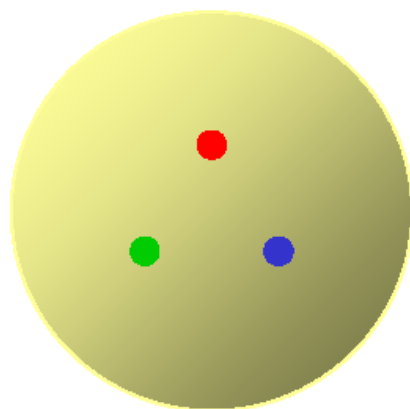
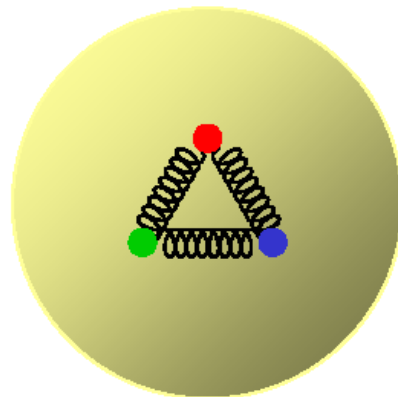
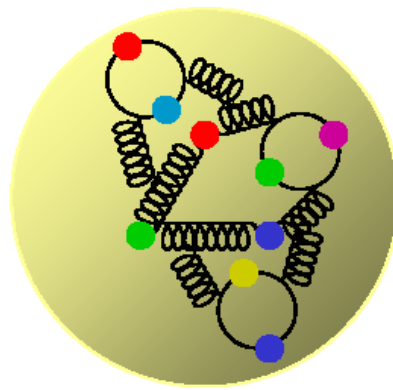


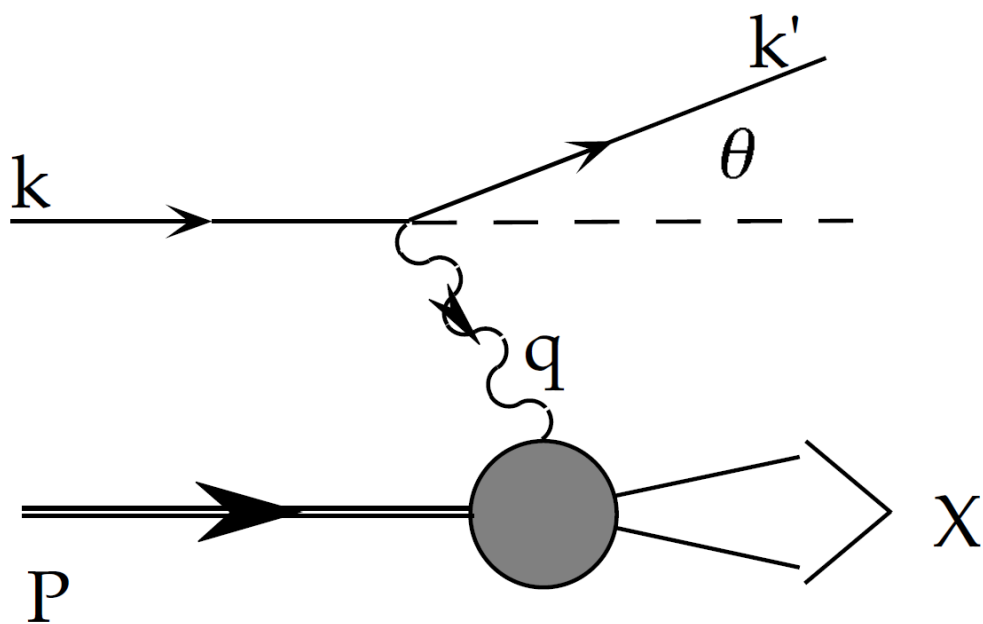
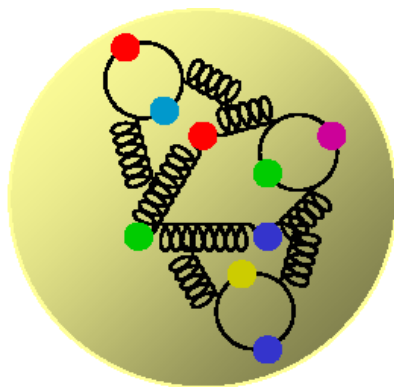
Inclusive and exclusive Drell-Yan at J-PARC

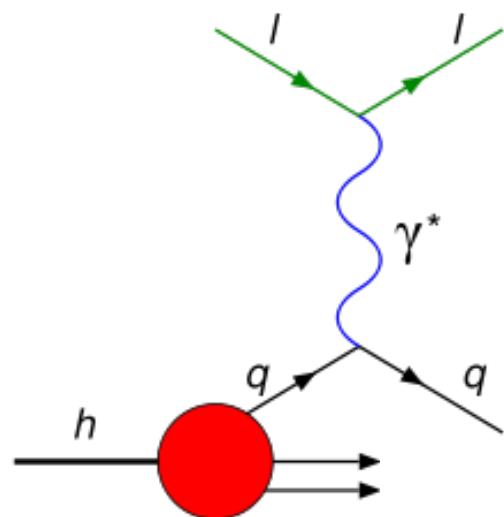
Kazuhiro Tanaka (Juntendo U/KEK)

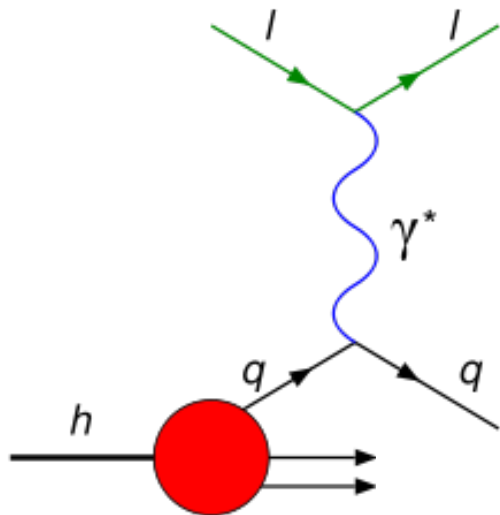










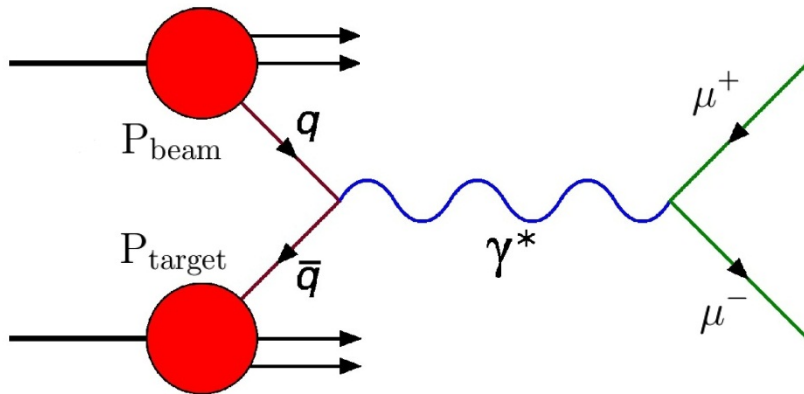
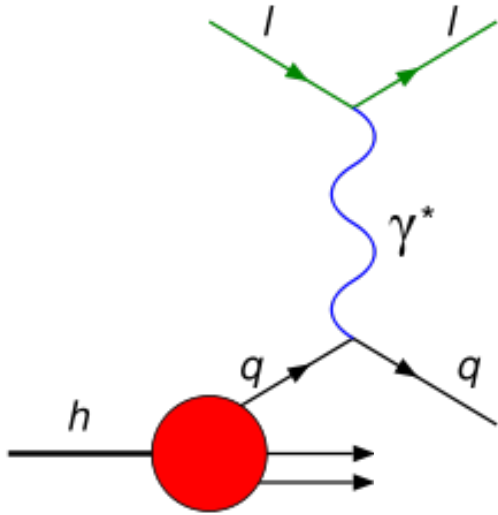


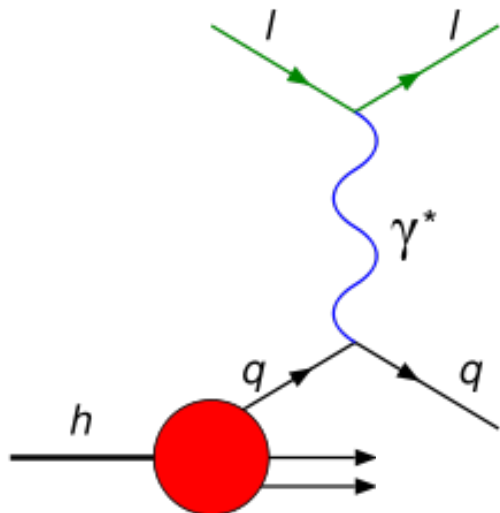
DIS

$$[q(x) + \bar{q}(x)], \quad G(x)$$

DIS

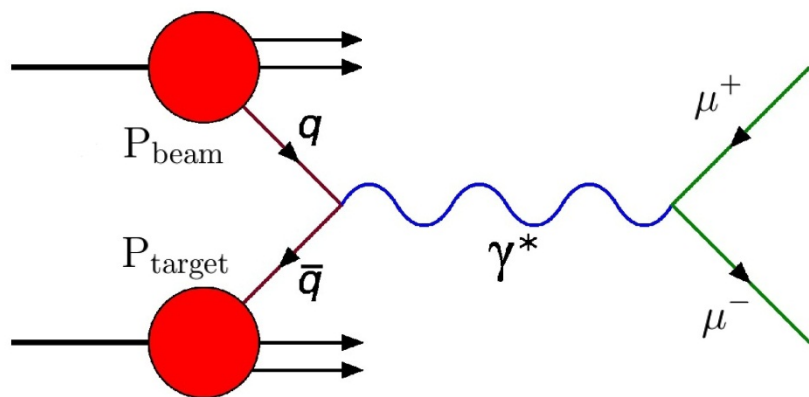
$$[q(x) + \bar{q}(x)], \quad G(x)$$





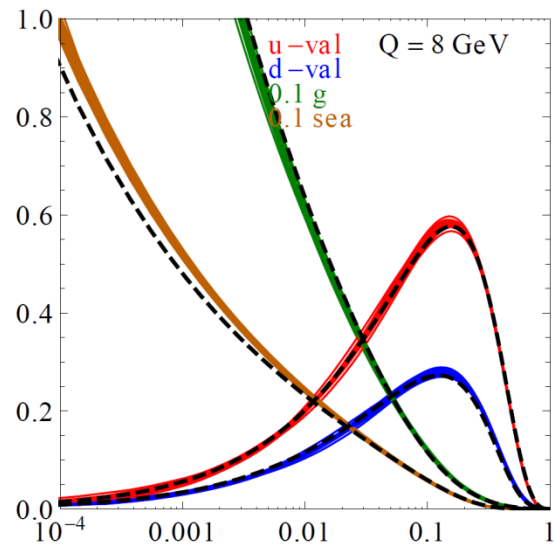
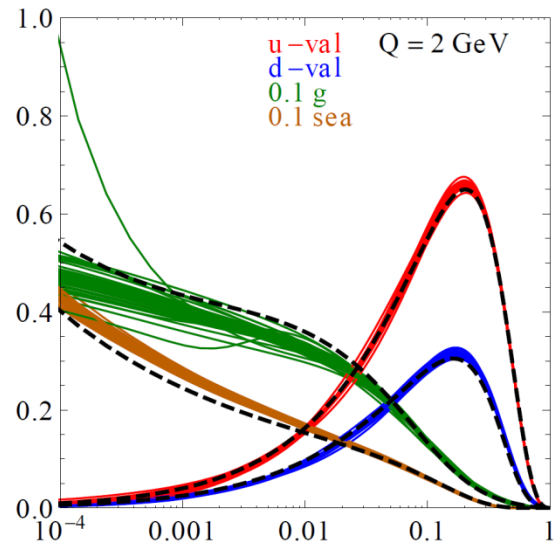
DIS

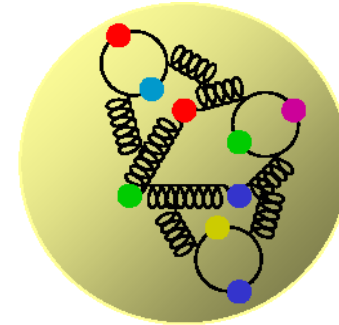
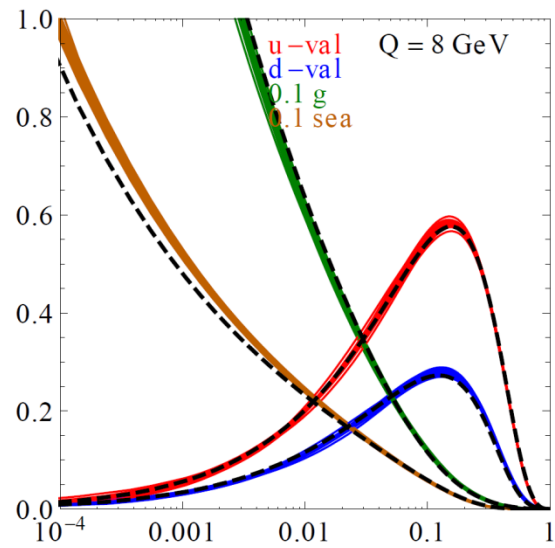
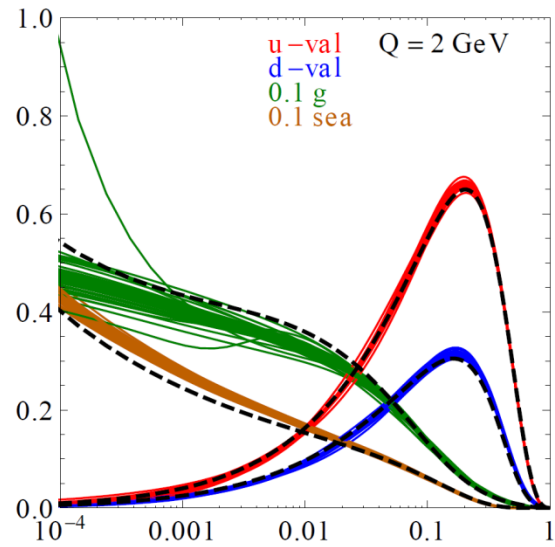
$$[q(x) + \bar{q}(x)], \quad G(x)$$

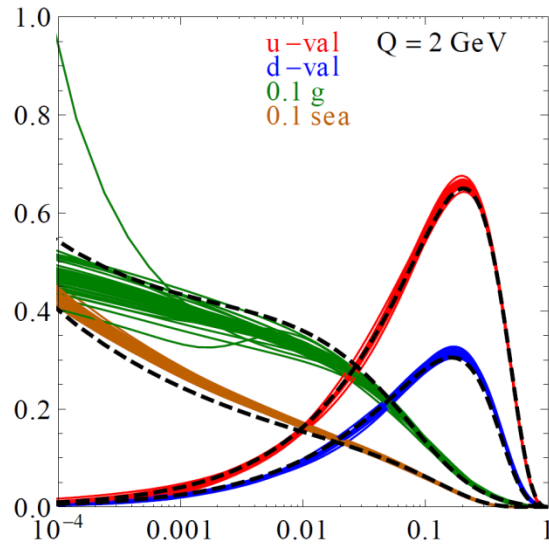
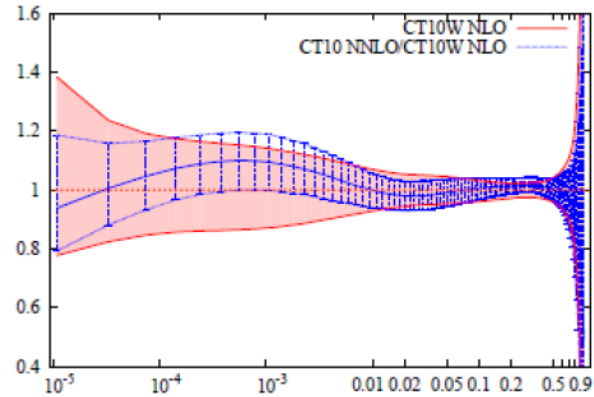
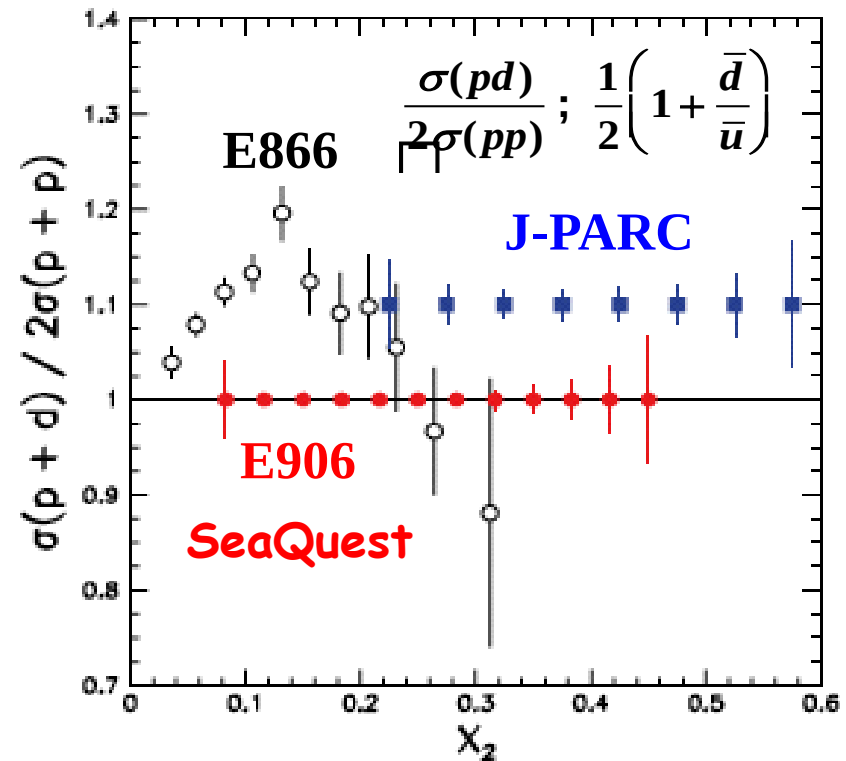
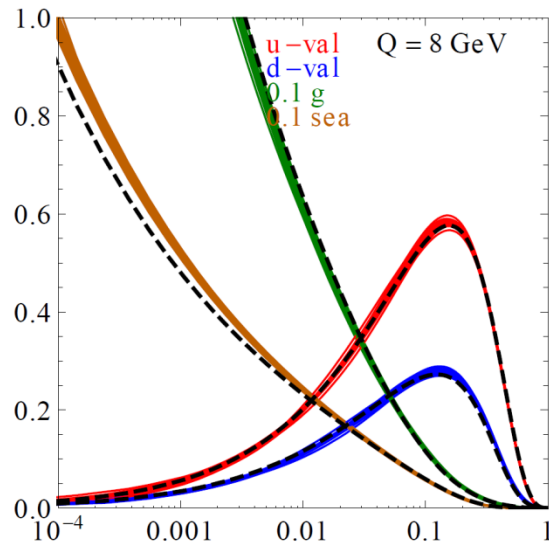
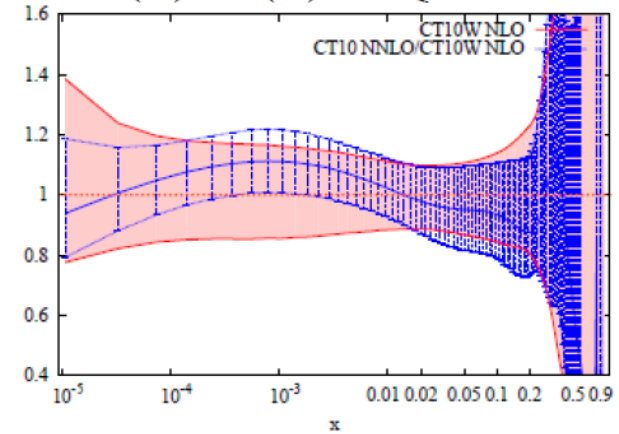


DY

$$q(x), \quad \bar{q}(x), \quad G(x)$$

$x f(x, Q)$ versus x 

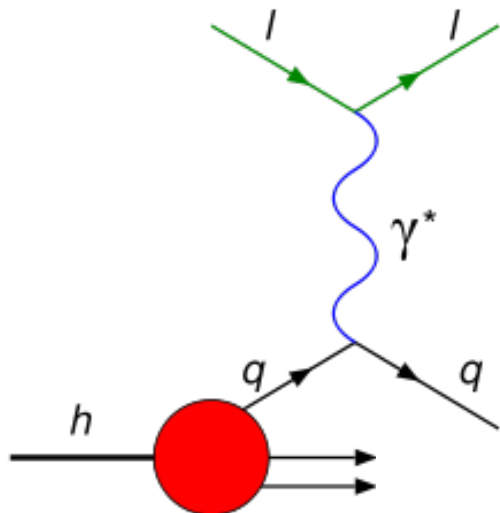
$x f(x, Q)$ versus x 

$x f(x, Q)$ versus x  $\delta u(x) / u(x)$ at $Q=2\text{GeV}$  $\delta \bar{u}(x) / \bar{u}(x)$ at $Q=2\text{GeV}$ 

High-momentum beamline

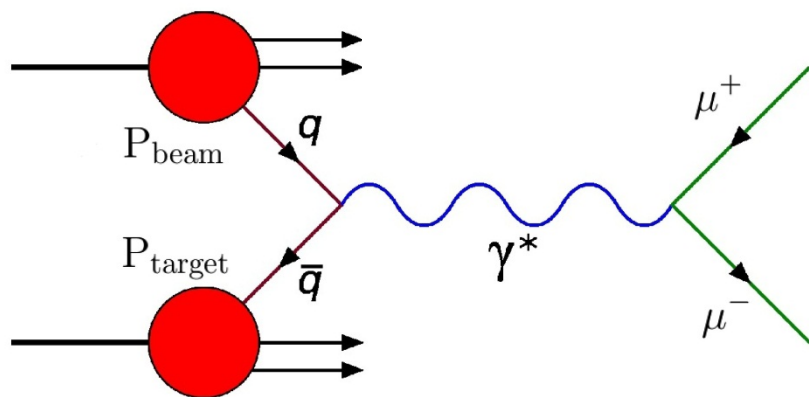
- 30 GeV proton

high intensity



DIS

$$[q(x) + \bar{q}(x)], \quad G(x)$$



DY

$$q(x), \quad \bar{q}(x), \quad G(x)$$

High-momentum beamline

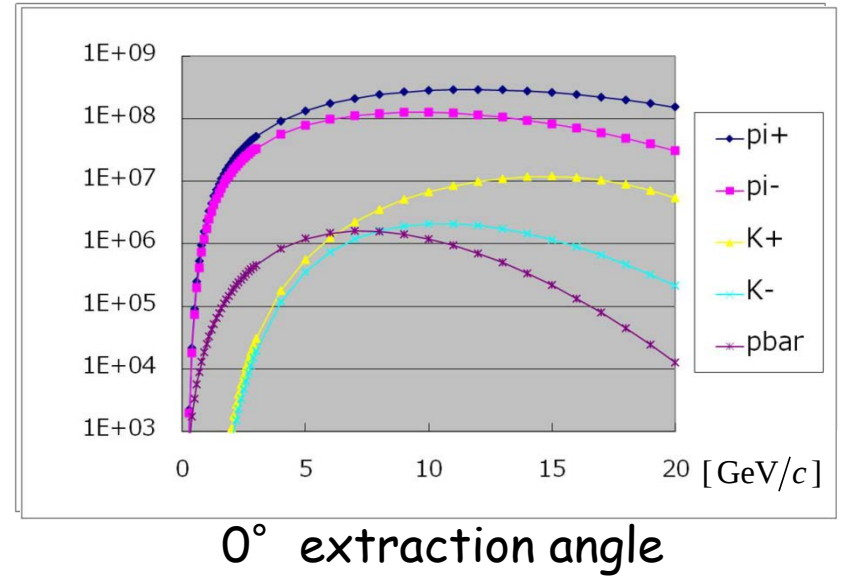
- 30 GeV proton

high intensity



beam loss limit @ SM1:15kW

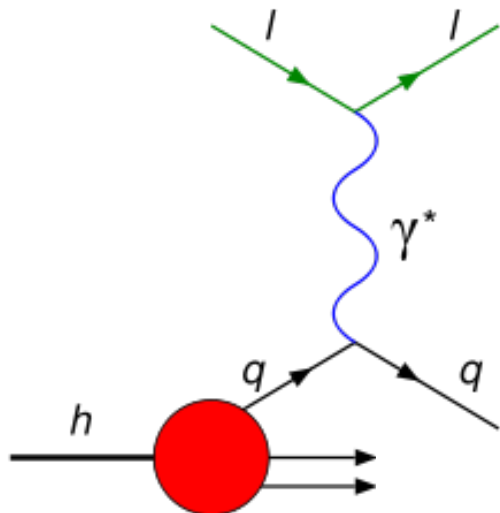
(limited by the thickness of the tunnel wall)



