Quark Structure of the Pion

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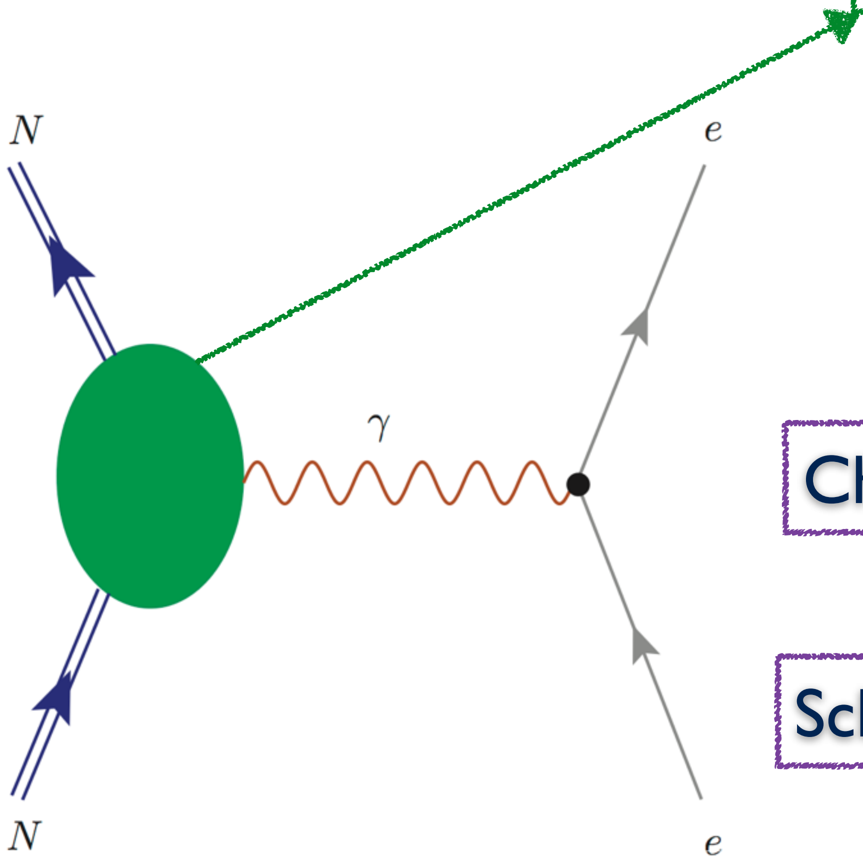
Progress of J-PARC Hadron Physics, Nov. 30-Dec. 01, 2014

Interpretation of the Form factors

Non-Relativistic picture of the EM form factors



Electromagnetic form factors

3-D Fourier Transform

Charge & Magnetisation Densities

Schroedinger Eq. & Wave functions

Interpretation of the EMFFs

Traditional interpretation of the nucleon form factors

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \boxed{\rho(\mathbf{r})} \longrightarrow \qquad \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

However, the initial and final momenta are different in a relativistic case. Thus, the initial and final wave functions are different.

$\Rightarrow$ **Probability interpretation is wrong in a relativistic case!**

$\Downarrow$

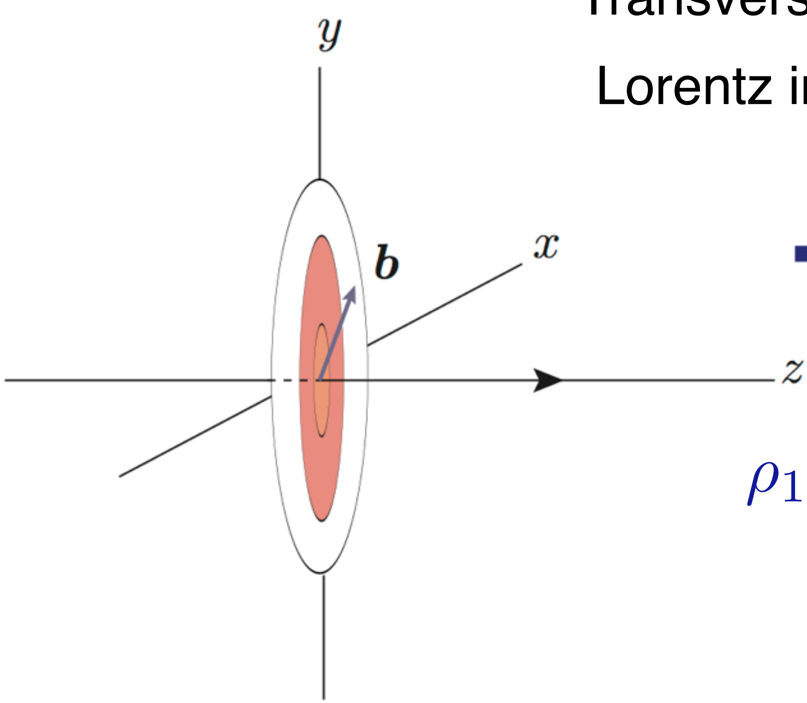
We need a correct interpretation of the form factors

Belitsky & Radyushkin, Phys.Rept. **418**, 1 (2005)

G.A. Miller, PRL **99,** 112001 (2007)

Interpretation of the EMFFs

Modern understanding of the form factors



Transverse Charge densities $\rho(\mathbf{b})$

Lorentz invariant: independent of any observer.

$\xrightarrow{p}$ Infinite momentum framework

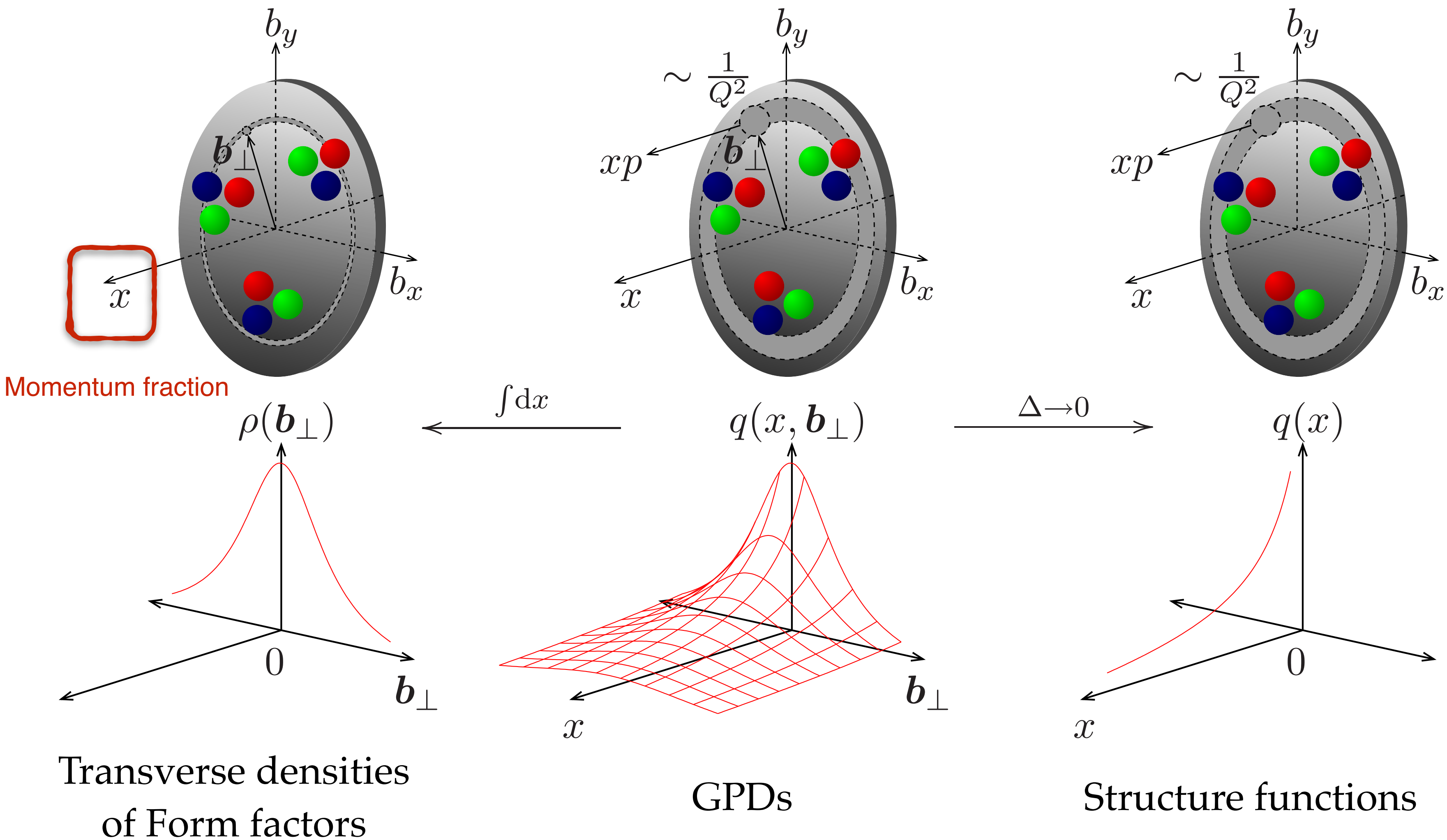
$$\rho_1(b) = \sum_q e_q^2 \int dx \boxed{f_{q-\bar{q}}(x,\, \mathbf{b})}$$

GPDs

Form factors

$$F(q^2) = \int d^2 b e^{i\mathbf{q}\cdot\mathbf{b}} \rho(\mathbf{b})$$

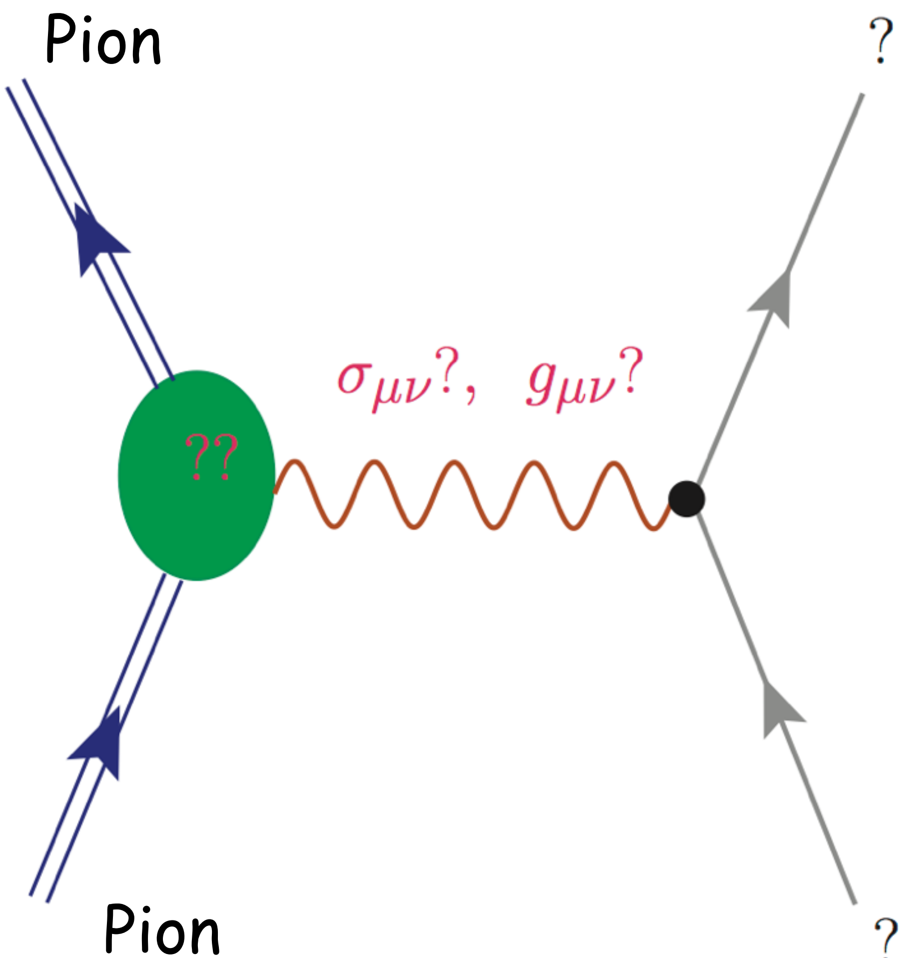
Hadron Tomography



D. Brömmel, Dissertation (Regensburg U.)

Generalised Parton Distributions

Probes are unknown for **Tensor form factors** and the **Energy-Momentum Tensor form factors!**



