Variational method for X(3872)

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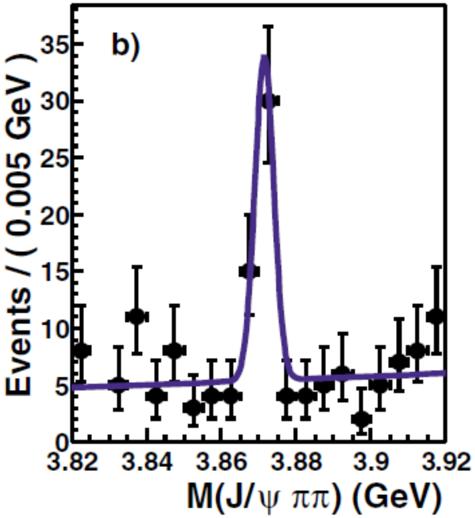
Introduction

Recently, a number of new states containing a $c\bar{c}$ quark pair have been observed.

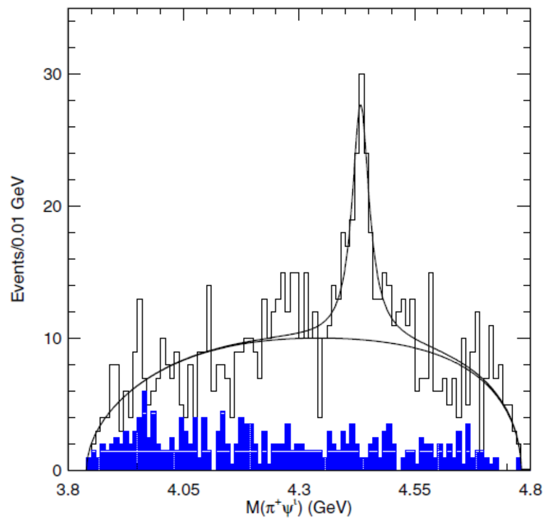
In particular, interesting channel is $J^P = 1^+$.

$X(3872)$



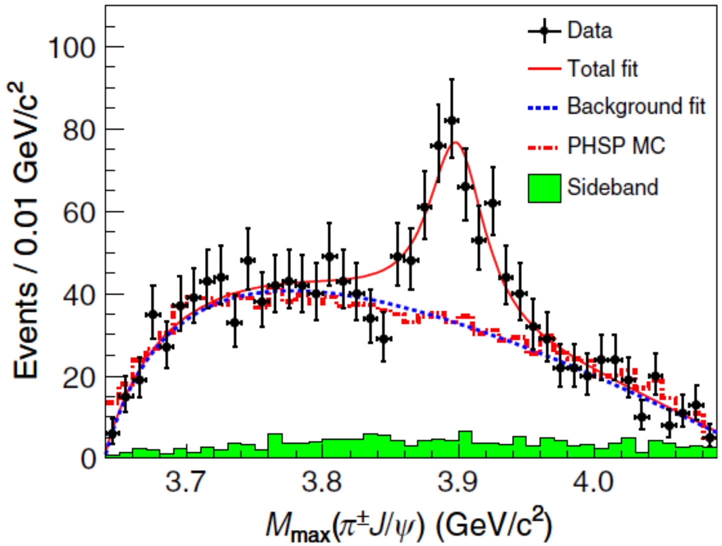
PRL 91, 262001 (2003)

$Z_c(4430)^+$



PRL 100, 142001 (2008)

$Z(3900)$



PRL 110, 252001 (2013)

Hamiltonian

$$H=\sum_{i=1}^{4}(m_i+\frac{\mathbf{p}_i^2}{2m_i})-\frac{3}{4}\sum_{i<j}^{4}\frac{\lambda_i^c}{2}\frac{\lambda_j^c}{2}(V_{ij}^C+V_{ij}^{SS})$$

$$V_{ij}^C=-\frac{\kappa}{r_{ij}}+\frac{r_{ij}}{a_0^2}-D,$$: confinement

$$V_{ij}^{SS}=\frac{\hbar^2c^2\kappa}{m_im_jc^4}\frac{1}{r_0^2r_{ij}}e^{-r_{ij}/r_0}\sigma_i\cdot\sigma_j$$: hyperfine

