$K \rightarrow \pi$ transition form factors and the transverse spin density

Hyeon-Dong Son

Inha University, Korea

@ Progress of J-PARC Hadron Physics,
Nov. 30. - Dec. 02. 2014.

in collaboration with
Seung-il Nam
Hyun-Chul Kim

Contents

Motivation

- Semileptonic decays (K_{l3})
  : CKM matrix elements |Vus|

- Tensor transition provides non-standard type interaction

  [I. Baum et al, Phys. Rev. D 84, 074503 (2011)]

- Transverse charge & spin densities of the transition

  [C. Carlson and M. Vanderhaeghen, Phys. Rev. Lett. 100, 032004 (2008)]

Kaon l3 decay

수식, Dirac structure, flavor 더 명확하게 적어서 improve 할 것.
su가 되려면 K0 \pi- 여야 함.
예를 들어 마지막 식에 F -> \Gamma

The decay amplitude

$$T_{K\to l\mu\pi} = \frac{G_F}{\sqrt{2}} \sin\theta_c \left[W^\mu(p_l, p_\nu) F_\mu(p_\pi, p_K)\right]$$

$G_F = 1.116 \times 10^{-5}\mathrm{GeV}^{-2}$

Weak leptonic element

$$W^\mu(p_l, p_\nu) = \bar{u}(p_\nu)\Gamma^\mu \nu(p_l)$$

Hadronic matrix element

$$F_\mu(p_\pi, p_K) = c\langle \pi(p_\pi)|\Gamma_\mu|K(p_K)\rangle$$

Hadronic Matrix Elements

- Vector transition

$$F_{\mu}^{K^0}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = (p_K + p_\pi)_\mu f_{l+}(t) + (p_K - p_\pi)_\mu f_{l-}(t)$$

- Tensor transition

$$F_{\mu\nu}^{K^0}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}\sigma_{\mu\nu} u|K^0(p_K)\rangle = \frac{p_{K\mu}p_{\pi\nu} - p_{K\nu}p_{\pi\mu}}{m_K} B_T^{K\pi}(t)$$

- Scalar transition

$$F^{K^0}(p_\pi, p_K) = \langle \pi^-(p_\pi)|\bar{s}u|K^0(p_K)\rangle = -\frac{m_K^2 - m_\pi^2}{m_s - m_u} f_0(t)$$

Nonlocal Chiral Quark Model

$$S_{\text{eff}} = -N_c \text{Tr} \log \left[i\not{\partial} + i\hat{m} + i\sqrt{M(i\partial)} U^{\gamma_5} \sqrt{M(i\partial)} \right]$$

- ✤ The chiral effective action derived from the instanton vacuum

$$U^{\gamma_5} = \exp\left[\frac{i\gamma_5}{f_\phi}(\lambda \cdot \phi)\right]$$

- ✤ No free parameter
  - Average Instanton size & separation

$$\bar{\rho} \approx \frac{1}{3}\ \text{fm} \quad \bar{\text{R}} \approx 1\ \text{fm}$$

- ✤ Nonlocality
  - Momentum-dependent dynamical quark mass
- ✤ Nicely reproduces pion properties: Fpi, EMFF
- ✤ Explicit SU(3) symmetry breaking

$$\hat{m} = \text{diag}(m_u, m_d, m_s),\ m_u = m_d = 5\ \text{MeV},\ m_s = 150\ \text{MeV}$$

[D. Diakonov, Instantons at work, arXiv:hep-ph/0212026v4]

Nonlocal Chiral Quark Model

modbess ff 써서 수치계산 하는데 무슨 문
가 있었지?

- Momentum-dependent dynamical quark mass

$$\sqrt{M(i\partial)} = \sqrt{M_0 f(m) F^2(i\partial)}$$

$$F(k) = \frac{k}{\Lambda}\left[I_0(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda}) - I_1(\frac{k}{2\Lambda})K_0(\frac{k}{2\Lambda}) - \frac{2\Lambda}{k}I_1(\frac{k}{2\Lambda})K_1(\frac{k}{2\Lambda})\right]$$

$$F_N(k) = \left(\frac{2N\Lambda^2}{2N\Lambda^2 + k^2}\right)^N \qquad \Lambda = 1/\bar{\rho} = 600\ \mathrm{MeV}$$

- Current quark mass correction f(m)

$$f(m) = \sqrt{1 + \frac{m^2}{d^2}} - \frac{m}{d}, d \approx 198MeV$$

- Zero-momentum dynamical quark mass M0

$$M_0 \approx 350\mathrm{MeV}$$

[M. Musakhanov Eur.Phys.J.C9,235(1999)]

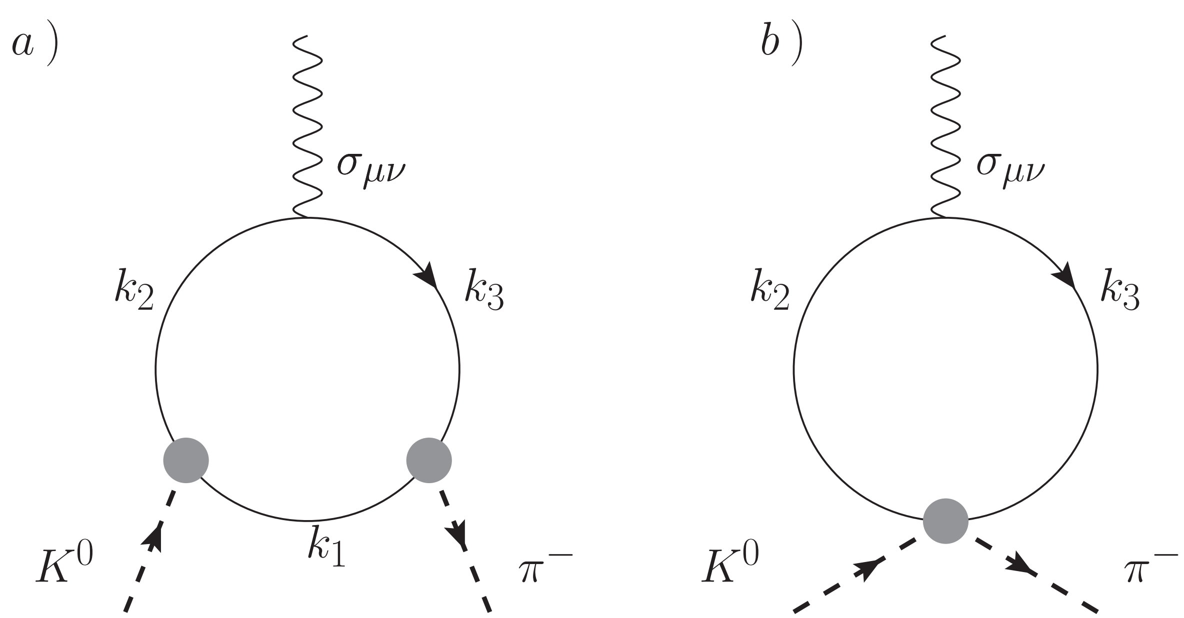
Pion Decay Constant $f_\pi \approx 93\mathrm{MeV}$