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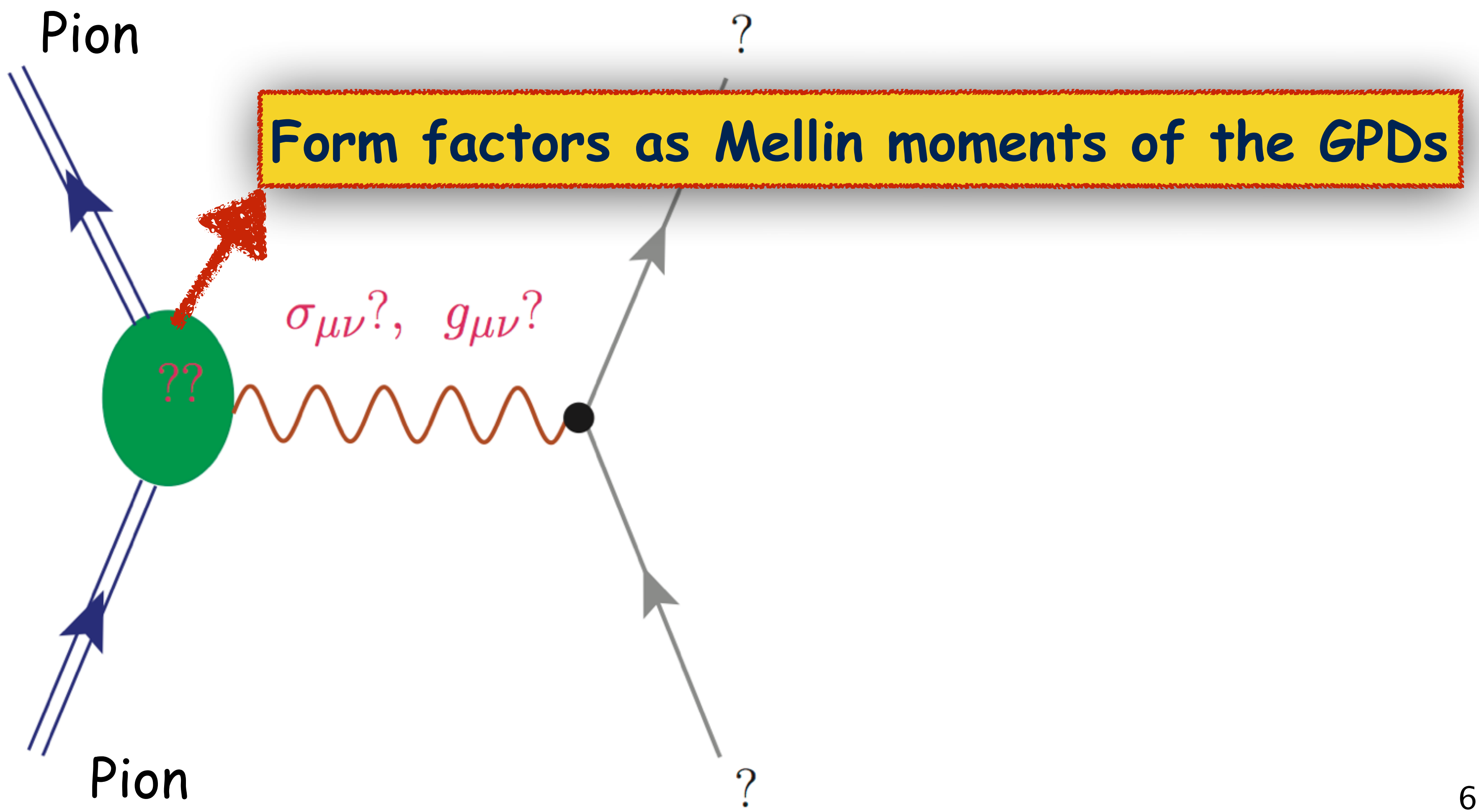


What we know about the Pion

Experimentally, we know about the pion

- Pion Mass = 139.57 MeV
- Pion Spin = 0

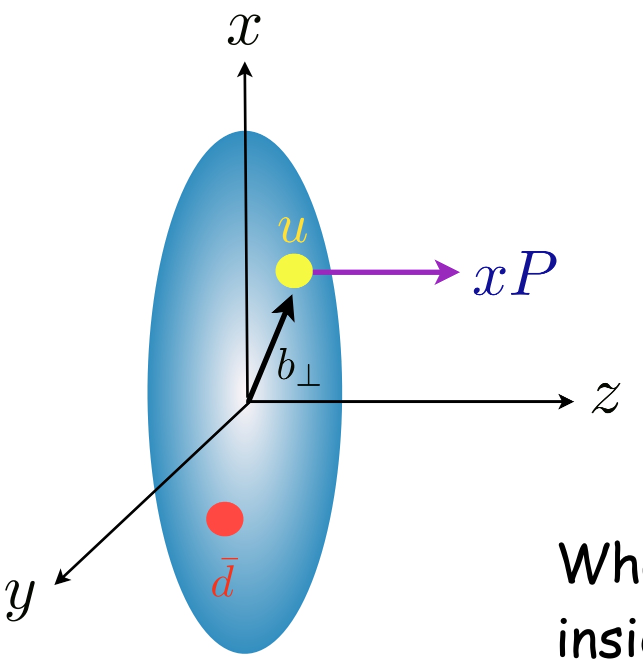
Theoretically

- pseudo-Goldstone boson
- The lowest-lying meson
  (1 q + 1anti-q + sea quarks + gluons + …)

The structure of the pion is still not trivial!

The spin structure of the Pion

Vector & Tensor Form factors of the pion



Pion: Spin S=0

Longitudinal spin structure is trivial.

$$\langle \pi(p')|\bar{\psi}\gamma_3\gamma^5\psi|\pi(p)\rangle \; = \; 0$$

What about the transversely polarized quarks inside a pion?

→ Internal spin structure of the pion

The spin distribution of the quark

$$\rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2}\right]$$

Spin probability densities in the transverse plane

A_{n0}: Vector densities of the pion, B_{n0}: Tensor densities of the pion

$$\int_{-1}^{1} dx\, x^{n-1} H(x, \xi = 0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx\, x^{n-1} E(x, \xi = 0, b_\perp^2) = B_{n0}(b_\perp^2)$$

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Vector and Tensor form factors of the pion

$$\langle \pi(p_f) | \psi^\dagger \gamma_\mu \hat{Q} \psi | \pi(p_i) \rangle = (p_i + p_f) A_{10}(q^2)$$

$$\langle \pi^+(p_f) | \mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} | \pi^+(p_i) \rangle = \mathcal{AS}\left[\frac{(p^\mu q^\nu - q^\mu p^\nu)}{m_\pi} \sum_{i=\text{even}}^{n-1} q^{\mu_1} \cdots q^{\mu_i} p^{\mu_{i+1}} \cdots p^{\mu_{n-1}} B_{ni}(Q^2)\right]$$

Nonlocal chiral quark model

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Gauged Effective Nonlocal Chiral Action

$$S_{\text{eff}} = -N_c \text{Tr} \ln \left[i\not{D} + im + i\sqrt{M(iD,m)}U^{\gamma_5}\sqrt{M(iD,m)} \right]$$

$$D_\mu = \partial_\mu - i\gamma_\mu V_\mu$$

The nonlocal chiral quark model from the instanton vacuum

- Fully relativistically field theoretic model.
- "Derived" from QCD via the Instanton vacuum.
- Renormalization scale is naturally given.
- No free parameter

$\rho \approx 0.3\,\text{fm},\ R \approx 1\,\text{fm}$

Dilute instanton liquid ensemble

$\mu \approx 600\,\text{MeV}$

D. Diakonov & V. Petrov Nucl.Phys. B272 (1986) 457

H.-Ch.K, M. Musakhanov, M. Siddikov Phys. Lett. B **608**, 95 (2005).

Musakhanov & H.-Ch. K, Phys. Lett. B **572**, 181-188 (2003)

Nonlocal chiral quark model

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$$S_{\text{eff}} = -N_c \mathrm{Tr}\ln\left[i\not{D} + im + i\sqrt{M(iD,m)}U^{\gamma_5}\sqrt{M(iD,m)}\right]$$

$$U^{\gamma_5} = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} = 1 + \frac{i}{f_\pi}\gamma_5(\pi^a\lambda^a) - \frac{1}{f_\pi^2}(\pi^a\lambda^a)^2 + \cdots$$

$$M(iD,m) = M_0F^2(iD)f(m) = M_0F^2(iD)\left[\sqrt{1+\frac{m^2}{d^2}} - \frac{m}{d}\right] \qquad d \approx 0.198\,\mathrm{GeV}$$

$$F(k) = 2t\left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t)\right]_{t=\frac{k\bar{\rho}}{2}}$$

$M_0 \approx 0.35\,\mathrm{GeV}$: It is determined by the gap equation.

