

Theoretical study of a prototype system of kaonic nuclei "K⁻pp"



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1. Introduction
2. Effective single-channel potential by “Feshbach method with coupled-channel Complex Scaling Method”
3. Result of “K⁻pp” calculated with ccCSM+Feshbach method
4. Other quantum number case – $J^{\pi}=1^{-} \dots S_{NN}=1$
5. Summary and future plan

Kaonic nuclei

1. Introduction

Prototype system = $K^- pp$

at J-PARC

大強度陽子加速器施設 (J-PARC)

K^-

$K^{\text{bar}}N$ two-body system

Proton

Low energy scattering data, 1s level shift of kaonic hydrogen atom

“Excited hyperon $\Lambda(1405) = K^-$ proton quasi-bound state”

Strongly attractive $K^{\text{bar}}N$ potential

Prototype system = $K^- pp$

- Doorway to **dense matter**[†]
→ Chiral symmetry restoration in dense matter
- Interesting structure[†]
- Neutron star

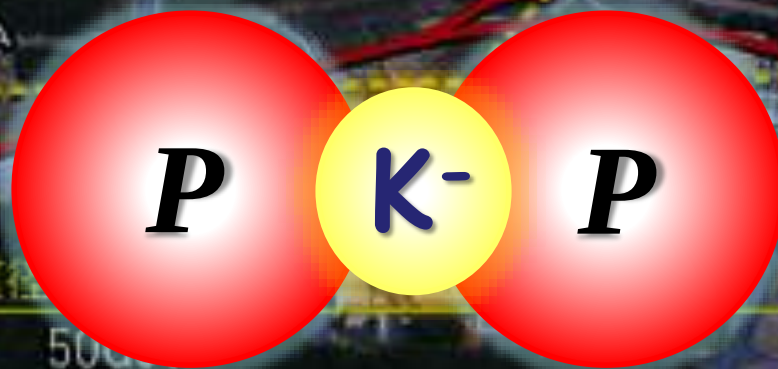
Nuclear many-body system with K^-



$^3\text{He}K^-$, $pppK^-$,
 $^4\text{He}K^-$, $pppnK^-$,
..., $^8\text{Be}K^-$, ...

[†] A. D., H. Horiuchi, Y. Akaishi and T. Yamazaki, PRC70, 044313 (2004)

Kaonic nuclei



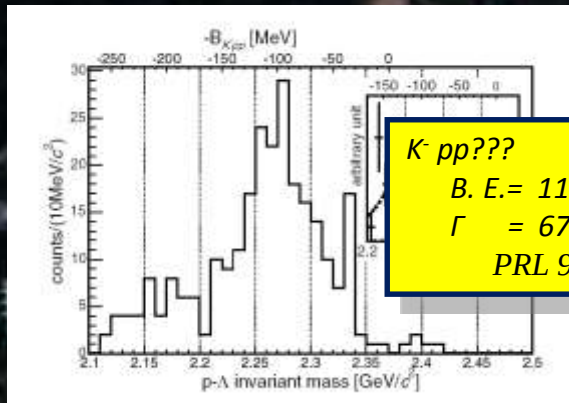
Prototype system = $K^- pp$

at J-PARC

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Experiments of K - pp search

FINUDA



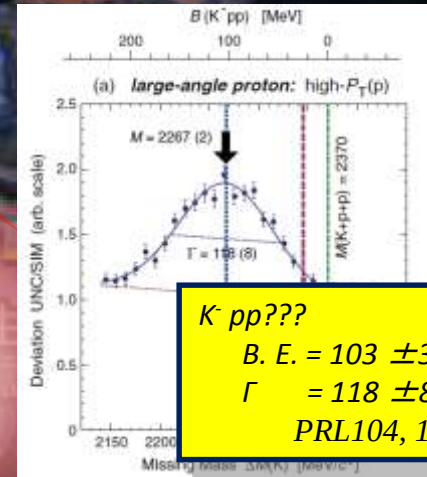
$K^- pp???$

$B. E. = 115 \text{ MeV}$

$\Gamma = 67 \text{ MeV}$

PRL 94, 212303 (2005)

DISTO



$K^- pp???$

$B. E. = 103 \pm 3 \pm 5 \text{ MeV}$

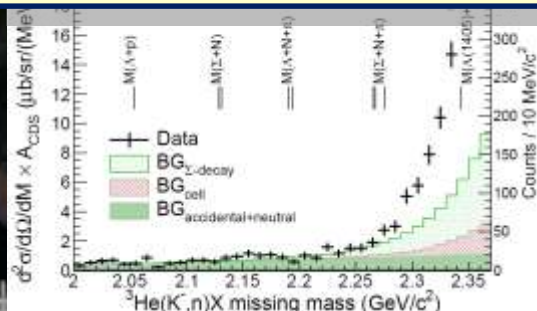
$\Gamma = 118 \pm 8 \pm 10 \text{ MeV}$

PRL 104, 132502 (2010)

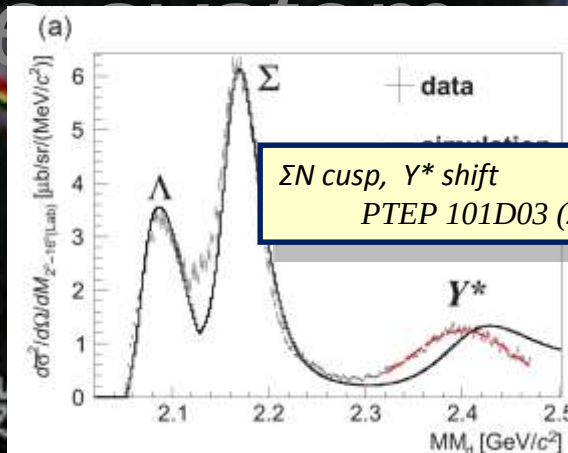
J-PARC E15

Attraction in K - pp subthreshold region

arXiv:1408.5637 [nucl-ex]



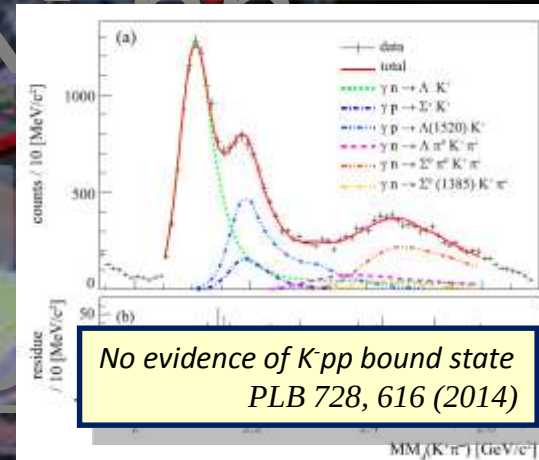
J-PARC E27



ΣN cusp, Y^* shift

PTEP 101D03 (2014)