Calculation

$$\langle \pi^-(p_f)|\bar{s}\sigma_{\mu\nu}u|K^0(p_i)\rangle =$$

$$\frac{8N_c}{f_\pi f_K}\int\frac{d^4l}{(2\pi)^4}\Bigg[\frac{M_{2d}\sqrt{M_{1u}}\sqrt{M_{3s}}}{G_{u1}G_{d2}G_{s3}}\Big(k_{1\mu}(k_{2\nu}\bar{M}_{3s}-k_{3\nu}\bar{M}_{2d})+k_{2\mu}(k_{3\nu}\bar{M}_{1u}-k_{1\nu}\bar{M}_{3s})$$

$$+k_{3\mu}(k_{1\nu}\bar{M}_{2d}-k_{2\nu}\bar{M}_{1u})\Big)-\frac{\sqrt{M_{1u}}\sqrt{M_{3s}}}{2G_{u1}\,G_{s3}}(k_{3\mu}k_{1\nu}-k_{3\nu}k_{1\mu})\Bigg]$$



$$G_{fi}=k_i^2+\bar{M}_{if}^2$$

$$\bar{M}_{if}=m_f+M(k_i,m_f)$$

$$k_1=l-\frac{p_i}{2}-\frac{q}{2}\qquad k_2=l+\frac{p_i}{2}-\frac{q}{2}$$

$$k_3=l+\frac{p_i}{2}+\frac{q}{2}$$

Experimental Values

$$f_\pi=93\ \text{MeV},\quad f_K=113\ \text{MeV}$$

$$m_\pi=140\ \text{MeV},\quad m_K=495\ \text{MeV}$$

QCD RG Evolution

- At the leading order

$$B_T(t,\mu) = B_T(t,\mu_0)\left[\frac{\alpha(\mu)}{\alpha(\mu_0)}\right]^{\gamma/(2\beta_0)}$$

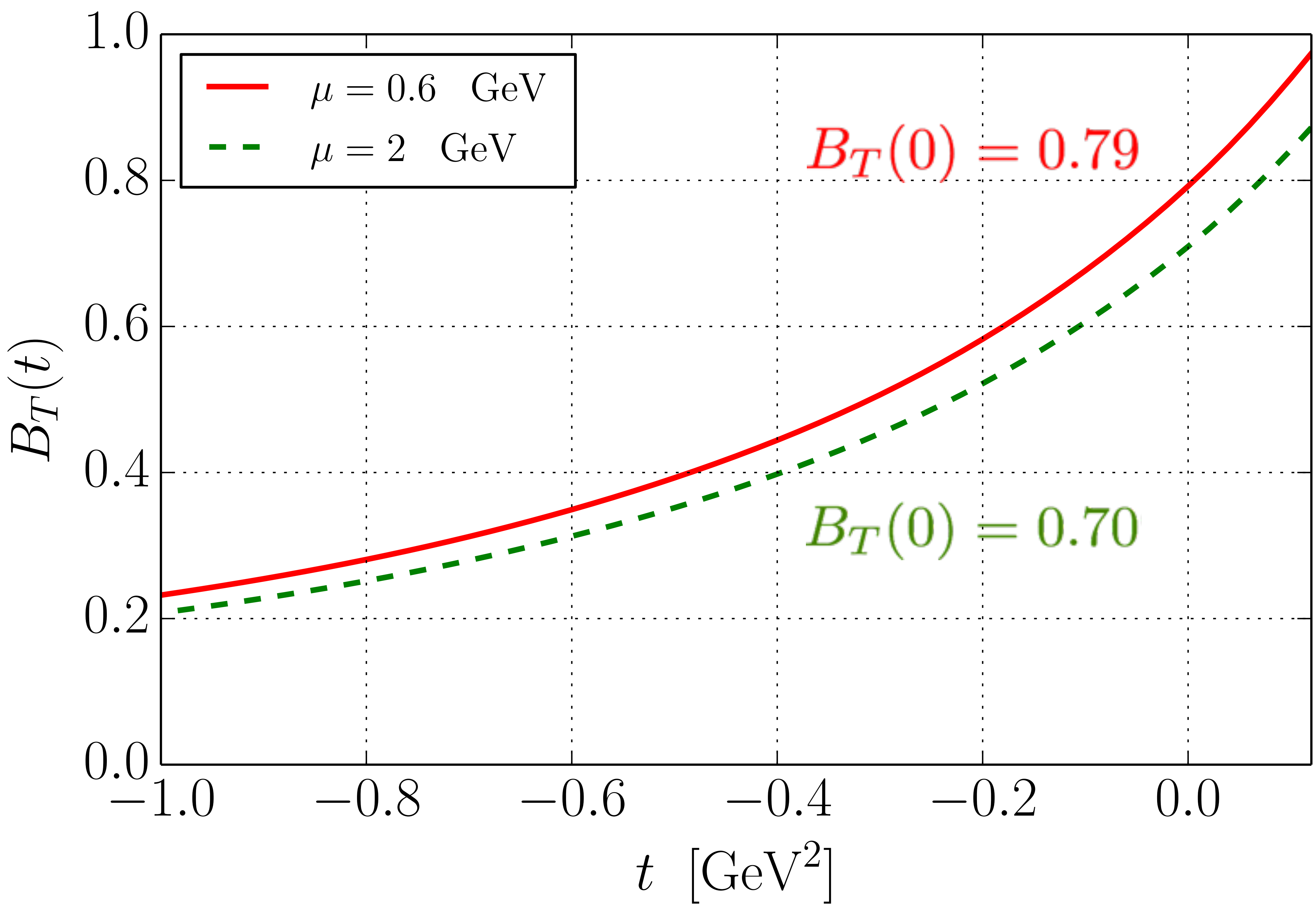
$$\gamma = \frac{8}{3}, \quad \beta_0 = 11\frac{N_c}{3} - 2\frac{N_f}{3} \quad (N_c = 3, N_f = 3)$$

$$\Lambda_{\rm QCD} = 0.25\ {\rm GeV}$$

[Glück et al. Zeits.Für.Phys. C.67(3),
Barone et al. Phys.Repts., 359(1-2), 1-168.]