D. Diakonov & V. Petrov Nucl.Phys. B272 (1986) 457

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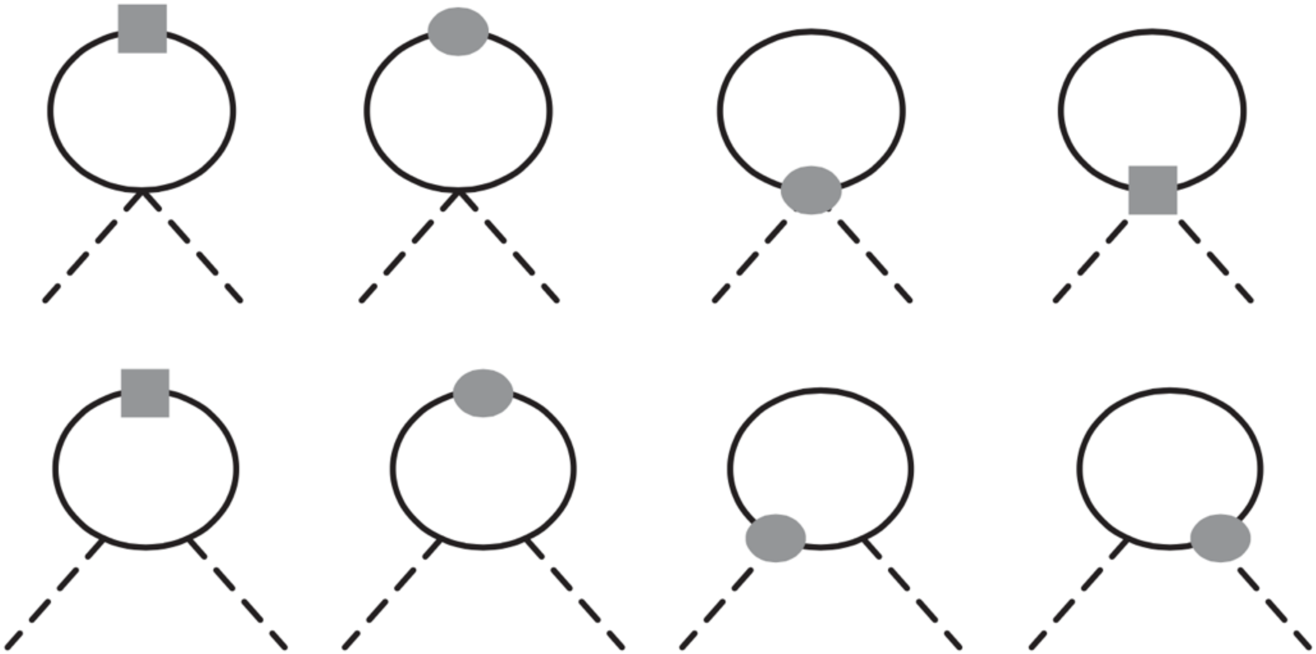
EM Form factor of the pion

EM form factor (A_{10})

$$\langle\pi(p_f)|\psi^\dagger\gamma_\mu\hat{Q}\psi|\pi(p_i)\rangle = (p_i + p_f)A_{10}(q^2)$$



S.i. Nam & HChK, Phys. Rev. D77 (2008) 094014

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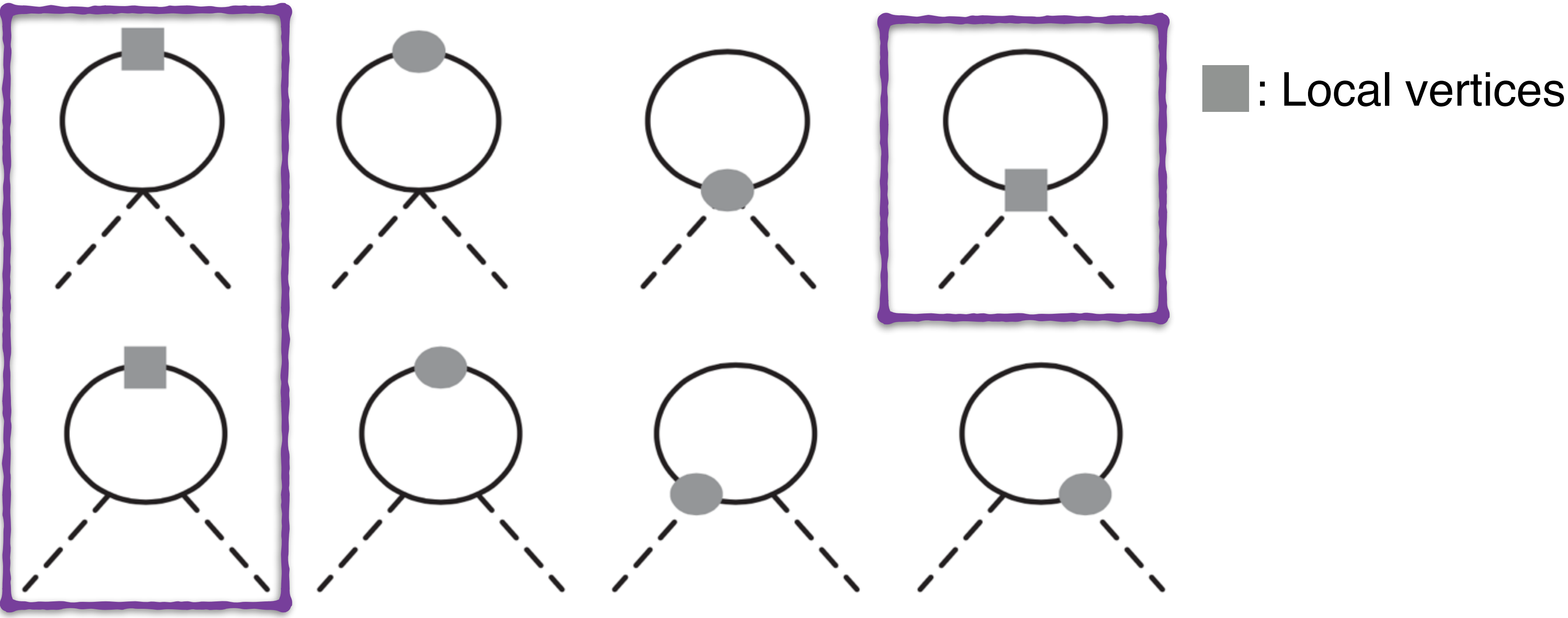

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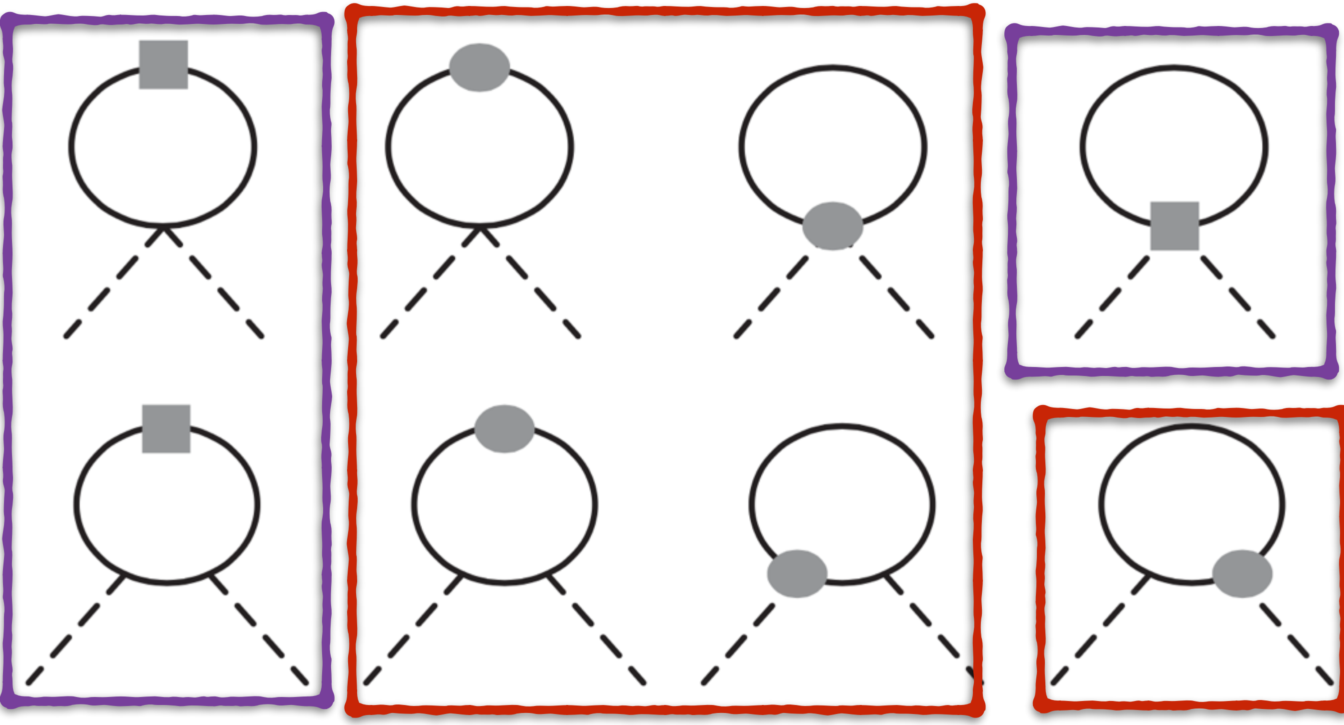
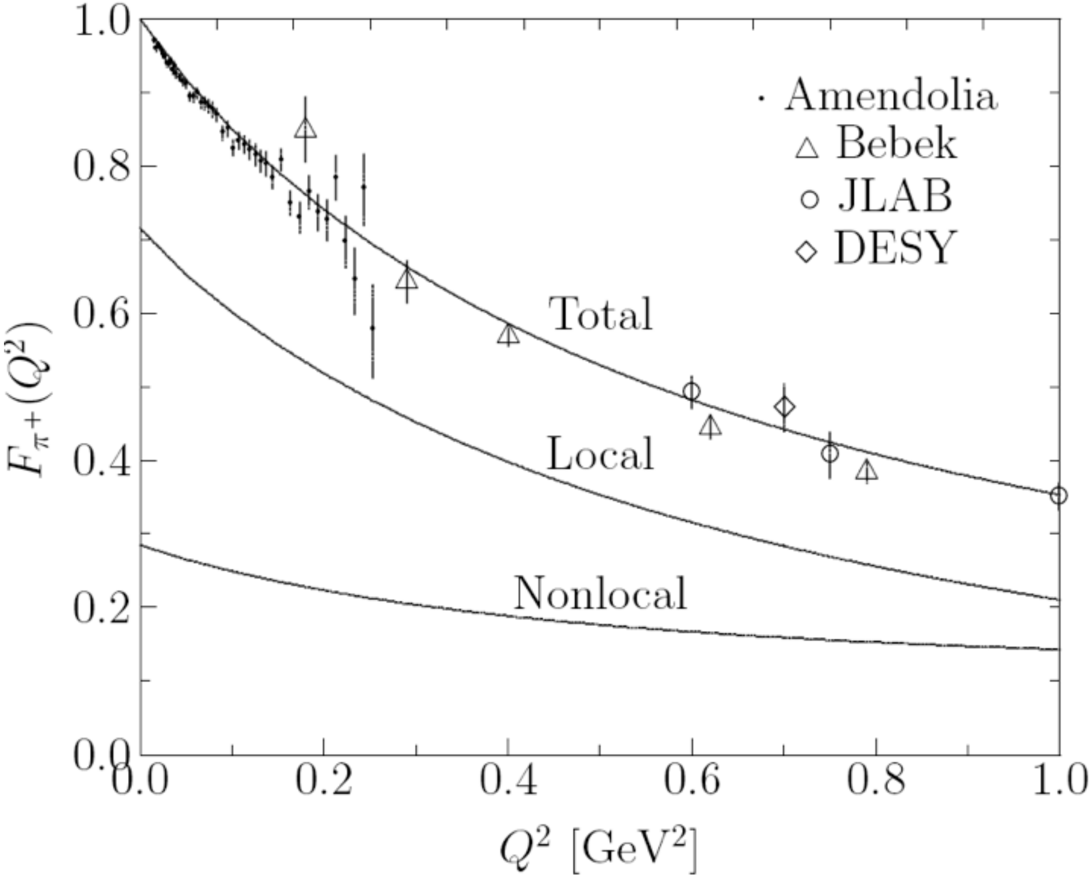