High-momentum beamline

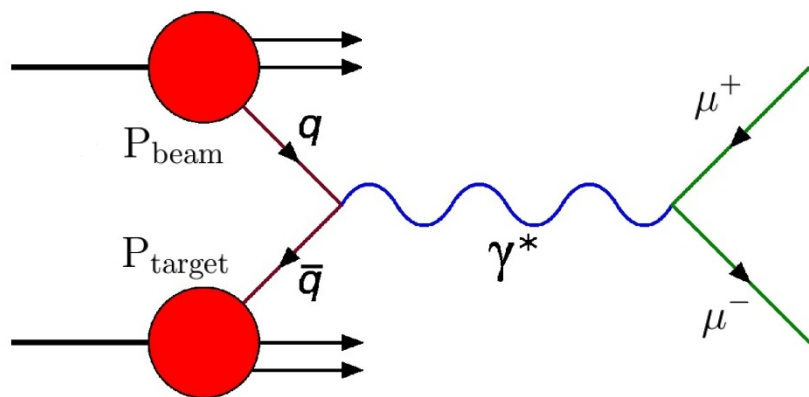
- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

high intensity



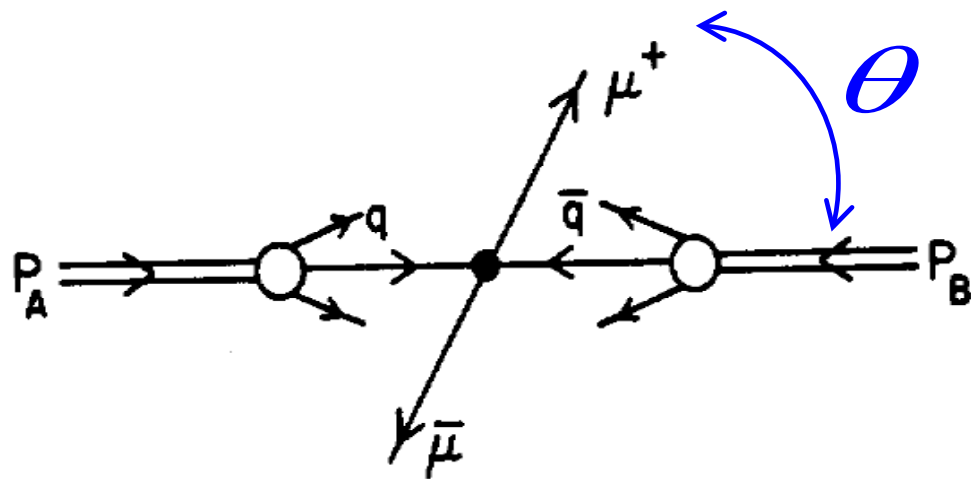
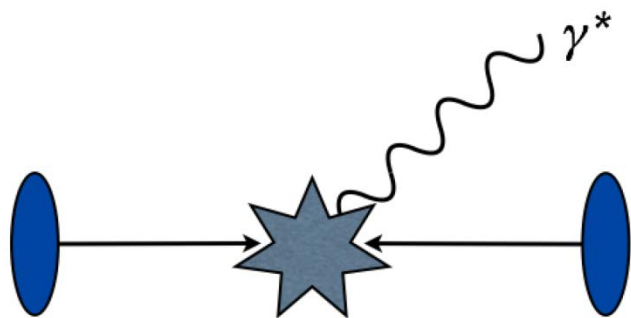
DIS

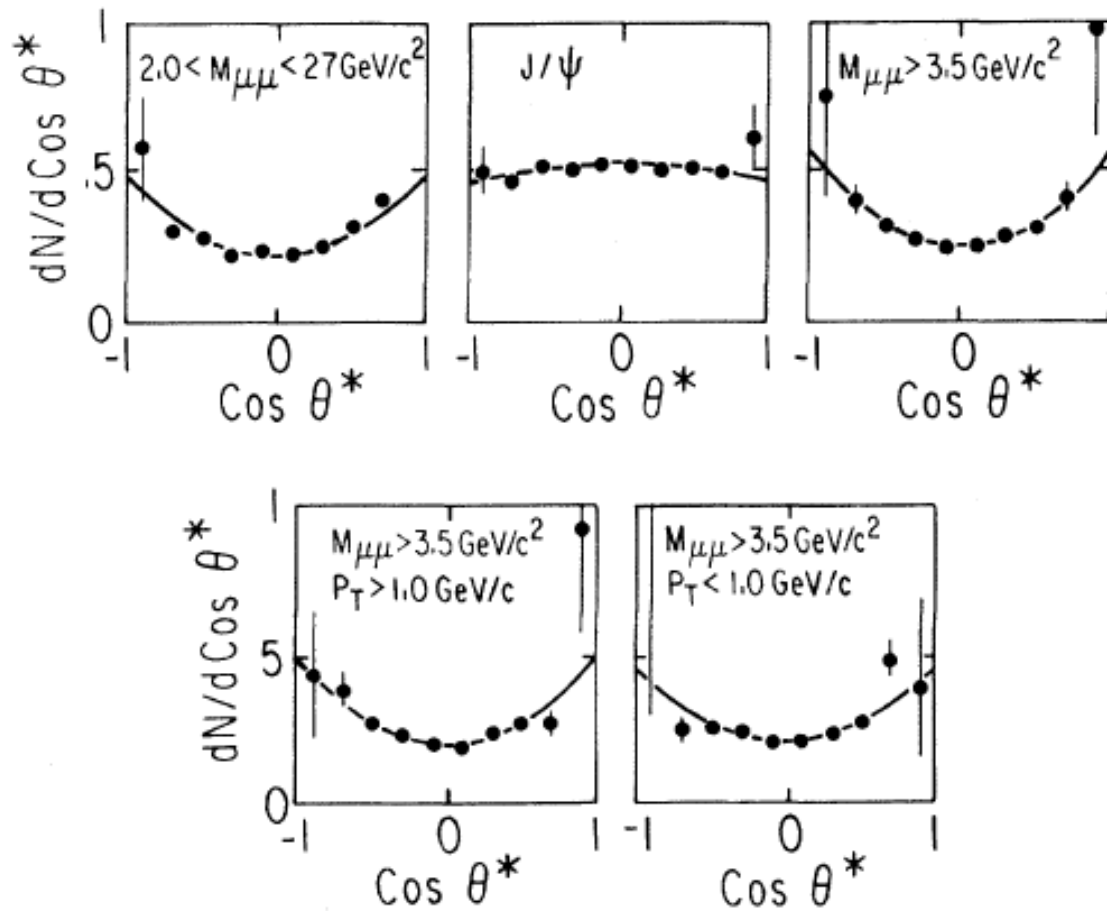
$$[q(x) + \bar{q}(x)], \quad G(x)$$



DY

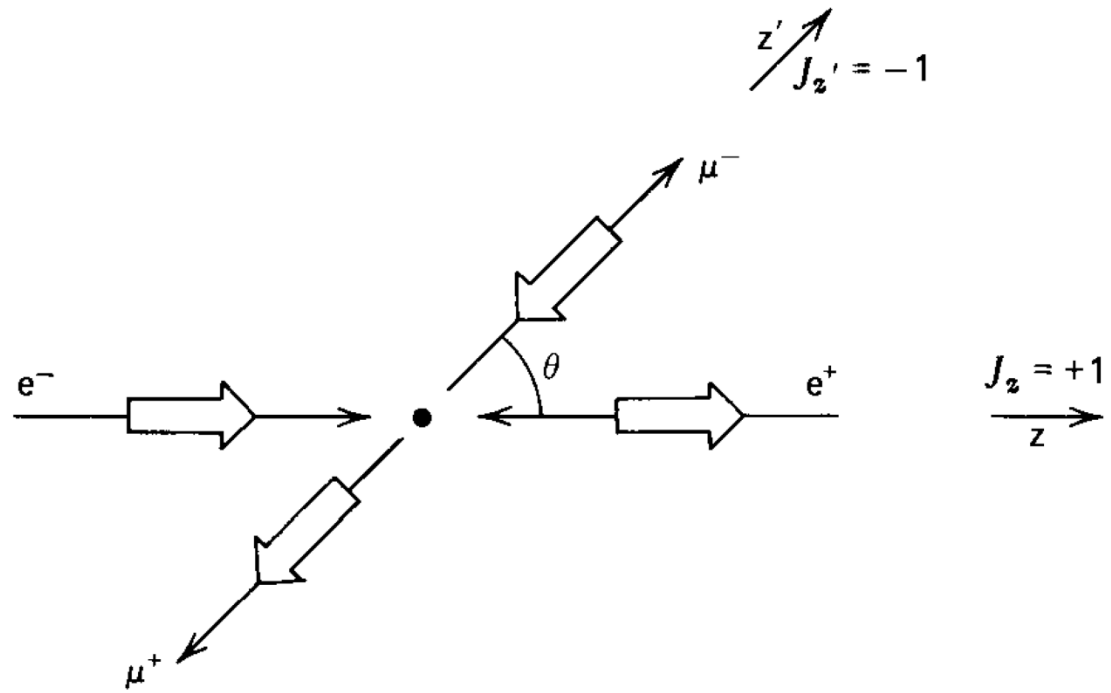
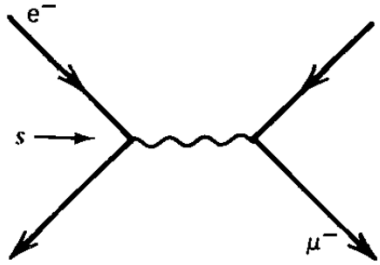
$$q(x), \quad \bar{q}(x), \quad G(x)$$





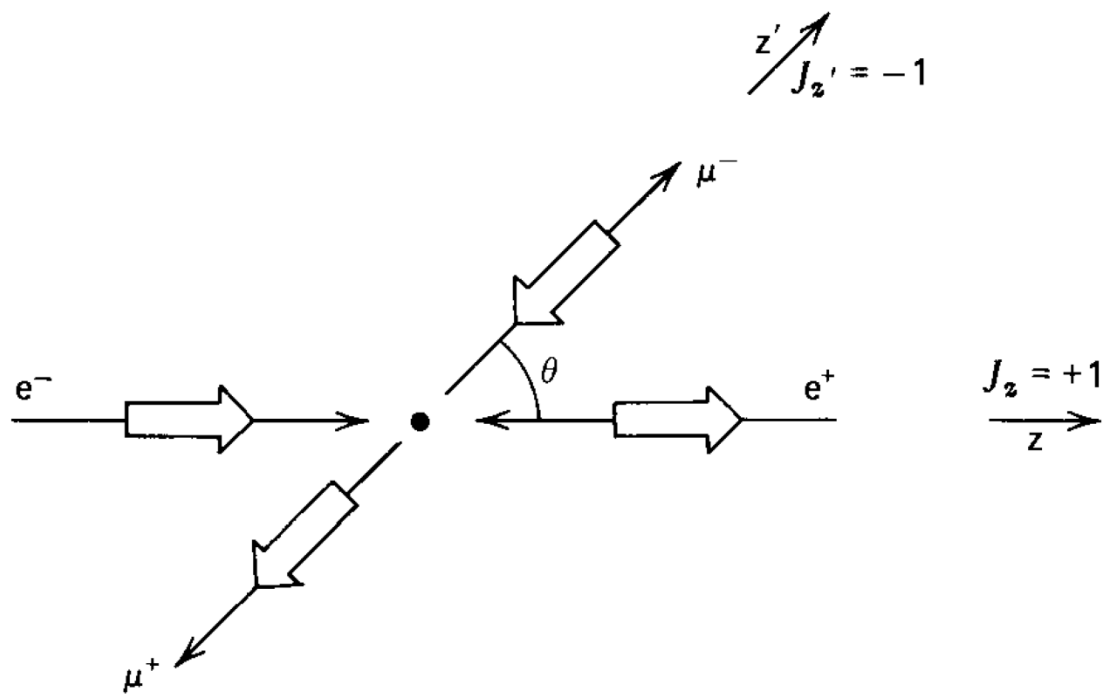
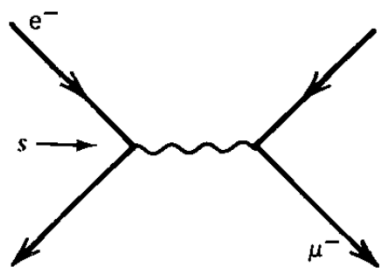
$$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta)$$

$$e^- e^+ \rightarrow \mu^+ \mu^-$$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)$$

$$e^- e^+ \rightarrow \mu^+ \mu^-$$



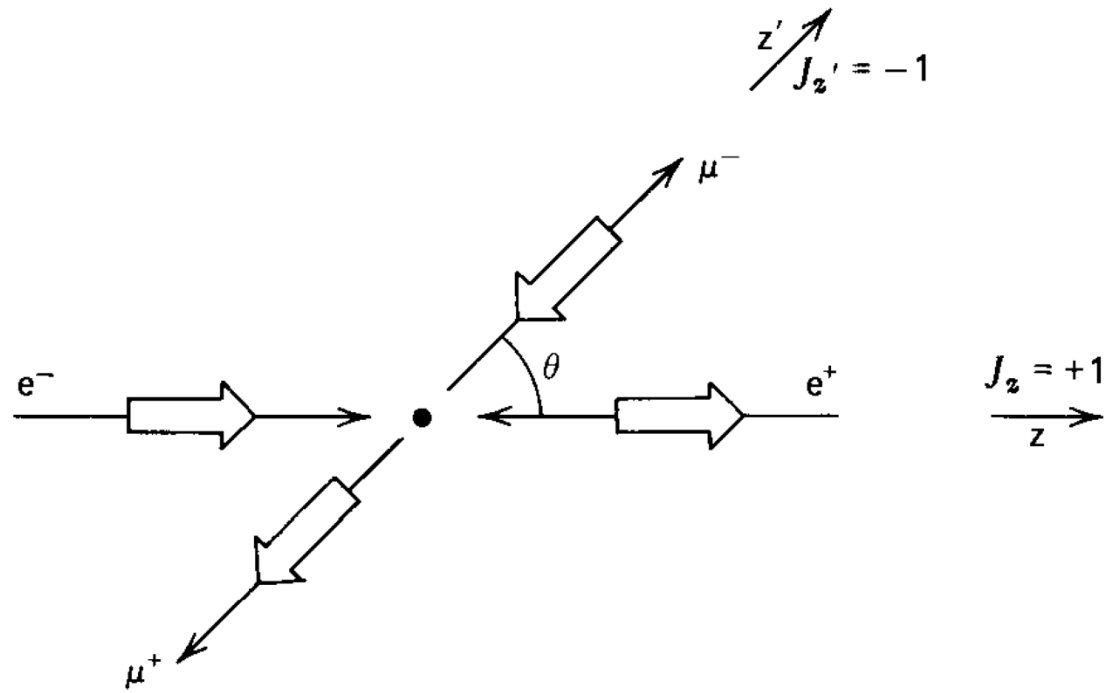
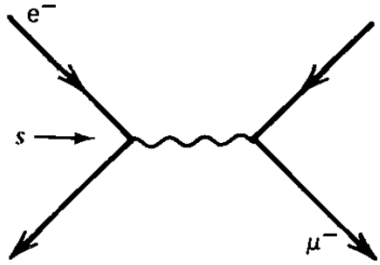
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left(1 + \cos^2 \theta\right) \propto \left|d_{-1\,1}^1(\theta)\right|^2 + \left|d_{1\,1}^1(\theta)\right|^2$$

$$d_{\lambda'\lambda}^j(\theta) = \langle j\lambda' | e^{-i\theta \hat{J}_y} | j\lambda \rangle$$

$$d_{-1\,1}^1(\theta) = d_{1\,-1}^1(\theta) = \frac{1 - \cos \theta}{2}$$

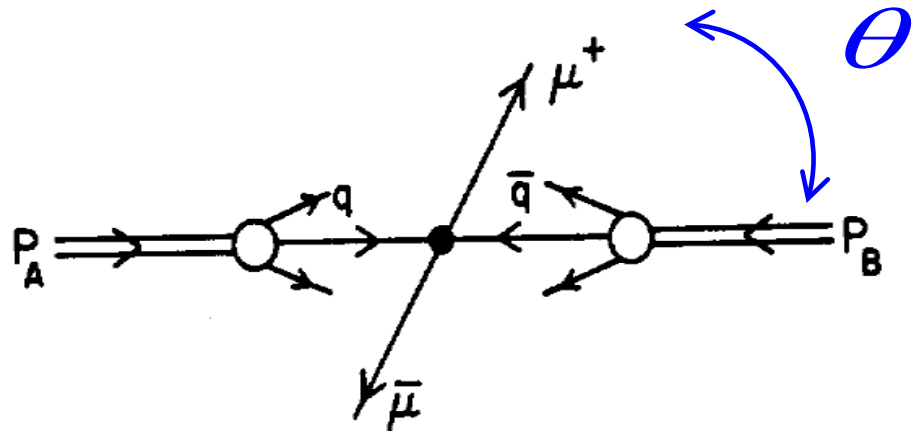
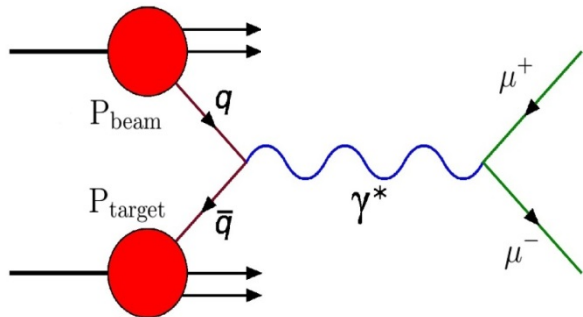
$$d_{1\,1}^1(\theta) = d_{-1\,-1}^1(\theta) = \frac{1 + \cos \theta}{2}$$

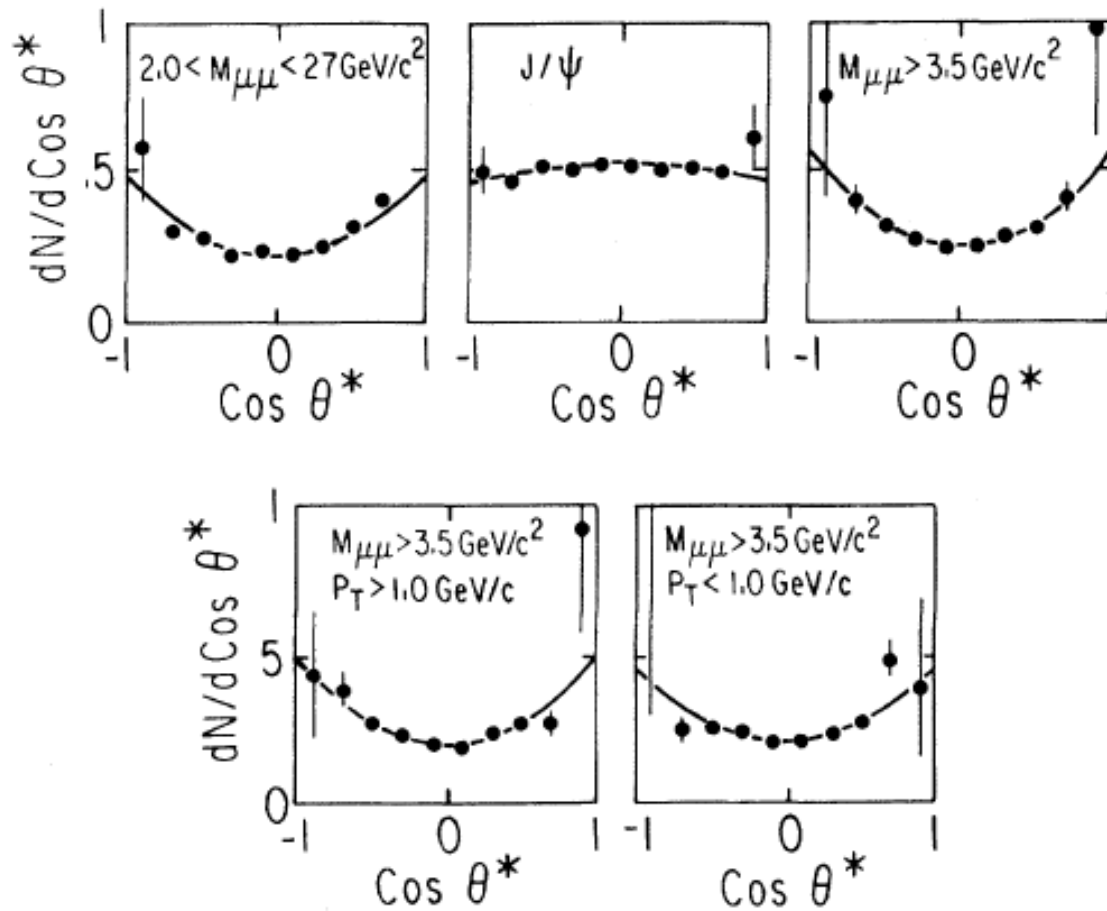
$$e^- e^+ \rightarrow \mu^+ \mu^-$$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) \propto |d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2$$

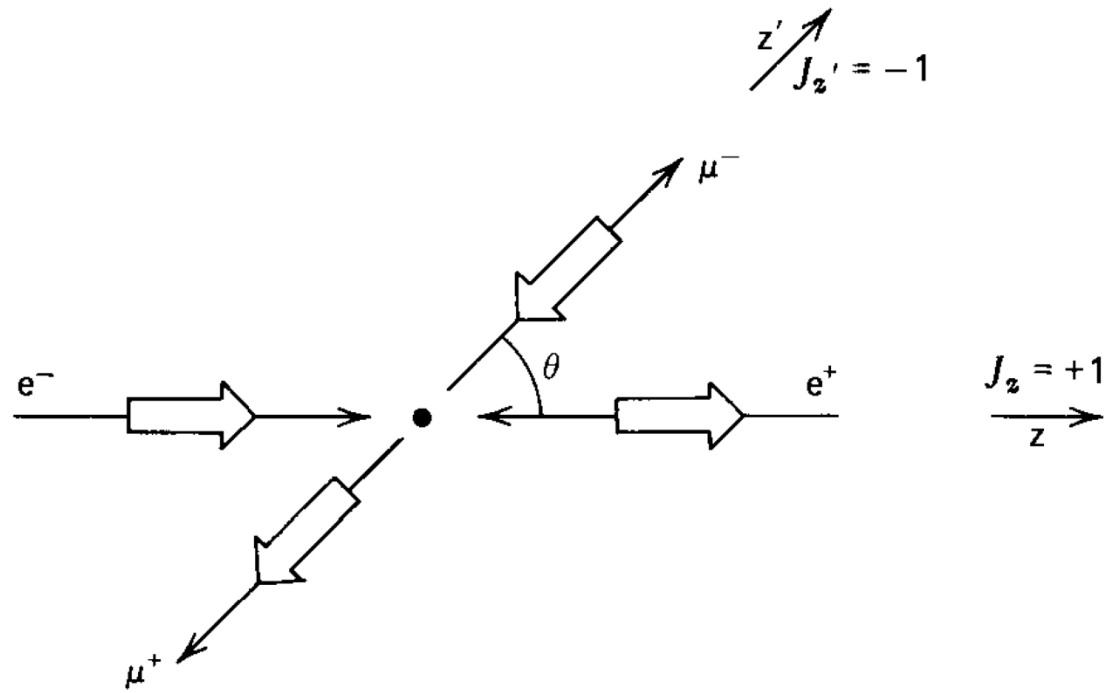
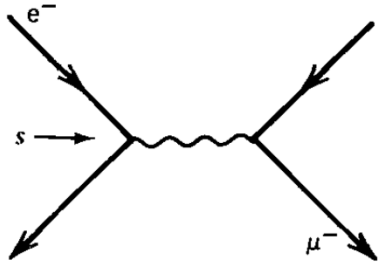
DY