SPring8/LEPS



No evidence of $K^- pp$ bound state

PLB 728, 616 (2014)

Theoretical studies of K -pp

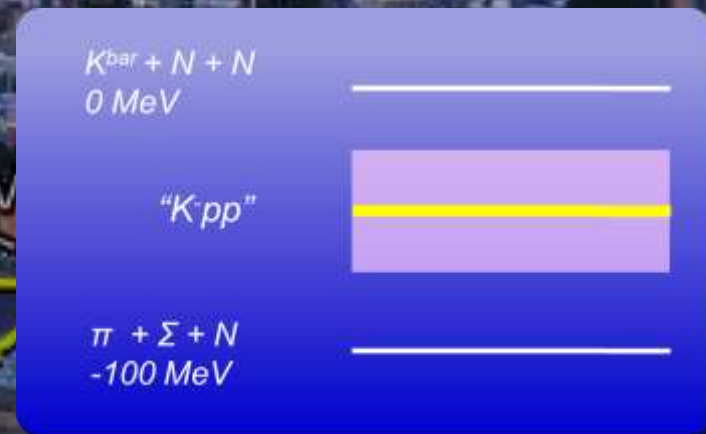
	<i>Dote-Hyodo-Weise</i>	<i>Barnea-Gal-Liverts</i>	<i>Akaishi-Yamazaki</i>	<i>Ikeda-Kamano-Sato</i>	<i>Shevchenko-Gal-Mares</i>
	PRC79, 014003 (2009)	PLB712, 132 (2012)	PRC76, 045201 (2007)	PTP124, 533 (2010)	PRC76, 044004 (2007)
$B(K\text{-}pp)$	20 ± 3	16	47	$9 \sim 16$	$50 \sim 70$
Γ	40 ~ 70	41	61	34 ~ 46	90 ~ 110
Method	Variational (Gauss)	Variational (H. H.)	Variational (Gauss)	Faddeev-AGS	Faddeev-AGS
Potential	<i>Chiral</i> (<i>E</i> -dep.)	<i>Chiral</i> (<i>E</i> -dep.)	<i>Pheno.</i>	<i>Chiral</i> (<i>E</i> -dep.)	<i>Pheno.</i>

- *Chiral pot. (E-dep.)* → Small binding
- *Phenomenological pot. (E-indep.)* → Large binding

$B(K\text{-}pp) < 100 \text{ MeV}$

K -pp should be bound, but exist as a resonance between $K^{\text{bar}}NN$ and $\pi\Sigma N$ thresholds.

- **Resonant state**
- **Coupled-channel system**



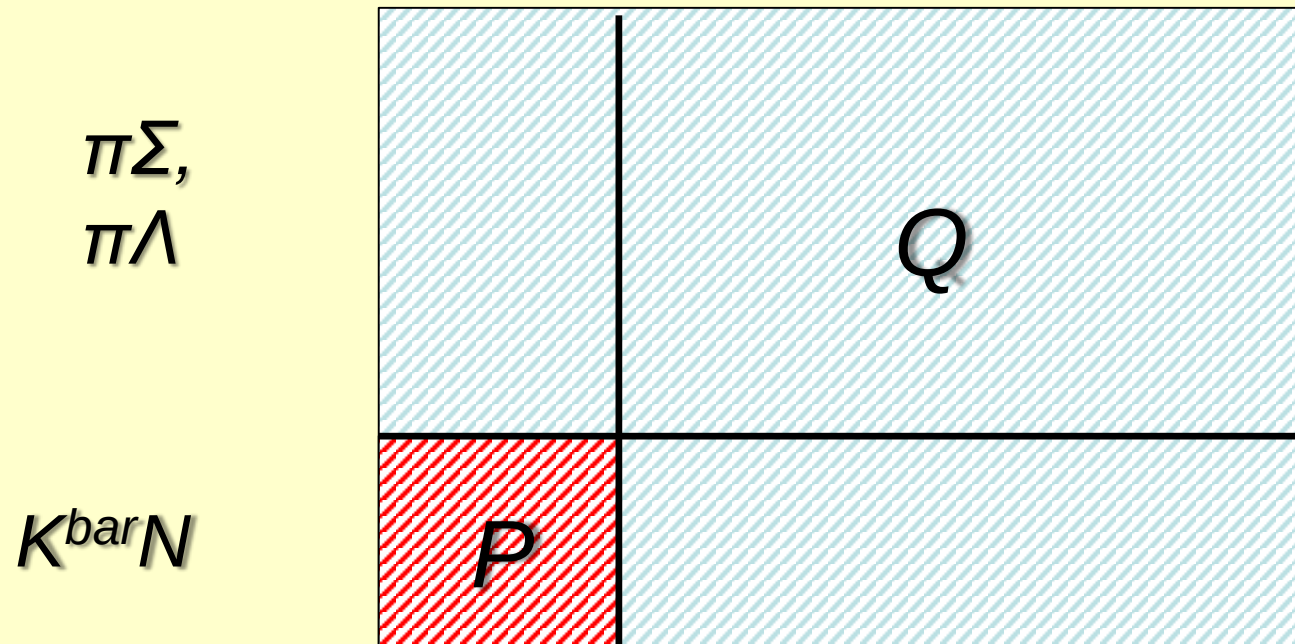
⇒ coupled-channel
Complex Scaling Method

Prototype system = $K^{\text{bar}}pp$

- **Anti-kaon = Nambu-Goldstone boson**

⇒ Chiral $SU(3)$ -based $K^{\text{bar}}N$ potential

2. Effective single-channel potential by “Feshbach projection with ccCSM”



**Reduce the coupled-channel problem
to a single channel problem**

Complex Scaling Method for Resonance

Complex rotation of coordinate (Complex scaling)