$m_u=m_d=340$ MeV
$m_c=1858$ MeV
$\kappa=104.5$ MeV fm
$a_0=0.0328\ (\mathrm{MeV}^{-1}\mathrm{fm})^{1/2}$
$D=913.5$ MeV
$r_1=0.4545$ fm
$\alpha=0.0015$

$$r_0=\frac{1}{\frac{1}{r_1}+\alpha\frac{m_im_j}{m_i+m_j}}$$

Mesons

	M_{theory}	M_{theory} (corrected)	M_{exp}	Δ
π	250.14	135.37	134.98	0.39
ρ	779.7	780.3	775.26	5.04
D	1912.93	1860.55	1864.84	-4.29
D^*	2030.14	2012.26	2006.96	5.3
η_c	3071.58	2990.18	2983.6	6.58
J/ψ	3121.6	3096.57	3096.92	-0.35

Unit : MeV

Coordinate Systems

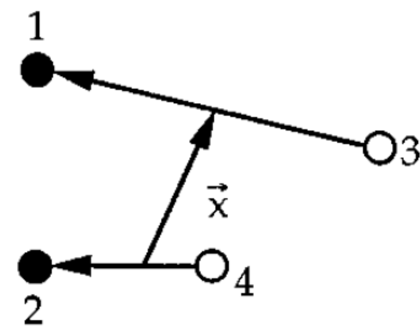
$c(1)q(2)\bar{c}(3)\bar{q}(4)$

D. M. Brink and F. Stancu, Phys. Rev. D 57, 6778 (1998)

- Coordinate I :

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathrm{r}_1 - \mathrm{r}_3), \qquad \boldsymbol{\rho}' = \frac{1}{\sqrt{2}}(\mathrm{r}_2 - \mathrm{r}_4),$$

$$\boldsymbol{x} = \frac{1}{2}(\mathrm{r}_1 - \mathrm{r}_2 + \mathrm{r}_3 - \mathrm{r}_4).$$

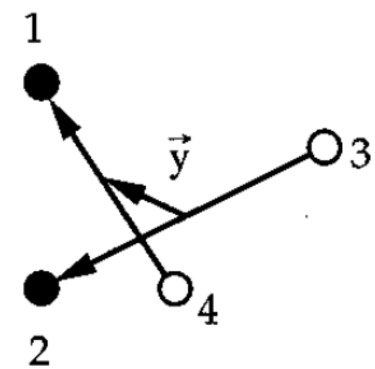


Asymptotic form (direct channel)

- Coordinate II :

$$\boldsymbol{\alpha} = \frac{1}{\sqrt{2}}(\mathrm{r}_1 - \mathrm{r}_4), \qquad \boldsymbol{\alpha}' = \frac{1}{\sqrt{2}}(\mathrm{r}_2 - \mathrm{r}_3),$$

$$\boldsymbol{y} = \frac{1}{2}(\mathrm{r}_1 - \mathrm{r}_2 - \mathrm{r}_3 + \mathrm{r}_4).$$

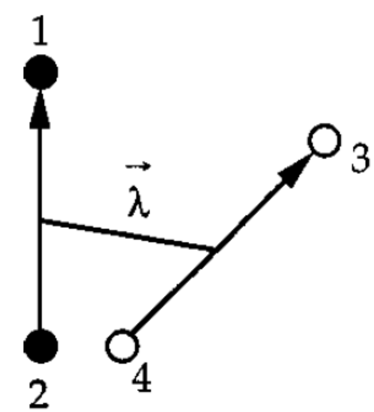


Asymptotic form (exchange channel)

- Coordinate III :

$$\boldsymbol{\sigma} = \frac{1}{\sqrt{2}}(\mathrm{r}_1 - \mathrm{r}_2), \qquad \boldsymbol{\sigma}' = \frac{1}{\sqrt{2}}(\mathrm{r}_3 - \mathrm{r}_4),$$

$$\boldsymbol{\lambda} = \frac{1}{2}(\mathrm{r}_1 + \mathrm{r}_2 - \mathrm{r}_3 - \mathrm{r}_4).$$



Closed form

Wave function (orbital part)

The most general Gaussian form with L=0 is a function of six scalar quantities expressed in either of the coordinate systems.