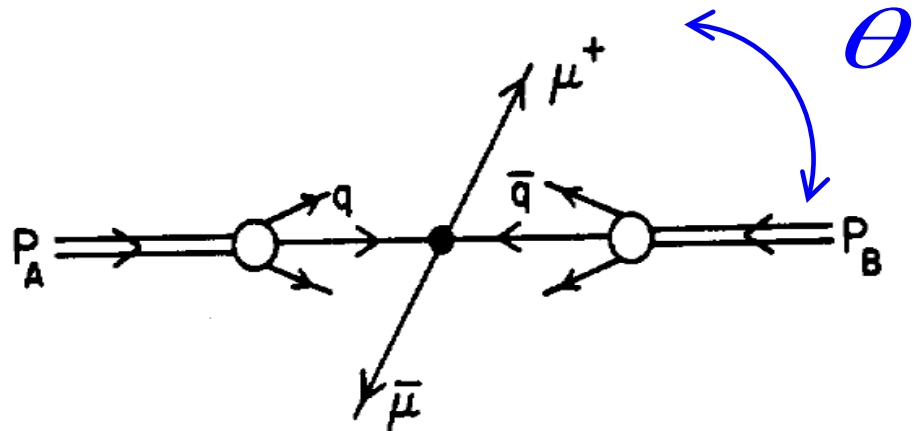
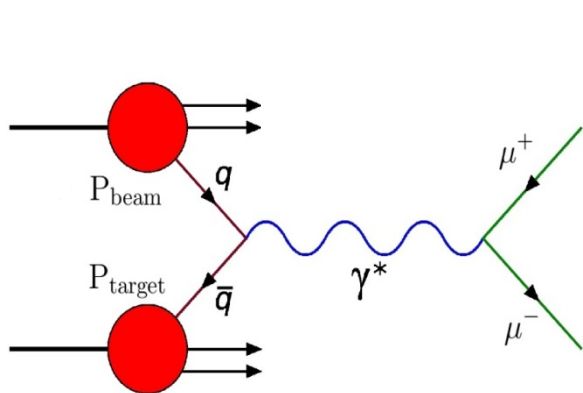
$$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta)$$

$$e^- e^+ \rightarrow \mu^+ \mu^-$$

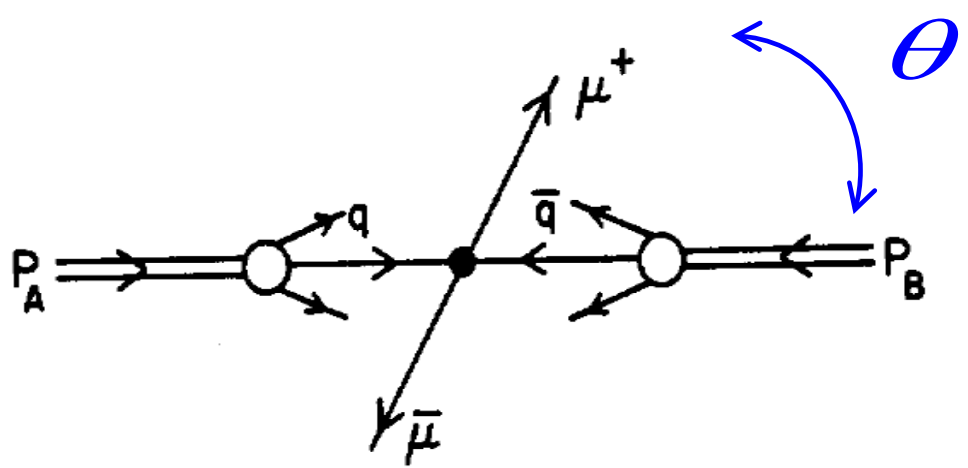
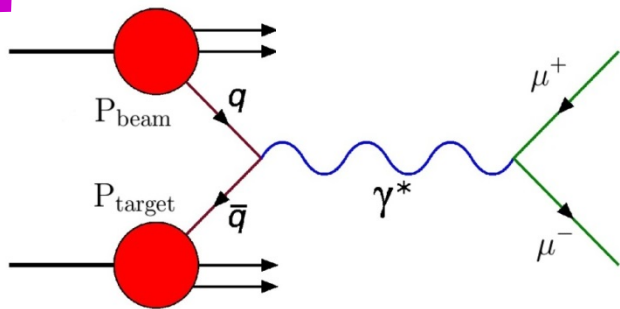


$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) \propto |d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2$$

DY



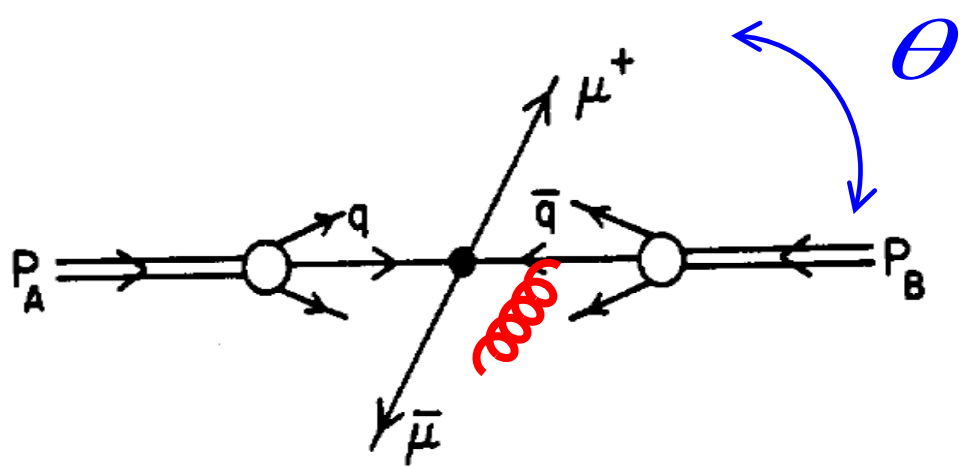
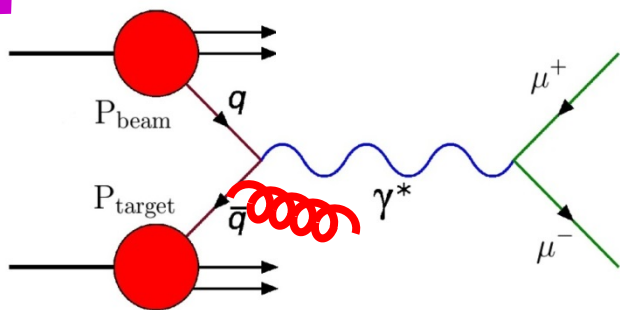
DY



$$\frac{d\sigma}{d\Omega} \propto \left| d_{-1\ 1}^1(\theta) \right|^2 + \left| d_{1\ 1}^1(\theta) \right|^2 \propto (1 + \cos^2 \theta)$$

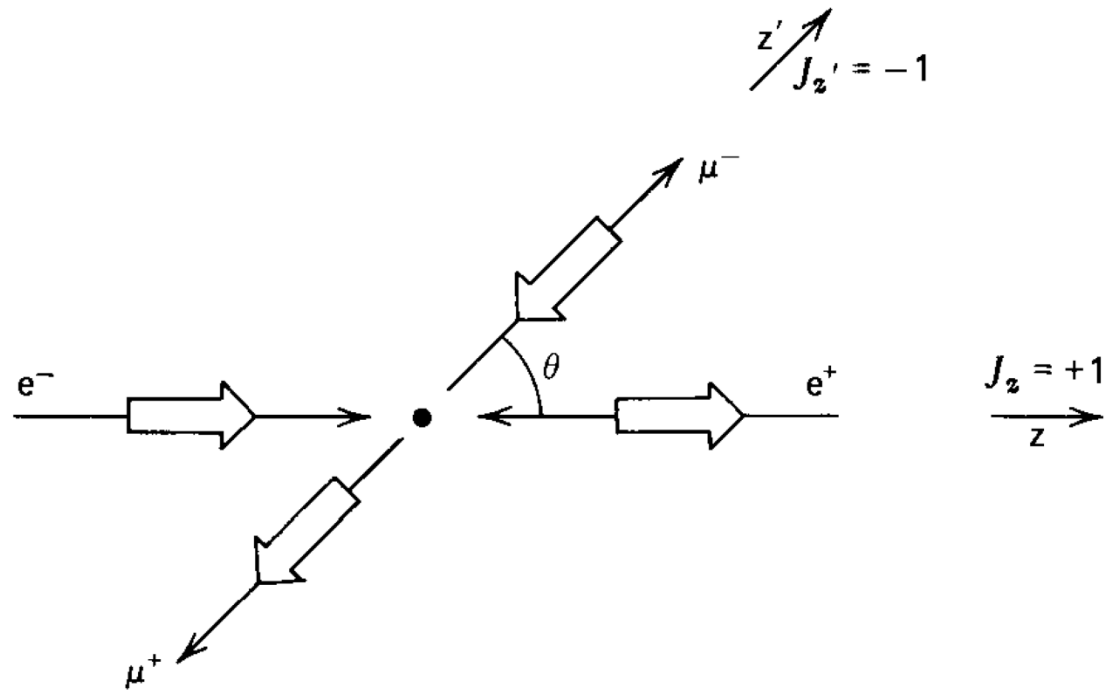
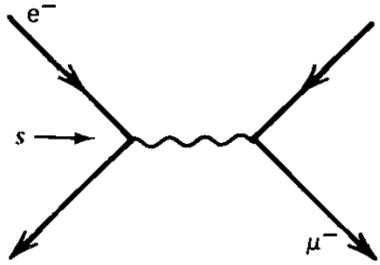
spin- $\frac{1}{2}$ quark

DY



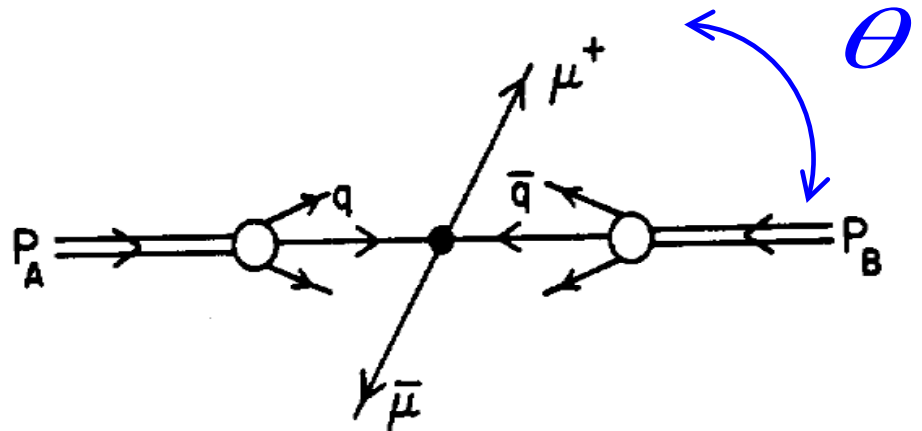
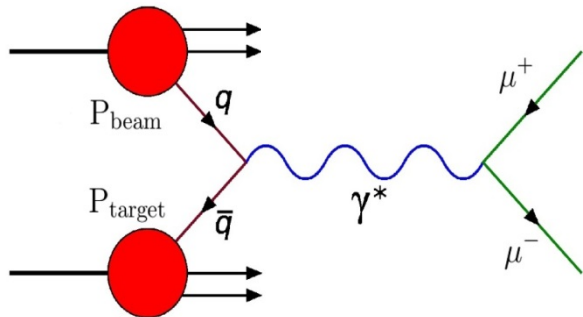
$$\frac{d\sigma}{d\Omega} \propto \left| d_{-1\ 1}^1(\theta) \right|^2 + \left| d_{1\ 1}^1(\theta) \right|^2 \propto (1 + \cos^2 \theta)$$

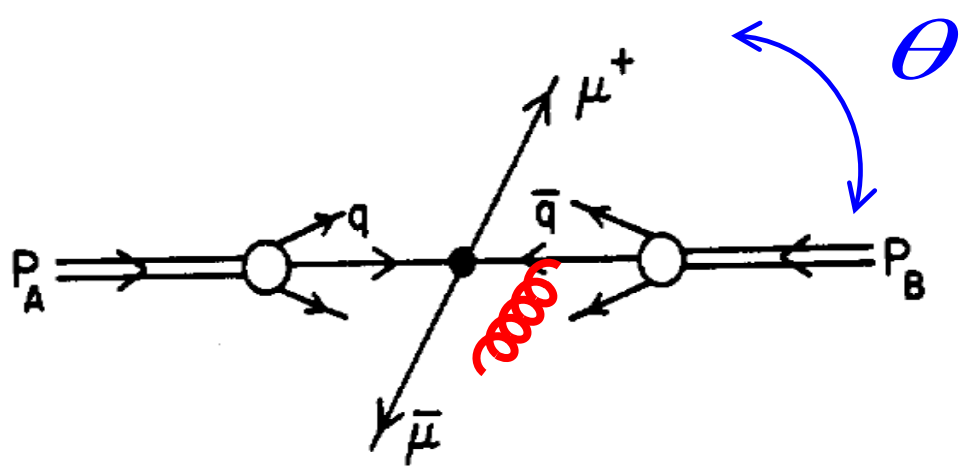
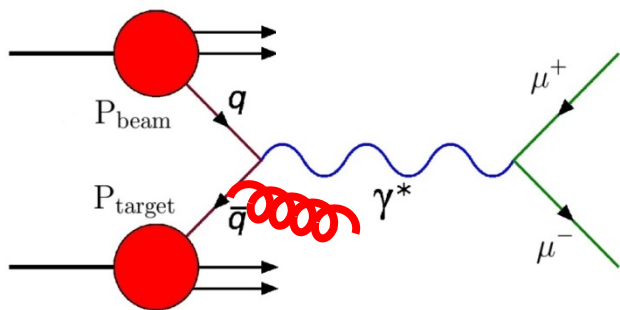
$$e^- e^+ \rightarrow \mu^+ \mu^-$$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta) \propto |d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2$$

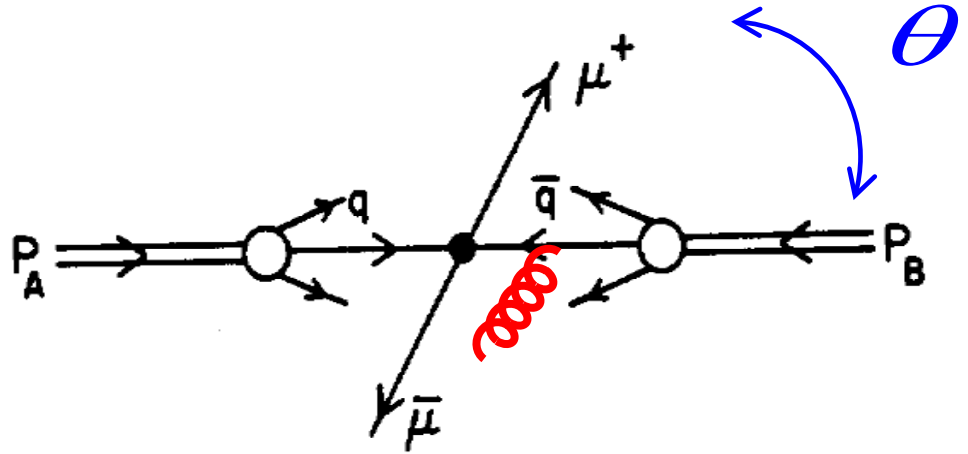
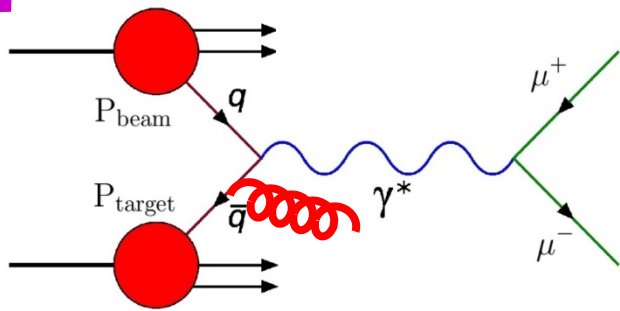
DY





$$\frac{d\sigma}{d\Omega} \propto \left| d_{-1\ 1}^1(\theta) \right|^2 + \left| d_{1\ 1}^1(\theta) \right|^2 \propto (1 + \cos^2 \theta)$$

DY



$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(\left| d_{-1\,1}^1(\theta) \right|^2 + \left| d_{1\,1}^1(\theta) \right|^2 \right) + \textcolor{red}{b} \left(\left| d_{-1\,0}^1(\theta) \right|^2 + \left| d_{1\,0}^1(\theta) \right|^2 \right)$$

$$D_{\lambda'\lambda}^j(\alpha, \theta, \phi) = e^{-i\lambda'\alpha} d_{\lambda'\lambda}^j(\theta) e^{-i\lambda\phi}$$

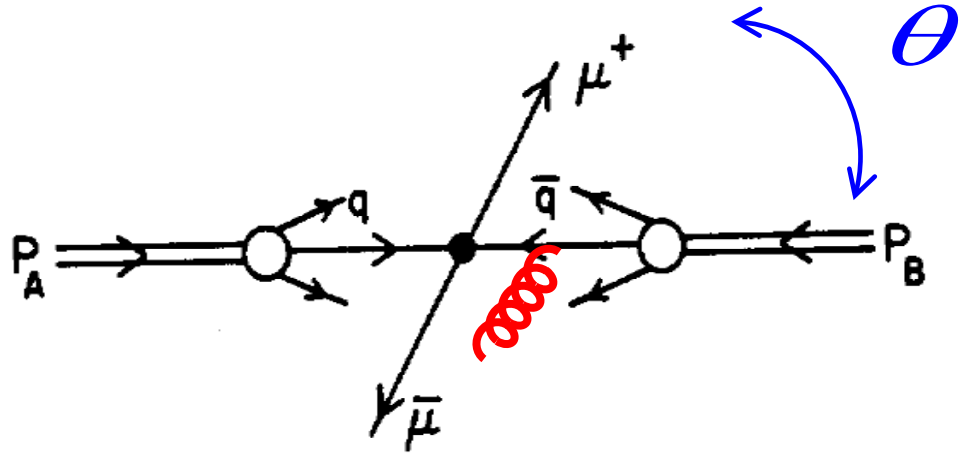
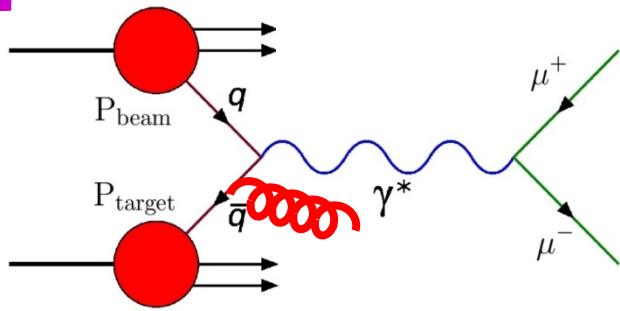
$$d_{\lambda'\lambda}^j(\theta) = \langle j\lambda' | e^{-i\theta \hat{J}_y} | j\lambda \rangle$$

$$d_{11}^1(\theta) = d_{-1\,-1}^1(\theta) = \frac{1 + \cos \theta}{2}$$

$$d_{-1\,1}^1(\theta) = d_{1\,-1}^1(\theta) = \frac{1 - \cos \theta}{2}$$

$$d_{-1\,0}^1(\theta) = -d_{1\,0}^1(\theta) = \frac{\sin \theta}{\sqrt{2}}$$

DY

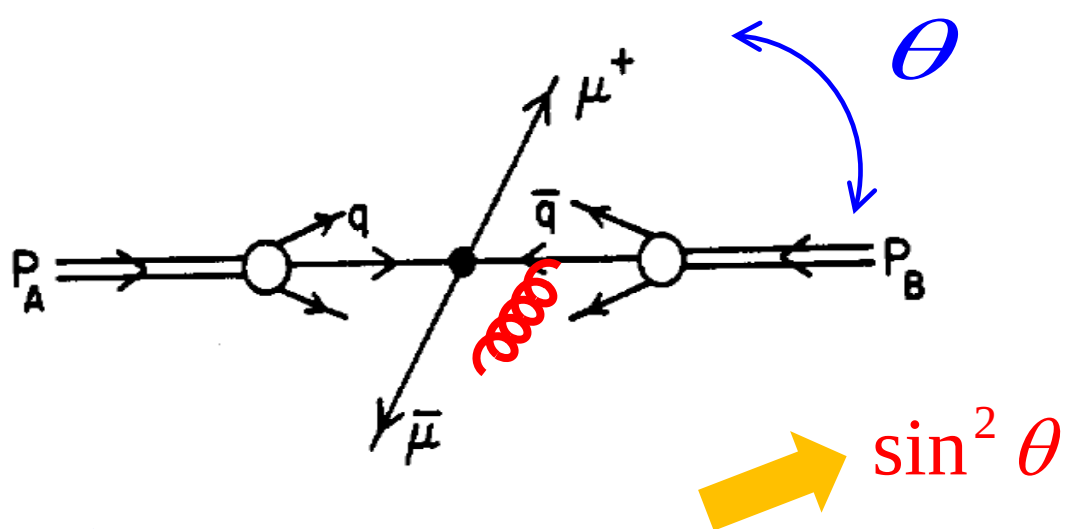
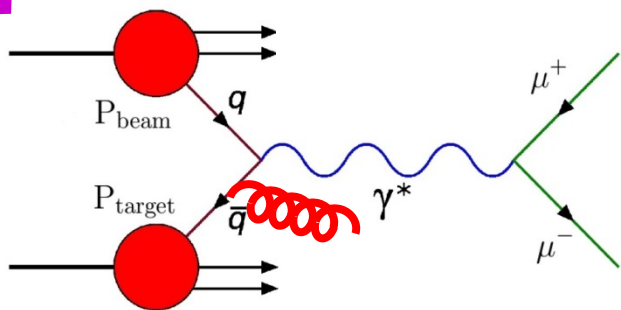


$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(\left| d_{-1\,1}^1(\theta) \right|^2 + \left| d_{1\,1}^1(\theta) \right|^2 \right) + \textcolor{red}{b} \left(\left| d_{-1\,0}^1(\theta) \right|^2 + \left| d_{1\,0}^1(\theta) \right|^2 \right) \\ + \textcolor{violet}{\Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

$$D_{\lambda'\lambda}^j(\alpha, \theta, \phi) = e^{-i\lambda'\alpha} d_{\lambda'\lambda}^j(\theta) e^{-i\lambda\phi} \\ d_{\lambda'\lambda}^j(\theta) = \langle j\lambda' | e^{-i\theta\hat{J}_y} | j\lambda \rangle$$

$$d_{1\,1}^1(\theta) = d_{-1\,-1}^1(\theta) = \frac{1 + \cos\theta}{2} \quad d_{-1\,0}^1(\theta) = -d_{1\,0}^1(\theta) = \frac{\sin\theta}{\sqrt{2}} \\ d_{-1\,1}^1(\theta) = d_{1\,-1}^1(\theta) = \frac{1 - \cos\theta}{2}$$

DY



$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(|d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2 \right) + \textcolor{red}{b} \left(|d_{-1\,0}^1(\theta)|^2 + |d_{1\,0}^1(\theta)|^2 \right)$$

$$+ \textcolor{violet}{\Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

$1 + \cos^2 \theta$
 $\sin \theta \cos \theta \cos \phi$
 $\sin^2 \theta \cos 2\phi$

$$D_{\lambda'\lambda}^j(\alpha, \theta, \phi) = e^{-i\lambda'\alpha} d_{\lambda'\lambda}^j(\theta) e^{-i\lambda\phi}$$