Numerical Results

Display the form factor only in the space like region.
Discuss the slope parametrisation and the values.



Lattice Results

- Present work

$$f_T^{K\pi}(t) = \frac{m_K + m_\pi}{2m_K} B_T^{K\pi}(t)$$

$$B_T^{K\pi}(0) = 0.70 \rightarrow \boxed{f_T^{K\pi}(0) = 0.45}$$

- I. Baum et al, μ = 2 GeV

$$\langle \pi^0 | \bar{s}\sigma^{\mu\nu} d | K^0 \rangle = (p_\pi^\mu p_K^\nu - p_\pi^\nu p_K^\mu) \frac{\sqrt{2} f_T^{K\pi}(q^2)}{M_K + M_\pi}$$

$$\boxed{f_T^{K\pi}(0) = 0.417\ (14_{\rm stat})\ (5_{\rm syst}) = 0.417\ (15)}$$

Extrapolated to Physical meson masses

- I. V. Ajinenko et al. Physics Letters B 574, 14 (2003).

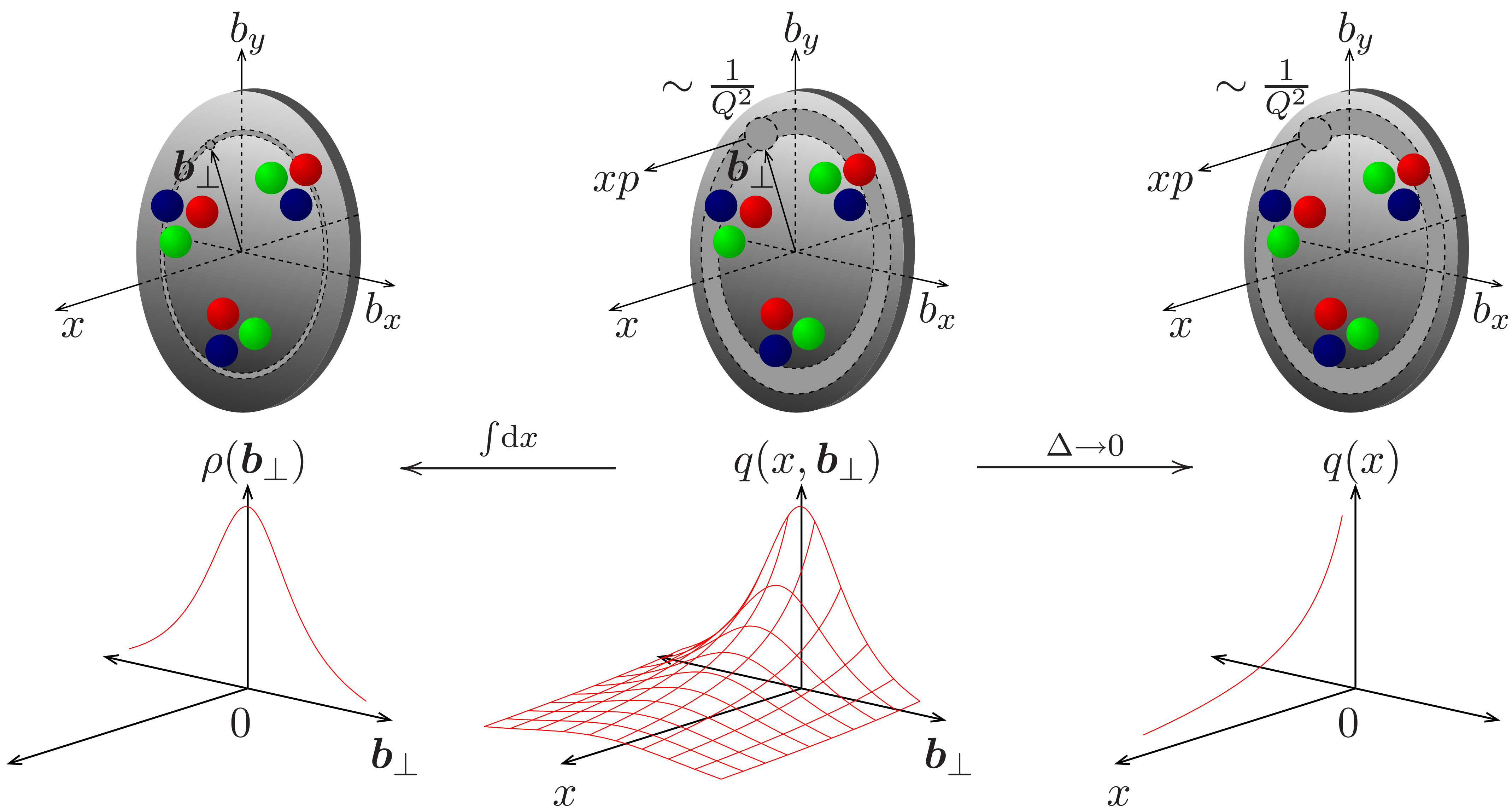
The limits on the possible tensor and scalar couplings are derived:

$$\boxed{f_T/f_+(0) = 0.021^{+0.064}_{-0.075}(\text{stat}) \pm 0.026(\text{syst}),}$$