$$
\begin{aligned}
R^s &= \exp[-(A^s_{11}\rho^2 + A^s_{22}\rho'^2 + A^s_{33}x^2 + 2A^s_{12}\boldsymbol{\rho}\cdot\boldsymbol{\rho}' + 2A^s_{13}\boldsymbol{\rho}\cdot\boldsymbol{x} + 2A^s_{23}\boldsymbol{\rho}'\cdot\boldsymbol{x})] \\
&= \exp[-(B^s_{11}\alpha^2 + B^s_{22}\alpha'^2 + B^s_{33}y^2 + 2B^s_{12}\boldsymbol{\alpha}\cdot\boldsymbol{\alpha}' + 2B^s_{13}\boldsymbol{\alpha}\cdot\boldsymbol{y} + 2B^s_{23}\boldsymbol{\alpha}'\cdot\boldsymbol{y})] \\
&= \exp[-(C^s_{11}\sigma^2 + C^s_{22}\sigma'^2 + C^s_{33}\lambda^2 + 2C^s_{12}\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}' + 2C^s_{13}\boldsymbol{\sigma}\cdot\boldsymbol{\lambda} + 2C^s_{23}\boldsymbol{\sigma}'\cdot\boldsymbol{\lambda})]
\end{aligned}
$$

$$R^s = e^{-X^T A^s X} = e^{-Y^T B^s Y} = e^{-L^T C^s L}$$

where $X = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\rho}' \\ \mathbf{x} \end{pmatrix}$; $Y = \begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\alpha}' \\ \mathbf{y} \end{pmatrix}$; $L = \begin{pmatrix} \boldsymbol{\sigma} \\ \boldsymbol{\sigma}' \\ \boldsymbol{\lambda} \end{pmatrix}$

Wave function (spin-color part)

$$[3]\otimes[3]=[6]\oplus[\bar{3}]$$

$$[\bar{3}]\otimes[\bar{3}]=[\bar{6}]\oplus[3]$$

$$[3]\otimes[\bar{3}]=[\bar{3}]\otimes[3]=[8]\oplus[1]$$

$$[3]\otimes[3]\otimes[\bar{3}]\otimes[\bar{3}]=([6]\oplus[\bar{3}])\otimes([\bar{6}]\oplus[3])$$
$$=\boxed{([6]\otimes[\bar{6}])}\oplus([6]\otimes[3])\oplus([\bar{3}]\otimes[\bar{6}])\oplus\boxed{([\bar{3}]\otimes[3])}$$

$$[3]\otimes[\bar{3}]\otimes[3]\otimes[\bar{3}]=([8]\oplus[1])\otimes([8]\oplus[1])$$
$$=\boxed{([8]\otimes[8])}\oplus([8]\otimes[1])\oplus([1]\otimes[8])\oplus\boxed{([1]\otimes[1])}$$

Coordinate 1 : $|1_{13}1_{24}>, |8_{13}8_{24}>$ → $|1_{c\bar{c}}1_{q\bar{q}}>, |8_{c\bar{c}}8_{q\bar{q}}>$

Coordinate 2 : $|1_{14}1_{23}>, |8_{14}8_{23}>$

Coordinate 3 : $|\bar{3}_{12}3_{34}>, |6_{12}\bar{6}_{34}>$

Wave function (spin-color part)

By coupling four fermions of spin 1/2, the total spin is S=0, 1, or 2.

For S=1, there are three spin independent states.

$$\chi_1 = |V_{13}P_{24}>, \chi_2 = |V_{13}V_{24}>, \chi_3 = |P_{13}V_{24}>$$

V: vector

P: pseudoscalar

Hence, for S=1, one deals with a six-dimensional color-spin basis formed of

$$|1_{13}1_{24}>\chi_1,\ \ |1_{13}1_{24}>\chi_2,\ \ |1_{13}1_{24}>\chi_3,$$
$$|8_{13}8_{24}>\chi_1,\ \ |8_{13}8_{24}>\chi_2,\ \ |8_{13}8_{24}>\chi_3,$$

Hamiltonian (spin matrix)

$$H = \sum_{i=1}^{4}(m_i + \frac{\mathbf{p}_i^2}{2m_i}) - \frac{3}{4}\sum_{i<j}^{4}\frac{\lambda_i^c}{2}\frac{\lambda_j^c}{2}(V_{ij}^C + V_{ij}^{SS})$$

$$S^2|sm>= s(s+1)|sm>$$
$$\sigma^2|sm>= 4s(s+1)|sm>$$
$$\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j = \frac{1}{2}(\sigma_{ij}^2 - \sigma_i^2 - \sigma_j^2)$$

$$V_{ij}^{SS} = \frac{\hbar^2 c^2 \kappa}{m_i m_j c^4}\frac{1}{r_0^2 r_{ij}} e^{-r_{ij}/r_0}\sigma_i \cdot \sigma_j$$