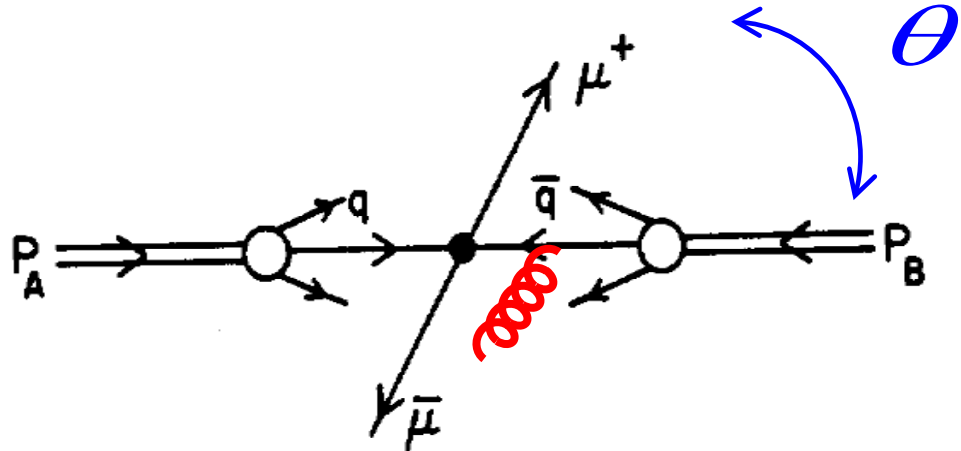
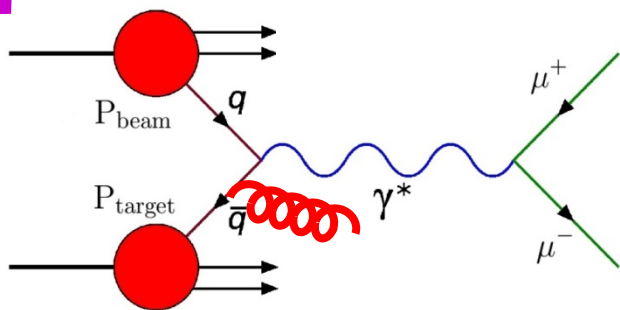
$$d_{\lambda'\lambda}^j(\theta) = \langle j\lambda' | e^{-i\theta \hat{J}_y} | j\lambda \rangle$$

$$d_{1\,1}^1(\theta) = d_{-1\,-1}^1(\theta) = \frac{1 + \cos \theta}{2}$$

$$d_{-1\,1}^1(\theta) = d_{1\,-1}^1(\theta) = \frac{1 - \cos \theta}{2}$$

$$d_{-1\,0}^1(\theta) = -d_{1\,0}^1(\theta) = \frac{\sin \theta}{\sqrt{2}}$$

DY

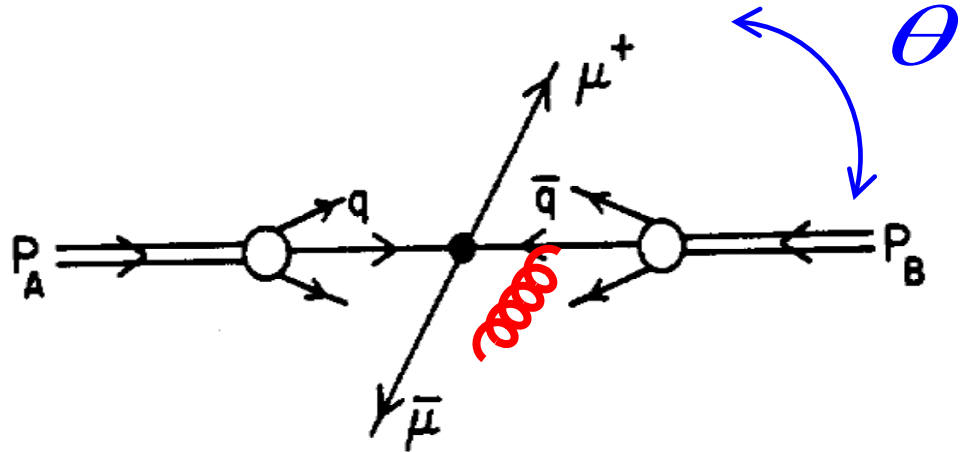
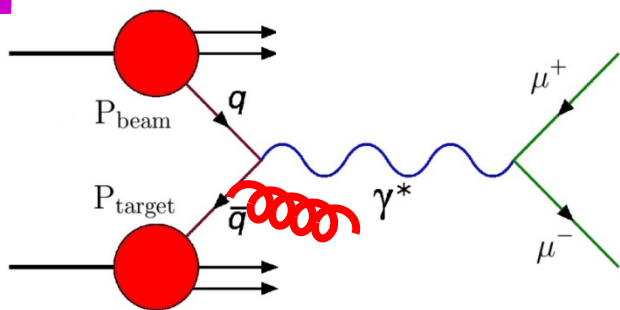


$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(\left| d_{-1\,1}^1(\theta) \right|^2 + \left| d_{1\,1}^1(\theta) \right|^2 \right) + \textcolor{red}{b} \left(\left| d_{-1\,0}^1(\theta) \right|^2 + \left| d_{1\,0}^1(\theta) \right|^2 \right) \\ + \textcolor{violet}{\Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

➡

$$\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \textcolor{red}{\lambda} \cos^2 \theta + \textcolor{red}{\mu} \sin 2\theta \cos \phi + \frac{\textcolor{red}{\nu}}{2} \sin^2 \theta \cos 2\phi$$

DY

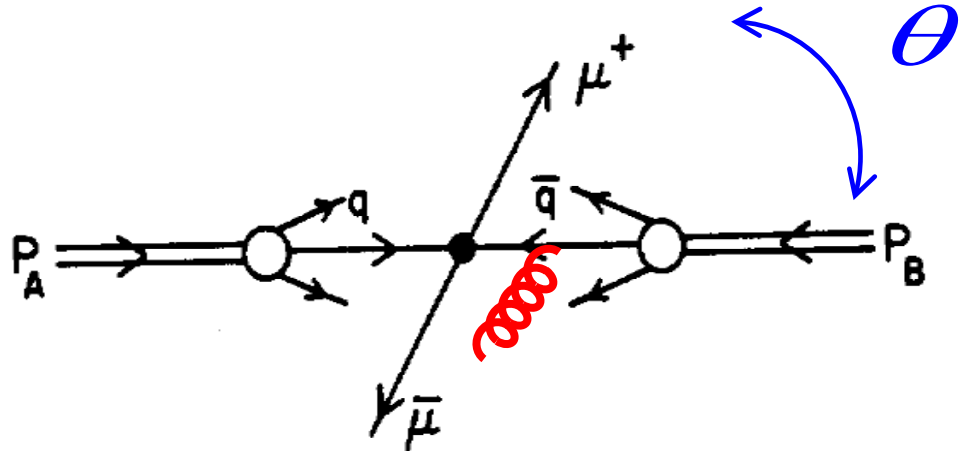
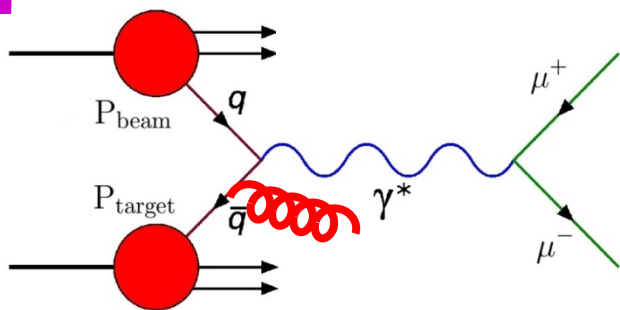


$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(|d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2 \right) + \textcolor{red}{b} \left(|d_{-1\,0}^1(\theta)|^2 + |d_{1\,0}^1(\theta)|^2 \right) \\ + \textcolor{violet}{\Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

➔ $\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \textcolor{red}{\lambda} \cos^2 \theta + \textcolor{red}{\mu} \sin 2\theta \cos \phi + \frac{\textcolor{red}{v}}{2} \sin^2 \theta \cos 2\phi$

$\textcolor{red}{1} - \textcolor{red}{\lambda}, \quad \textcolor{red}{\mu}, \quad \textcolor{red}{v} : \propto Q_T \text{ at } O(\alpha_s)$

DY



$$\frac{d\sigma}{d\Omega} \propto (1 + \lambda) \left(|d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2 \right) + \nu \left(|d_{-1\,0}^1(\theta)|^2 + |d_{1\,0}^1(\theta)|^2 \right) + \Re \left[c_1 d_{1\,0}^1(\theta) (d_{1\,1}^1(\theta) e^{-i\phi})^* + c_2 d_{1\,-1}^1(\theta) e^{i\phi} (d_{1\,1}^1(\theta) e^{-i\phi})^* + \dots \right]$$

$$\Rightarrow \frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

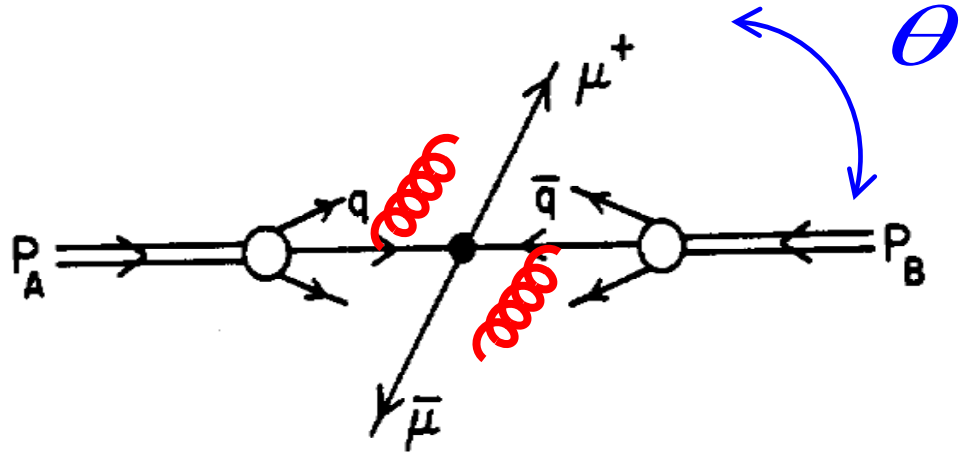
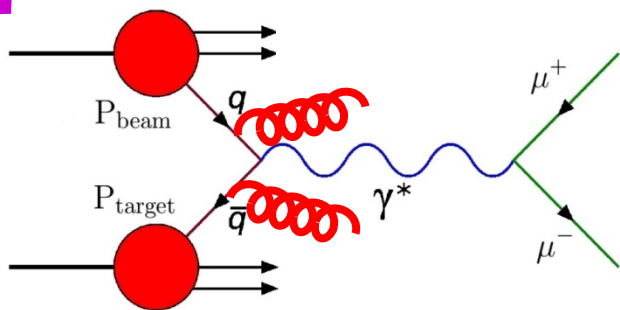
$$1 - \lambda, \quad \mu, \quad \nu : \propto Q_T \text{ at } O(\alpha_s)$$

$$1 - \lambda - 2\nu = 0$$

Lam-Tung relation ('78)

spin-1 gluon

DY



$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(|d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2 \right) + \textcolor{red}{b} \left(|d_{-1\,0}^1(\theta)|^2 + |d_{1\,0}^1(\theta)|^2 \right) \\ + \textcolor{violet}{\Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

➔ $\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \textcolor{red}{\lambda} \cos^2 \theta + \textcolor{red}{\mu} \sin 2\theta \cos \phi + \frac{\textcolor{red}{v}}{2} \sin^2 \theta \cos 2\phi$

$\textcolor{red}{1} - \textcolor{red}{\lambda}, \quad \textcolor{red}{\mu}, \quad \textcolor{red}{v} : \propto Q_T \text{ at } O(\alpha_s)$

$\textcolor{blue}{1} - \textcolor{blue}{\lambda} - 2\textcolor{blue}{v} \simeq 0$

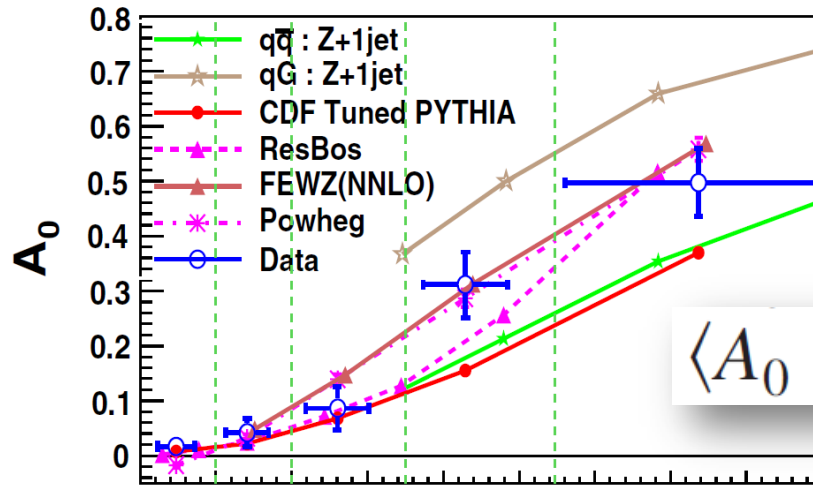
Lam-Tung relation ('78)

spin-1 gluon

CDF (PRL 106, 241801 (2011))

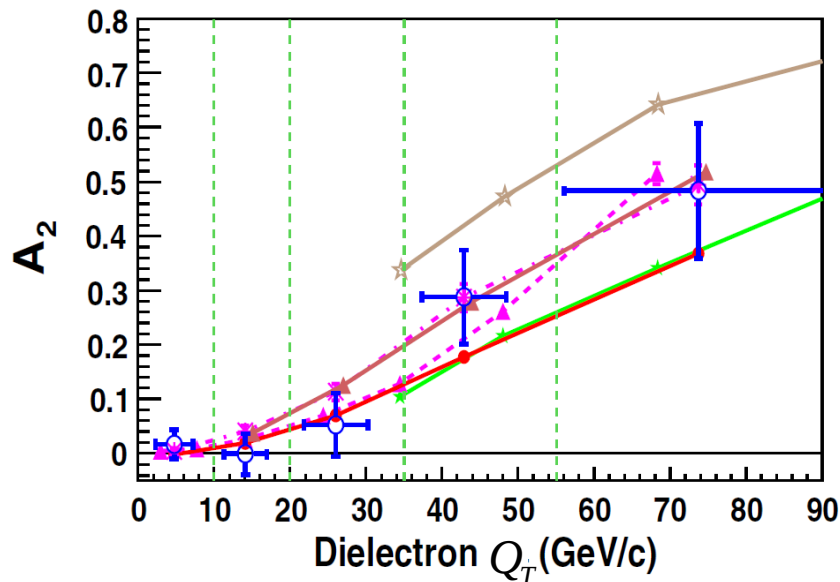
Angular Distribution of p-pbar DY at Z pole

$$A_0 = \frac{2(1-\lambda)}{3+\lambda}$$

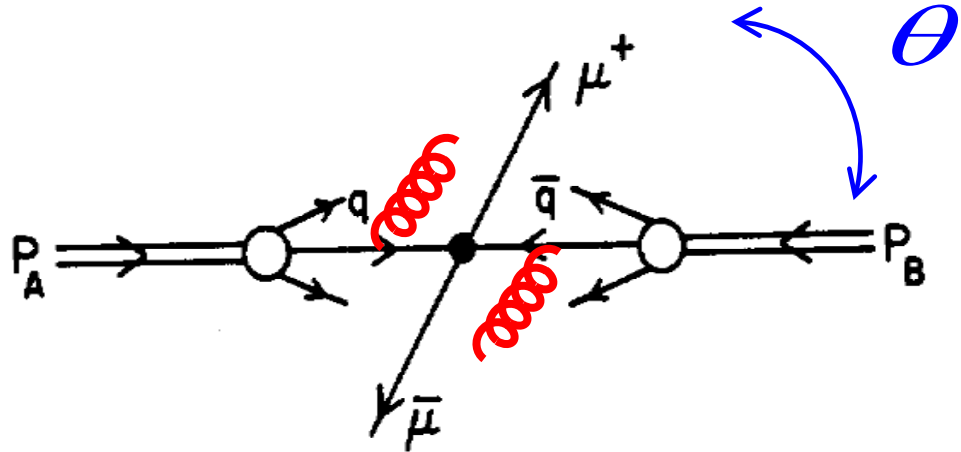
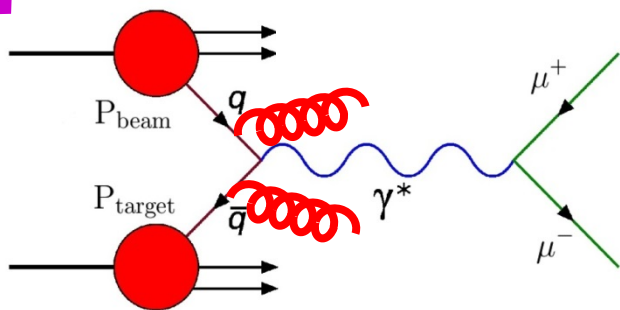


$$\langle A_0 - A_2 \rangle = 0.02 \pm 0.02$$

$$A_2 = \frac{4\nu}{3+\lambda}$$



DY



$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(|d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2 \right) + \textcolor{red}{b} \left(|d_{-1\,0}^1(\theta)|^2 + |d_{1\,0}^1(\theta)|^2 \right) \\ + \textcolor{violet}{Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

 $\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \textcolor{red}{\lambda} \cos^2 \theta + \textcolor{red}{\mu} \sin 2\theta \cos \phi + \frac{\textcolor{red}{\nu}}{2} \sin^2 \theta \cos 2\phi$

$\textcolor{red}{1} - \textcolor{red}{\lambda}, \quad \textcolor{red}{\mu}, \quad \textcolor{red}{\nu} : \propto Q_T \text{ at } O(\alpha_s)$

$\textcolor{blue}{1} - \textcolor{blue}{\lambda} - 2\textcolor{blue}{\nu} \simeq 0$

Lam-Tung relation ('78)

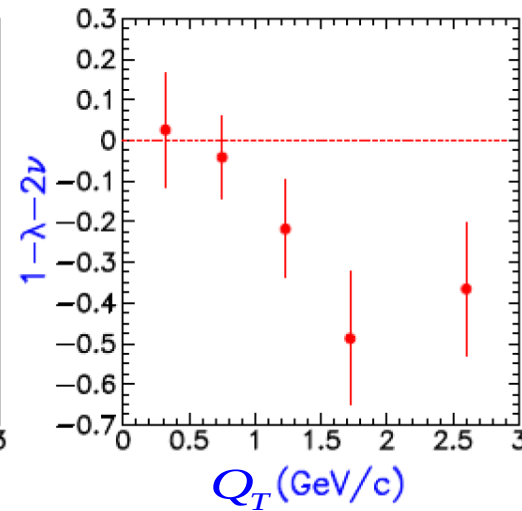
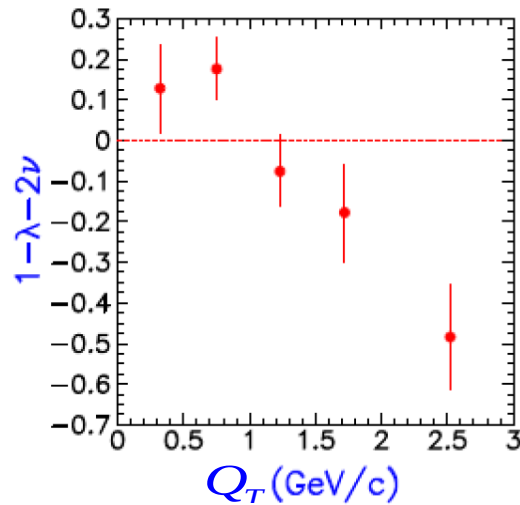
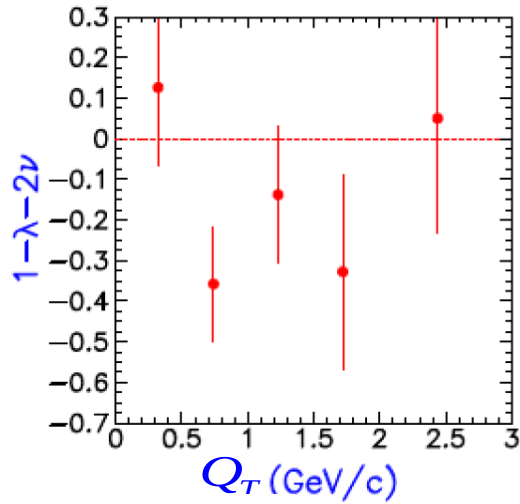
spin-1 gluon

NA10 (Z. Phys. C 37, 545 (1988))

140 GeV/c

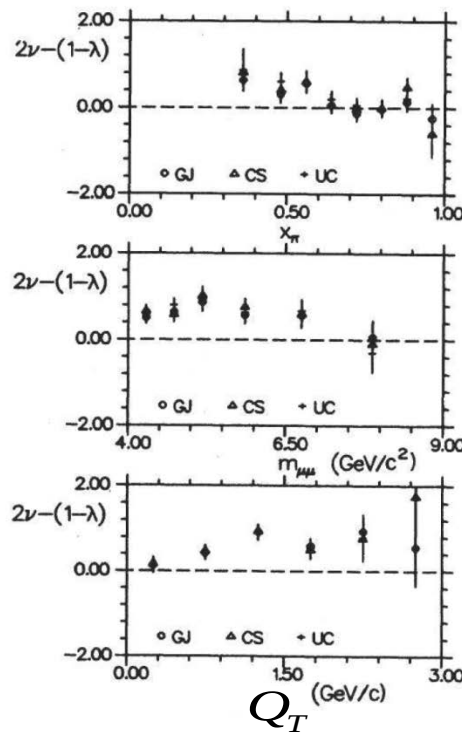
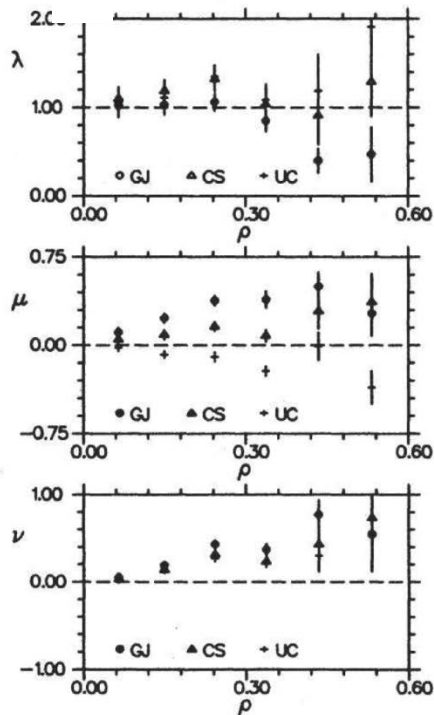
194 GeV/c

286 GeV/c



π^- -tungsten

π^- -deuterium

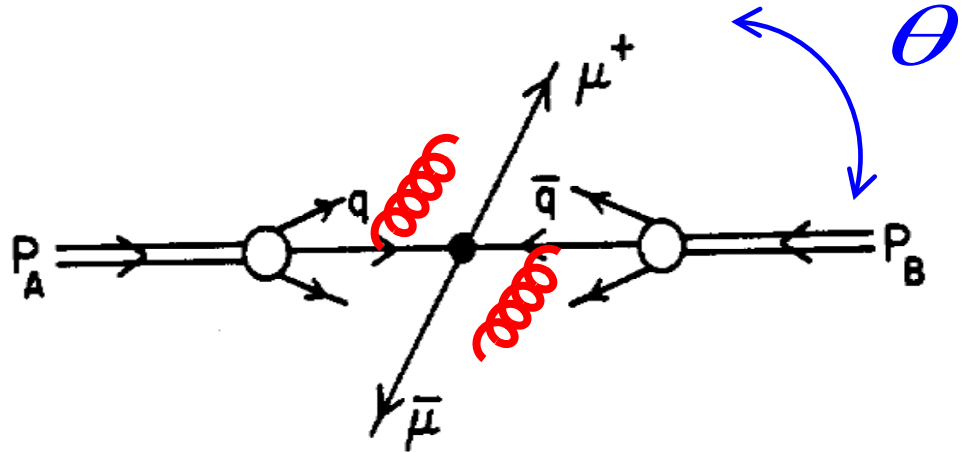
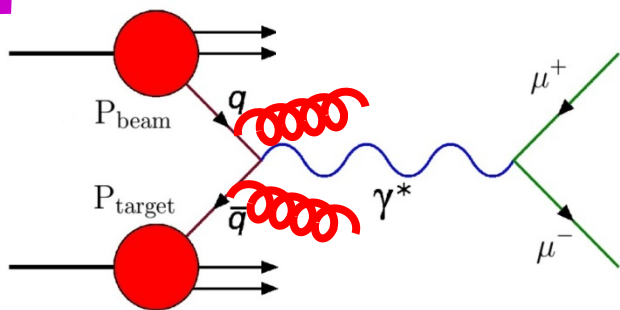


E615 (PRD 39, 92 (1989))

π^- -tungsten

$$\rho = \frac{Q_T}{Q}$$

DY



$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(|d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2 \right) + \textcolor{red}{b} \left(|d_{-1\,0}^1(\theta)|^2 + |d_{1\,0}^1(\theta)|^2 \right) \\ + \textcolor{violet}{\Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

➔ $\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \textcolor{red}{\lambda} \cos^2 \theta + \textcolor{red}{\mu} \sin 2\theta \cos \phi + \frac{\textcolor{red}{\nu}}{2} \sin^2 \theta \cos 2\phi$

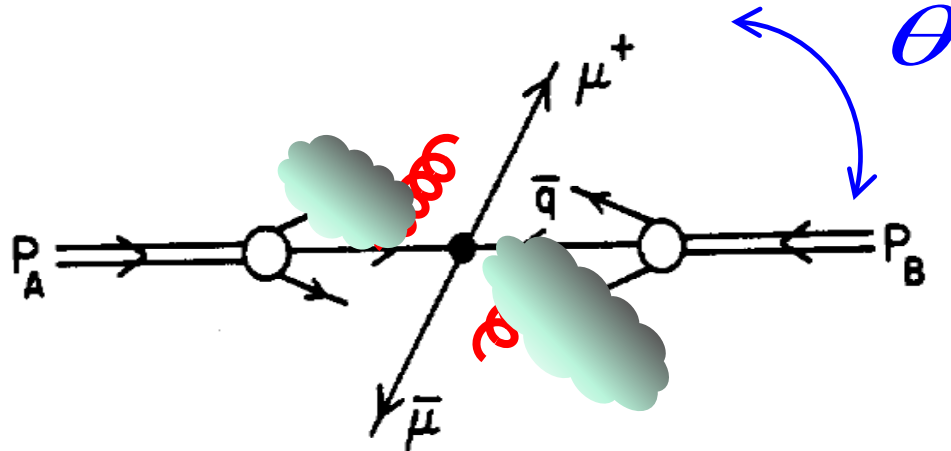
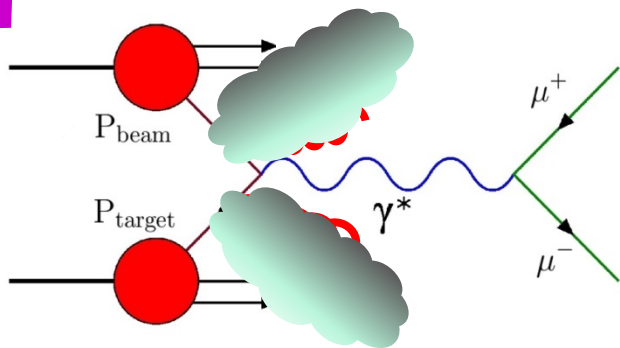
$\textcolor{red}{1} - \textcolor{red}{\lambda}, \quad \textcolor{red}{\mu}, \quad \textcolor{red}{\nu} : \propto Q_T \text{ at } O(\alpha_s)$