$$f_S/f_+(0) = 0.002^{+0.020}_{-0.022}(\text{stat}) \pm 0.003(\text{syst}).$$

Generalised Parton Distributions

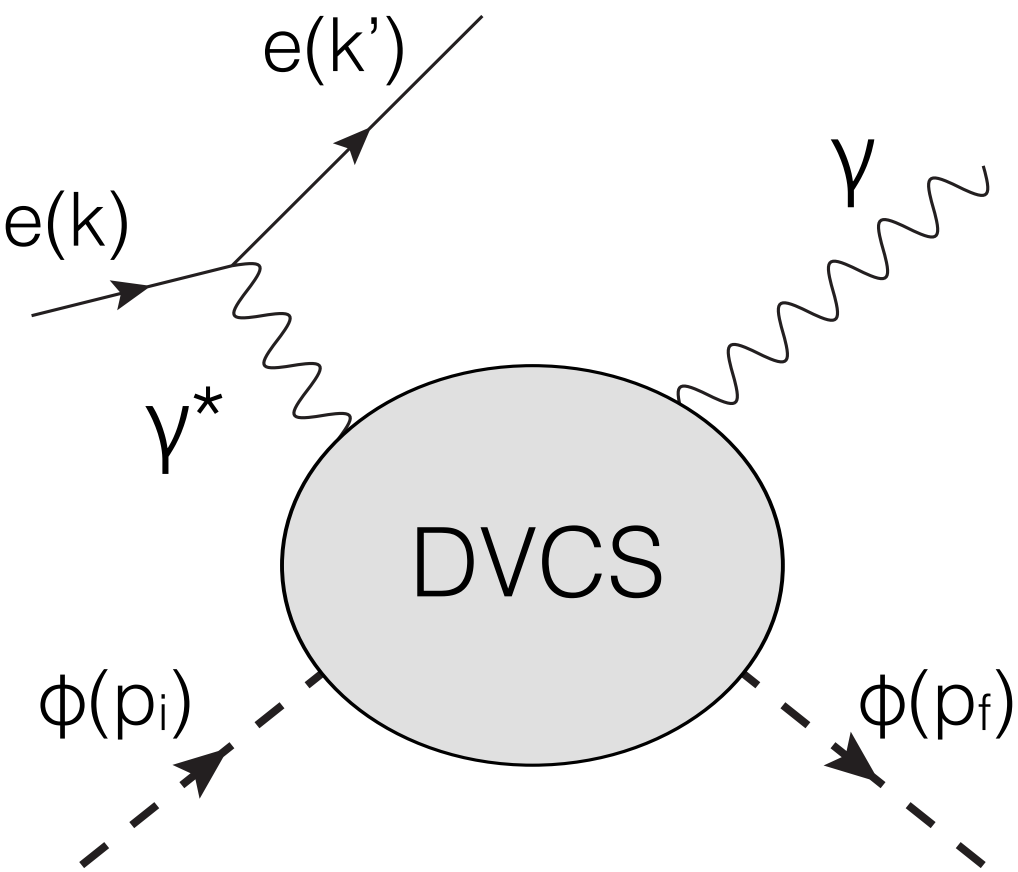
[D. Brömmel, Pion Structure Frome the Lattice, Regensburg Univ., Thesis]

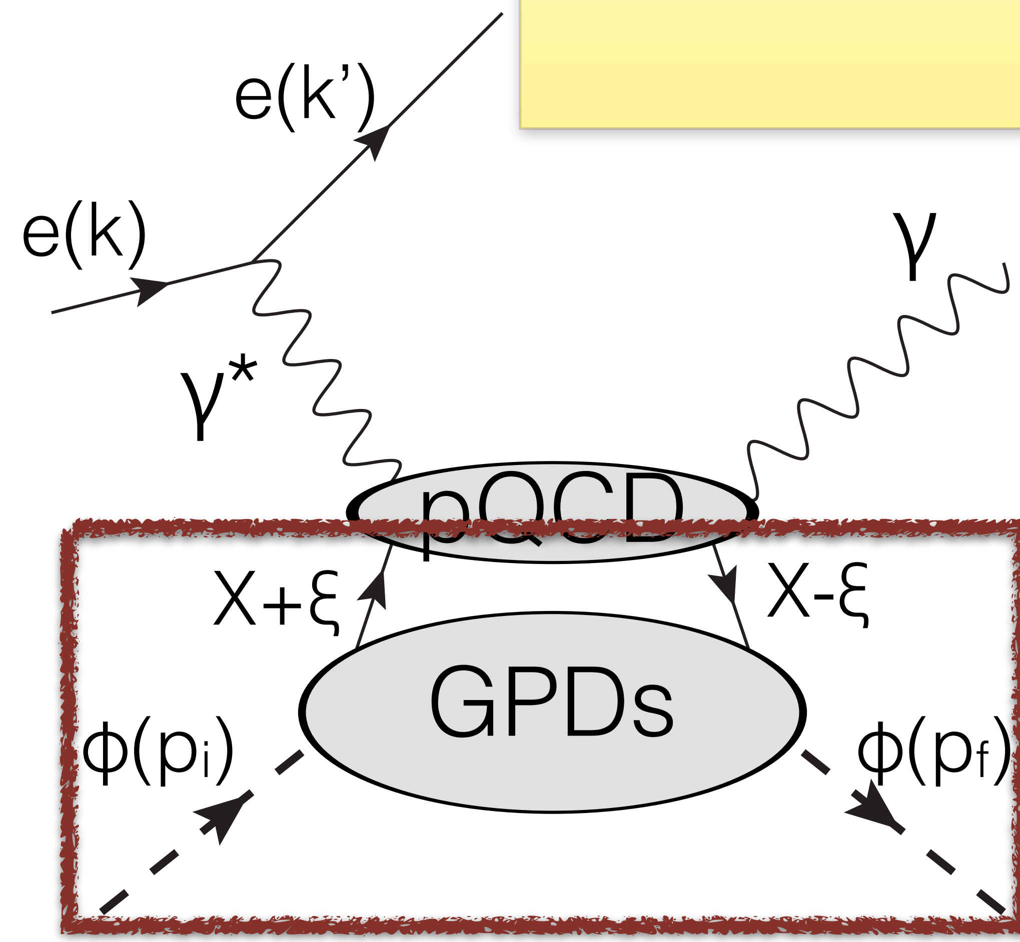


Form factors GPD, ξ=0 Parton distribution

DVCS & GPDs

How about the corresponding processes to the weak transitions? are they measurable from an experiment?





Factorization : [Collins, J., & Freund, A. (1999).Phys. Rev. D, 59(7), 074009.]