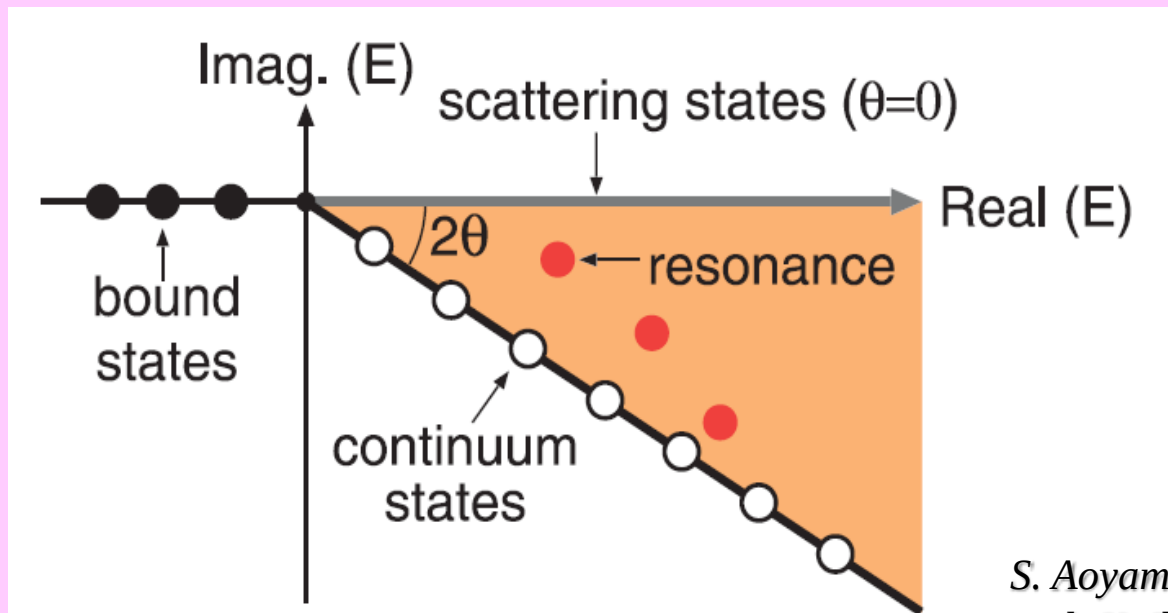
$$U(\theta): \mathbf{r} \rightarrow \mathbf{r} e^{i\theta}, \quad \mathbf{k} \rightarrow \mathbf{k} e^{-i\theta}$$



$$H_\theta \equiv U(\theta) H U^{-1}(\theta), \quad |\Phi_\theta\rangle \equiv U(\theta) |\Phi\rangle$$

Diagonalize H_θ with Gaussian base,

we can obtain resonant states, in the same way as bound states!



*S. Aoyama, T. Myo, K. Kato
and K. Ikeda, PTP116, 1 (2006)*

➤ *Continuum state appears on 2ϑ line.*

➤ *Resonance pole is off from 2ϑ line, and independent of ϑ . (ABC theorem[†])*

[†] J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971),269.
E. Balslev and J. M. Combes, Commun. Math. Phys. 22 (1971),280

Formalism of ccCSM + Feshbach method

Elimination of channels by Feshbach method

Schrödinger eq.
in model space “P” and out of model space “Q”

$$\begin{pmatrix} T_P + v_P & V_{PQ} \\ V_{QP} & T_Q + v_Q \end{pmatrix} \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix} = E \begin{pmatrix} \Phi_P \\ \Phi_Q \end{pmatrix}$$

Schrödinger eq. in P-space : $(T_P + U_P^{\text{Eff}}(E))\Phi_P = E\Phi_P$

Effective potential for P-space

$$U_P^{\text{Eff}}(E) = v_P + V_{PQ} G_Q(E) V_{QP}$$

Q-space Green function:

$$G_Q(E) = \frac{1}{E - H_{QQ}}$$

Extended Closure Relation in Complex Scaling Method

$$\begin{aligned} H_{QQ}^\theta |\chi_n^\theta\rangle &= \varepsilon_n^\theta |\chi_n^\theta\rangle \\ H_{QQ}^\theta &= U(\theta) H_{QQ} U^{-1}(\theta) \end{aligned} \quad \int_C \sum_{R+B} |\chi_n^\theta\rangle \langle \chi_n^\theta| = 1$$



Diagonalize H_{QQ}^θ with Gaussian base,

$$\sum_n |\chi_n^\theta\rangle \langle \chi_n^\theta| \approx 1 \quad \text{Well approximated}$$

T. Myo, A. Ohnishi and K. Kato, PTP99, 801 (1998)

Express the $G_Q(E)$ with Gaussian base using ECR

$$G_Q^\theta(E) = \frac{1}{E - H_{QQ}^\theta} \approx \sum_n |\chi_n^\theta\rangle \frac{1}{E - \varepsilon_n^\theta} \langle \chi_n^\theta|$$



$$U_P^{\text{Eff}}(E) = v_P + V_{PQ} \underbrace{U^{-1}(\theta) G_Q^\theta(E) U(\theta)}_{G_Q(E)} V_{QP}$$

$\{ |\chi_n^\theta\rangle \}$: expanded with Gaussian base.

*3. Result of K^-pp calculated with
ccCSM + Feshbach method*

Apply ccCSM + Feshbach method to K^-pp

“ K^-pp ” ... $K^{bar}NN - \pi\Sigma N - \pi\Lambda N$ ($J^\pi=0^-, T=1/2$)

For the two-body system, $P = K^{bar}N$, $Q = \pi Y$

$$\begin{matrix} V(K^{bar}N - \pi Y; I=0,1) \\ V(\pi Y - \pi Y'; I=0,1) \end{matrix} \xrightarrow{\text{Feshbach + ccCSM}} U_{K^{bar}N(I=0,1)}^{Eff}(E)$$

- Schrödinger eq. for $K^{bar}NN$ channel :

$$\left(T_{K^{bar}NN} + V_{NN} + \sum_{i=1,2} U_{K^{bar}N_i(I)}^{Eff}(E_{K^{bar}N}) \right) \Phi_{K^{bar}NN} = E \Phi_{K^{bar}NN}$$

- Trial wave function

$$\begin{aligned} |"K^-pp"> &= \sum_a C_a^{(KNN,1)} \left\{ G_a^{(KNN,1)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) + G_a^{(KNN,1)}(-\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) \right\} |S_{NN}=0> \left| [K[NN]_1]_{T=1/2} \right> && \text{Ch. 1: } K^{bar}NN, \quad NN: ^1E \\ &+ \sum_a C_a^{(KNN,2)} \left\{ G_a^{(KNN,2)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) - G_a^{(KNN,2)}(-\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) \right\} |S_{NN}=0> \left| [K[NN]_0]_{T=1/2} \right> && \text{Ch. 2: } K^{bar}NN, \quad NN: ^1O \end{aligned}$$