: Local vertices

: Nonlocal vertices

S.i. Nam & HChK, Phys. Rev. D77 (2008) 094014

EM Form factor of the pion



$$\sqrt{\langle r^2 \rangle} = 0.675\,\mathrm{fm}$$

$$\sqrt{\langle r^2 \rangle} = 0.672 \pm 0.008\,\mathrm{fm}\ (\mathrm{Exp})$$

$$F_\pi(Q^2) = A_{10}(Q^2) = \frac{1}{1 + Q^2/M^2}$$

M(Phen.): 0.714 GeV

M(Lattice): 0.727 GeV

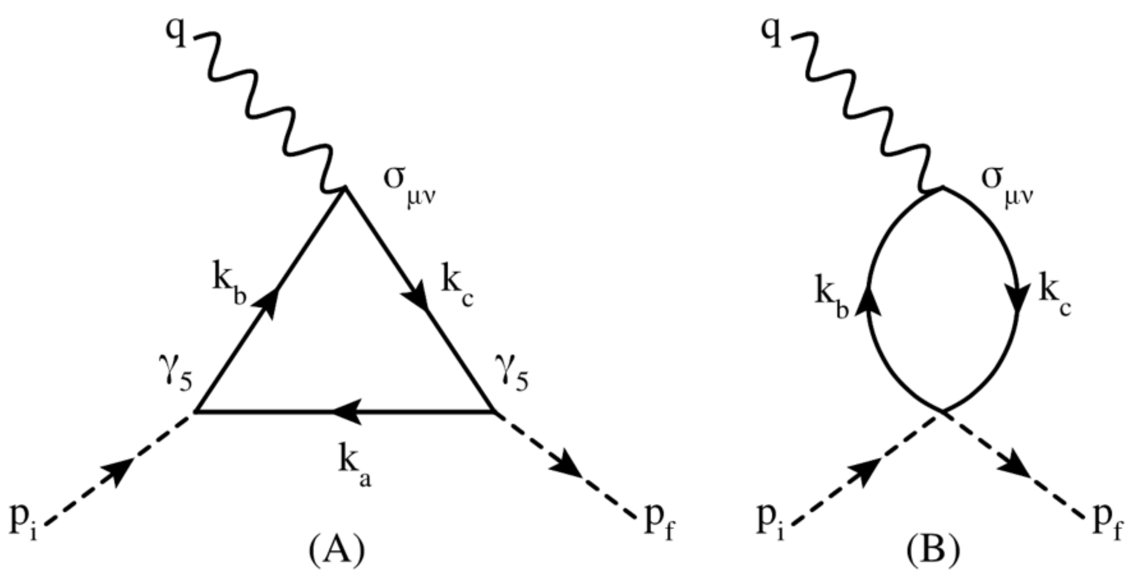
M(XQM): 0.738 GeV

S.i. Nam & HChK, Phys. Rev. D77 (2008) 094014

Tensor Form factor of the pion

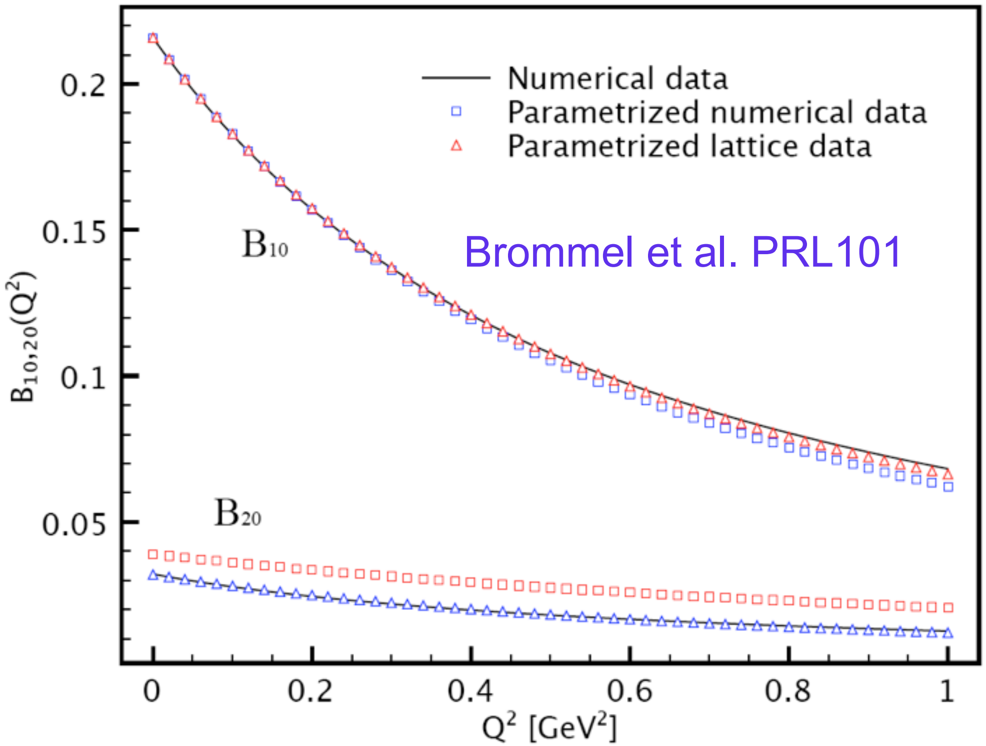
$$\left\langle \pi^+(p_f) \left| \psi^\dagger(0)\sigma_{\mu\nu}\psi(0) \right| \pi^+(p_i) \right\rangle = \frac{p^\mu q^\nu - q^\mu p^\nu}{m_\pi} B_{10}(Q^2)$$

$$p = (p_f + p_i)/2, \; q = p_f - p_i$$



S.i. Nam & H.-Ch.K, Phys. Lett. B **700**, 305 (2011).

Tensor Form factor of the pion



RG equation for the tensor form factor

$$B_{10}(Q^2,\mu) = B_{10}(Q^2,\mu_0)\left[\frac{\alpha(\mu)}{\alpha(\mu_0)}\right]^{\frac{4}{33-2N_f}}$$

p-pole parametrization for the form factor

$$B_{10}(Q^2) \; = \; B_{10}(0)\left[1+\frac{Q^2}{pm_p^2}\right]^{-p}$$

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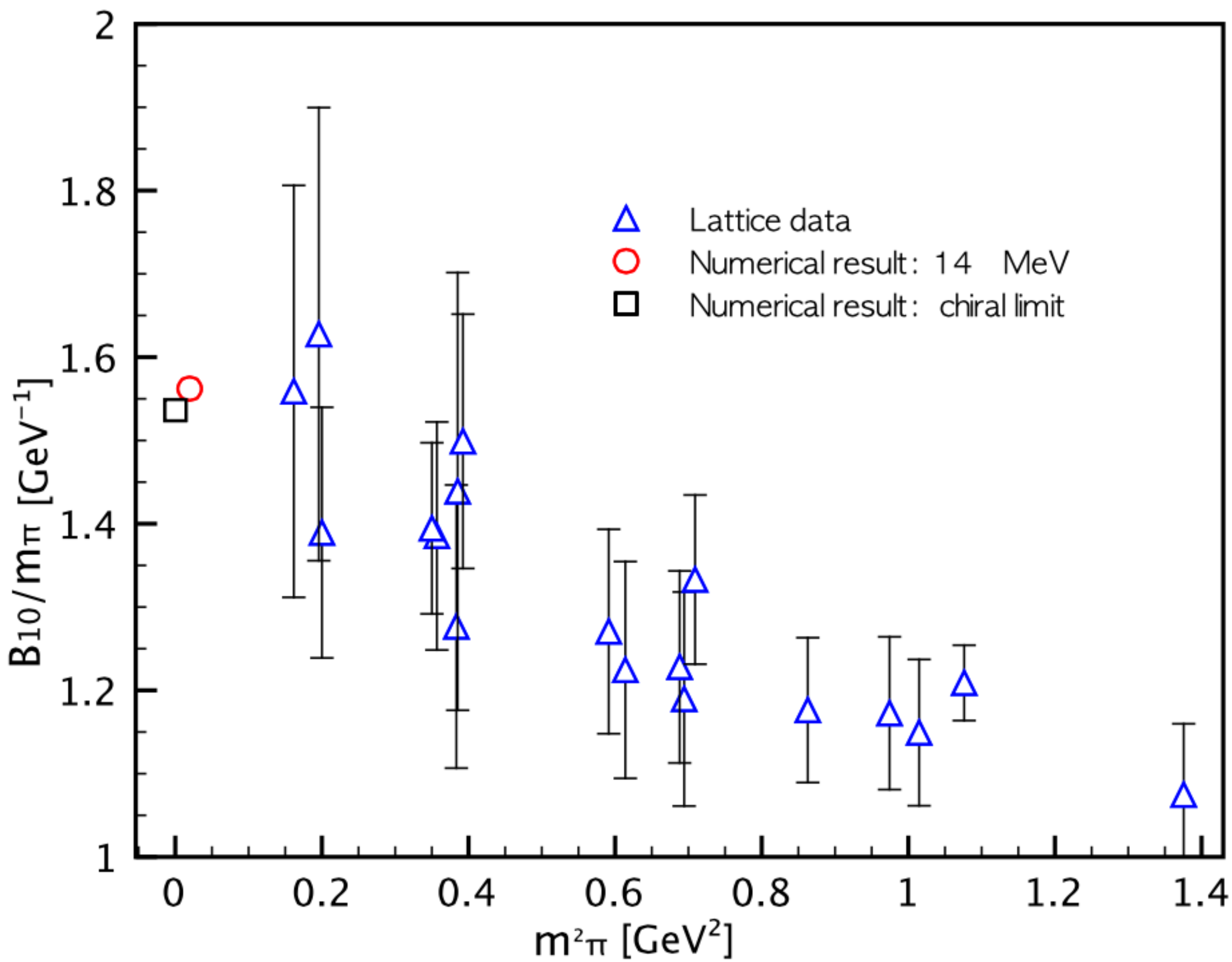