- Light cone frame

$$P^\mu = \frac{1}{2}\left(p_i^\mu + p_f^\mu\right), \quad q^\mu = p_f^\mu - p_i^\mu, \quad t = q^2 \qquad n_\pm = \frac{1}{\sqrt{2}}(1,0,0,\pm 1)$$

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3), \quad v_\perp = (v^1, v^2) \qquad \xi = \frac{p_i^+ - p_f^+}{p_i^+ + p_f^+}$$

Generalised Parton Distributio

lightcone gauge: n \cdot A =0

- Generalised parton distributions (GPDs)

$$2P^+ H^q_\phi(X,\xi,t) = \int \frac{dz^-}{2\pi} e^{iXP^+z^-} \langle \phi(p_f)|\bar{\psi}^q(-\frac{z^-}{2})\gamma^+ \left[-\frac{z^-}{2},\frac{z^-}{2}\right] \psi^q(\frac{z^-}{2})|\phi(p_i)\rangle\Big|_{z^+=z_\perp=0}$$

$$\frac{P^{[+}t^{j]}}{m_\phi} E^q_\phi(X,\xi,t) = \int \frac{dz^-}{2\pi} e^{iXP^+z^-} \langle \phi(p_f)|\bar{\psi}^q(-\frac{z^-}{2})i\sigma^{+j} \left[-\frac{z^-}{2},\frac{z^-}{2}\right] \psi^q(\frac{z^-}{2})|\phi(p_i)\rangle\Big|_{z^+=z_\perp=0}$$

- Kaon Transition GPDs for $K^0 \to \pi^-$

$$2P^+ H^{K\pi}_\phi(X,\xi,t) = \int \frac{dz^-}{2\pi} e^{iXP^+z^-} \langle \pi^-(p_f)|\bar{s}(-\frac{z^-}{2})\gamma^+ \left[-\frac{z^-}{2},\frac{z^-}{2}\right] u(\frac{z^-}{2})|K^0(p_i)\rangle\Big|_{z^+=z_\perp=0}$$

$$\frac{P^{[+}t^{j]}}{m_K} E^{K\pi}_\phi(X,\xi,t) = \int \frac{dz^-}{2\pi} e^{iXP^+z^-} \langle \pi^-(p_f)|\bar{s}(-\frac{z^-}{2})i\sigma^{+j} \left[-\frac{z^-}{2},\frac{z^-}{2}\right] u(\frac{z^-}{2})|K^0(p_i)\rangle\Big|_{z^+=z_\perp=0}$$

Expanding the nonlocal operators → Generalised Form Factors

Generalised Form Factors

- Generalised form factors

Vector

$$\langle \phi^a(p_f)|\psi^\dagger(0)\gamma_{\{\mu} i \overleftrightarrow{D}_{\mu_1} \ldots \overleftrightarrow{D}_{\mu_{n-1}\}}\psi(0)|\phi^b(p_i)\rangle$$

$$= 2P_{\{\mu}P_{\mu_1}\ldots P_{\mu_{n-1}\}}A_{n0}(t)$$

$$+ \sum_{k=2\ even}^{n} q_{\{\mu}q_{\mu_1}\cdots q_{\mu_{k-1}}P_{\mu_k}P_{\mu_{n-1}\}}2^{-k}A_{nk}(t)$$

Tensor

$$\langle \phi(p_f)|\psi^\dagger(0)\sigma_{[\mu\nu} i \overleftrightarrow{D}_{\mu_1} \ldots i \overleftrightarrow{D}_{\mu_{n-1}]}\psi(0)|\phi(p_i)\rangle$$

$$= \frac{p_{[\mu}q_\nu - q_\mu p_\nu}{m_\phi} \sum_{i=even}^{n-1} q_{\mu_1}\ldots q_{\mu_i}P_{\mu_{i+1}}P_{\mu_{n-1}]}B_{ni}(t)$$

Polynomiality & Transverse charge density

- Polynomiality: Mellin moments of the GPDs

$$\int dX X^{n-1} H(X,\xi,t)$$ ⇒ Finite polynomial of ξ

$$\int dX H(X,\xi,t) = A_{10}(t)$$ ➔ Vector FF

$$\int dX X H(X,\xi,t) = A_{20}(t) + \xi^2 A_{22}(t)$$ ➔ EMT FFs

- Transverse densities:
  2D Fourier transformation into the impact parameter $b_\perp$ at $\xi=0$

$$F(b_\perp^2) = \int dq_\perp^2 e^{-ib_\perp \cdot q_\perp} \int dX H(X, q_\perp^2, \xi = 0)$$
$$= \int dq_\perp^2 e^{-ib_\perp \cdot q_\perp} F(q_\perp^2)$$

Probability distribution of the partons inside the hadrons in the transverse impact parameter plane

GFFs & Transverse Densities for the Ka

K^0 스테이트에 (p_i)가 빠져있다 ㅠㅠㅠㅠㅠ

- Generalised form factors for kaon transitions: n =1

$$\langle \pi^-(p_f)|\bar{s}(0)\gamma_\mu u(0)|K^0\rangle = 2P_\mu A_{10}^{K\pi}(t) + q_\mu C_{10}^{K\pi}(t)$$

$$\langle \pi^-(p_f)|\bar{s}(0)\sigma_{\mu\nu} u(0)|K^0\rangle = \frac{p_{i\mu}p_{f\nu} - p_{i\nu}p_{f\mu}}{m_K} B_{10}^{K\pi}(t)$$

- Quarks with definite transverse polarisation **s**

$$\frac{1}{2}\bar{\psi}\left[\gamma^+ - s^j i\sigma^{+j}\gamma_5\right]\psi$$

$$\sigma^{\mu\nu}\gamma_5 = -\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} i\sigma_{\alpha\beta}$$