- Basis function = Correlated Gaussian
...including 3-types Jacobi-coordinates

$$G_a^{(KNN,i)}(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) = N_a^{(KNN,i)} \exp \left[-(\mathbf{x}_1^{(3)}, \mathbf{x}_2^{(3)}) A_a^{(KNN,i)} \begin{pmatrix} \mathbf{x}_1^{(3)} \\ \mathbf{x}_2^{(3)} \end{pmatrix} \right]$$

Chiral SU(3) potential with a Gaussian form

- Weinberg-Tomozawa term of effective chiral Lagrangian
- Gaussian form in r -space
- Semi-rela. / Non-rela.
- Based on Chiral SU(3) theory
→ **Energy dependence**

A non-relativistic potential (NRv2c)

$$V_{ij}^{(I=0,1)}(r) = -\frac{C_{ij}^{(I=0,1)}}{8f_\pi^2} (\omega_i + \omega_j) \sqrt{\frac{1}{m_i m_j}} g_{ij}(r)$$

$$g_{ij}(r) = \frac{1}{\pi^{3/2} d_{ij}^3} \exp\left[-(r/d_{ij})^2\right] : \text{Gaussian form}$$

ω_i : meson energy

Constrained by $K^{\text{bar}}N$ scattering length

$$a_{KN(I=0)} = -1.70 + i0.67 \text{ fm}, \quad a_{KN(I=1)} = 0.37 + i0.60 \text{ fm} \quad \text{A. D. Martin, NPB179, 33(1979)}$$

➡ A resonance pole corresponding to $\Lambda(1405)$ at
 $(M, -\Gamma/2) \sim (1420, -20) \text{ MeV}$ $B_{KN} \sim 15 \text{ MeV}$

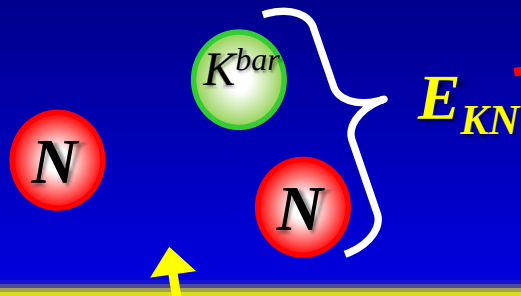
A. D., T. Inoue, T. Myo, Nucl. Phys. A 912, 66 (2013)

➡ **Double-pole structure is confirmed.**

Lower pole $\sim (1395, -138) \text{ MeV}$ at $f_\pi = 110 \text{ MeV}$

A. D., T. Myo, Nucl. Phys. A 930, 86 (2014)

Self-consistency for **complex** $K^{\text{bar}}N$ energy



$$U_{K^{\text{bar}} N_i(I)}^{\text{Eff}} \left(E_{K^{\text{bar}} N} \right)$$

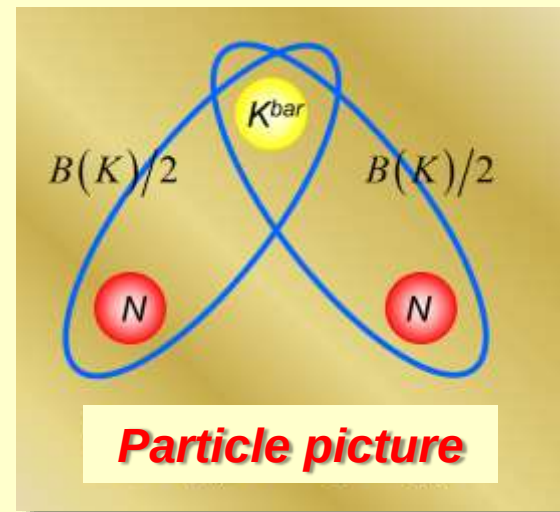
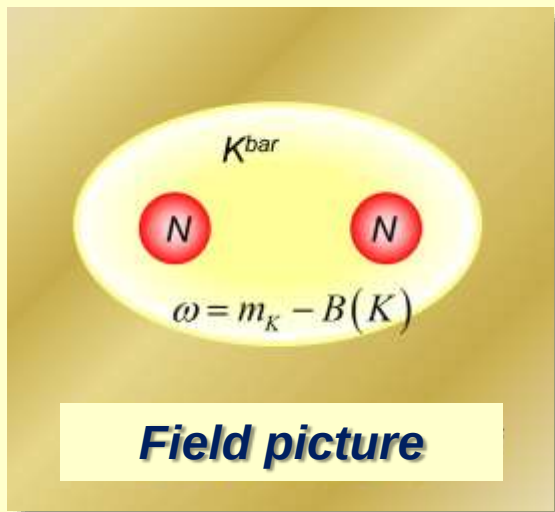
How to determine the two-body energy in the three-body system?

A. D., T. Hyodo, W. Weise,
PRC79, 014003 (2009)

1. Kaon's binding energy: $B(K) \equiv -\left\{ \langle H \rangle - \langle H_{NN} \rangle \right\}$ H_{NN} : Hamiltonian of two nucleons

2. Define a $K^{\text{bar}}N$ -bond energy in two ways

$$E_{KN} = M_N + \omega = \begin{cases} M_N + m_K - B(K) & : \text{Field picture} \\ M_N + m_K - B(K)/2 & : \text{Particle picture} \end{cases}$$