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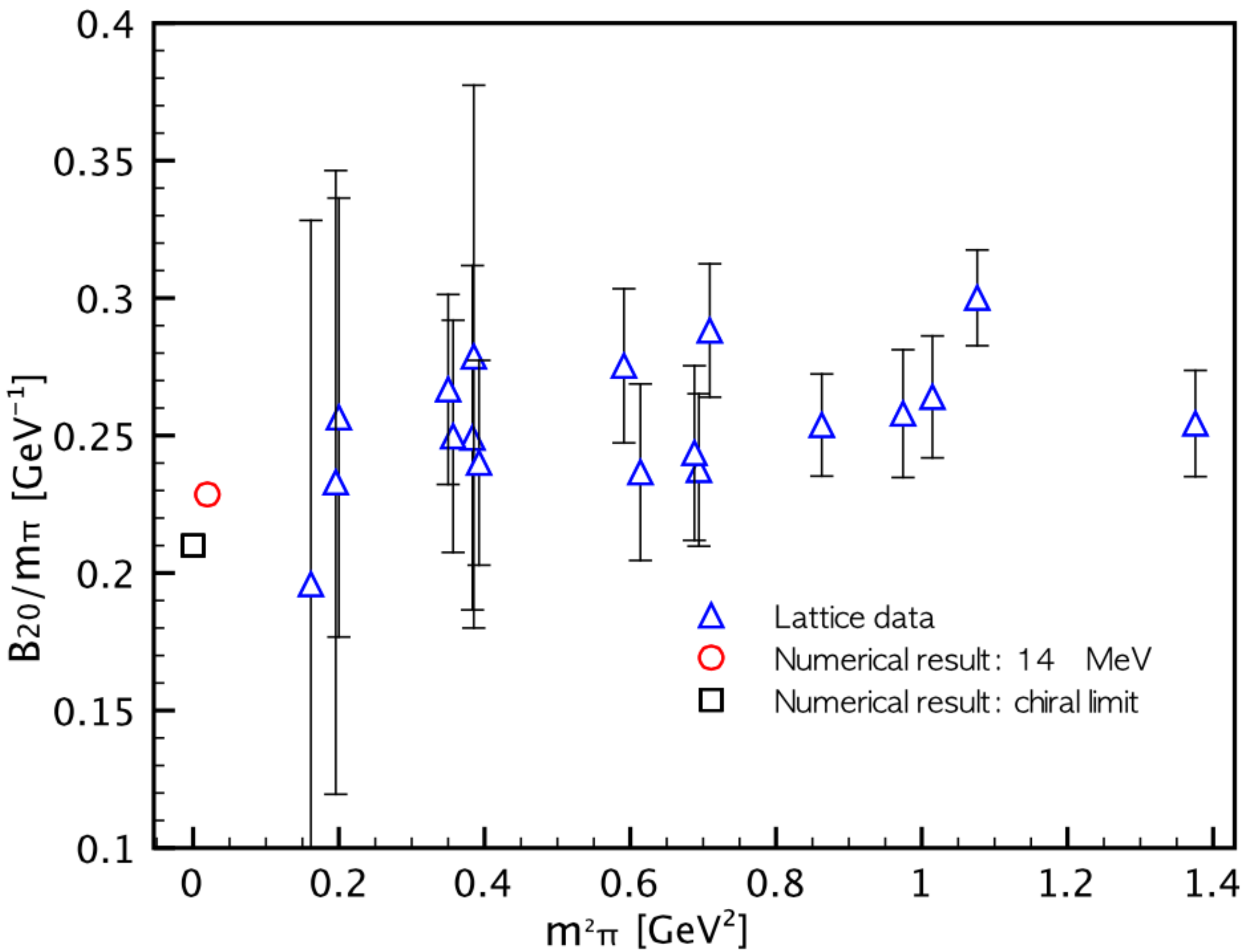

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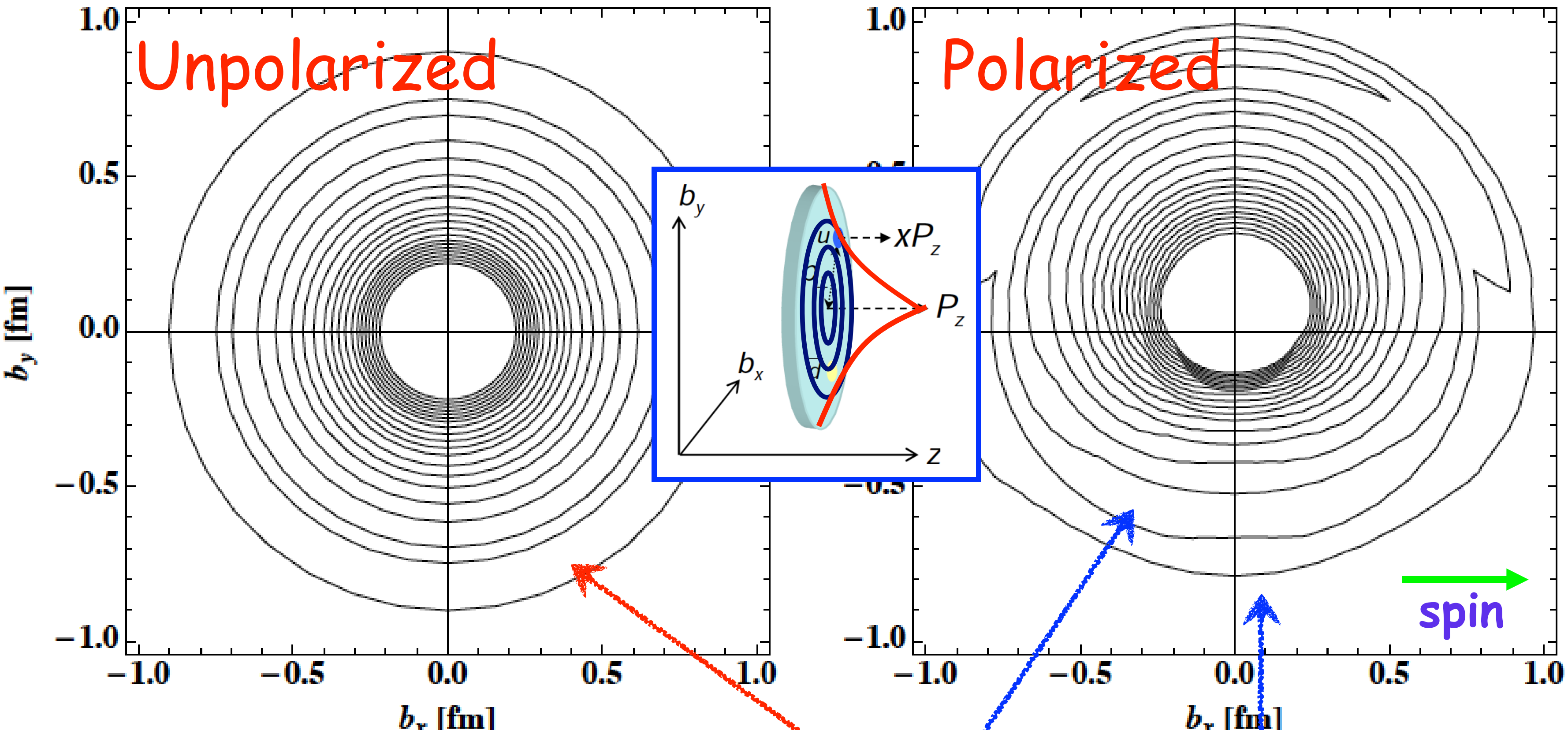
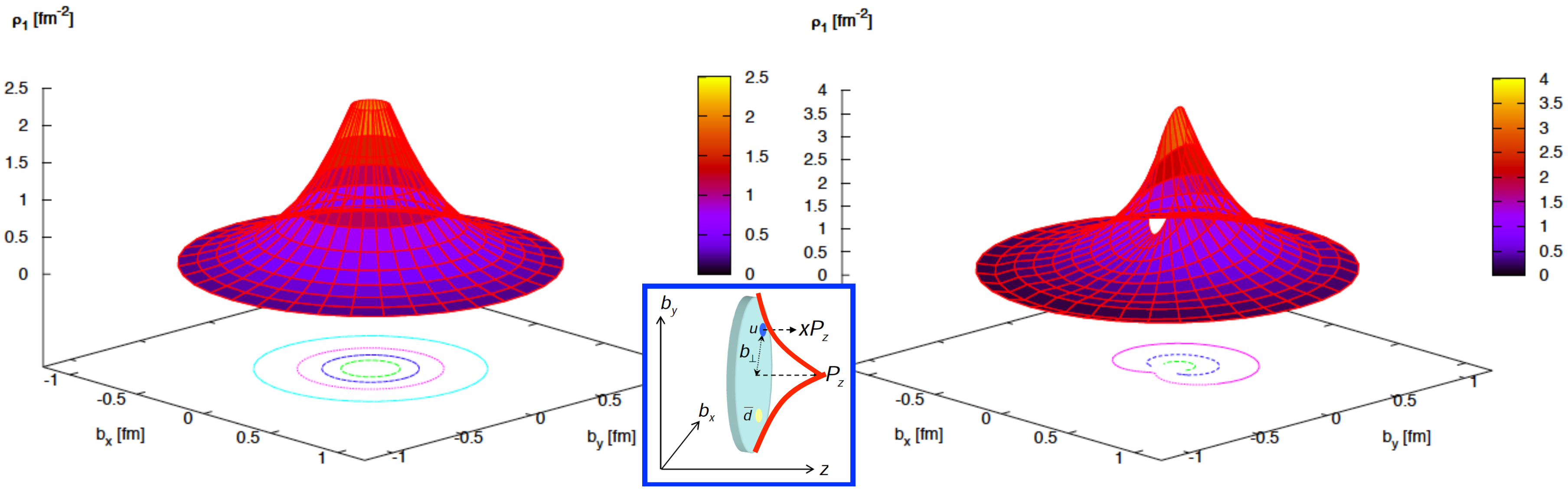
Spin density of the quark



$$\rho_1\left(b_\perp, s_x = \pm\frac{1}{2}\right) = \frac{1}{2}\left[A_{10}(b^2) \mp \frac{b\sin\theta}{m_\pi} B'_{10}(b^2)\right]$$