- Transverse transition density, ξ=0

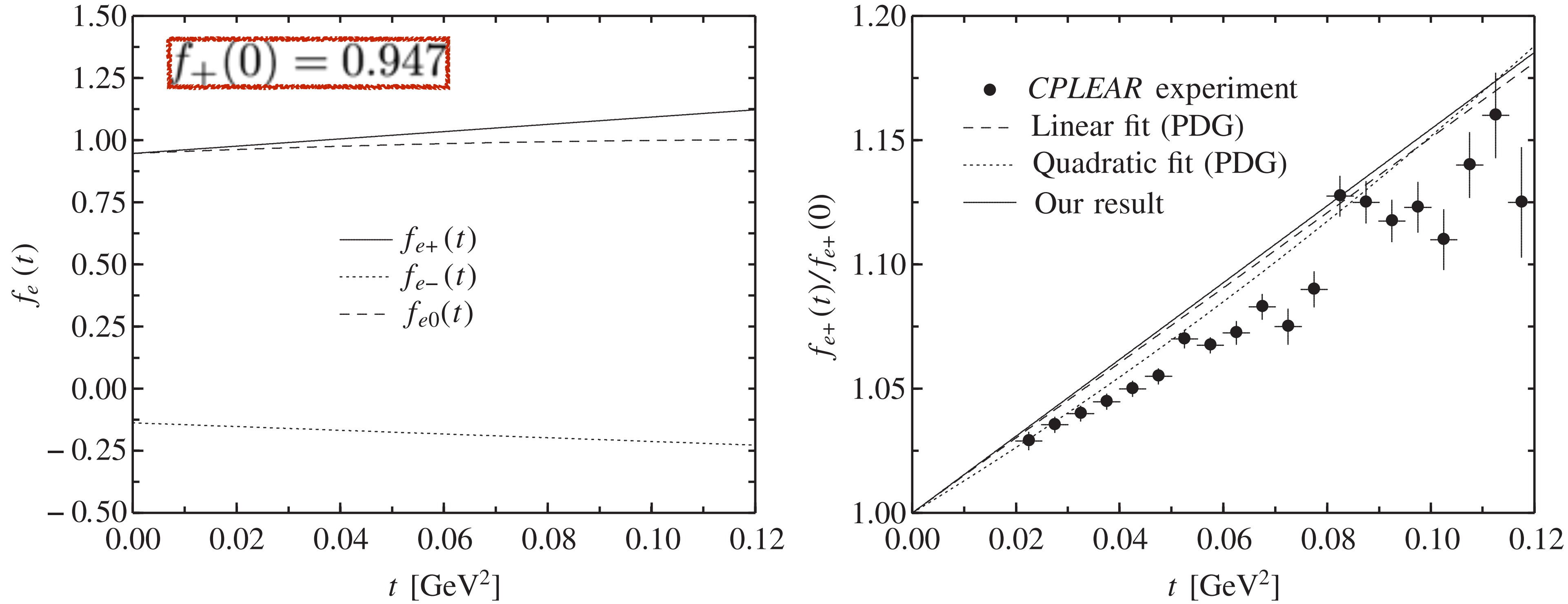
$$\rho^{K\pi}(b_\perp, s_\perp) = \int dX \rho(X, b_\perp, s_\perp) = \frac{1}{2}\left[A_{10}^{K\pi}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_K}\frac{\partial B_{10}^{K\pi}(b_\perp^2)}{\partial b_\perp^2}\right]$$

Transverse charge & spin density

How the quark spin is distributed
in the transverse plane during the K-π transition process

$$\rho^{K\pi}(b_\perp, s_\perp) = \int dX \rho(X, b_\perp, s_\perp) = \frac{1}{2}\left[A_{10}^{K\pi}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_K} \frac{\partial B_{10}^{K\pi}(b_\perp^2)}{\partial b_\perp^2}\right]$$

$$A_{10}^{K\pi}(b_\perp^2) \to f_+(b_\perp^2), \quad B_{10}^{K\pi}(b_\perp^2) \to B_T^{K\pi}(b_\perp^2)$$

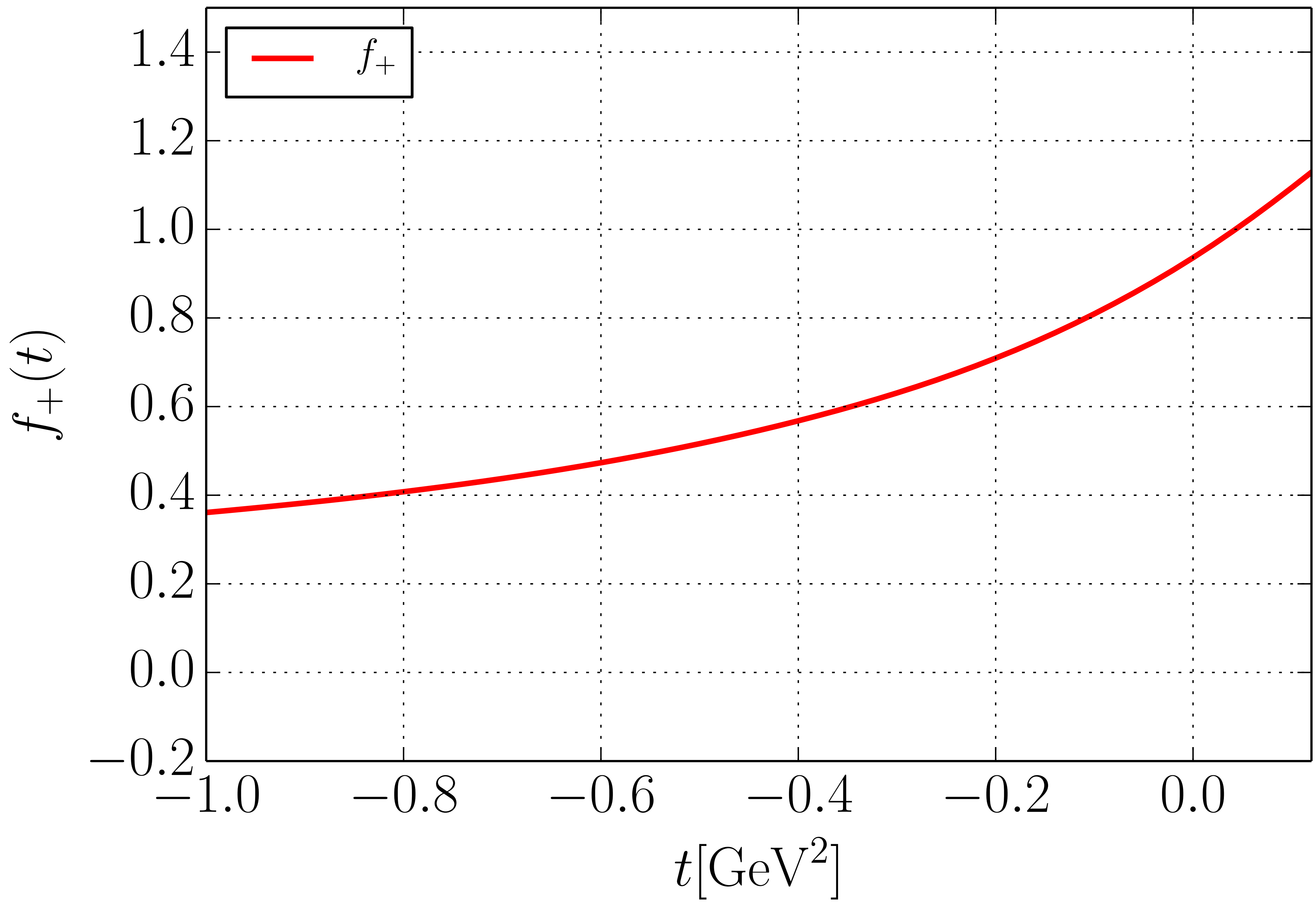


$$m_l^2 < t < (m_K - m_\pi)^2 \approx 0.12 \text{ GeV}^2$$

[S.-i. Nam and H.-Ch. Kim, Phys. Rev. D 75, 094011 (2007).]