$\textcolor{blue}{1} - \textcolor{blue}{\lambda} - 2\textcolor{blue}{\nu} \simeq 0$

Lam-Tung relation ('78)

spin-1 gluon

DY



$$\frac{d\sigma}{d\Omega} \propto (1 + \textcolor{red}{a}) \left(|d_{-1\,1}^1(\theta)|^2 + |d_{1\,1}^1(\theta)|^2 \right) + \textcolor{red}{b} \left(|d_{-1\,0}^1(\theta)|^2 + |d_{1\,0}^1(\theta)|^2 \right) \\ + \textcolor{violet}{Re} \left[\textcolor{red}{c}_1 d_{1\,0}^1(\theta) \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \textcolor{red}{c}_2 d_{1\,-1}^1(\theta) e^{i\phi} \left(d_{1\,1}^1(\theta) e^{-i\phi} \right)^* + \dots \right]$$

➡ $\frac{1}{\sigma_0} \frac{d\sigma}{d\Omega} = 1 + \textcolor{red}{\lambda} \cos^2 \theta + \textcolor{red}{\mu} \sin 2\theta \cos \phi + \frac{\textcolor{red}{v}}{2} \sin^2 \theta \cos 2\phi$

$\textcolor{red}{1} - \textcolor{red}{\lambda}, \quad \textcolor{red}{\mu}, \quad \textcolor{red}{v} : \propto Q_T \text{ at } O(\alpha_s)$

$\textcolor{blue}{1} - \textcolor{blue}{\lambda} - 2\textcolor{red}{v} \neq 0$

Lam-Tung relation VIOLATED!
 $\textcolor{red}{k}_T$ & $\textcolor{violet}{nonpert. spin flip}$

BM, vacuum effect, and Glauber gluons

	$h_1^\perp \neq 0$	QCD vacuum effect	Glauber gluon
$\rho^{(q,\bar{q})}$	$\rho^{(q)} \otimes \rho^{(\bar{q})}$	possibly entangled	possibly entangled
Q dependence	$\kappa \sim 1/Q$	?	$1/Q^2$
large Q_T limit	$\kappa \rightarrow 0$	need not disappear ($\kappa \rightarrow \kappa_0$)	need not disappear
flavor dependence	yes	flavor blind	yes
x dependence	yes	if yes, then not hadron blind	yes

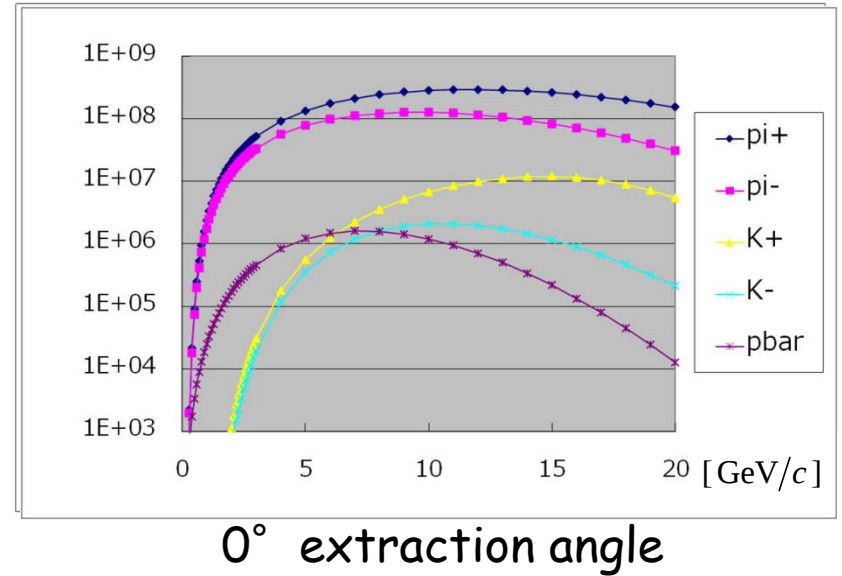
D.B., Brandenburg, Nachtmann & Utermann, EPJC 40 (2005) 55

Different experiments ($\pi^\pm, p, \bar{p}, \dots$ beams) are needed in different kinematical regimes



beam loss limit @ SM1:15kW

(limited by the thickness of the tunnel wall)



High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

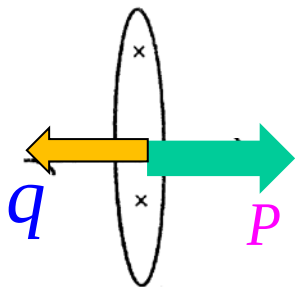
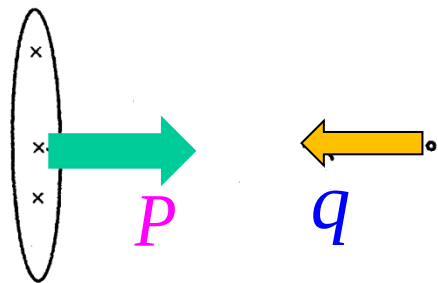
high intensity

BM, vacuum effect, and Glauber gluons

	$h_1^\perp \neq 0$	QCD vacuum effect	Glauber gluon
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D.B., Brandenburg, Nachtmann & Utermann, EPJC 40 (2005) 55

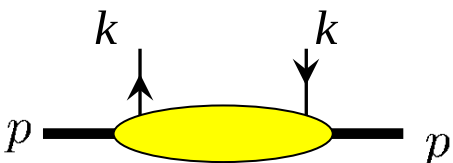
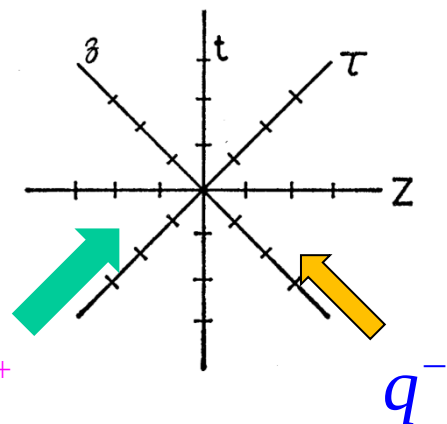
Different experiments ($\pi^\pm, p, \bar{p}, \dots$ beams) are needed in different kinematical regimes



$$z^{\pm} = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$$P^{\pm} = \frac{P^0 \pm P^3}{\sqrt{2}}$$

$(P^- \approx 0)$



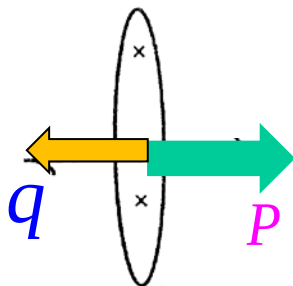
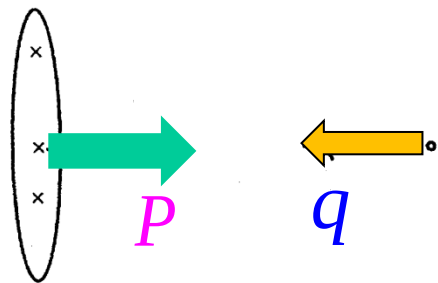
$$\int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{ik^+ z^- - i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \langle P | \psi^{\dagger}(0) \psi(z^+ = 0, z^-, \mathbf{z}_{\perp}) | P \rangle$$

$$U(0; z^-, \mathbf{z}_{\perp}) = P \exp \left(ig \int_{(z^-, \mathbf{z}_{\perp})}^0 d\xi_{\mu} A^{\mu}(\xi) \right) \quad \text{TMD}$$

$$f(x) \sim \int \frac{dz^-}{4\pi} e^{i(xP^+)z^-} \langle P | \psi^{\dagger}(0) \psi(z^+ = 0, z^-, \mathbf{z}_{\perp} = \mathbf{0}_{\perp}) | P \rangle \quad \text{PDF}$$

$$U(0; z^-, \mathbf{z}_{\perp} = 0) = P \exp \left(ig \int_{z^-}^0 d\xi^- A^+(\xi^-) \right)$$

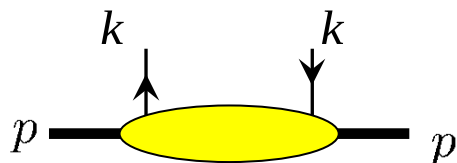
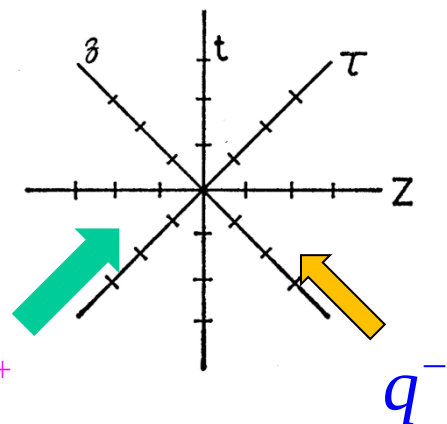
Collins, Soper ('82)



$$z^{\pm} = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$$P^{\pm} = \frac{P^0 \pm P^3}{\sqrt{2}}$$

$$(P^- \approx 0)$$



$$\int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{ik^+ z^- - i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \langle P | \psi^{\dagger}(0) \psi(z^+ = 0, z^-, \mathbf{z}_{\perp}) | P \rangle$$

TMD

$$U(0; z^-, \mathbf{z}_{\perp}) = P \exp \left(ig \int_{(z^-, \mathbf{z}_{\perp})}^0 d\xi_{\mu} A^{\mu}(\xi) \right)$$

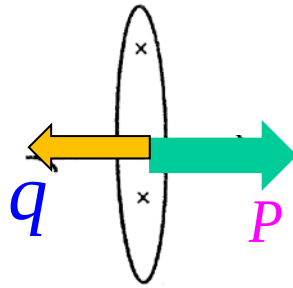
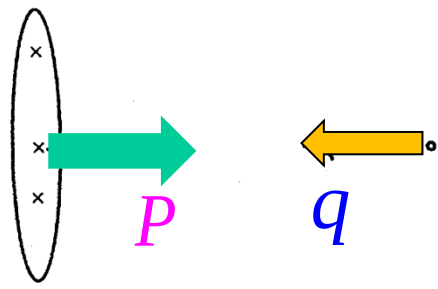
$$h_1^{\perp}(x, k_{\perp}) \sim \int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{i(xP^+)z^- - i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \langle P | \bar{\psi}(0) \gamma^+ \gamma^{\perp} U(0; z^-, \mathbf{z}_{\perp}) \psi(z^-, \mathbf{z}_{\perp}) | P \rangle$$

$$f(x) \sim \int \frac{dz^-}{4\pi} e^{i(xP^+)z^-} \langle P | \psi^{\dagger}(0) \psi(z^+ = 0, z^-, \mathbf{z}_{\perp} = \mathbf{0}_{\perp}) | P \rangle$$

PDF

$$U(0; z^-, \mathbf{z}_{\perp} = 0) = P \exp \left(ig \int_{z^-}^0 d\xi^- A^+(\xi^-) \right)$$

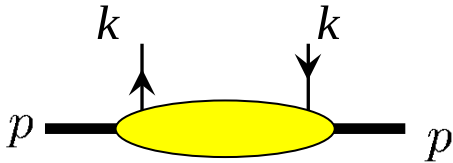
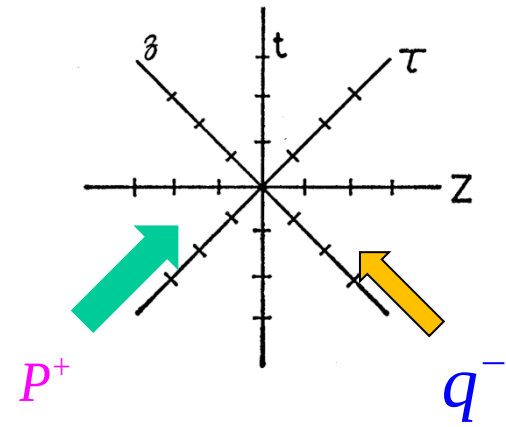
Collins, Soper ('82)



$$z^{\pm} = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$$P^{\pm} = \frac{P^0 \pm P^3}{\sqrt{2}}$$

($P^- \approx 0$)



$$\int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{ik^+ z^- - i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \langle P | \psi^{\dagger}(0) \underbrace{\psi(z^+ = 0, z^-, \mathbf{z}_{\perp})}_{\text{TMD}} | P \rangle$$

$$U(0; z^-, \mathbf{z}_{\perp}) = P \exp \left(ig \int_{(z^-, \mathbf{z}_{\perp})}^0 d\xi_{\mu} A^{\mu}(\xi) \right)$$

$$h_1^{\perp}(x, k_{\perp}) \sim \int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{i(xP^+)z^- - i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \langle P | \bar{\psi}(0) \gamma^+ \gamma^{\perp} U(0; z^-, \mathbf{z}_{\perp}) \psi(z^-, \mathbf{z}_{\perp}) | P \rangle$$

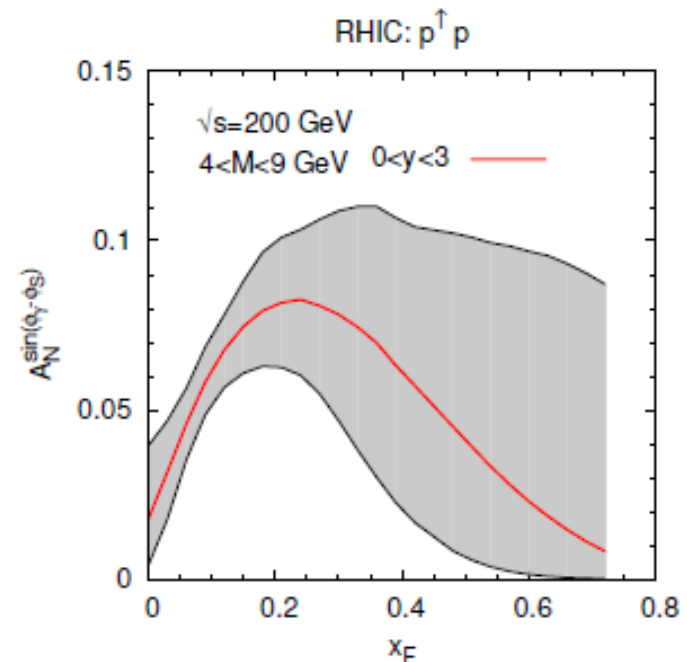
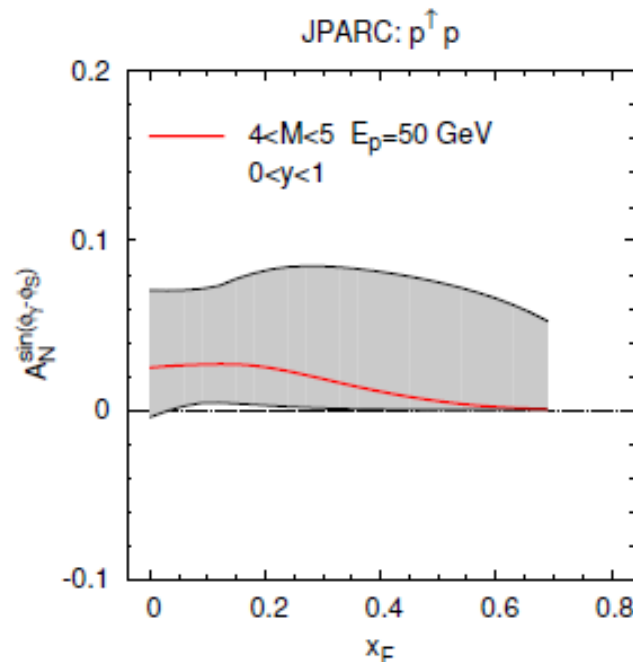
$$f_1^{\perp}(x, k_{\perp}) \sim \int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{i(xP^+)z^- - i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \langle PS_{\perp} | \bar{\psi}(0) \gamma^+ U(0; z^-, \mathbf{z}_{\perp}) \psi(z^-, \mathbf{z}_{\perp}) | PS_{\perp} \rangle$$

$$h_1^{\perp}(x, k_{\perp}) \Big|_{DY} = -h_1^{\perp}(x, k_{\perp}) \Big|_{DIS}$$

$$f_1^{\perp}(x, k_{\perp}) \Big|_{DY} = -f_1^{\perp}(x, k_{\perp}) \Big|_{DIS}$$

Sivers effect in Drell-Yan

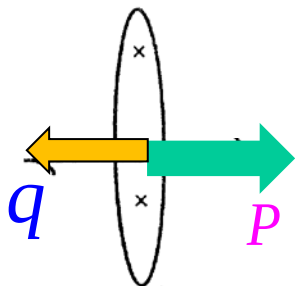
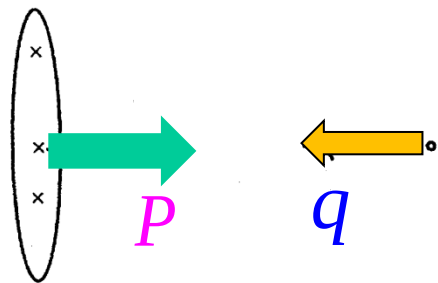
Experimental test of Sivers effect in Drell-Yan is highly desired ($\sin(\phi - \phi_S) f_{1T}^\perp \bar{f}_1$)



Anselmino *et al.* '09

$p^\uparrow p$ DY studies kinematically largely complementary to SIDIS data (careful about nodes)

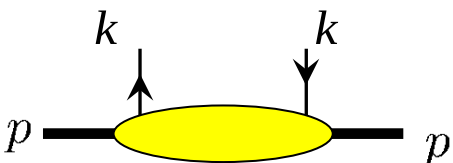
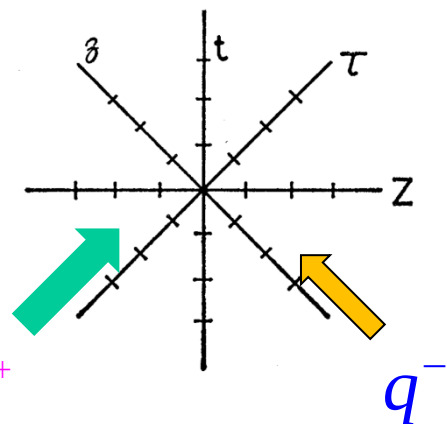
These predictions take into account the *process dependence* of the Sivers function



$$z^{\pm} = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$$P^{\pm} = \frac{P^0 \pm P^3}{\sqrt{2}}$$

$$(P^- \approx 0)$$



$$\int \frac{dz^- d^2 z_{\perp}}{(2\pi)^3} e^{ik^+ z^- - i\mathbf{k}_{\perp} \cdot \mathbf{z}_{\perp}} \langle P | \psi^{\dagger}(0) \psi(z^+ = 0, z^-, \mathbf{z}_{\perp}) | P \rangle$$

$$U(0; z^-, \mathbf{z}_{\perp}) = P \exp \left(ig \int_{(z^-, \mathbf{z}_{\perp})}^0 d\xi_{\mu} A^{\mu}(\xi) \right)$$

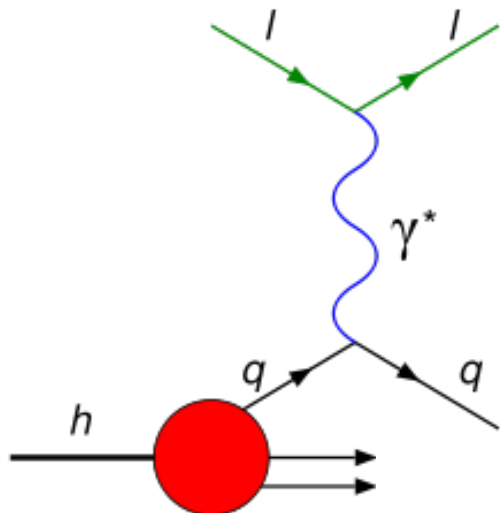
TMD

$$f(x) \sim \int \frac{dz^-}{4\pi} e^{i(xP^+)z^-} \langle P | \psi^{\dagger}(0) \psi(z^+ = 0, z^-, \mathbf{z}_{\perp} = \mathbf{0}_{\perp}) | P \rangle$$

PDF

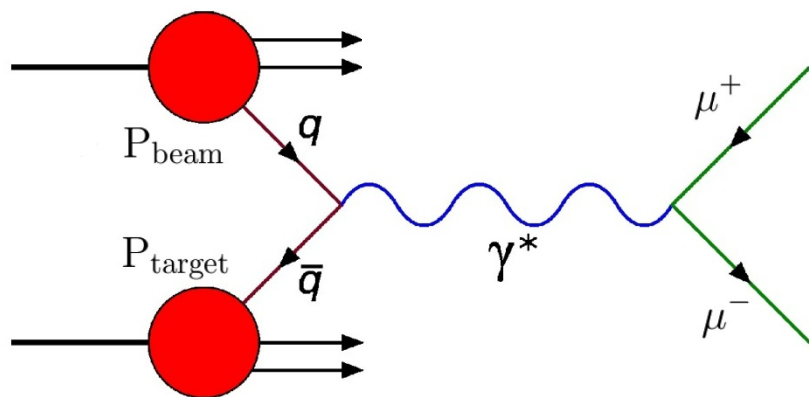
$$U(0; z^-, \mathbf{z}_{\perp} = 0) = P \exp \left(ig \int_{z^-}^0 d\xi^- A^+(\xi^-) \right)$$

Collins, Soper ('82)



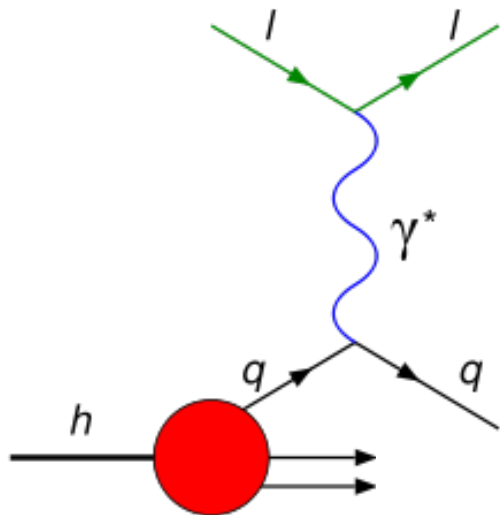
DIS

$$[q(x) + \bar{q}(x)], \quad G(x)$$



DY

$$q(x), \quad \bar{q}(x), \quad G(x)$$

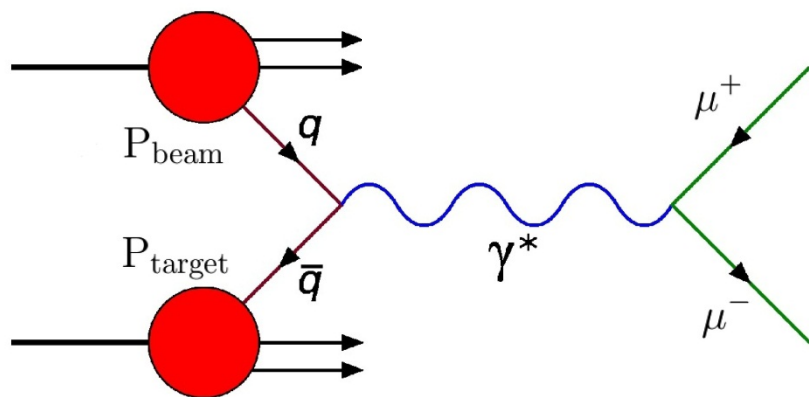


DIS

$$[q(x) + \bar{q}(x)], \quad G(x)$$

$$[\Delta q(x) + \Delta \bar{q}(x)], \quad \Delta G(x)$$

$$[g_T(x) + \bar{g}_T(x)]$$



DY

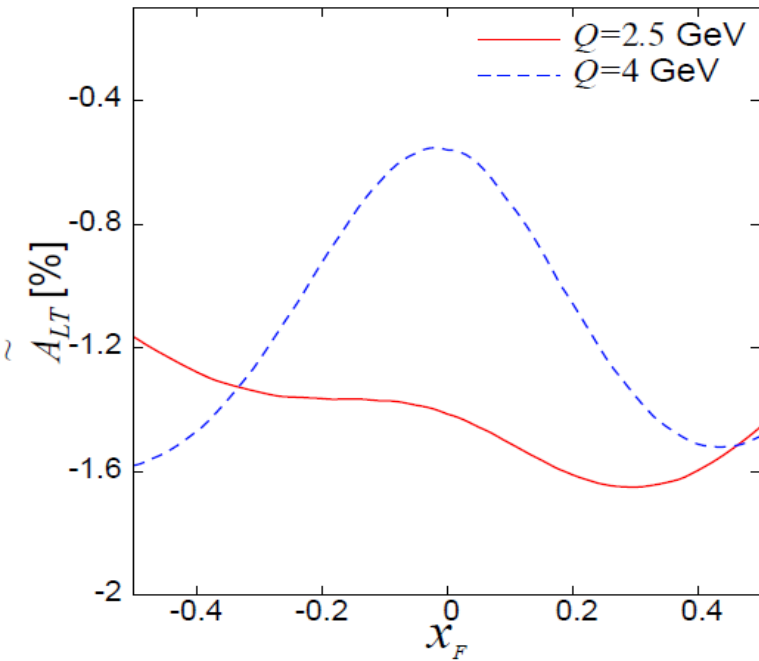
$$q(x), \quad \bar{q}(x), \quad G(x)$$

$$\Delta q(x), \quad \Delta \bar{q}(x), \quad \Delta G(x)$$

$$g_T(x), \quad \bar{g}_T(x)$$

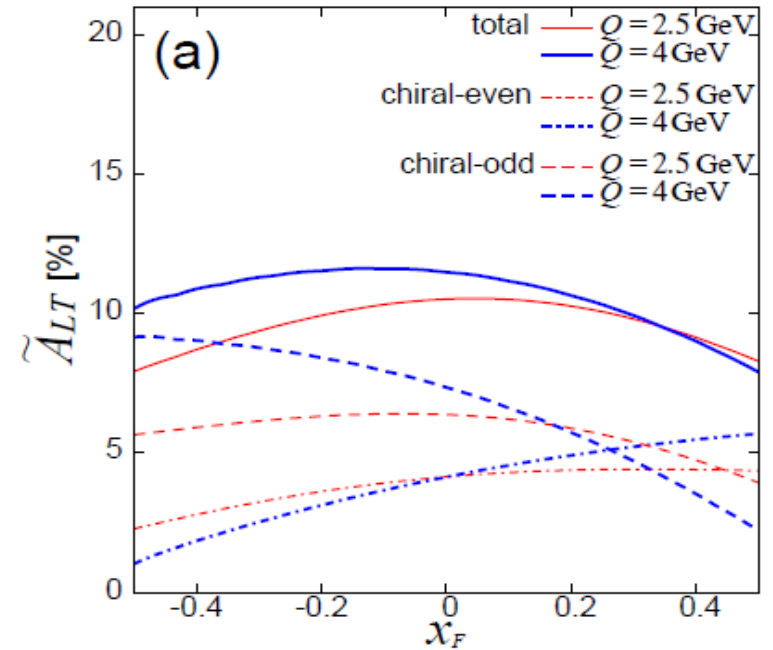
$$\delta q(x), \quad \delta \bar{q}(x), \quad h_L(x), \quad \bar{h}_L(x)$$