Polarization

Spin density of the quark

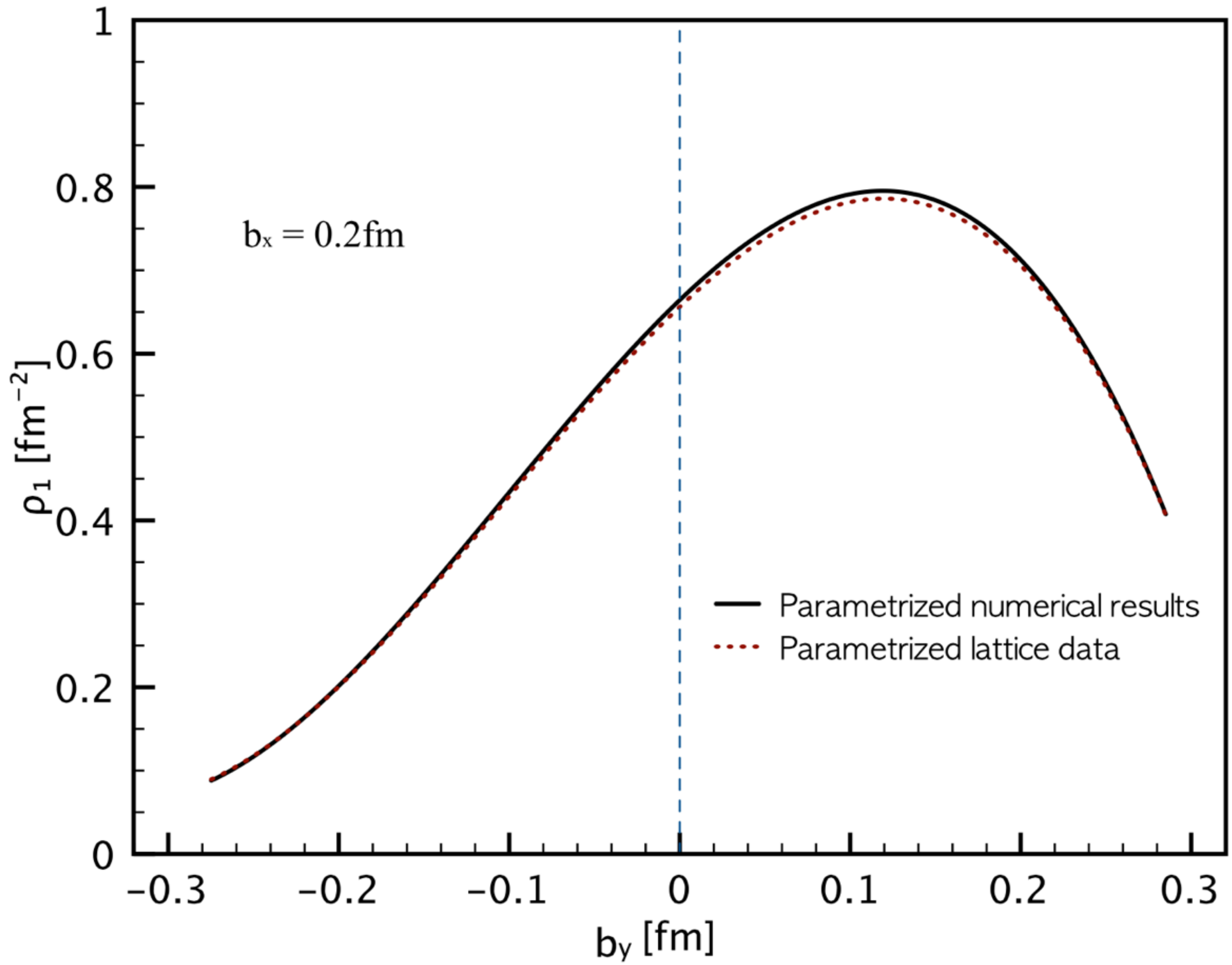


Significant distortion appears for the polarized quark!

$m_\pi = 140$ MeV	$B_{10}(0)$	m_{p_1} [GeV]	$\langle b_y \rangle$ [fm]	$B_{20}(0)$	m_{p_2} [GeV]
Present work	0.216	0.762	0.152	0.032	0.864
Lattice QCD [7]	0.216 ± 0.034	0.756 ± 0.095	0.151	0.039 ± 0.099	1.130 ± 0.265

Results are in a good agreement with the lattice calculation!

Spin density of the quark



Stability of the pion

Isoscalar vector GPDs of the pion

$$2\delta^{ab}H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda(P\cdot n)}\langle\pi^a(p')|\bar{\psi}(-\lambda n/2)\not{n}[-\lambda n/2,\lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

The second moment of the GPD

$$\int dx\, x H_\pi^{I=0}(x,\xi,t) = A_{20}(t) + 4\xi^2 A_{22}(t)$$

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: Generalized form factors of the pion

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Energy-momentum Tensor Form factors (Pagels, 1966)

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_n u)\Theta_1(t) + 2P_\mu P_\nu \Theta_2(t)\right]$$

$$T_{\mu\nu}(x) = \frac{1}{2}\bar{\psi}(x)\gamma_{\{}i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$: QCD EMT operator

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EMTFFs (Gravitational FFs)

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$$\boxed{\Theta_1 = -4A_{22}^{I=0},\ \Theta_2 = A_{20}^{I=0}}$$

Stability of the pion

Time component of the EMT matrix element gives the pion mass.

$$\langle \pi^a(p)|T_{44}(0)|\pi^b(p)\rangle\Big|_{t=0} = -2m_\pi^2\Theta_2(0)\delta^{ab}$$

The sum of the spatial component of the EMT matrix element gives the pressure of the pion, which should vanish!

$$\langle \pi^a(p)|T_{ii}(0)|\pi^b(p)\rangle\Big|_{t=0} = \frac{3}{2}\delta^{ab}t\,\Theta_1(t)\Big|_{t=0}$$ Zero in the chiral limit

$$\mathcal{P} = \langle \pi^a(p)|T_{ii}(0)|\pi^a(p)\rangle$$
$$= \boxed{\frac{12N_c mM}{f_\pi^2}\int d\tilde{l}\,\frac{-l^2}{[l^2+\overline{M}^2]^2}} + \boxed{\frac{12N_c M^2}{f_\pi^2}\int d\tilde{l}\int_0^1 dx\,\frac{-p^2l^2}{[l^2+x(1-x)p^2+\overline{M}^2]^3}}$$

(Based on the local model)

H.D. Son & H.-Ch.K, to be published in PRD Rapid Comm.

Stability of the pion

Pressure of the pion beyond the chiral limit

$$\mathcal{P} = \frac{12N_c m M}{f_\pi^2}\int d\tilde{l}\,\frac{-l^2}{[l^2+\overline{M}^2]^2} + \frac{12N_c M^2}{f_\pi^2}\int d\tilde{l}\int_0^1 dx\,\frac{-p^2 l^2}{[l^2+x(1-x)p^2+\overline{M}^2]^3}$$

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$$i\langle\psi^\dagger\psi\rangle = 8N_c\int d\tilde{l}\,\frac{\overline{M}}{[l^2+\overline{M}^2]}$$

Quark condensate

H.D. Son & H.-Ch.K, to be published in PRD Rapid Comm.

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Pressure of the pion beyond the chiral limit

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$$i\langle\psi^\dagger\psi\rangle = 8N_c\int d\tilde{l}\,\frac{\overline{M}}{[l^2+\overline{M}^2]}$$

Quark condensate

$$f_\pi^2 = 4N_c\int_0^1 dx\int d\tilde{l}\,\frac{M\overline{M}}{[l^2+\overline{M}^2+x(1-x)p^2]^2}$$

Pion decay constant

H.D. Son & H.-Ch.K, to be published in PRD Rapid Comm.

Stability of the pion

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Pressure of the pion beyond the chiral limit

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Quark condensate

$$f_\pi^2 = 4N_c\int_0^1 dx\int d\tilde{l}\,\frac{M\overline{M}}{[l^2+\overline{M}^2+x(1-x)p^2]^2}$$

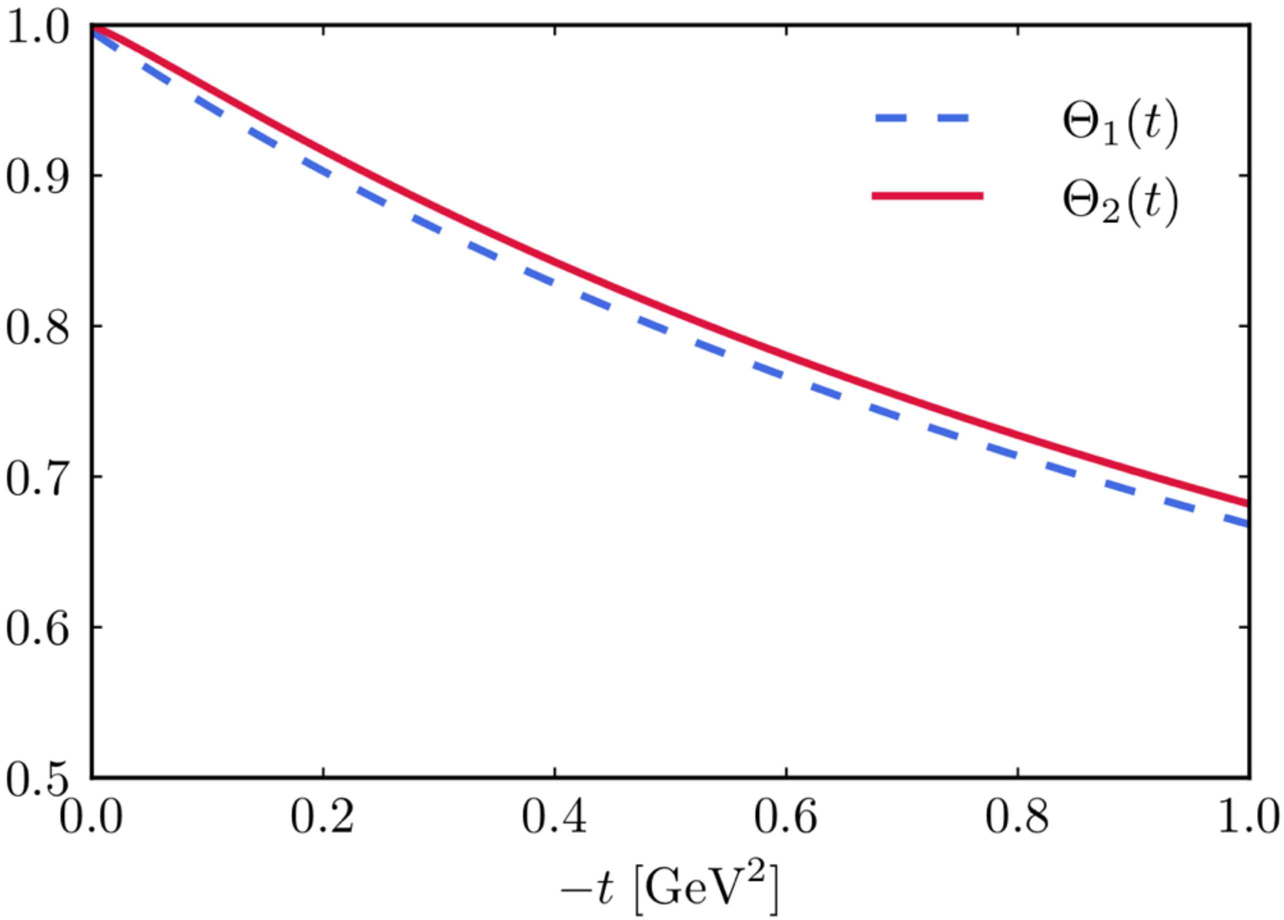
Pion decay constant

$$\mathcal{P} = \frac{3M}{f_\pi^2\overline{M}}\left(m\left\langle\bar{\psi}\psi\right\rangle + m_\pi^2 f_\pi^2\right) = 0\ !$$

by the Gell-Mann-Oakes-Renner relation to linear m order

H.D. Son & H.-Ch.K, to be published in PRD Rapid Comm.