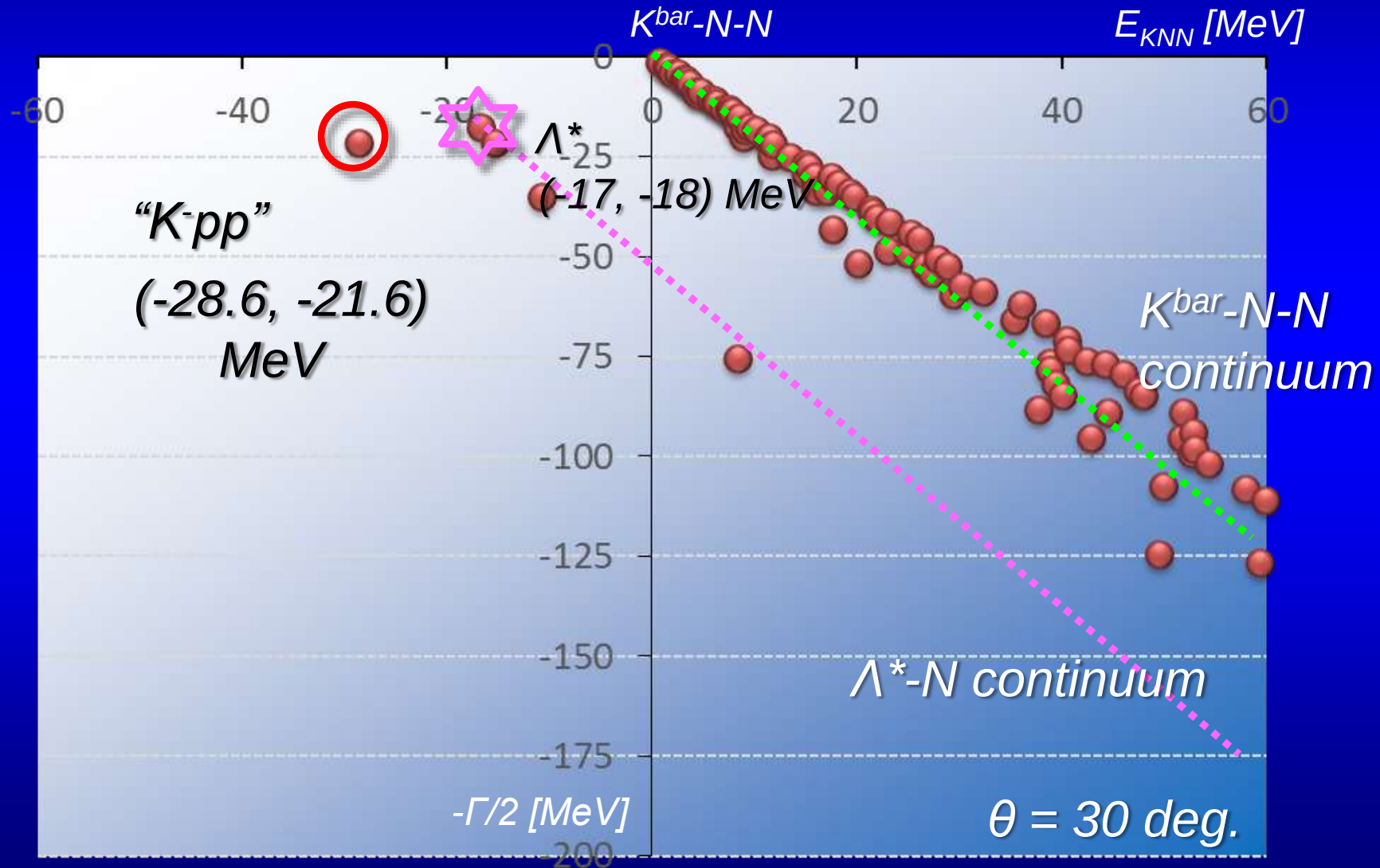


Result

Fix the $K^{\text{bar}}N$ energy at Λ^*

... self-consistent for Λ^* in free space

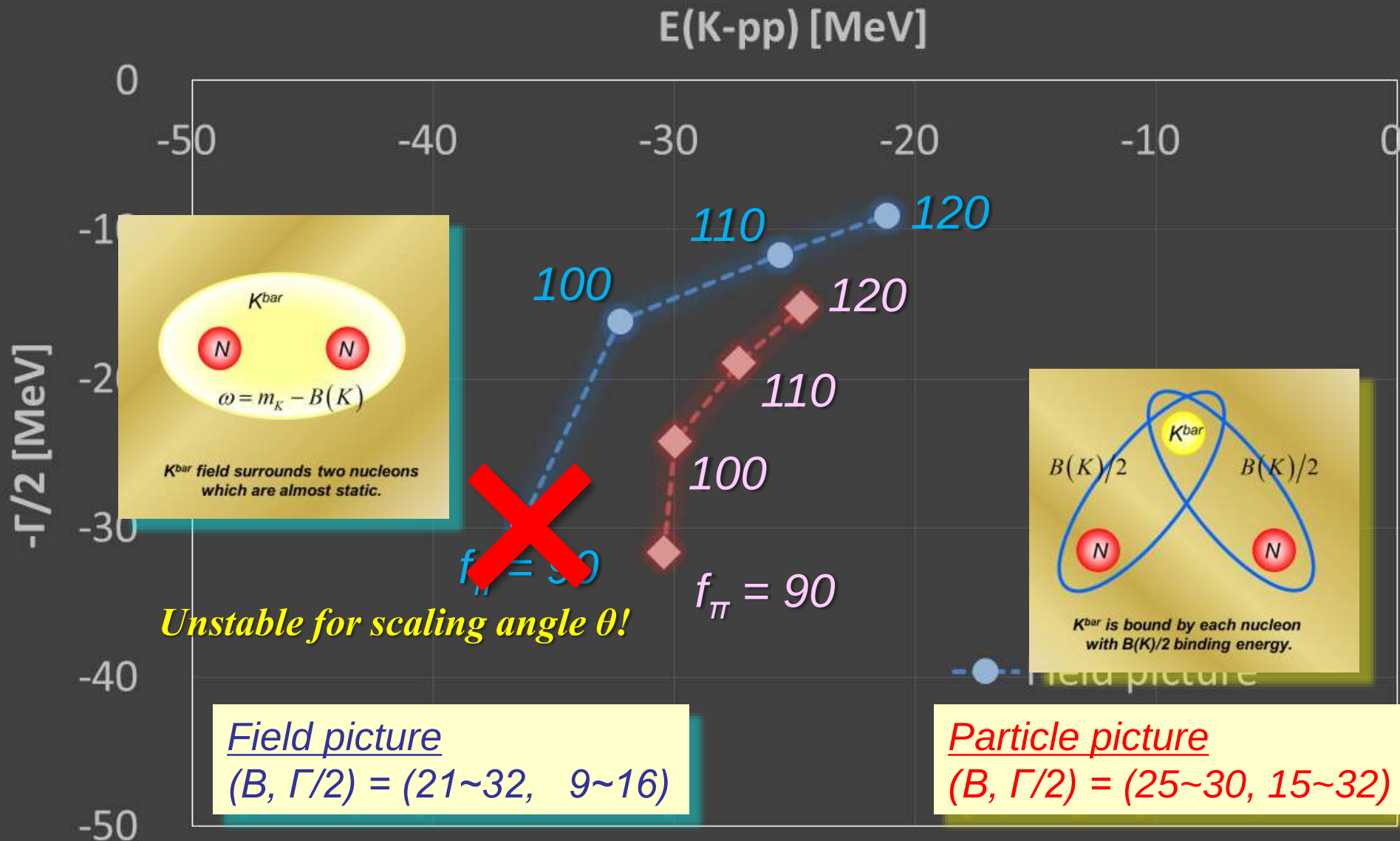
NN pot. : Av18 (Central)
 $K^{\text{bar}}N$ pot. : NRv2c potential
($f_\pi=110\text{MeV}$)