$|V_{12}P_{34}>, |V_{12}V_{34}>, |P_{12}V_{34}>$

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \quad \boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_4 = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -\sqrt{2} & 1 \\ -\sqrt{2} & -1 & \sqrt{2} \\ 1 & \sqrt{2} & 0 \end{pmatrix}$$

$$\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_4 = \begin{pmatrix} 0 & \sqrt{2} & 1 \\ \sqrt{2} & -1 & -\sqrt{2} \\ 1 & -\sqrt{2} & 0 \end{pmatrix} \quad \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_4 = \begin{pmatrix} 0 & \sqrt{2} & -1 \\ \sqrt{2} & -1 & \sqrt{2} \\ -1 & \sqrt{2} & 0 \end{pmatrix} \quad \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -\sqrt{2} & -1 \\ -\sqrt{2} & -1 & -\sqrt{2} \\ -1 & -\sqrt{2} & 0 \end{pmatrix}$$

Hamiltonian (color matrix)

$[1]=D(0,0)$ $[3]=D(1,0)=\begin{array}{|c|}\hline \ \\ \hline\end{array}$ $[\bar{3}]=D(0,1)=\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$

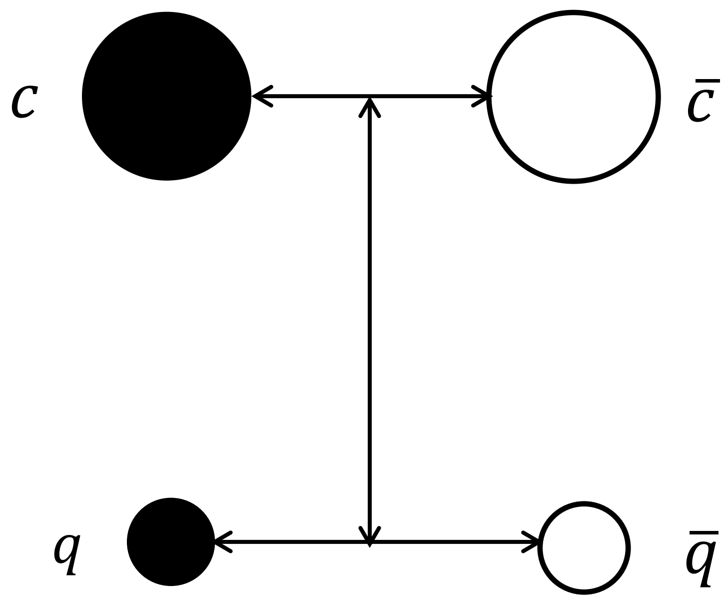
$[6]=D(2,0)=\begin{array}{|c|c|}\hline \ & \ \\ \hline\end{array}$ $[\bar{6}]=D(0,2)=\begin{array}{|c|c|}\hline \ & \ \\ \hline \ & \ \\ \hline\end{array}$

$$\lambda^c\lambda^c\psi(D(p_1,p_2)) = \frac{4}{3}(p_1^2 + p_1p_2 + p_2^2 + 3p_1 + 3p_2)\psi(D(p_1,p_2))$$

	$6_{12}\bar{6}_{34}, \bar{3}_{12}3_{34}$	$8_{13}8_{24}, 1_{13}1_{24}$	$8_{14}8_{23}, 1_{14}1_{23}$
$\lambda_1^c\lambda_2^c = \lambda_3^c\lambda_4^c$	$\begin{pmatrix} \frac{4}{3} & 0 \\ 0 & -\frac{8}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ \frac{4\sqrt{2}}{3} & 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{4}{3} & \frac{4\sqrt{2}}{3} \\ \frac{4\sqrt{2}}{3} & 0 \end{pmatrix}$
$\lambda_1^c\lambda_3^c = \lambda_2^c\lambda_4^c$	$\begin{pmatrix} -\frac{10}{3} & -2\sqrt{2} \\ -2\sqrt{2} & -\frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{16}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{14}{3} & -\frac{4\sqrt{2}}{3} \\ -\frac{4\sqrt{2}}{3} & 0 \end{pmatrix}$
$\lambda_1^c\lambda_4^c = \lambda_2^c\lambda_3^c$	$\begin{pmatrix} -\frac{10}{3} & 2\sqrt{2} \\ 2\sqrt{2} & -\frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{14}{3} & -\frac{4\sqrt{2}}{3} \\ -\frac{4\sqrt{2}}{3} & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{16}{3} \end{pmatrix}$