@J-PARC $\sqrt{S} = 10$ GeV



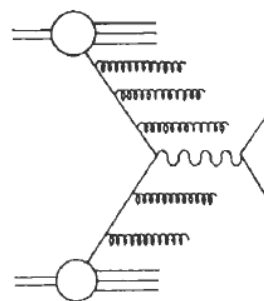
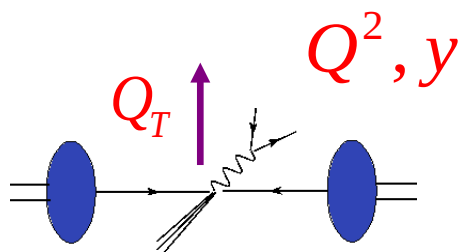
S. Yoshida

@GSI $\sqrt{S} = 6.7$ GeV



Y. Koike, KT, S. Yoshida, PLB668('08)286

$$x_F = x_1 - x_2$$



$$\alpha_s \ln^2(Q^2/Q_T^2)$$

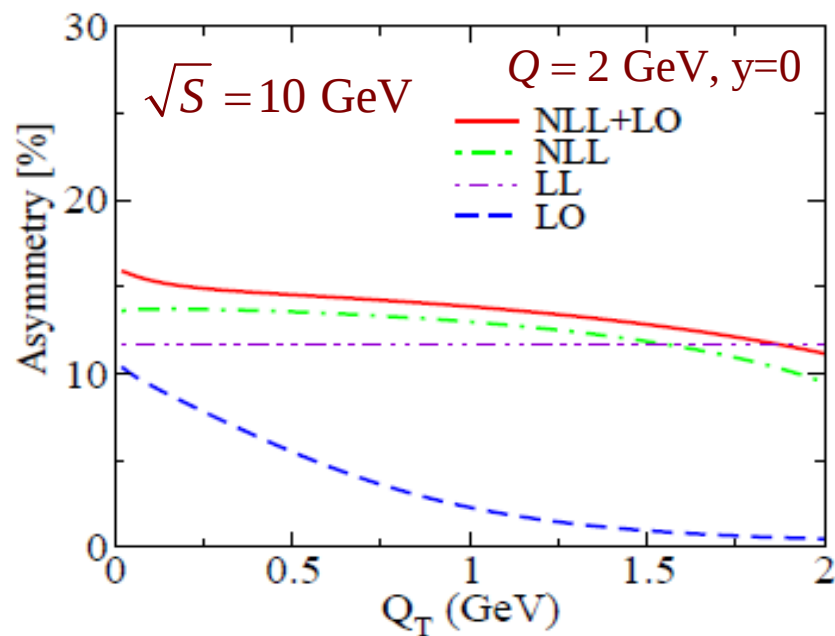
$$\alpha_s \ln(Q^2/Q_T^2)$$

resummation

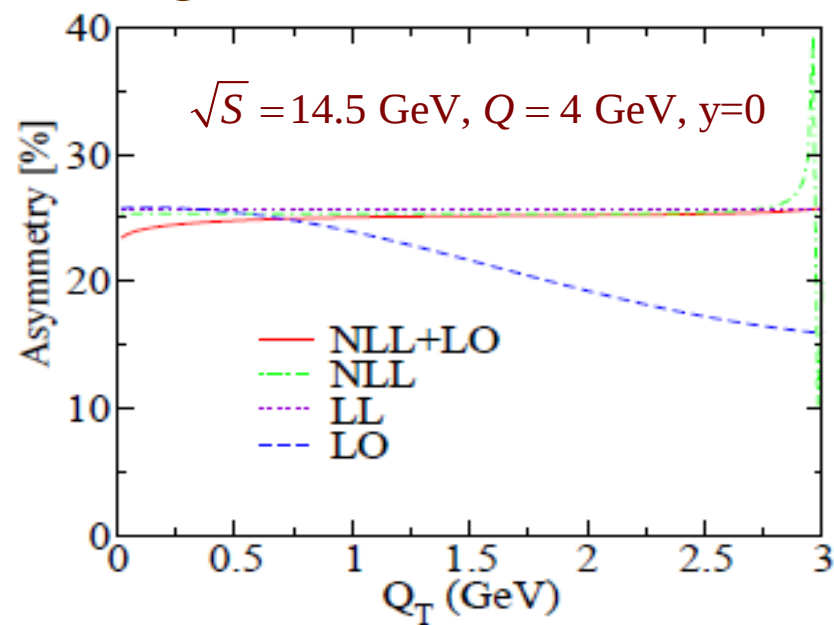
H. Kawamura, J. Kodaira, KT,

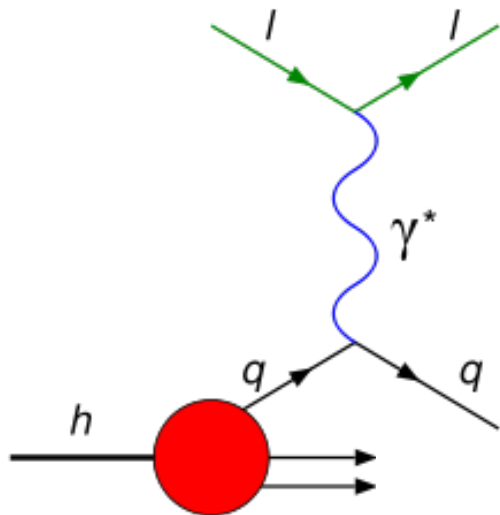
NPB777 ('07) 203, PTP 118 ('07) 581
PLB662 ('08) 139

@ J-PARC



@ GSI



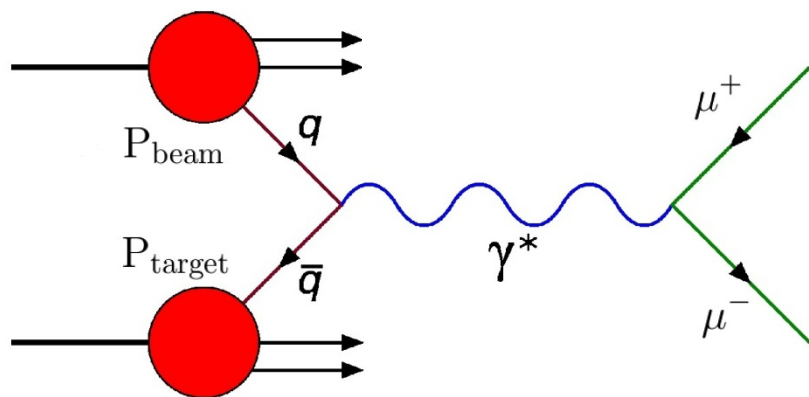


DIS

$$[q(x) + \bar{q}(x)], \quad G(x)$$

$$[\Delta q(x) + \Delta \bar{q}(x)], \quad \Delta G(x)$$

$$[g_T(x) + \bar{g}_T(x)]$$



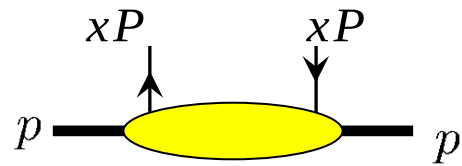
DY

$$q(x), \quad \bar{q}(x), \quad G(x)$$

$$\Delta q(x), \quad \Delta \bar{q}(x), \quad \Delta G(x)$$

$$g_T(x), \quad \bar{g}_T(x)$$

$$\delta q(x), \quad \delta \bar{q}(x), \quad h_L(x), \quad \bar{h}_L(x)$$



$$f(x) \sim \int \frac{dz^-}{4\pi} e^{i(xP^+)z^-} \langle P | \psi^\dagger(0) \psi(z^-) | P \rangle$$

PDF

$$z^- = \lambda n^- \quad P \cdot n = P^+ n^- = 1$$

Jaffe, Ji ('92)

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \bar{\psi}(0) \gamma^\sigma \psi(\lambda n) | P \ S \rangle = q(x) P^\sigma$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \bar{\psi}(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | P \ S \rangle = \Delta q(x) (S \cdot n) P^\sigma + g_T(x) S_\perp^\sigma$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \bar{\psi}(0) \sigma^{\mu\nu} i \gamma_5 \psi(\lambda n) | P \ S \rangle = \delta q(x) \frac{S_\perp^\mu P^\nu - S_\perp^\nu P^\mu}{M_N} + h_L(x) (P^\mu n^\nu - P^\nu n^\mu) M_N (S \cdot n)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \bar{\psi}(0) \psi(\lambda n) | P \ S \rangle = M_N e(x)$$

Ji, PLB289 ('92) 137

$$-\frac{(n^-)^2}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | F^{+\nu}(0) F^{+\sigma}(\lambda n) | P \ S \rangle$$

$$= \frac{1}{4} \left[G(x) g_\perp^{\nu\sigma} + \Delta G(x) i \epsilon^{\nu\sigma P n} (S \cdot n) + 2 G_{3E}(x) i \epsilon^{\nu\sigma \alpha n} S_{\perp \alpha} \right]$$

$$M^{\mu\nu} \quad M^{03} \quad M^{12} = J^3$$

$$\left[M^{\mu\nu}, \psi(x) \right] = - \left(i \left(x^\mu \partial^\nu - x^\nu \partial^\mu \right) + \frac{1}{2} \sigma^{\mu\nu} \right) \psi(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \psi^\dagger(0) \mathbf{1} \psi(\lambda n) | P \ S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \psi(\lambda n) | P \ S \rangle \sim q(x)$$

$$\hat{s}^i = \frac{1}{2} \varepsilon^{ijk} \frac{\sigma^{jk}}{2} \quad \left[\hat{s}^1, \hat{s}^2 \right] = i \hat{s}^3$$

$$2 \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \psi^\dagger(0) \hat{s}^3 \psi(\lambda n) | P \ S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \gamma_5 \psi(\lambda n) | P \ S \rangle \sim \Delta q(x)$$

$$2 \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \psi^\dagger(0) \hat{s}^\perp \psi(\lambda n) | P \ S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P \ S | \bar{\psi}(0) \gamma^\perp \gamma_5 \psi(\lambda n) | P \ S \rangle \sim g_T(x)$$

KT (2014)

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x) \right] = - \left(i \left(x^\mu \partial^\nu - x^\nu \partial^\mu \right) + \Sigma^{\mu\nu} \right) F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu} F^{\alpha\beta} = i \left(g^{\mu\alpha} F^{\nu\beta} - g^{\nu\alpha} F^{\mu\beta} - (\alpha \leftrightarrow \beta) \right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P S | F^{\alpha\beta}(0)^\dagger \mathbf{1} F^{\alpha\beta}(\lambda n) | P S \rangle = - \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P S | F^{+\beta}(0) F^+_{\beta}(\lambda n) | P S \rangle \sim x G(x)$$

$$\hat{S}^i = \frac{1}{2} \varepsilon^{ijk} \Sigma^{jk} \quad \left[\hat{S}^1, \hat{S}^2 \right] = i \hat{S}^3$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P S | F^{\alpha\beta}(0)^\dagger \hat{S}^3 F^{\alpha\beta}(\lambda n) | P S \rangle = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P S | i F^{+\beta}(0) \tilde{F}^+_{\beta}(\lambda n) | P S \rangle \sim x \Delta G(x)$$

$$\begin{aligned} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P S | F^{\alpha\beta}(0)^\dagger \hat{S}^\perp F^{\alpha\beta}(\lambda n) | P S \rangle &= \frac{-i}{\sqrt{2}} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P S | \tilde{F}^{+\perp}(0) F^{+-}(\lambda n) + F^{+\perp}(0) F^{12}(\lambda n) + \dots | P S \rangle \\ &\sim x \left(G_{3E}(x) + G_{3H}(x) \right) \equiv x G_T(x) \end{aligned}$$

KT (2014)

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P S | \Phi^\dagger(0) \begin{Bmatrix} \mathbf{1} \\ \hat{S}^i \end{Bmatrix} \Phi(\lambda n) | P S \rangle$$

$$\Phi = \psi, F^{\mu\nu}$$

$$q(x) \quad \Delta q(x) \quad g_T(x)$$

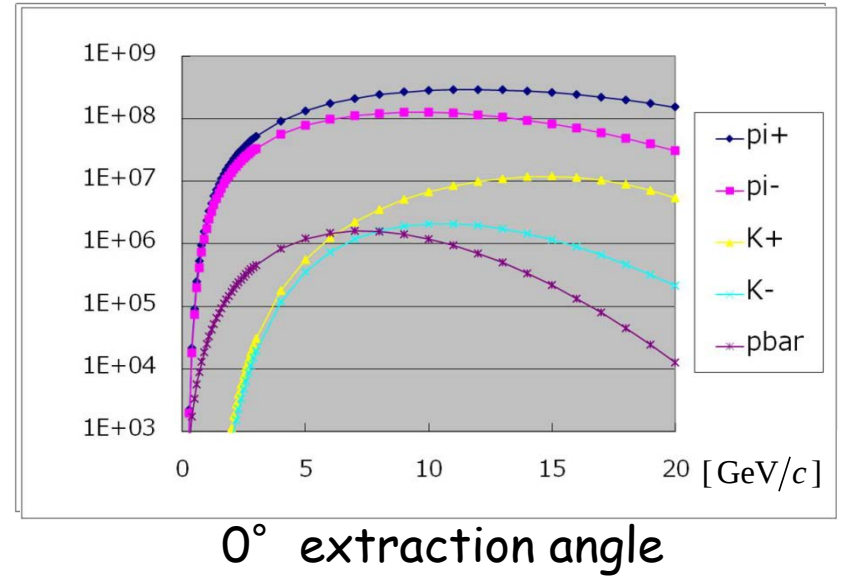
$$G(x) \quad \Delta G(x) \quad G_T(x)$$

density helicity flip



beam loss limit @ SM1:15kW

(limited by the thickness of the tunnel wall)



High-momentum beamline

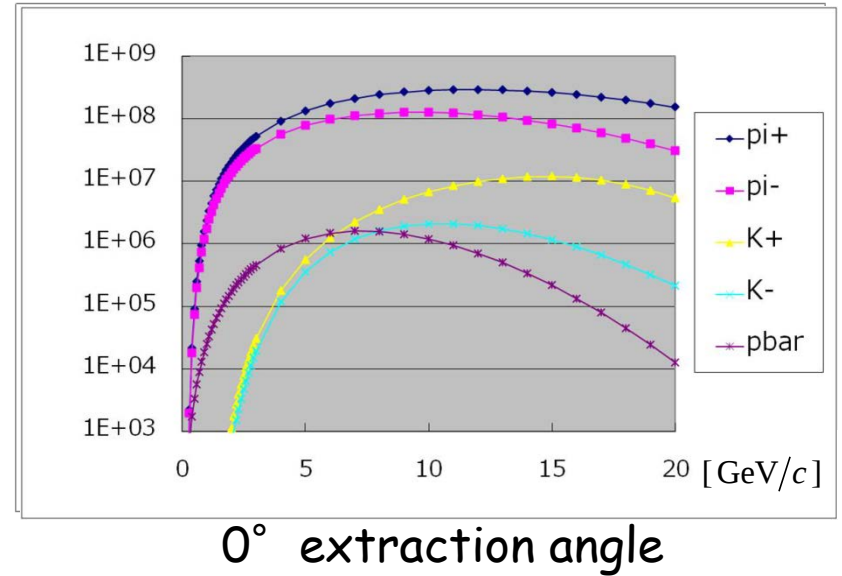
- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

high intensity



beam loss limit @ SM1:15kW

(limited by the thickness of the tunnel wall)



High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

high intensity

not too high energy

$$d\sigma \sim 1/s^a$$

best suited to study meson-induced
hard exclusive processes

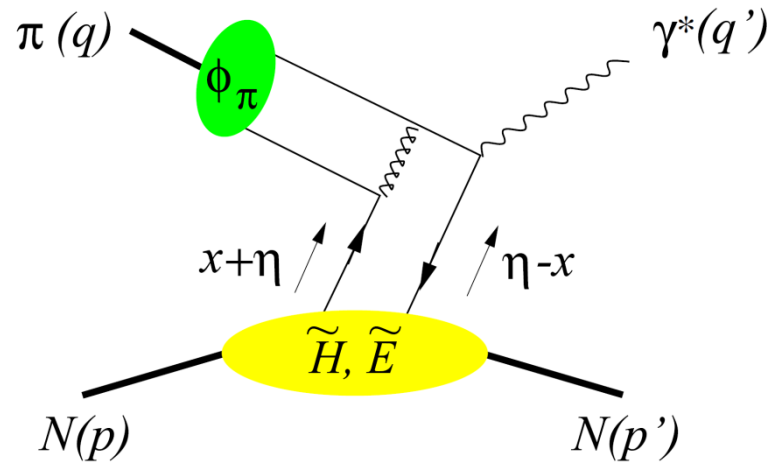
Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

Berger, Diehl, Pire, PLB523(2001)265

“exclusive limit of DY”

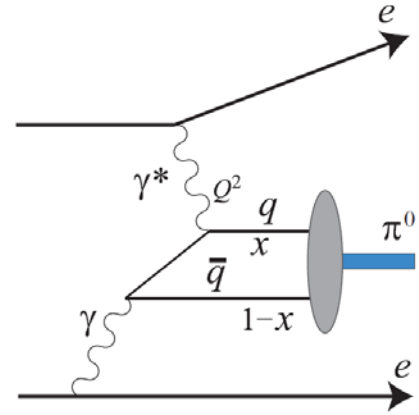
small $t = (q - q')^2$



Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

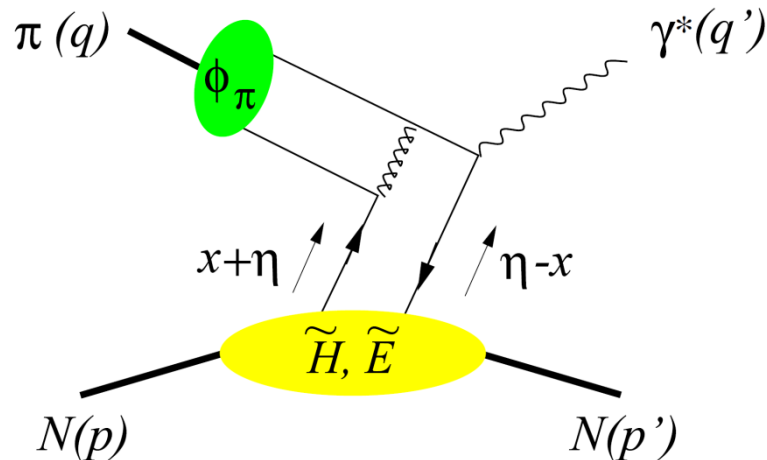
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

"exclusive limit of DY"

small $t = (q - q')^2$



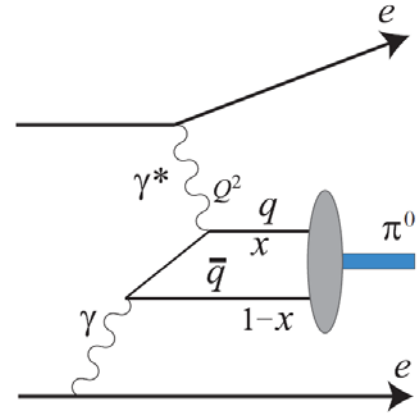
Exclusive lepton pair production in πN scattering

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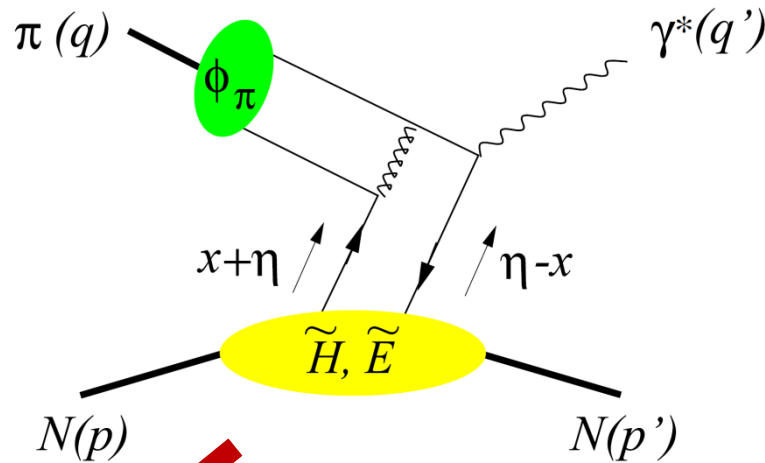
Berger, Diehl, Pire, PLB523(2001)265

@Belle, Babar

“exclusive limit of DY”



small $t = (q - q')^2$



$\Delta q(x)$ $\xrightarrow{t \rightarrow 0}$

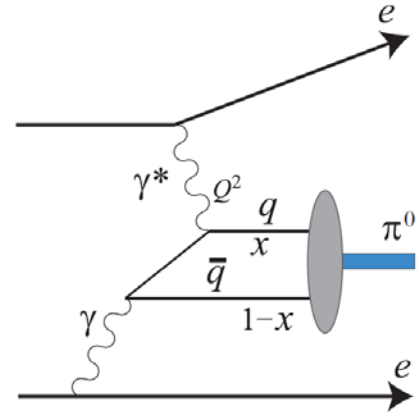
Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