Self-consistent results

$f_\pi = 90 \sim 120 \text{ MeV}$

NN pot. : Av18 (Central)
 K^{bar} N pot. : NRv2c potential
 ($f_\pi = 90 - 120 \text{ MeV}$)

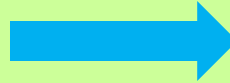


NN correlation density

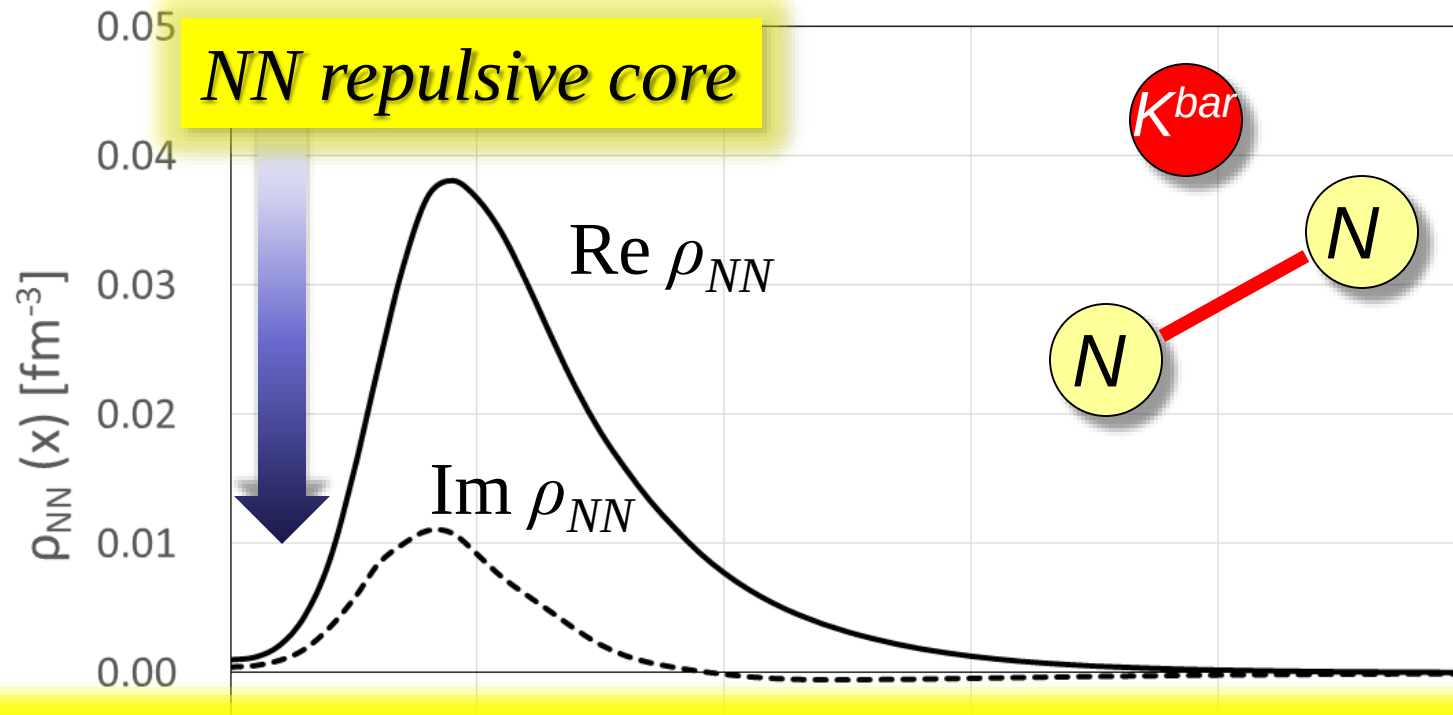
NN pot. : Av18 (Central)
 $K^{bar}N$ pot. : NRv2c potential
 $f_\pi=110$, Particle pict.

Correlation density in Complex Scaling Method

$$\rho_{NN,\theta}(\mathbf{x}) = \delta^3(\hat{\mathbf{r}}_{NN,\theta} - \mathbf{x})$$
$$\hat{\mathbf{r}}_{NN,\theta} = \hat{\mathbf{r}}_{NN} e^{i\theta}$$



$$\rho_{NN}(\mathbf{x}) \equiv \langle \Phi_\theta | \rho_{NN,\theta}(\mathbf{x}) | \Phi_\theta \rangle$$
$$= e^{-3i\theta} \int d^3\mathbf{R} \Phi_\theta^2(\mathbf{x}e^{-i\theta}, \mathbf{R})$$