Energy-momentum Tensor FFs

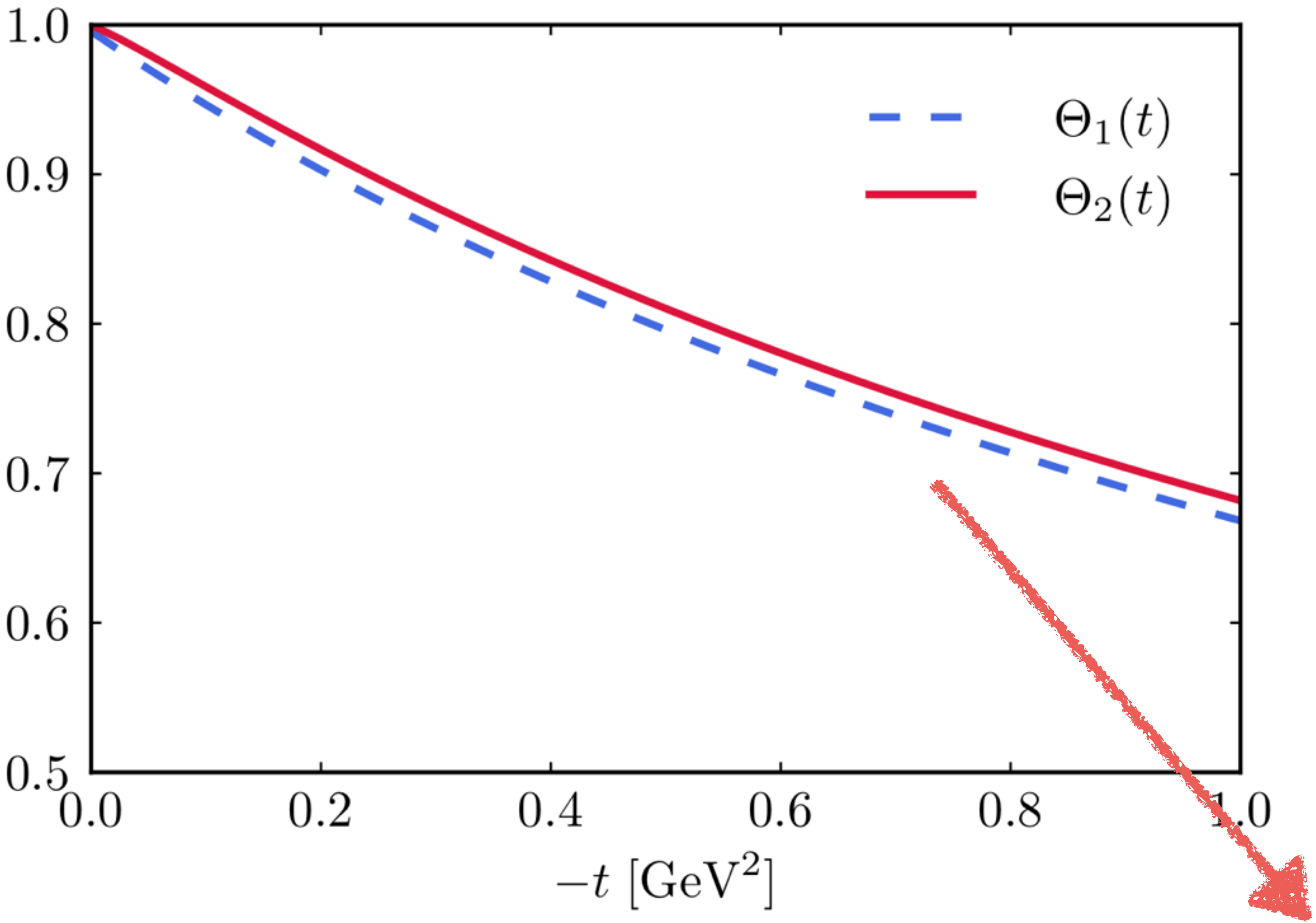


$$\Theta_1 = \Theta_2$$

in the chiral limit

H.D. Son & H.-Ch.K, to be published in PRD Rapid Comm.

Energy-momentum Tensor FFs



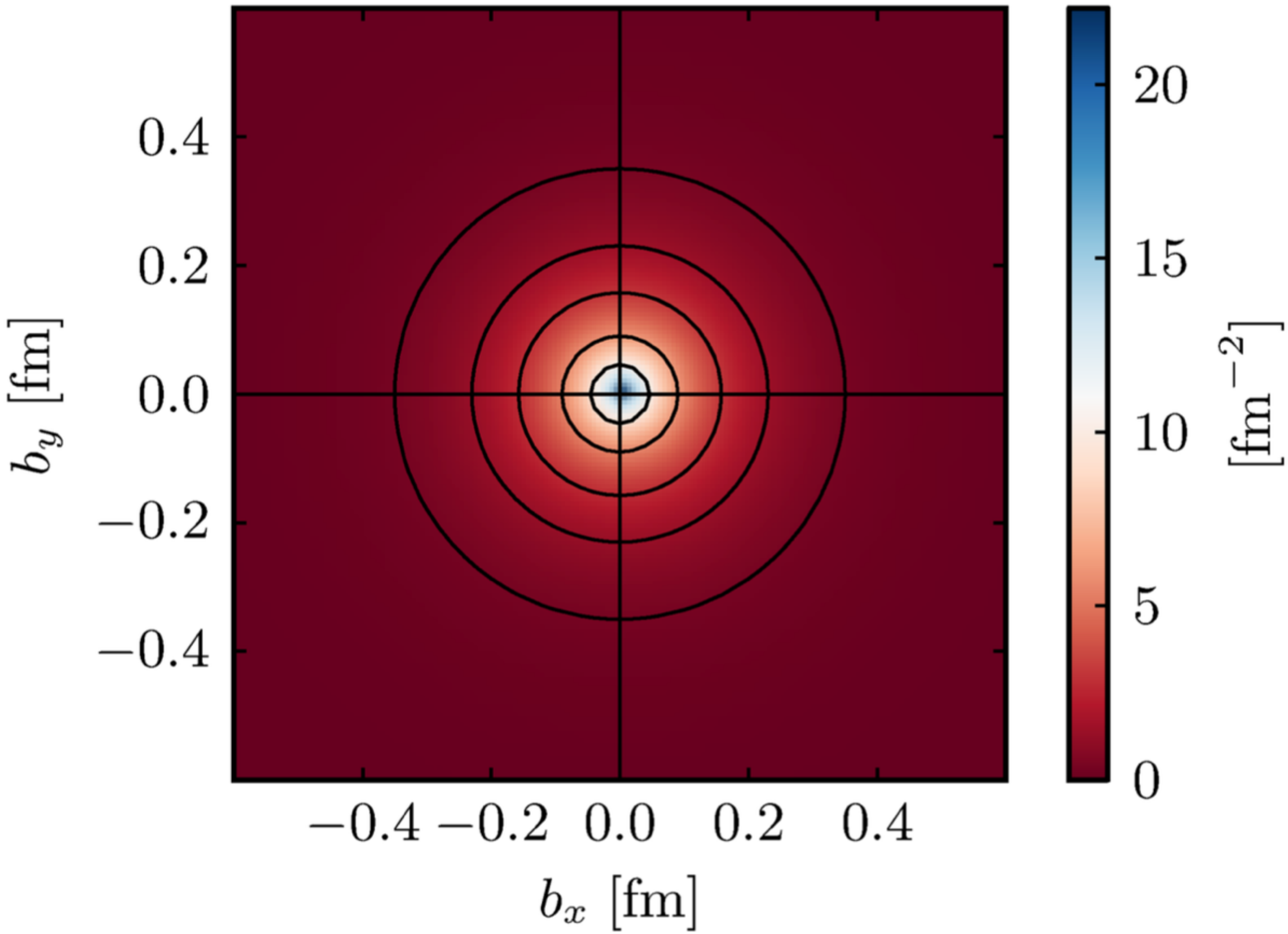
$$\Theta_1 = \Theta_2$$

in the chiral limit

The difference arises from the explicit chiral symmetry breaking.

H.D. Son & H.-Ch.K, to be published in PRD Rapid Comm.

Transverse charge density of the pion



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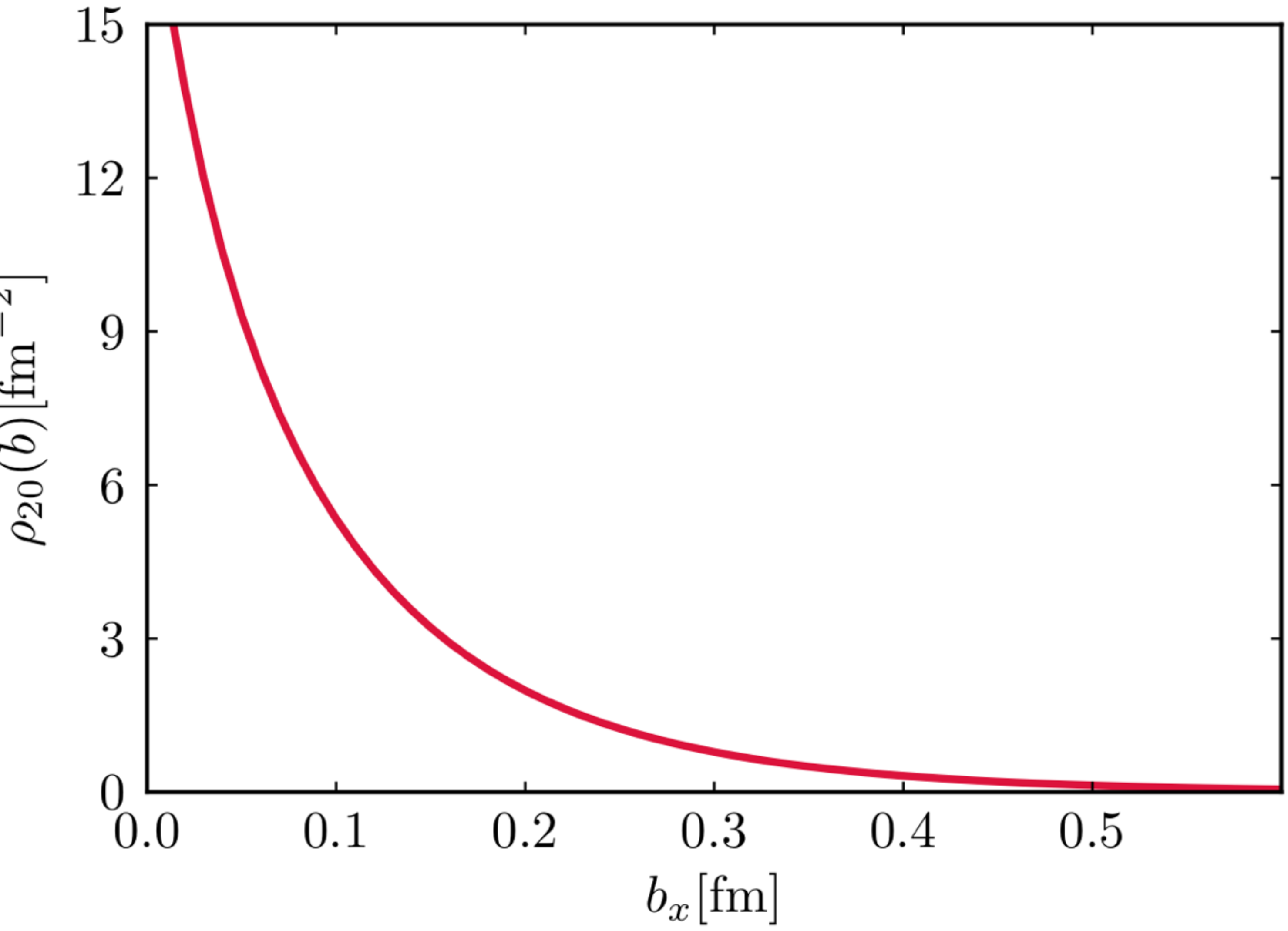