X(3872) in $(J/\psi + \rho)$ configuration

$\mathrm{c}(1)q(2)\bar{c}(3)\bar{q}(4)$



C = +

$$\left|1_{13}1_{24}\right\rangle\left|V_{13}V_{24}\right\rangle$$
$$\left|8_{13}8_{24}\right\rangle\left|V_{13}V_{24}\right\rangle$$

C = -

$$\left|8_{13}8_{24}\right\rangle\left|P_{13}V_{24}\right\rangle$$
$$\left|1_{13}1_{24}\right\rangle\left|V_{13}P_{24}\right\rangle$$
$$\left|8_{13}8_{24}\right\rangle\left|V_{13}P_{24}\right\rangle$$
$$\left|1_{13}1_{24}\right\rangle\left|P_{13}V_{24}\right\rangle$$

Charge conjugation = +

Ground state in $(J/\psi + \rho)$ configuration

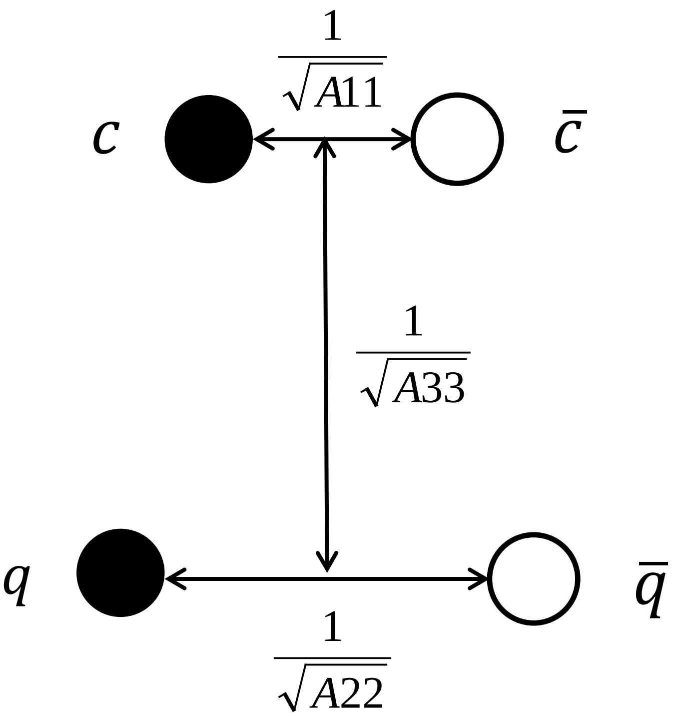
$$R^s = \exp[-(A_{11}^s\rho^2 + A_{22}^s\rho'^2 + A_{33}^s x^2 + 2A_{12}^s \boldsymbol{\rho}\cdot\boldsymbol{\rho}' + 2A_{13}^s \boldsymbol{\rho}\cdot\boldsymbol{x} + 2A_{23}^s \boldsymbol{\rho}'\cdot\boldsymbol{x})]$$

For C=+, ground state is $\left|1_{13}1_{24}\right\rangle\left|V_{13}V_{24}\right\rangle$

A11=10.6, A22=2, A33 $\cong$ 0, A12=0

E=3876.98 MeV

This is the sum of two mesons(J/ψ + ρ).



Excited state in $(J/\psi + \rho)$ configuration

$$R^s = \exp[-(A^s_{11}\rho^2 + A^s_{22}\rho'^2 + A^s_{33}x^2 + 2A^s_{12}\boldsymbol{\rho}\cdot\boldsymbol{\rho}' + 2A^s_{13}\boldsymbol{\rho}\cdot\boldsymbol{x} + 2A^s_{23}\boldsymbol{\rho}'\cdot\boldsymbol{x})]$$