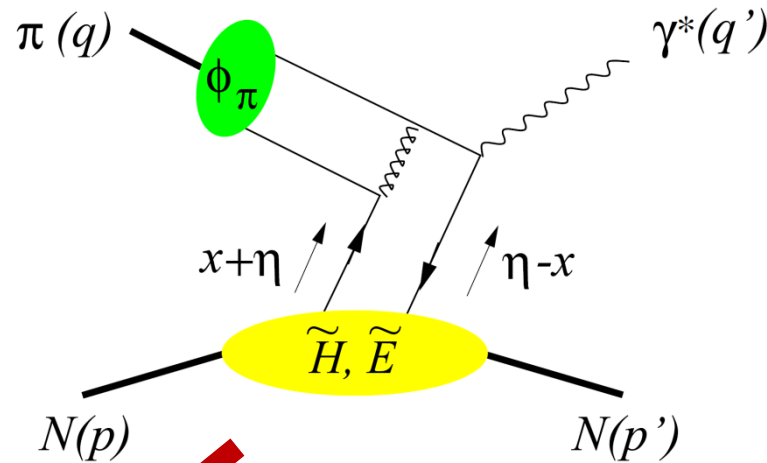
Berger, Diehl, Pire, PLB523(2001)265

@Belle, Babar

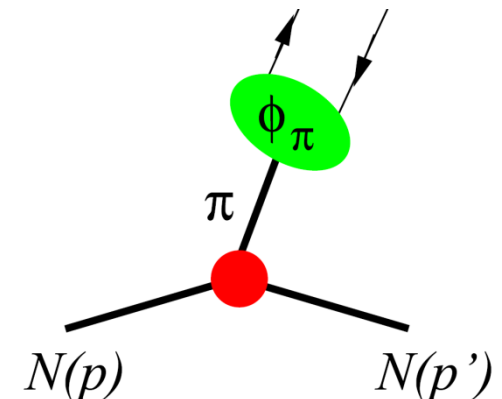
“exclusive limit of DY”



small $t = (q - q')^2$



$\Delta q(x)$ $t \rightarrow 0$



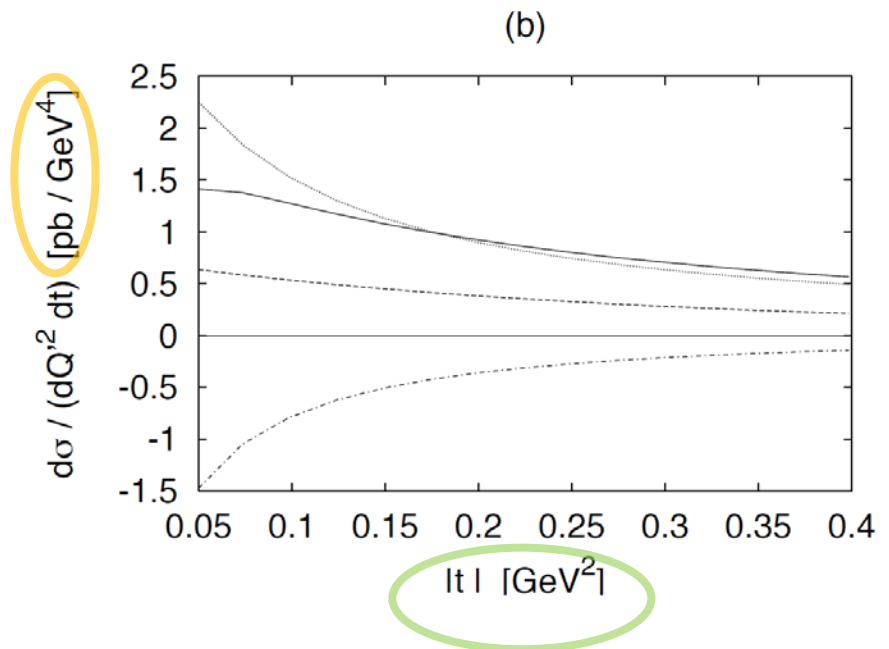
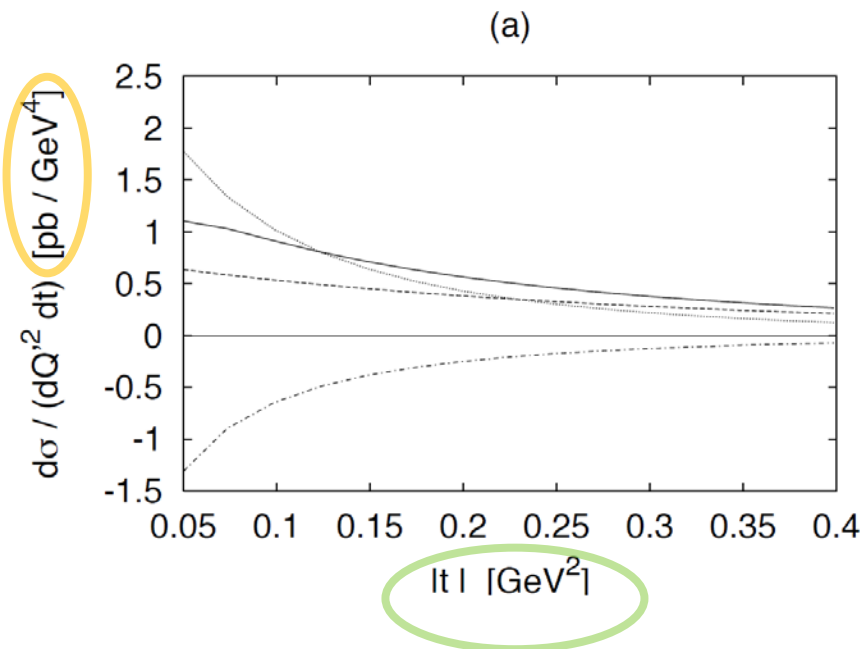
LO Estimates

Bjorken variable $\tau = \frac{Q'^2}{s-M^2}$

Berger, Diehl, Pire, PLB523(2001)265

$$Q'^2 = 5 \text{ GeV}^2$$

$$\tau = 0.2$$

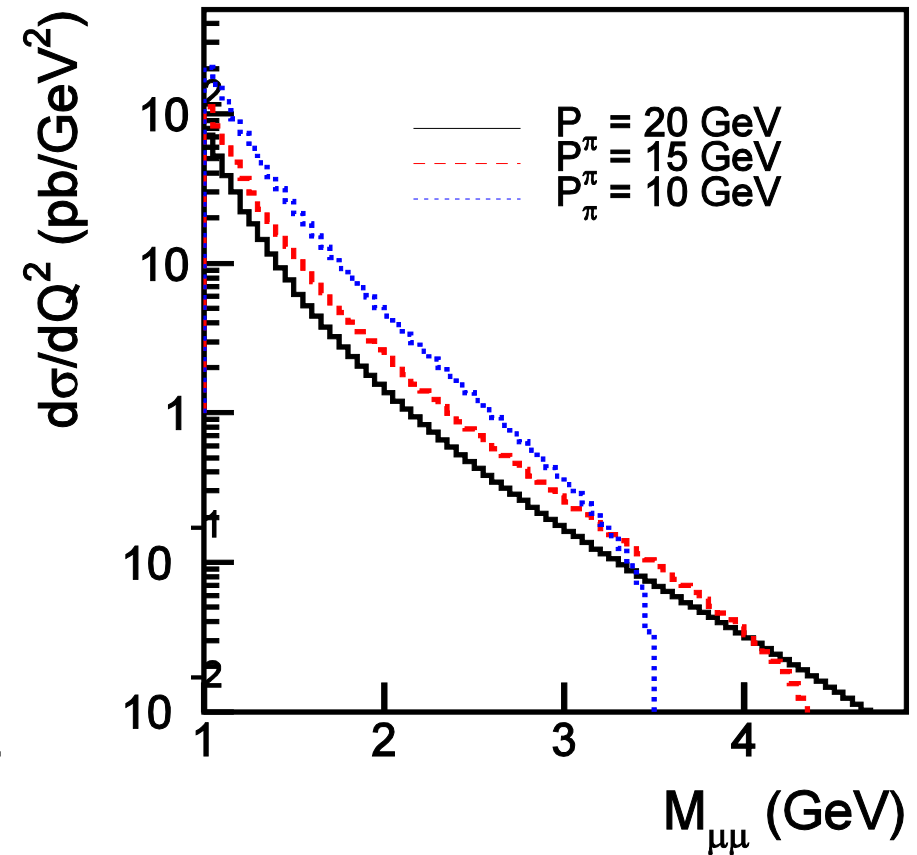
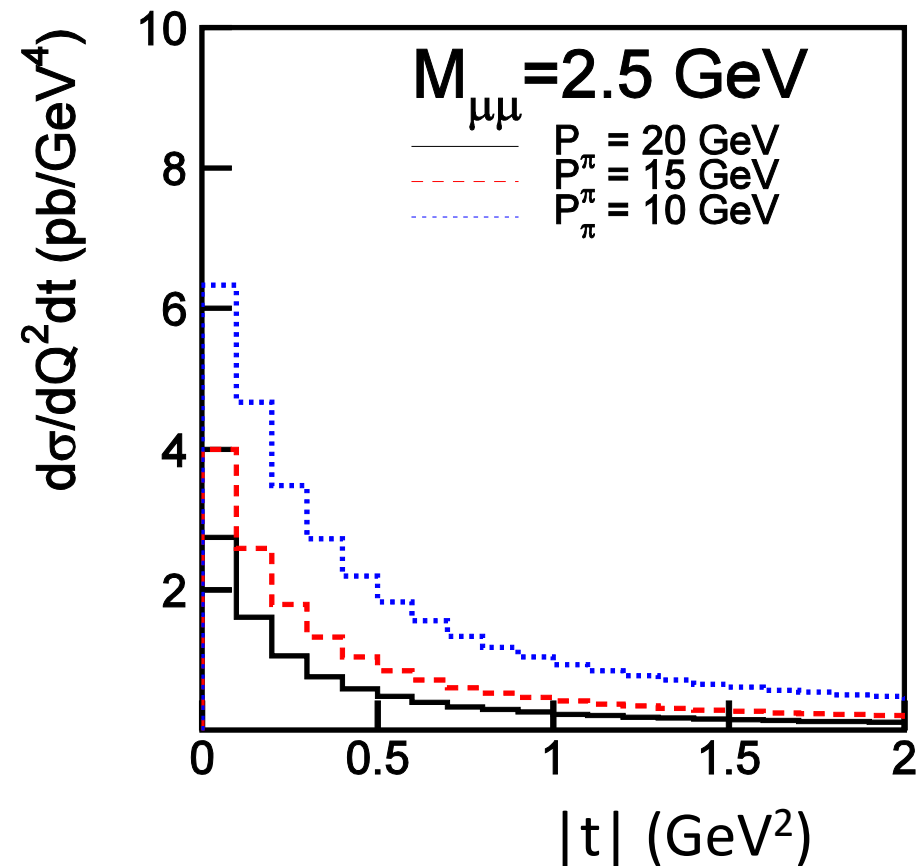


(dashed) = $|\tilde{\mathcal{H}}|^2$; (dash-dotted) = $\text{Re}(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; (dotted) = $|\tilde{\mathcal{E}}|^2$

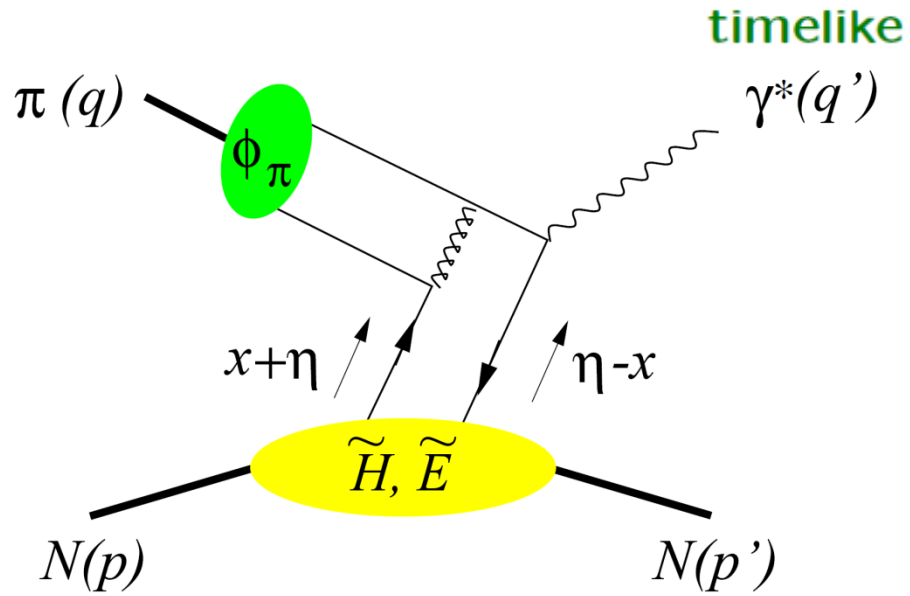
$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \rightarrow \gamma^* n) = \frac{4\pi\alpha_{\text{em}}^2}{27} \frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2) |\tilde{H}^{du}|^2 - 2\eta^2 \text{Re}(\tilde{H}^{du*} \tilde{E}^{du}) - \eta^2 \frac{t}{4M^2} |\tilde{E}^{du}|^2 \right]$$

$$\pi N \rightarrow \mu^+ \mu^- N :$$

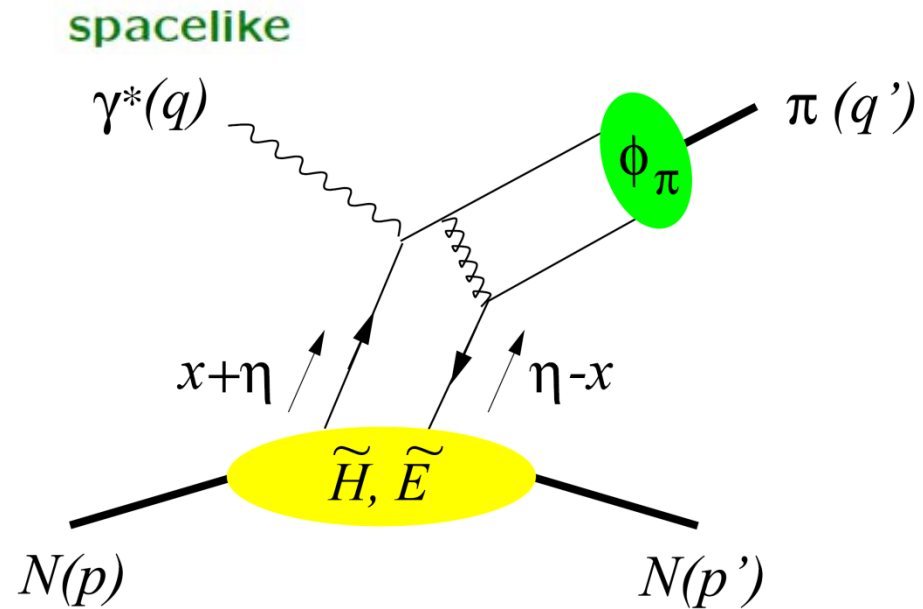
$|t|$ and Q ($M_{\mu\mu}$) dependence



Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions

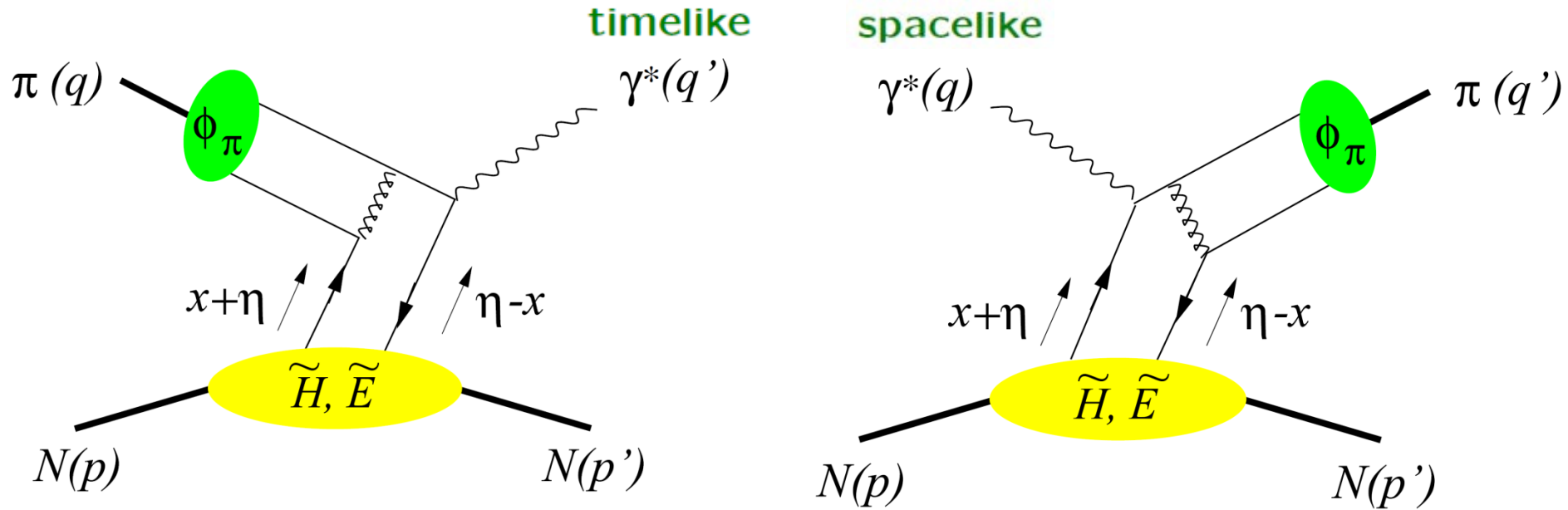


exDY@J-PARC



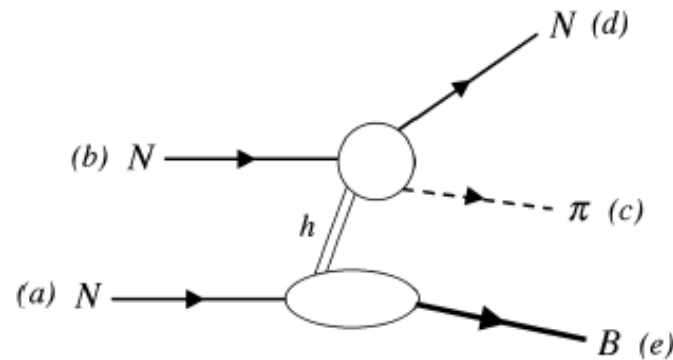
DVMP@JLab

Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions

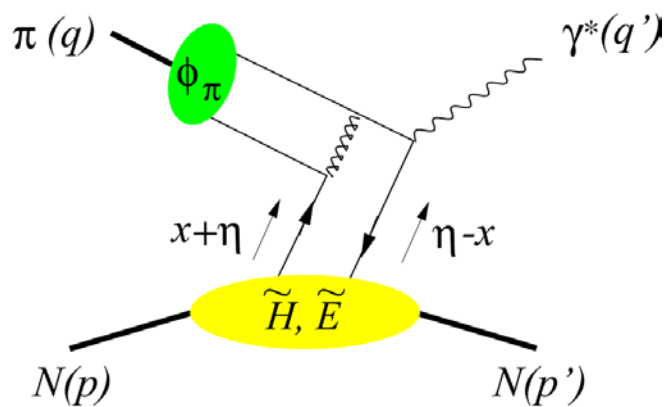


exDY@J-PARC

DVMP@JLab



Kumano, Sudoh, Strikman, PRD80(2000)074003.



Bjorken variable: $\tau = \frac{Q'^2}{2p \cdot q}$

Skewness: $\eta = \frac{p^+ - p'^+}{p^+ + p'^+}$

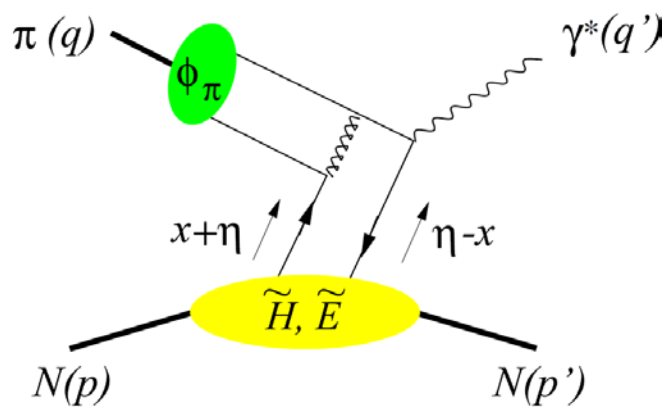
long. photon $\rightarrow$

$$\frac{d\sigma}{dQ'^2 dt d(\cos\theta) d\varphi} = \frac{\alpha_{\text{em}}}{256 \pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda', \lambda} |M^{0\lambda', \lambda}|^2 \sin^2 \theta$$

$$M^{0\lambda', \lambda}(\pi^- p \rightarrow \gamma^* n) = -ie \frac{4\pi}{3} \frac{f_\pi}{Q'} \frac{1}{(p+p')^+} \bar{u}(p', \lambda') \left[\gamma^+ \gamma_5 \tilde{\mathcal{H}}^{du}(\eta, t) + \gamma_5 \frac{(p'-p)^+}{2M} \tilde{\mathcal{E}}^{du}(\eta, t) \right] u(p, \lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta, t) = \frac{8\alpha_S}{3} \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx \left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon} \right] [\tilde{H}^d(x, \eta, t) - \tilde{H}^u(x, \eta, t)]$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2m} u(p) \right]$$

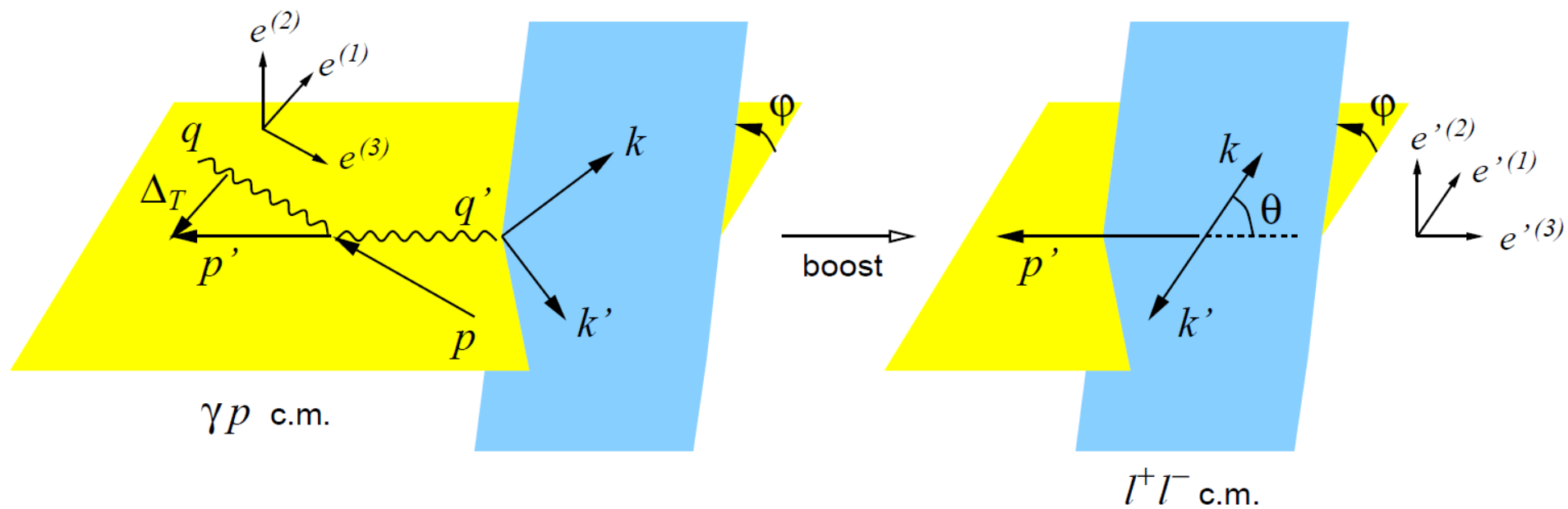


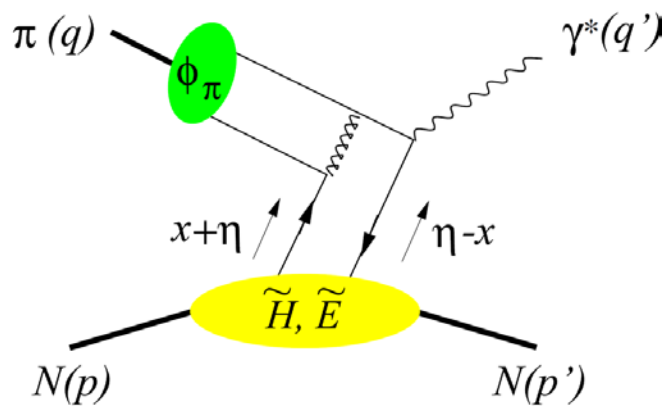
Bjorken variable: $\tau = \frac{Q'^2}{2p \cdot q}$

Skewness: $\eta = \frac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

$$\frac{d\sigma}{dQ'^2 dt d(\cos\theta) d\varphi} = \frac{\alpha_{\text{em}}}{256 \pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda', \lambda} |M^{0\lambda', \lambda}|^2 \sin^2 \theta$$