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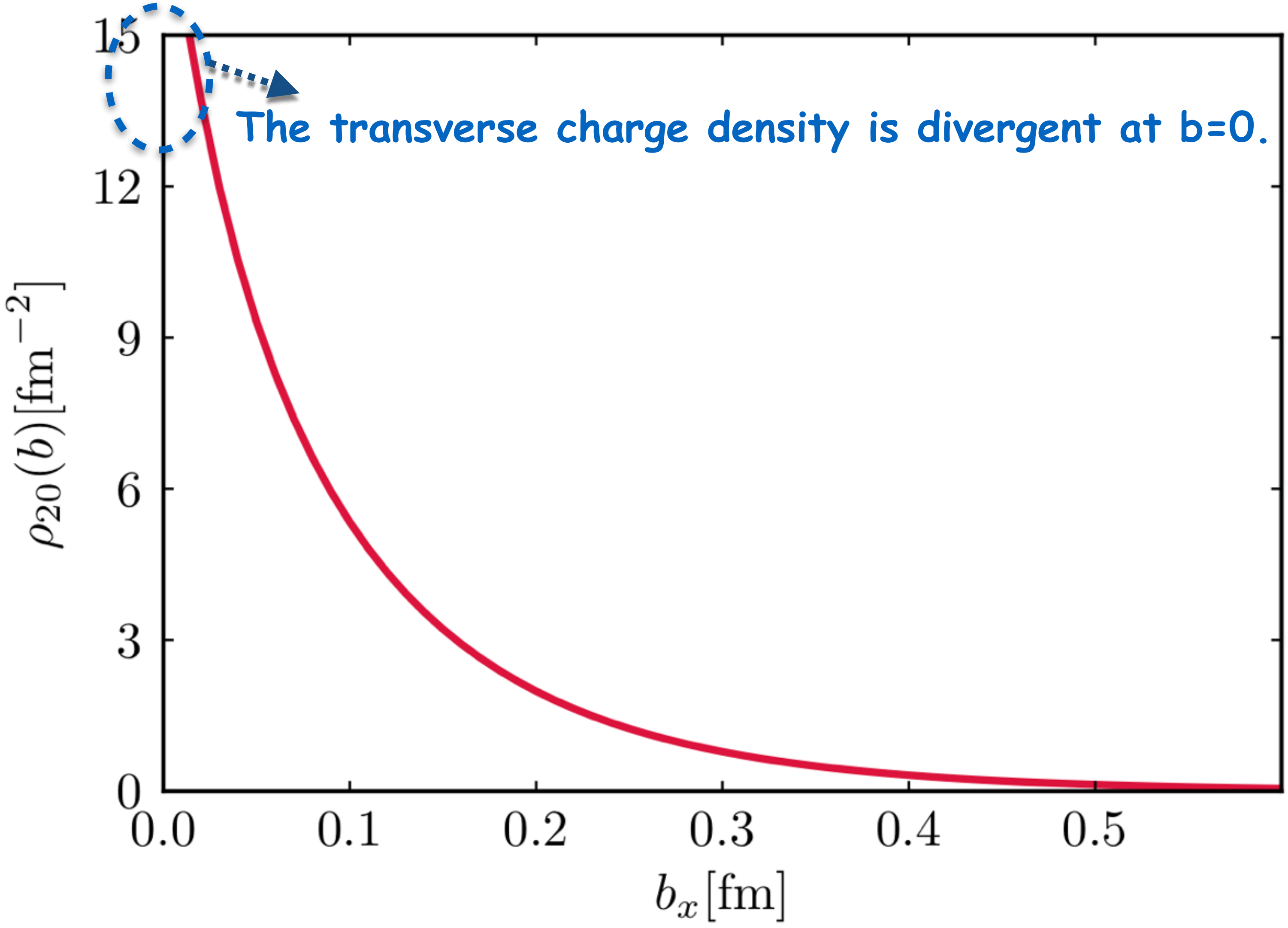


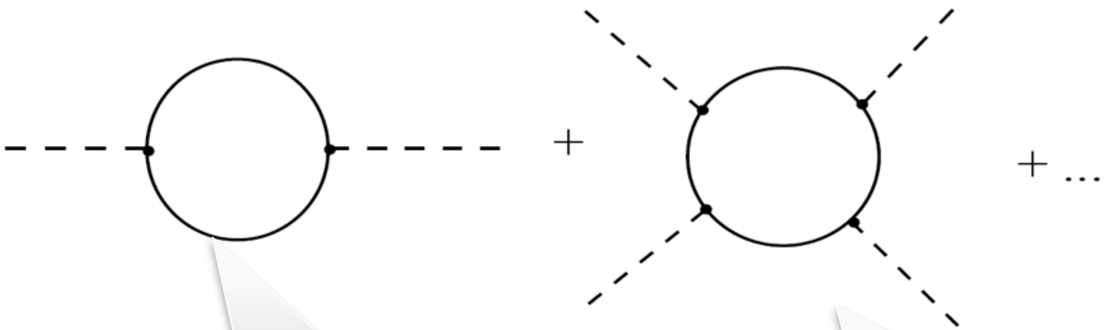
H.D. Son & H.-Ch.K, to be published in PRD Rapid Comm.

Effective chiral Lagrangian

Low-Energy Constants

$$S_{\rm eff} = -N_c {\rm Tr} \ln(i\partial\!\!\!/ + i\sqrt{M(i\partial)}U^{\gamma_5}\sqrt{M(i\partial)})$$

Derivative expansions: pion momentum as an expansion parameter



Weinberg term

Gasser-Leutwyler terms

HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

Effective chiral Lagrangian

Weinberg Lagrangian

$$\boxed{\mathcal{O}(p^2)}$$

$$\mathrm{Re}S_{\mathrm{eff}}^{(2)}[\pi^a] - \mathrm{Re}S_{\mathrm{eff}}^{(2)}[0] = \int d^4x \mathcal{L}^{(2)}$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4}\left\langle D^\mu U^\dagger D_\mu U \right\rangle + \frac{F_\pi^2}{4}\left\langle \mathcal{X}^\dagger U + \mathcal{X} U^\dagger \right\rangle$$

Gasser-Leutwyler Lagrangian

$$\boxed{\mathcal{O}(p^4)}$$

$$\mathcal{L}^{(4)} = L_1 \langle L_\mu L_\mu \rangle^2 + L_2 \langle L_\mu L_\nu \rangle^2 + L_3 \langle L_\mu L_\mu L_\nu L_\nu \rangle$$

H.A. Choi and HChK, PRD 69, 054004 (2004)

Low-energy constants in flat space

Gasser-Leutwyler Lagrangian

	M_0(MeV)	Λ(MeV)	$L_1(\times 10^{-3})$	$L_2(\times 10^{-3})$	$L_3(\times 10^{-3})$
local χQM	350	1905.5	0.79	1.58	−3.17
DP	350	611.7	0.82	1.63	−3.09
Dipole	350	611.2	0.82	1.63	−2.97
Gaussian	350	627.4	0.81	1.62	−2.88
GL			0.9 ± 0.3	1.7 ± 0.7	-4.4 ± 2.5
Bijnens			0.6 ± 0.2	1.2 ± 0.4	-3.6 ± 1.3
Arriola			0.96	1.95	−5.21
VMD			1.1	2.2	−5.5
Holdom(1)			0.97	1.95	−4.20
Holdom(2)			0.90	1.80	−3.90
Bolokhov et al.			0.63	1.25	2.50
Alfaro et al.			0.45	0.9	−1.8

H.A. Choi and HChK, PRD 69, 054004 (2004)

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Effective chiral Lagrangian in curved space

$$\mathcal{Z}_{\rm eff} = \int D\psi D\psi^\dagger D\pi^a \exp\left[i\int d^4x\sqrt{|g|}\mathcal{L}(\psi,\psi^\dagger,\pi^a)\right]$$

Gasser-Leutwyler Lagrangian

$$\mathcal{L}^{(4,R)} = L_{11}R\mathrm{Tr}(D_\mu U D^\mu U^\dagger) + L_{12}R^{\mu\nu}\mathrm{Tr}(D_\mu D_\nu U^\dagger) + L_{13}R\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger) + \cdots$$

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Low-energy constants in curved space

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Low-energy constants in curved space

They can be derived either by expanding the action by the heat-kernel method or expand the EMTffs with respect to *t*.

$$\Theta_1(t) = 1 + \frac{2}{f_\pi^2}\left[t(4L_{11}+L_{12}) - 8m_\pi^2(L_{11}-L_{13})\right]$$

$$\Theta_2(t) = 1 - \frac{2t}{f_\pi^2}L_{12}$$

Low-energy constants in curved space

Present Results

$$L_{11} = \frac{N_c}{192\pi^2} = 1.6 \times 10^{-3}$$

$$L_{12} = -2L_{11} = -3.2 \times 10^{-3}$$

$$L_{13} = \frac{N_c}{96\pi^2} \frac{M f_\pi^2}{\langle \bar{\psi}\psi \rangle} \Gamma(0, M^2/\Lambda^2) = 0.84 \times 10^{-3}$$

XPT Results (Gasser & Leutwyler)

$$L_{11} = 1.4 \times 10^{-3},\ L_{12} = -2.7 \times 10^{-3},\ L_{13} = 0.9 \times 10^{-3} \text{ at } \mu = 1\,\mathrm{GeV}$$

Summary & Conclusion

Summary

- We have reviewed recent investigations on the quark structures of the pion, based on the nonlocal chiral quark model from the instanton vacuum.

- We have derived the EM and tensor form factors of the pion, from which we have obtained the quark transverse spin densities inside a pion.

- We also have shown the energy-momentum tensor (gravitational or generalised) form factors of the pion.

- The pressure of the pion nontrivially vanishes because of the Gell-Mann-Oakes-Renner relation.

- We also have presented the higher-order transverse charge density of the pion.

Outlook

- The transverse charge and spin densities for the transition processes can be studied (K-pi transition is done, see the next talk by Hyeon-Dong Son).

- The excited states for the nucleon and the hyperon can be investigated (Generalisation of the XQSM is under way).

- **Internal structure of Heavy-light quark systems (Derivation of the Partition function is close to the final result.)**

- **New perspective on hadron tomography**

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!