For C=+, excited state is

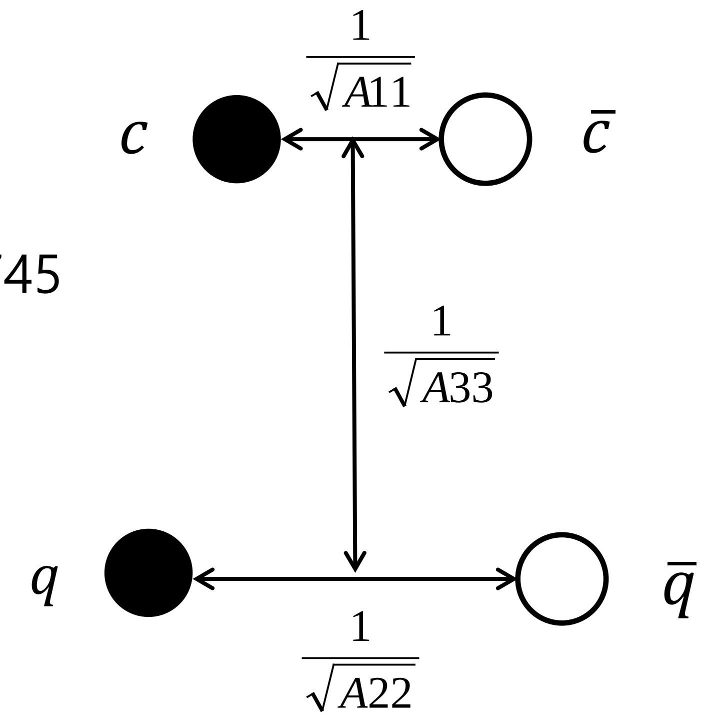
$0.333|1_{13}1_{24}\rangle|V_{13}V_{24}\rangle + 0.943|8_{13}8_{24}\rangle|V_{13}V_{24}\rangle$

A11=2.0755, A22=2.0755, A33=4.15, A12=2.0745

E=3883.71 MeV

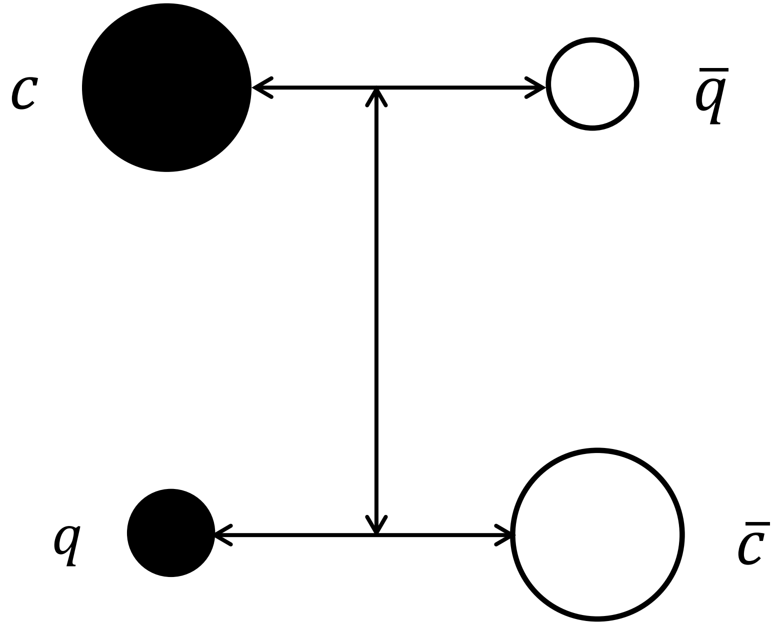
It contains octet-octet state.

And A33 is not zero.



X(3872) in $(D + D^*)$ configuration

$\mathrm{c}(1)q(2)\bar{c}(3)\bar{q}(4)$



Coordinate Transformation

$c(1)q(2)\bar{c}(3)\bar{q}(4)$

From coordinate transformation,

$$|8_{13}8_{24}\rangle|V_{13}P_{24}\rangle$$
$$|8_{13}8_{24}\rangle|V_{13}V_{24}\rangle$$
$$|8_{13}8_{24}\rangle|P_{13}V_{24}\rangle$$
$$|1_{13}1_{24}\rangle|V_{13}P_{24}\rangle$$
$$|1_{13}1_{24}\rangle|V_{13}V_{24}\rangle$$
$$|1_{13}1_{24}\rangle|P_{13}V_{24}\rangle$$