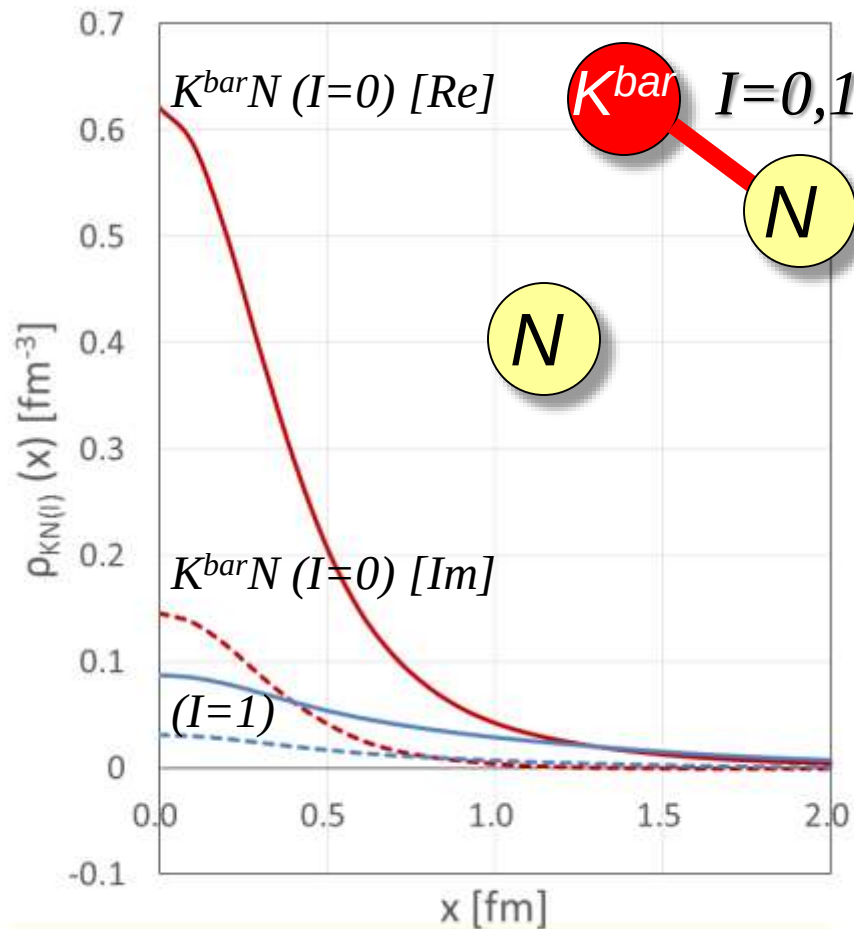


NN distance = $2.1 - i 0.3 \text{ fm}$

$\sim$ Mean distance of 2N in nuclear matter at **normal density!**

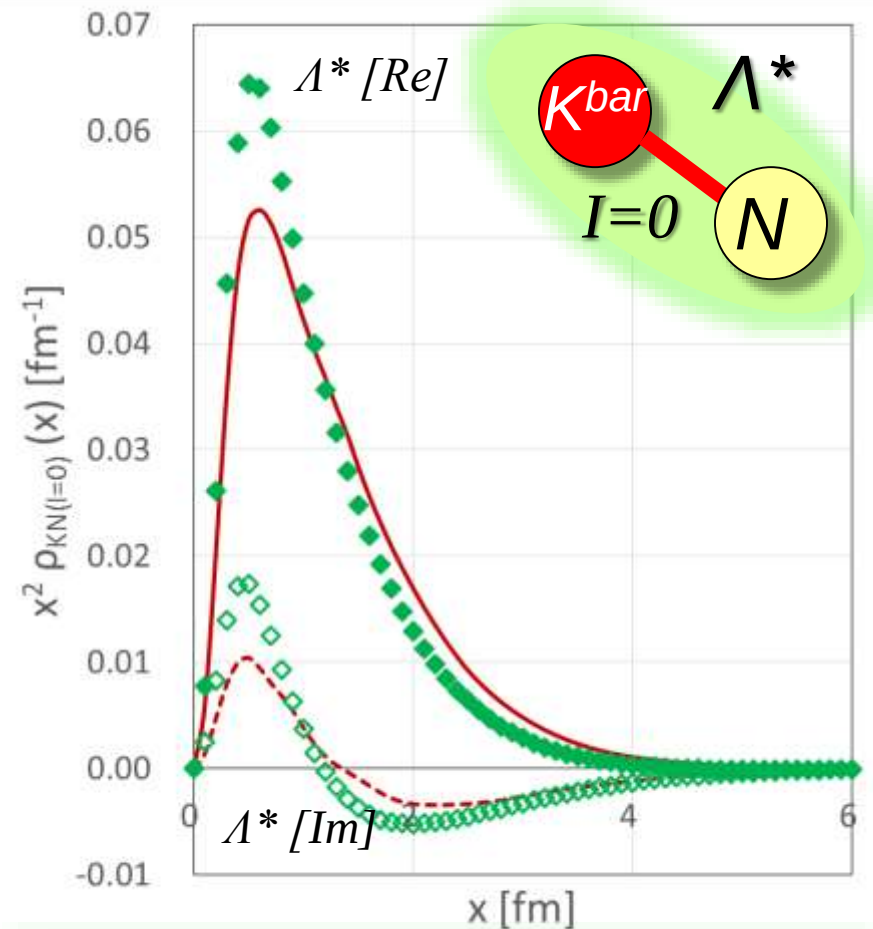
$K^{\text{bar}}N$ correlation density

NN pot. : Av18 (Central)
 $K^{\text{bar}}N$ pot. : NRv2c potential
 $f_{\pi}=110$, Particle pict.



$I=0$ $K^{\text{bar}}N$ compacter than $I=1$ one

← Strong $K^{\text{bar}}N$ attraction in $I=0$



$I=0$ $K^{\text{bar}}N$ seems similar to Λ^*

→ Λ^* survives in K -pp

4. Other quantum-number case

$$\underline{J^\pi = 1^- \text{ state} \cdots S_{\underline{NN}} = 1}$$

J-PARC E27 experiment

K - pp search by d (π^+ , K^+) reaction at 1.69 GeV/c

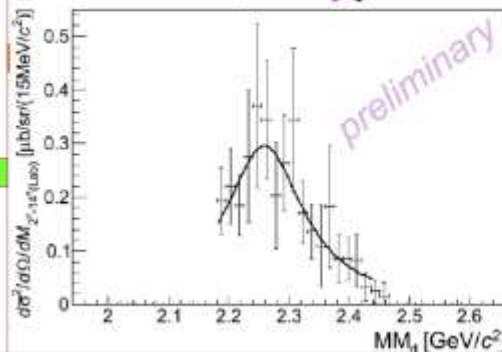
Inclusive spectrum: Ichikawa et al., PTEP 101D03 (2014)

2p-coincidence analysis

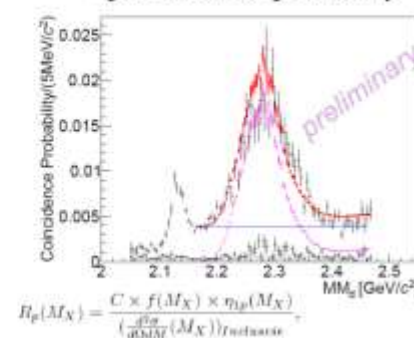
“ K^-pp ”-like structure

- The broad structure around 2.26 GeV/c² have been observed in the $\Sigma^0 p$ final state events.
- We fitted with a Lorentzian function smeared with the resolution.
 - $M_0 \sim 2260$ MeV/c² (B.E. ~ 110 MeV)
 - This distribution can reproduce the 1p coincidence probability spectrum (pink). Blue and red lines show the flat component and summed one.

