hadron quantities to quark condensate

Order Parameters of Chiral Symmetry

phase transition phenomena have their order parameters

quark condensate is just one of the order parameters of SChSB, not unique order parameter

$$\langle [Q_5, \Pi] \rangle = \langle \Sigma \rangle \neq 0$$

ex.) $\Pi^a = \bar{q} i \gamma_5 t^a q$ $\quad$ $\Sigma = \frac{1}{2} \bar{q} q$

$\Pi^a = \epsilon^{abc} V_\mu^b A_\mu^c$ $\quad$ $\Sigma = V_\mu^b V_\mu^b - A_\mu^b A_\mu^b$

quark condensate $\langle \bar{q} q \rangle$ is **a good quantity to measure SChSB**

we have simple ways to translate the quark condensate in QCD to other effective models

chiral condensate $\langle \sigma \rangle$ in linear sigma model

quark condensate in NJL model

order parameters can change as environment changes

we discuss partial restoration of chiral symmetry in nuclear medium

partial restoration of chiral symmetry is

incomplete restoration of symmetry

with **significant reduction of order parameters** of SChSB

Partial Restoration of Chiral Symmetry

PRChS in nucleus has been proved phenomenologically

we need several steps to connect observables to quark condensate

K. Suzuki et al. PRL92, 072302, (04); Friedman et al., PRL93, 122302 (04); DJ, Hatsuda, Kunihiro, PLB 670, 109 (08).

Yamazaki, Hirenzaki, Hayano, Toki, Phys. Rept. 514 (2012) 1

experimental observations:

energy levels of pionic atoms, πN scattering

compile data to pion optical potential

s-wave $$2m_\pi U_S = -4\pi \left[1+\frac{m_\pi}{m_N}\right](b_0^*(\rho)\rho - b_1^*(\rho)\delta\rho)$$ **linear approx. in δρ**

isoscalar density $\rho = \rho_p + \rho_n$ **isovector density** $\delta\rho = \rho_p - \rho_n$

theoretical relation **in-medium Weinberg-Tomozawa relation**

$$4\pi \left[1+\frac{m_\pi}{m_N}\right] b_1^*(\rho) = \mathcal{T}^{(-)*}(m_\pi) = \frac{m_\pi}{2(F_\pi^t)^2}$$

valid at the leading order of chiral expansion and any density

if one can identify the isovector part, one does not have to remove the higher orders of δρ.

$$\frac{b_1}{b_1^*} = \left(\frac{F_\pi^t}{F_\pi}\right)^2$$ **missing repulsion** $b_1 < b_1^*$ **reduction of F_π** $F_\pi > F_\pi^t$

Partial Restoration of Chiral Symmetry

one more theoretical relation at chiral limit

$$\left(\frac{F_\pi^t}{F_\pi}\right) Z_\pi^{*1/2} = \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}$$

in-medium sum rule
in linear density approximation

$$Z_\pi^{*1/2} = 1 - \gamma \frac{\rho}{\rho_0}$$

wavefunction renorm.

K. Suzuki et al. PRL92, 072302, (04); Friedman et al., PRL93, 122302 (04); DJ, Hatsuda, Kunihiro, PLB 670, 109 (08).

Yamazaki, Hirenzaki, Hayano, Toki, Phys. Rept. 514 (2012) 1

quark condensate can be expressed by observables
within linear density approximation

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \simeq \left(\frac{b_1}{b_1^*}\right)^{1/2} \left(1 - \gamma \frac{\rho}{\rho_0}\right)$$

γ can be obtained from πN scattering data

using observed values

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} = 1 - 0.37 \frac{\rho}{\rho_0}$$

30-40% reduction at saturation density

this conclusion has been obtained through **the physical observables** and theoretical consideration

Where is ChSB realized ?

We suppose that **ChSB is realized by vacuum condensates.**

Another theory tells us that it is **realized in hadrons**….

Brodsky, et al., PRC82 (2010) 022201(R)

- vacuum condensates are not physical observables
- quark condensate would lead to a cosmological constant some 45 orders of magnitude larger than observation
- hadrons are well-defined asymptotic states in QCD
- "strong interaction condensates are properties of rigorously well-defined wave function of the hadrons, rather than the hadronless ground state of QCD"

They say that quark condensate is not proper quantity for SChSB
but hadronic quantity is appropriate

What is it ???

Where is ChSB realized ?

Dyson-Schwinger-Bethe-Salpeter approach

Maris, Roberts, Tandy, PLB420 (1998) 267
Brodsky, et al., PRC82 (2010) 022201(R)

treat quark dynamics in a non-perturbative way

calculating two matrix elements of pion

$$i f_\pi P_\mu = \langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle$$

$$i \rho_\pi = - \langle 0 | \bar{q} i \gamma_5 q | \pi \rangle$$

they find **an exact relation in QCD**

$$f_\pi m_\pi^2 = 2 m_q \rho_\pi$$

same as the relation obtained from PCAC

if chiral symmetry is dynamically broken

at chiral limit

they find $-\langle \bar{q} q \rangle = f_\pi \rho_\pi$ **same as GW relation**

what they have obtained is completely same as ours

just they do not say that quark condensate is a fundamental quantity

Do we really need the quark condensate ?

good points

- we have obtained the in-medium quark condensate from the physical quantity

    this proves partial restoration of chiral symmetry

    quark condensate is a good quantity to measure SChSB

- it is easy to translate it to other effective models

    help us model calculations

- to estimate in-medium quark condensate makes sense

bad points

- in order to say that quark condensate is more fundamental quantity in QCD, we need more relations connecting quark condensate with hadron quantities

- but we do not have such relations in QCD yet

- as chiral symmetry is restored, the mass differences of ρ-a_1, N-N*, η-η' should decrease there are no clear relations between these mass differences and quark condensate

    no QCD predictions of in-medium hadron quantities from in-medium condensate yet

- quark condensate is not unique order parameter, and only quark condensate does not have to explain partial restoration of chiral symmetry

Do we really need the quark condensate ?

I think that…

the theory of Brodsky et al. is consistent with what we think

we calculate same things, but interpretation is different

I would like to emphasize that…

partial restoration of chiral symmetry does take place,
no matter how we describe its order parameter

with partial restoration

fpi decreases

the wave function renormalization decreases

the mass differences of ρ-a1, N-N*, η-η’ decrease

they are caused by partial restoration and can be evidences of partial restoration