$\Rightarrow$

$$|8_{14}8_{23}\rangle|V_{14}P_{23}\rangle$$
$$|8_{14}8_{23}\rangle|V_{14}V_{23}\rangle$$
$$|8_{14}8_{23}\rangle|P_{14}V_{23}\rangle$$
$$|1_{14}1_{23}\rangle|V_{14}P_{23}\rangle$$
$$|1_{14}1_{23}\rangle|V_{14}V_{23}\rangle$$
$$|1_{14}1_{23}\rangle|P_{14}V_{23}\rangle$$

$\Rightarrow$

C = +

$$\boxed{\begin{array}{c} \frac{1}{\sqrt{2}}(|1_{14}1_{23}\rangle|P_{14}V_{23}\rangle-|1_{14}1_{23}\rangle|V_{14}P_{23}\rangle) \\ \frac{1}{\sqrt{2}}(|8_{14}8_{23}\rangle|P_{14}V_{23}\rangle-|8_{14}8_{23}\rangle|V_{14}P_{23}\rangle) \end{array}}$$

C = -

$$\boxed{\begin{array}{c} \frac{1}{\sqrt{2}}(|1_{14}1_{23}\rangle|P_{14}V_{23}\rangle+|1_{14}1_{23}\rangle|V_{14}P_{23}\rangle) \\ \frac{1}{\sqrt{2}}(|8_{14}8_{23}\rangle|P_{14}V_{23}\rangle+|8_{14}8_{23}\rangle|V_{14}P_{23}\rangle) \\ |8_{14}8_{23}\rangle|V_{14}V_{23}\rangle \\ |1_{14}1_{23}\rangle|V_{14}V_{23}\rangle \end{array}}$$

But some of them are not charge conjugation eigenstate.

So we need to rearrange the basis.

Corresponding state in $(D + D^*)$ configuration

$$R^s = \exp[-(B^s_{11}\alpha^2 + B^s_{22}\alpha'^2 + B^s_{33}y^2 + 2B^s_{12}\boldsymbol{\alpha}\cdot\boldsymbol{\alpha}' + 2B^s_{13}\boldsymbol{\alpha}\cdot\boldsymbol{y} + 2B^s_{23}\boldsymbol{\alpha}'\cdot\boldsymbol{y})]$$

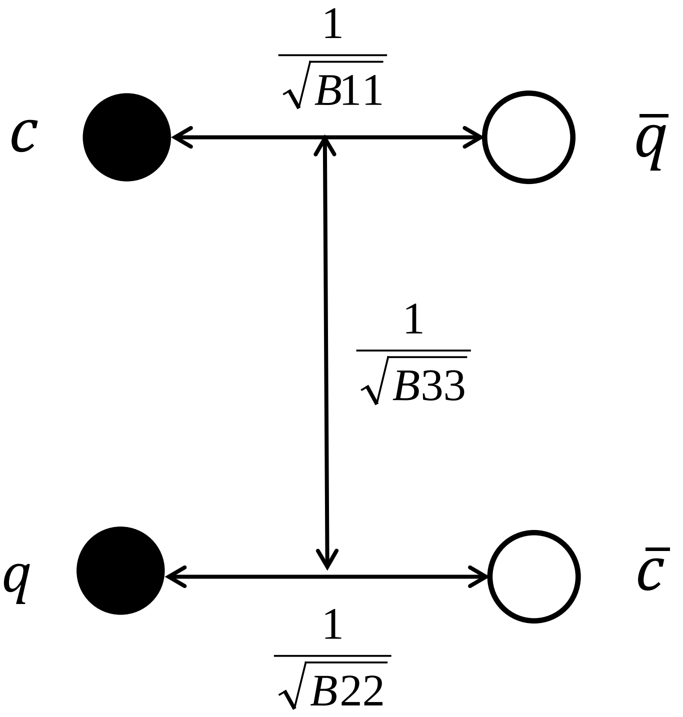
For C=+, local minimum state is

$$\frac{1}{\sqrt{2}}(|1_{14}1_{23}\rangle|P_{14}V_{23}\rangle - |1_{14}1_{23}\rangle|V_{14}P_{23}\rangle)$$

B11=4.15, B22=4.15, B33 ≅ 0, B12=0

E=3883.71 MeV

This is the sum of two mesons$(D + D^*)$.



→ Excited state in $(J/\psi + \rho)$ configuration is also the sum of two mesons.