Vector form factor $f_+(t)$

The local & nonlocal things might be misleading. It's better to show only the total result.



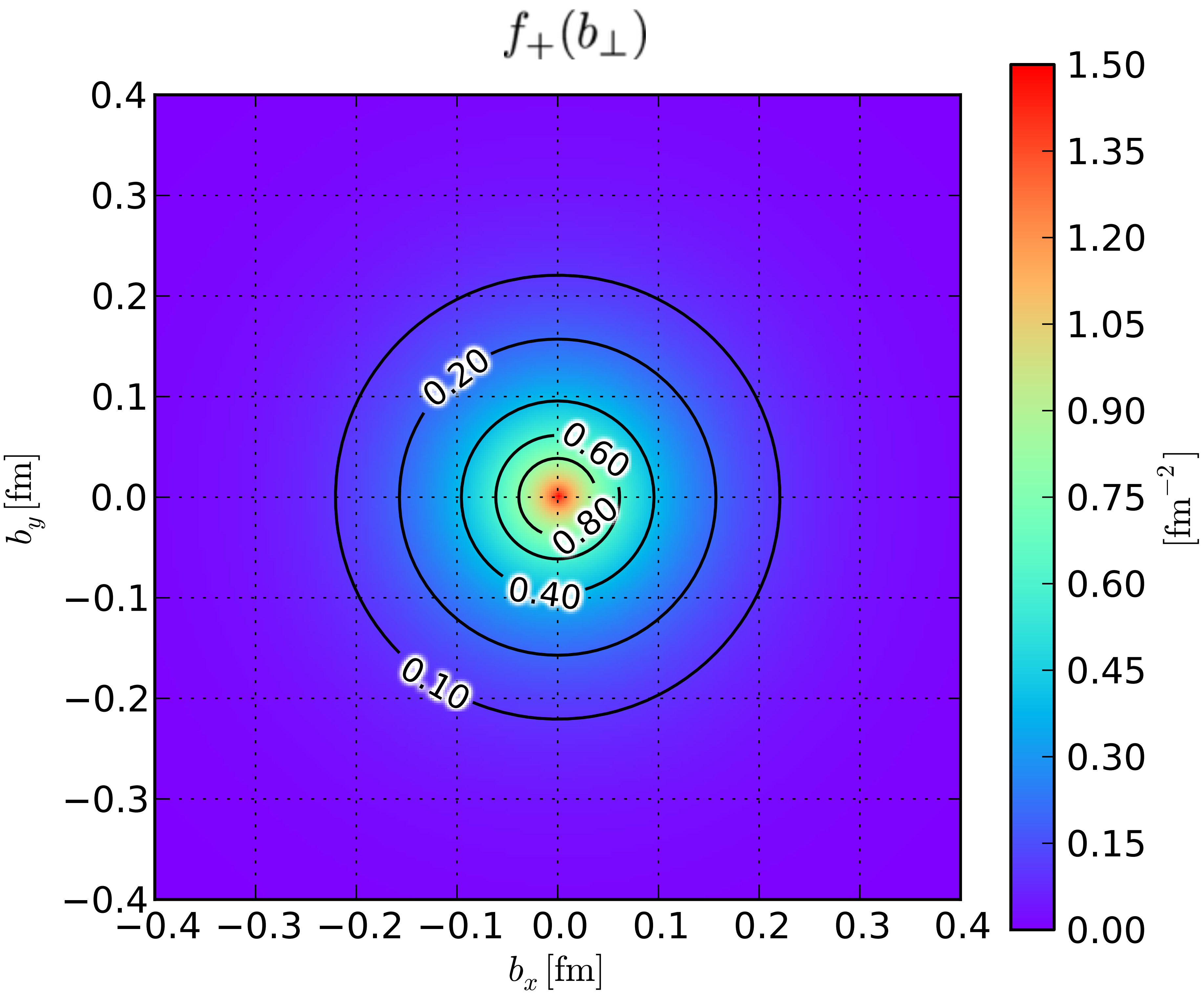
P-pole parametrisation

- **p-pole parametrisation**

is this slide necessary?

$F(t) = \frac{F(0)}{\left(1 - \frac{t}{pM^2}\right)^p}$	$f_+(t)$	$B_T^{K\pi}(t)$
p	1.31	2.2
M [GeV]	0.85	0.78
t=0	0.95	0.70