Bjorken variable: $\tau = \frac{Q'^2}{2p \cdot q}$

Skewness: $\eta = \frac{p^+ - p'^+}{p^+ + p'^+}$

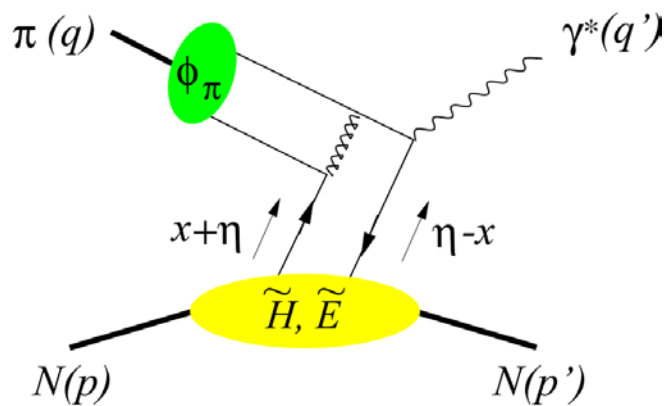
long. photon $\rightarrow$

$$\frac{d\sigma}{dQ'^2 dt d(\cos\theta) d\varphi} = \frac{\alpha_{\text{em}}}{256 \pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda', \lambda} |M^{0\lambda', \lambda}|^2 \sin^2 \theta$$

$$M^{0\lambda', \lambda}(\pi^- p \rightarrow \gamma^* n) = -ie \frac{4\pi}{3} \frac{f_\pi}{Q'} \frac{1}{(p+p')^+} \bar{u}(p', \lambda') \left[\gamma^+ \gamma_5 \tilde{\mathcal{H}}^{du}(\eta, t) + \gamma_5 \frac{(p'-p)^+}{2M} \tilde{\mathcal{E}}^{du}(\eta, t) \right] u(p, \lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta, t) = \frac{8\alpha_S}{3} \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx \left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon} \right] [\tilde{H}^d(x, \eta, t) - \tilde{H}^u(x, \eta, t)]$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2m} u(p) \right]$$



Bjorken variable: $\tau = \frac{Q'^2}{2p \cdot q}$

Skewness: $\eta = \frac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

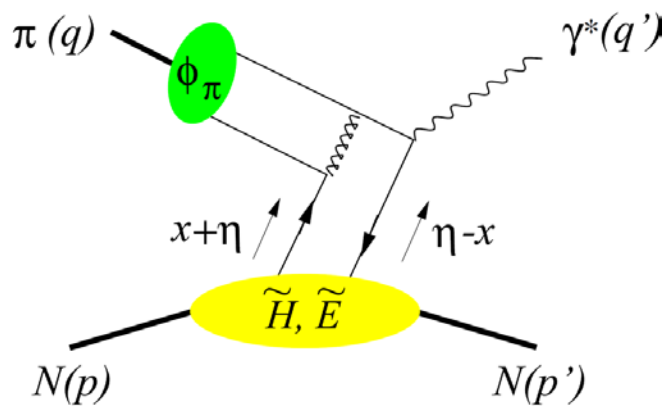
$$|d_{-10}^1(\theta)|^2 + |d_{10}^1(\theta)|^2$$

$$\frac{d\sigma}{dQ'^2 dt d(\cos\theta) d\varphi} = \frac{\alpha_{em}}{256\pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda',\lambda} |M^{0\lambda',\lambda}|^2 \sin^2\theta$$

$$M^{0\lambda',\lambda}(\pi^- p \rightarrow \gamma^* n) = -ie \frac{4\pi}{3} \frac{f_\pi}{Q'} \frac{1}{(p+p')^+} \bar{u}(p', \lambda') \left[\gamma^+ \gamma_5 \tilde{\mathcal{H}}^{du}(\eta, t) + \gamma_5 \frac{(p'-p)^+}{2M} \tilde{\mathcal{E}}^{du}(\eta, t) \right] u(p, \lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta, t) = \frac{8\alpha_S}{3} \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx \left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon} \right] [\tilde{H}^d(x, \eta, t) - \tilde{H}^u(x, \eta, t)]$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2m} u(p) \right]$$



Bjorken variable: $\tau = \frac{Q'^2}{2p \cdot q}$

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long. photon

$$|d_{-10}^1(\theta)|^2 + |d_{10}^1(\theta)|^2$$

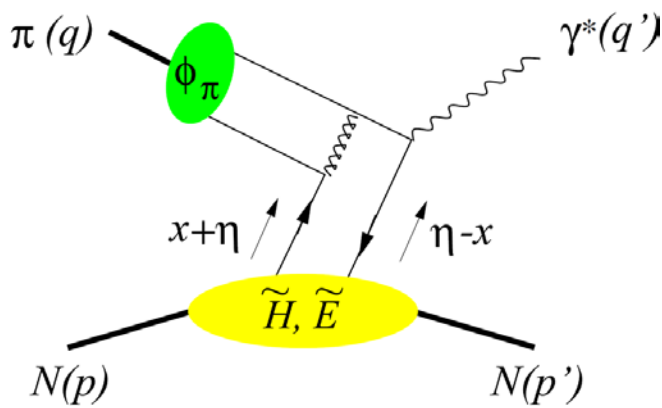
$$\frac{d\sigma}{dQ'^2 dt d(\cos \theta) d\varphi} = \frac{\alpha_{em}}{256 \pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda', \lambda} |M^{0\lambda', \lambda}|^2 \sin^2 \theta$$

$$M^{0\lambda', \lambda}(\pi^- p \rightarrow \gamma^* n) = -ie \frac{4\pi}{3} \frac{f_\pi}{Q'} \frac{1}{(p+p')^+} \bar{u}(p', \lambda') \left[\gamma^+ \gamma_5 \tilde{\mathcal{H}}^{du}(\eta, t) + \gamma_5 \frac{(p'-p)^+}{2M} \tilde{\mathcal{E}}^{du}(\eta, t) \right] u(p, \lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta, t) = \frac{8\alpha_S}{3} \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx \left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon} \right] [\tilde{H}^d(x, \eta, t) - \tilde{H}^u(x, \eta, t)]$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{P^+} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2m} u(p) \right]$$

$$M^{\perp, \lambda; \lambda'}(\pi^- p \rightarrow \gamma^* n) \sim \frac{1}{Q'^2} \quad \frac{1}{Q'} \text{ correction to } \frac{d\sigma}{dQ'^2 dt d(\cos \theta) d\varphi}$$



Bjorken variable: $\tau = \frac{Q'^2}{2p \cdot q}$

Skewness: $\eta = \frac{p^+ - p'^+}{p^+ + p'^+}$

long. photon

$$|d_{-10}^1(\theta)|^2 + |d_{10}^1(\theta)|^2$$

$$\frac{d\sigma}{dQ'^2 dt d(\cos\theta) d\varphi} = \frac{\alpha_{\text{em}}}{256 \pi^3} \frac{\tau^2}{Q'^6} \sum_{\lambda', \lambda} |M^{0\lambda', \lambda}|^2 \sin^2 \theta$$

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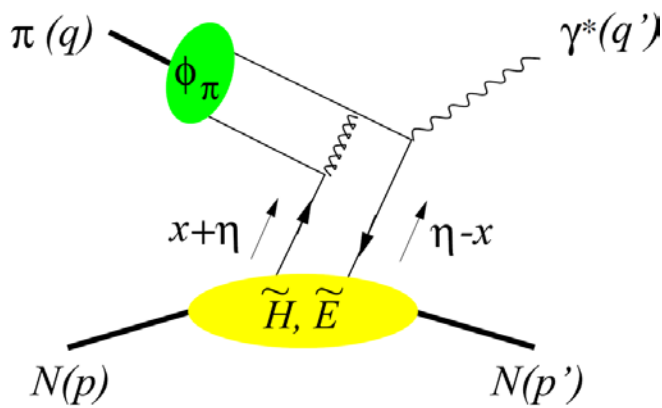
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$$M^{\perp, \lambda; \lambda'}(\pi^- p \rightarrow \gamma^* n) \sim \frac{1}{Q'^2}$$

$\frac{1}{Q'}$ correction to $\frac{d\sigma}{dQ'^2 dt d(\cos\theta) d\varphi}$

angular distribution $\propto \sin 2\theta \cos \varphi$

$$\sim \Re \left[d_{10}^1(\theta) (d_{11}^1(\theta) e^{-i\phi})^* \right]$$



Bjorken variable: $\tau = \frac{Q'^2}{2p \cdot q}$

Skewness: $\eta = \frac{p^+ - p'^+}{p^+ + p'^+}$

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angular distribution $\propto \sin 2\theta \cos \varphi$

need quantitative estimate!

$$\sim \text{Re} \left[d_{10}^1(\theta) (d_{11}^1(\theta) e^{-i\phi})^* \right]$$

Target Transverse Spin asymmetry

At the twist 2 level : $\frac{d^\uparrow\sigma - d^\downarrow\sigma}{d^\uparrow\sigma + d^\downarrow\sigma} = A_{UT}^{\sin(\phi - \phi_S)} \sin(\phi - \phi_S) + \text{other harmonics}$

$$A_{UT} = \frac{-2 \sqrt{\frac{t-t_{min}}{t_{min}}} \eta^2 \text{Im}(\tilde{\mathcal{H}} \tilde{\mathcal{E}}^*)}{(1-\eta^2)|\tilde{\mathcal{H}}|^2 - \frac{t}{4M^2}|\eta\tilde{\mathcal{E}}|^2 - 2\eta^2\text{Re}(\tilde{\mathcal{H}}\tilde{\mathcal{E}}^*)}$$

⇒ New information on GPDs.

e.g. if $\tilde{E}$ is well modeled by pion pole, $\tilde{\mathcal{E}}$ is real $\rightarrow A_{UT} \sim \tilde{H}(x, \xi = x, t)$

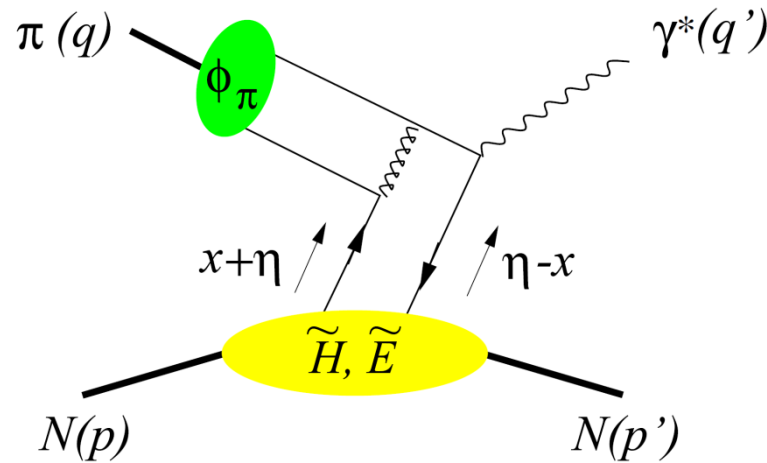
Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

Berger, Diehl, Pire, PLB523(2001)265

“exclusive limit of DY”

small $t = (q - q')^2$



$$\pi^- p \rightarrow \rho^- p \quad \rho^- \rightarrow \pi^- \pi^0$$

$$(4\pi/3) W(\theta, \phi) = r_{0,0} \cos^2(\theta) + r_{1,1} \sin^2(\theta) - r_{1,-1} \sin^2(\theta) \cos(2\phi) - \sqrt{2} \operatorname{Re}(r_{1,0}) \sin(2\theta) \cos(\phi),$$

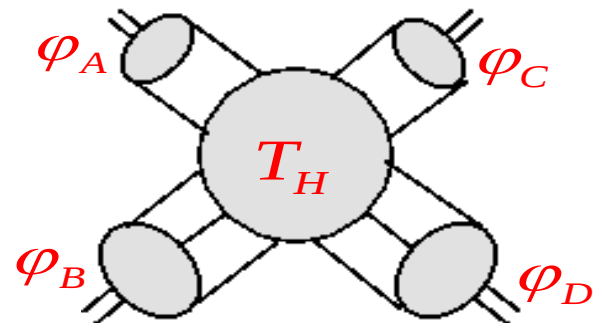
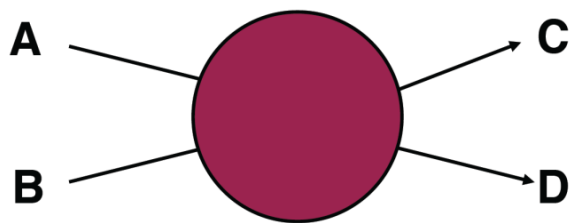
$$r_{i,i'} = \sum_{m,n} A_{m,n}^i A_{m,n}^{i'*}$$

$$m = i + n = i' + n$$

BNL AGS E755('88) 9.9 GeV

$$r_{0,0} = 0.12 \pm 0.30, \quad r_{1,1} = 0.44 \pm 0.15,$$

$$r_{1,-1} = 0.32 \pm 0.10, \quad \operatorname{Re}(r_{1,0}) = -0.01 \pm 0.05.$$



Hadron helicity conservation

$$\sum_i \lambda_{H^i} = \sum_f \lambda_{H^f}$$

$$\pi^- p \rightarrow \rho^- p \quad \rho^- \rightarrow \pi^- \pi^0$$

$$(4\pi/3) W(\theta, \phi) = r_{0,0} \cos^2(\theta) + r_{1,1} \sin^2(\theta) - r_{1,-1} \sin^2(\theta) \cos(2\phi) - \sqrt{2} \operatorname{Re}(r_{1,0}) \sin(2\theta) \cos(\phi),$$

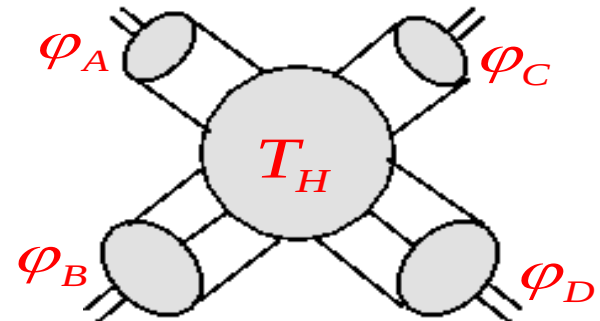
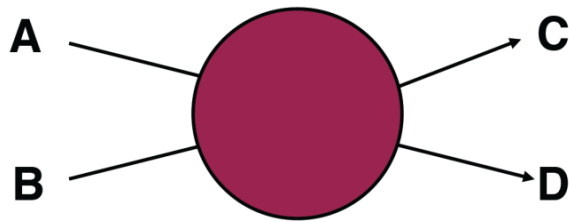
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Hadron helicity conservation

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violated!

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Higher twist effect?

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21 APRIL 1986

Polarization Effects in Exclusive Hadron Scattering

Glennys R. Farrar

Department of Physics, Rutgers University, Piscataway, New Jersey 08854

(Received 5 March 1986)

Measured helicity nonconservation in $\pi^- p \rightarrow \rho^- p$ and in pp elastic scattering indicates that higher-twist contributions are $\frac{1}{10} - \frac{1}{3}$ the size of the leading-twist amplitudes, and that the relative phase between certain pp amplitudes is at least 16° . The reported levels of helicity nonconservation are therefore consistent with leading-twist perturbative QCD.

PACS numbers: 13.85.Dz, 12.38.Bx, 12.38.Qk, 13.85.Fb

$$\pi^- p \rightarrow \rho^- p \quad \rho^- \rightarrow \pi^- \pi^0$$

$$(4\pi/3) W(\theta, \phi) = r_{0,0} \cos^2(\theta) + r_{1,1} \sin^2(\theta) - r_{1,-1} \sin^2(\theta) \cos(2\phi) - \sqrt{2} \operatorname{Re}(r_{1,0}) \sin(2\theta) \cos(\phi),$$

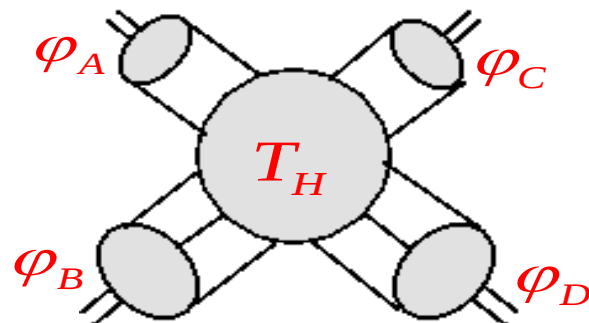
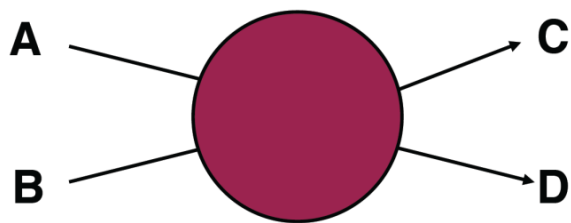
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Hadron helicity conservation

$$\sum_i \lambda_{H^i} = \sum_f \lambda_{H^f}$$

violated!

persist at J-PARC?

15-20 GeV

Systematic study of hard exclusive meson-nucleon reactions

$$\pi^{\pm} p \rightarrow p \pi^{\pm},$$

$$K^{\pm} p \rightarrow p K^{\pm},$$

$$\pi^{\pm} p \rightarrow p \rho^{\pm},$$

$$\pi^{\pm} p \rightarrow \pi^{\pm} \Delta^{\pm},$$

$$\pi^{\pm} p \rightarrow K^{\pm} \Sigma^{\pm},$$

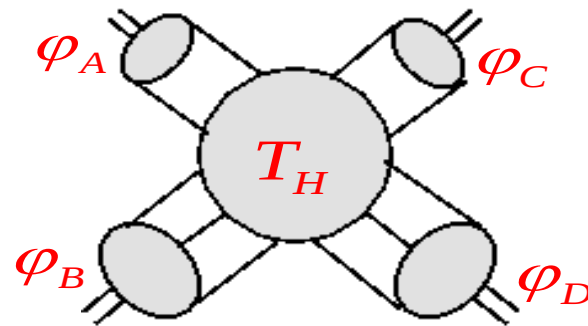
$$\pi^{-} p \rightarrow \Lambda^0 K^0, \Sigma^0 K^0,$$

$$p^{\pm} p \rightarrow p p^{\pm}.$$

BNL AGS E755('88) 9.9 GeV

E838('94) 5.9 GeV

J-PARC 15-20 GeV



$$= \int dx_a dx_b dx_c dx_d$$

$$\times \varphi_D^*(x_d) \varphi_C^*(x_c) T_H(x_i, s, \theta_{\text{CM}}) \varphi_B(x_b) \varphi_A(x_a)$$

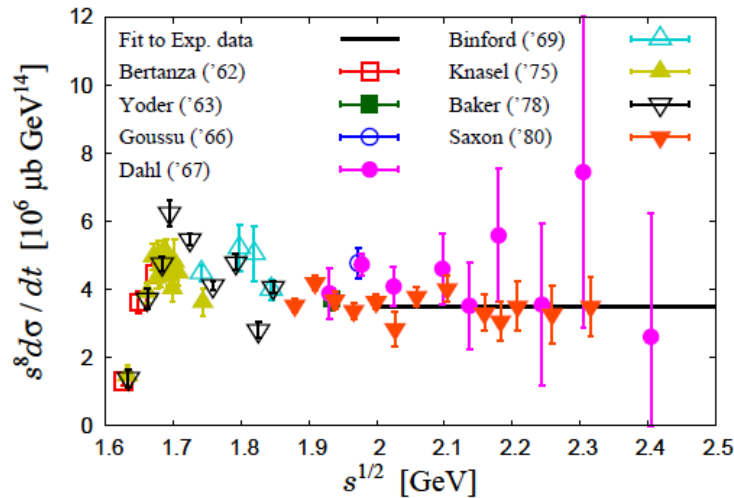
$$\frac{d\sigma}{dt} \sim \frac{1}{s^{n-2}}$$

$$n = n_A + n_B + n_C + n_D$$

Lambda & Lambda(1405)

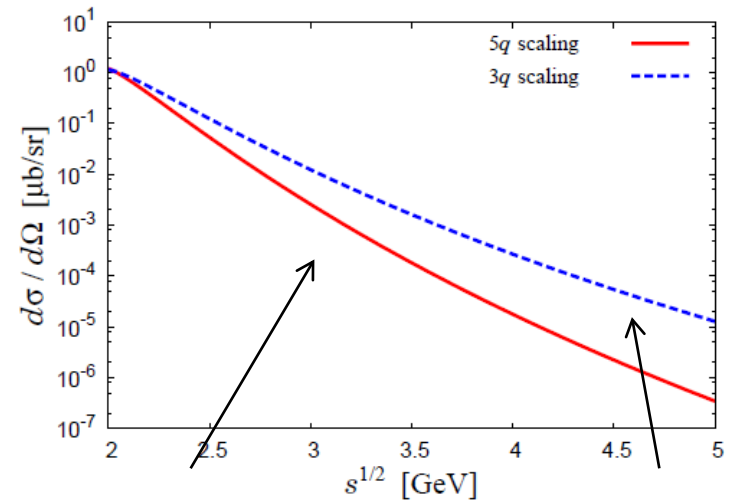
Kawamura, Kumano, Sekihara, PRD88 ('13) 034010

$$\pi^- + p \rightarrow K + \Lambda$$



$n = 10$

$$\pi^- + p \rightarrow K + \Lambda, K + \Lambda(1405)$$



5-quark

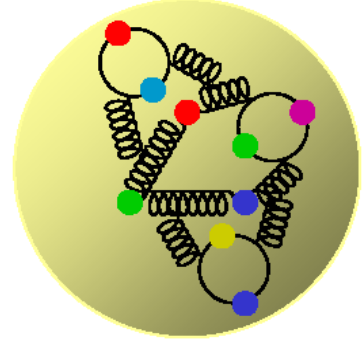
3-quark

$n = 10$ or 12 ?

Summary

unpol. beam $p, \pi^{\pm}, K^{\pm}, \dots$

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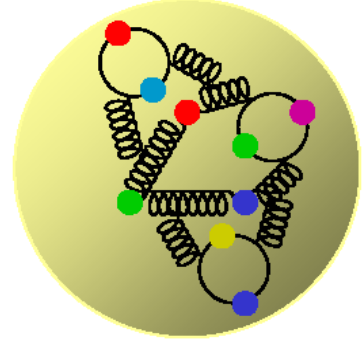
inclusive DY $q(x), \quad \bar{q}(x)$

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distributions for (θ, ϕ) & Q_T

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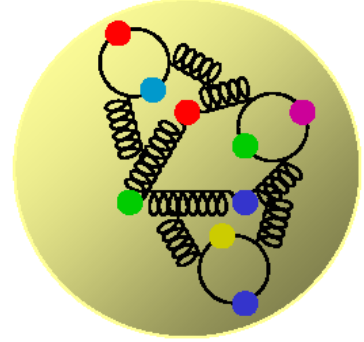
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Lam-Tung violation

k_T & nonpert. spin flip



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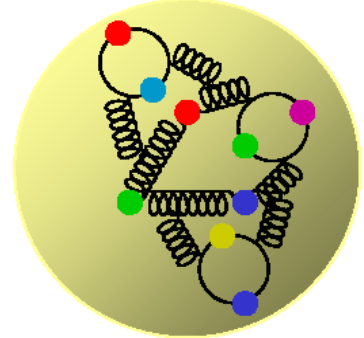
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SSA $f_1^\perp(x, k_\perp) \Big|_{DY} = -f_1^\perp(x, k_\perp) \Big|_{DIS}$



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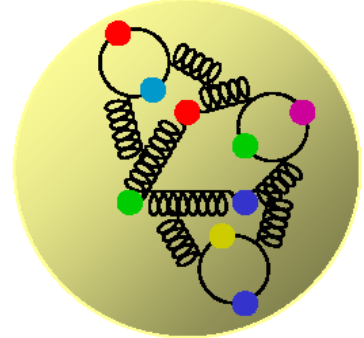
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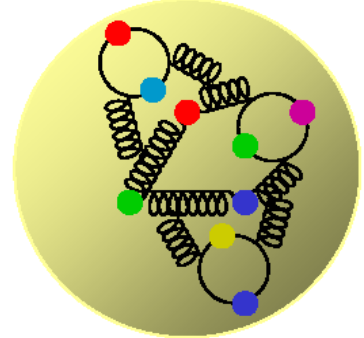
need to estimate $1/Q'$ power correction



Summary

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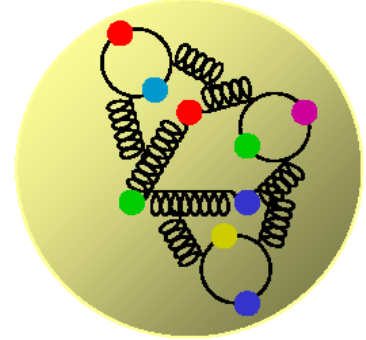
exclusive $2 \rightarrow 2$ ($\pi^\pm p \rightarrow p \rho^\pm$, etc)

~~helicity conserv.~~

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interplay of soft/hard QCD mechanism

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SSA

$$f_1^\perp(x, k_\perp) \Big|_{DY} = -f_1^\perp(x, k_\perp) \Big|_{DIS}$$

exclusive DY

GPDs

3D imaging

need to estimate $1/Q'$ power correction

exclusive $2 \rightarrow 2$ ($\pi^\pm p \rightarrow p \rho^\pm$, etc)

~~helicity conserv.~~

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