Differential cross section of
 $\pi^+ d \rightarrow K^+ X$, $X \rightarrow \Sigma^0 p$ process



1p coincidence probability



Ichikawa's talk
at EXA2104

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If the observed state is really the K - pp and **if spin-flip is not so strong in the reaction**, spin of K - pp formed from deuteron should be 1 because **the deuteron has the spin 1**.
(Prof. T. Harada, Osaka E.C. university)

How is $J^\pi = 1^-$ state ... $S_{NN}=1$?

$$| "K^- pp" \rangle \approx | L_{KNN} = 0, NN : s - wave \rangle | S_{NN} = 0 \rangle \left| [K[NN]_1]_{T=1/2} \right\rangle \quad J^\pi=0^-, T=1/2$$

$$| "K^- d" \rangle \approx | L_{KNN} = 0, NN : s - wave \rangle | \mathbf{S}_{NN} = \mathbf{1} \rangle \left| [K[NN]_0]_{T=1/2} \right\rangle \quad J^\pi=\mathbf{1}^-, T=1/2$$

“ K^- + Deuteron”-like channel

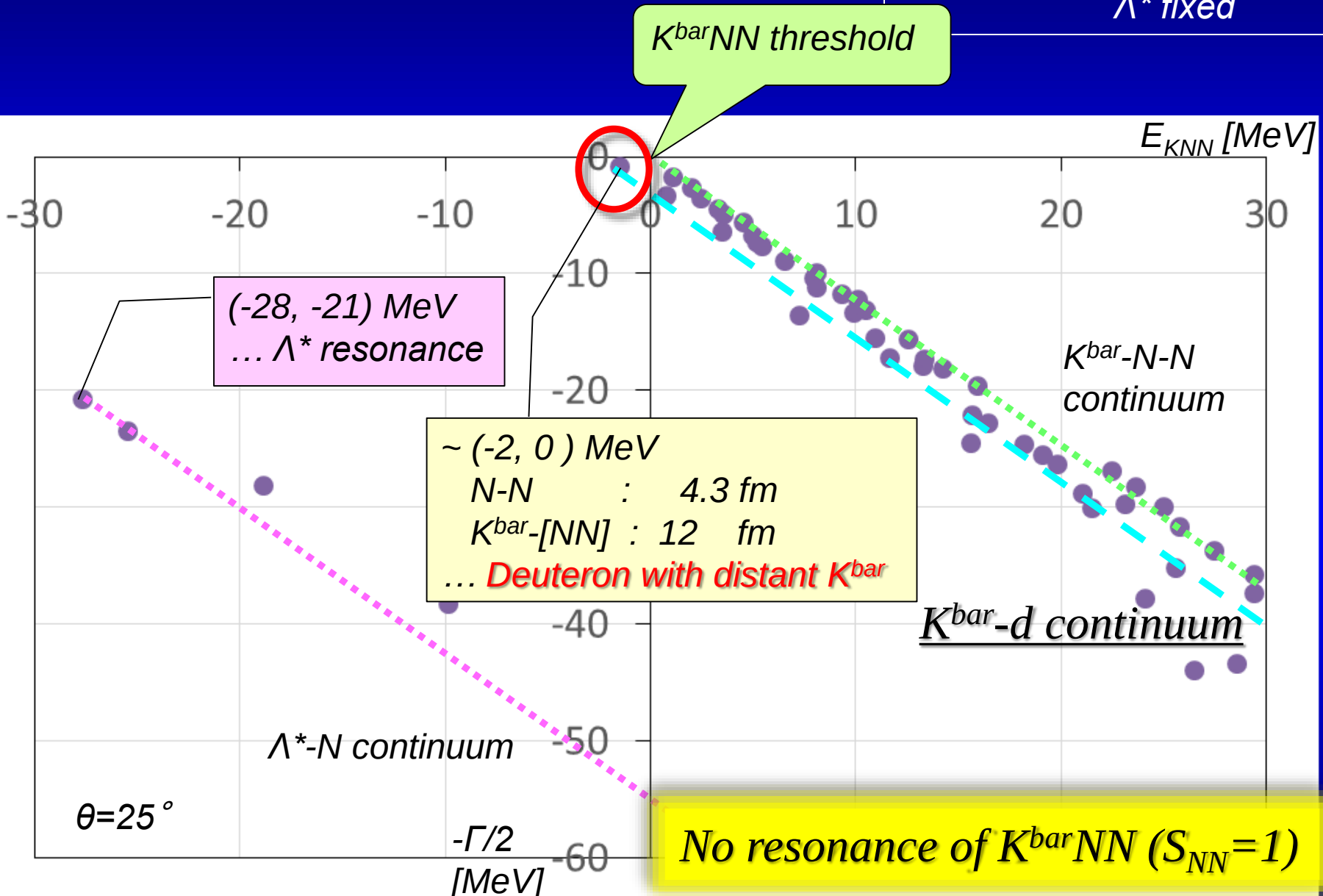
“ $K^- d$ ” studied **simply** with ...

- NN potential: $Av4'$ (3E , 3O) (fitted with 5-range Gaussian functions)
... **Tensor force is incorporated into central potential.**
- $K^{\text{bar}}N$ potential: A phenomenological potential
... **Energy independent**
- Λ^* -fixed ansatz
... $E(K^{\text{bar}}N)$ in effective $K^{\text{bar}}N$ potential is fixed to the Λ^* energy.

Y. Akaishi and T. Yamazaki,
PRC 52 (2002) 044005

How is $J^\pi = 1^-$ state ... $S_{NN}=1$?

NN pot. : Av4' (5 Gauss)
 K^{bar} N pot. : AY potential
 Λ^* fixed



No resonance of K^{bar} NN ($S_{NN}=1$)

***5. Summary
and future plans***

5. Summary and future plans

A prototype of K^{bar} nuclei “ K -pp” = Resonance state of K^{bar} NN- π YN coupled system

“coupled-channel Complex Scaling Method + Feshbach projection”

... Represent the *Q-space Green function* with the *Extended Complete Set* well approximated by *Gaussian base*

⇒ Eliminate π Y channels to reduce the problem to a K^{bar} NN single channel problem.

K -pp studied with ccCSM+Feshbch method

- Used a Chiral SU(3)-based potential (Gaussian form in r -space)
- Self-consistency for kaon's *complex* energy
- Correlation density in CSM shows effect of NN repulsive core and Λ^* survival in K -pp resonance.
- $J^\pi=1^-$ state (“Deuteron+ K^- ”-like channel) seems not to exist as a resonance state.

NRv2c potential case

$(B, \Gamma/2) = (21\sim 31, 9\sim 16)$ MeV : “Field picture”
(25~30, 15~32) MeV : “Particle pict.”

Mean NN distance ~ 2.2 fm → Normal density

Future plans

- Full-coupled channel calculation of K -pp
- Consider with the K^{bar} N potential constrained by the latest data of SIDDHARTA
- Application to resonances of other hadronic systems

Kaonic nuclei

Thank you very much!

ニュートリノビーム

P

K⁻

P

直径 500m

50GeV

シンクロトロン

Prototype system = $K^- pp$
Reference)

A. D., T. Inoue, T. Myo,

arXiv: 1411.0348 [nucl-th]