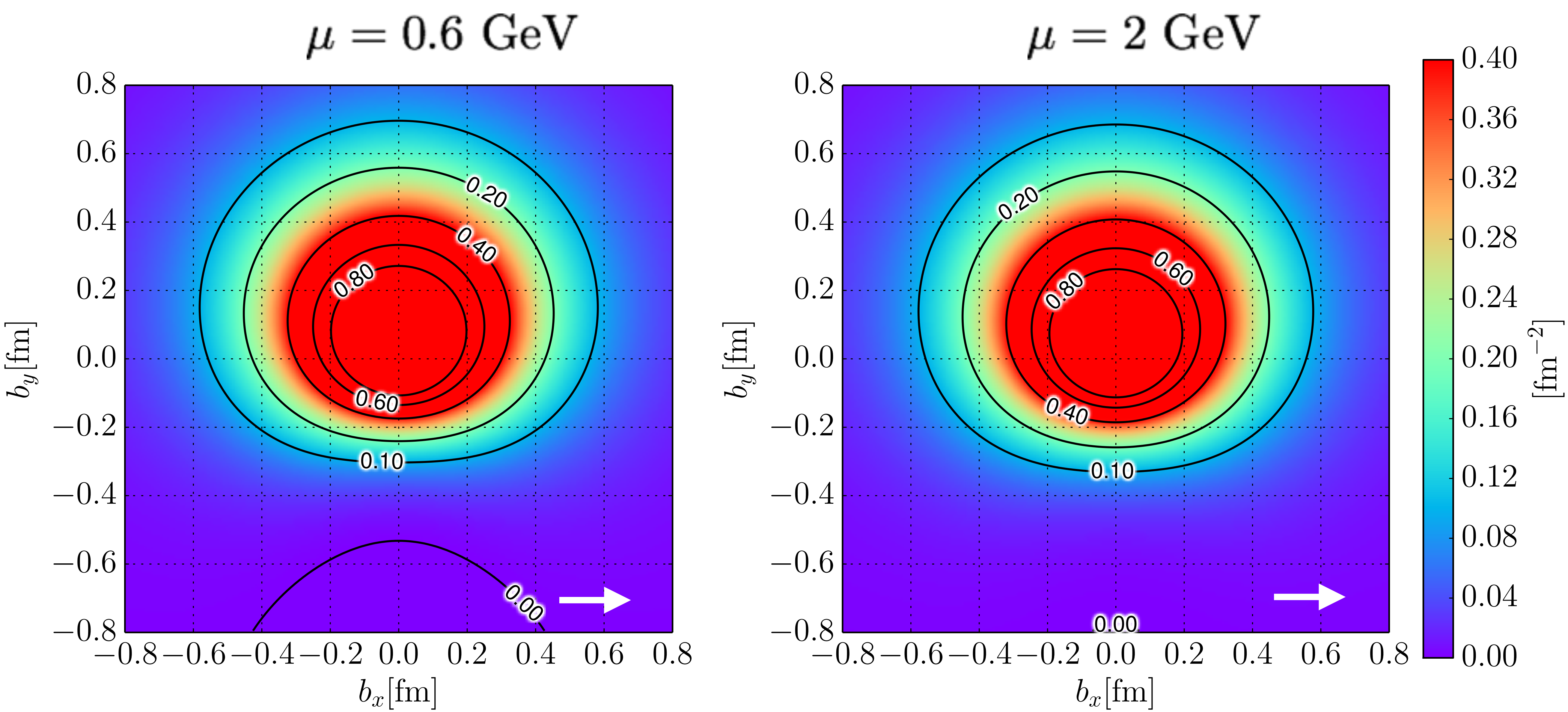
Transverse Charge Density



Transverse Spin Density

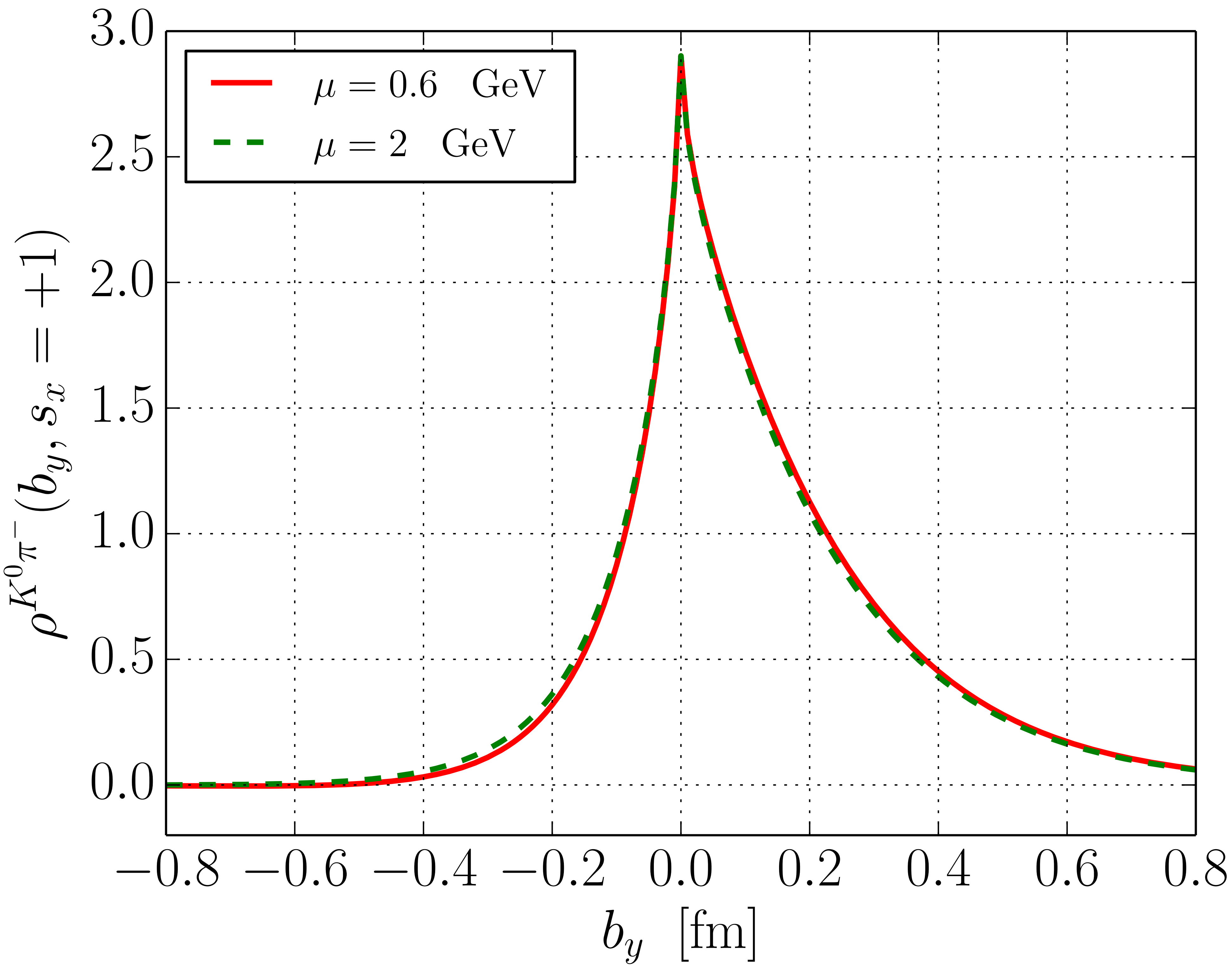
그림 업데이트할 것.
이 슬라이드랑 이 다음 슬라이드를 합칠 수 있을 것 같다.

$$\rho^{K\pi}(b_\perp, s_{b_x} = 1)$$



$$\langle b_y \rangle^{K\pi} = \frac{\int d^2 b_\perp \; b_y \; \rho^{K\pi}(b_\perp, s_\perp)}{\int d^2 \; b_\perp \; \rho^{K\pi}(b_\perp, s_\perp)} = \frac{1}{2m_K} \frac{B_T^{K\pi}(t=0)}{f_+(t=0)} = (0.17, 0.15)\text{fm}$$

Transverse Spin Density



Summary & Outlook

- K to pi tensor transition form factor
- In good agreement with the lattice result
- Distorted spin structure during the transition process when the quark spin is polarized
- Transition GPDs of the pseudo scalar mesons

Thank you very much!