Charge conjugation = -

Ground state in $(J/\psi + \rho)$ configuration

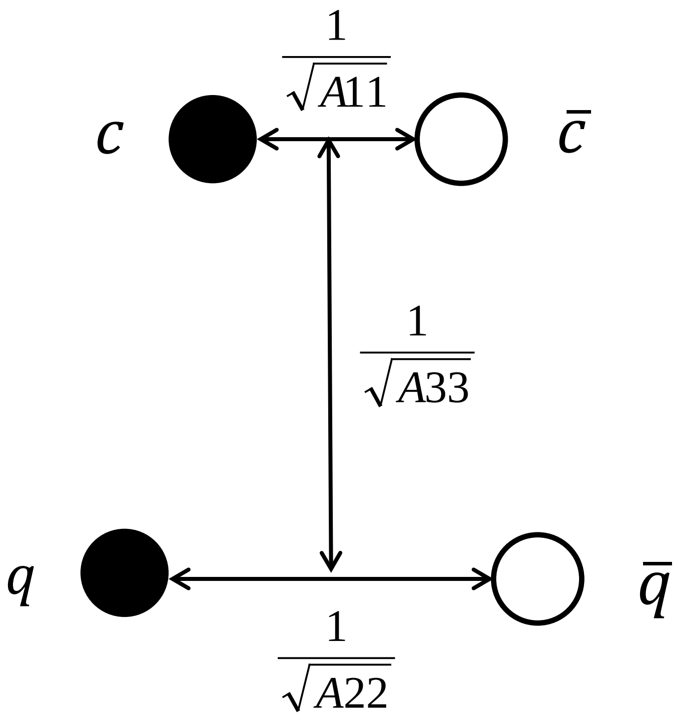
$$R^s = \exp[-(A^s_{11}\rho^2 + A^s_{22}\rho'^2 + A^s_{33}x^2 + 2A^s_{12}\boldsymbol{\rho}\cdot\boldsymbol{\rho}' + 2A^s_{13}\boldsymbol{\rho}\cdot\boldsymbol{x} + 2A^s_{23}\boldsymbol{\rho}'\cdot\boldsymbol{x})]$$

For C=-, ground state is $\left|1_{13}1_{24}\right\rangle\left|V_{13}P_{24}\right\rangle$

A11=10.6, A22=6.1, A33 $\cong$ 0, A12=0

E=3231.95 MeV

This is the sum of two mesons(J/ψ + π).



Excited states in $(J/\psi+\rho)$ configuration

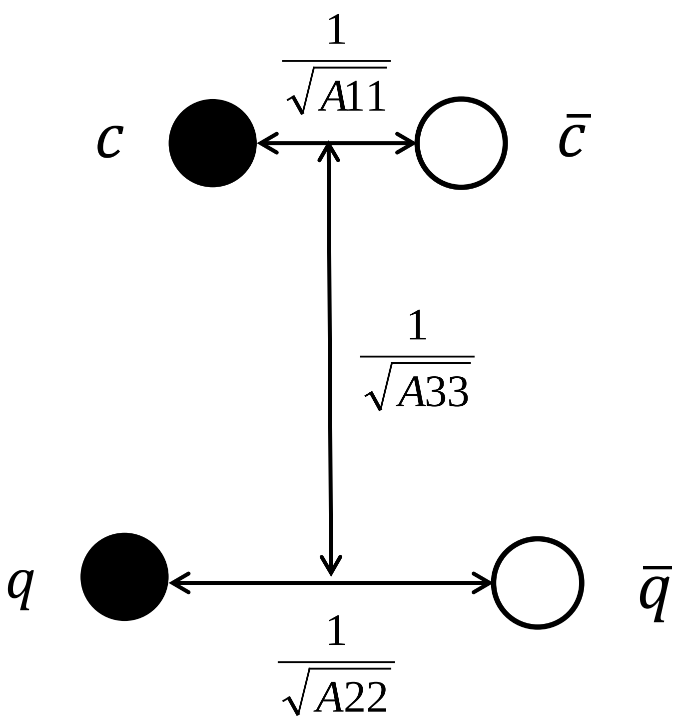
$$R^s = \exp[-(A^s_{11}\rho^2 + A^s_{22}\rho'^2 + A^s_{33}x^2 + 2A^s_{12}\boldsymbol{\rho}\cdot\boldsymbol{\rho}' + 2A^s_{13}\boldsymbol{\rho}\cdot\boldsymbol{x} + 2A^s_{23}\boldsymbol{\rho}'\cdot\boldsymbol{x})]$$

For C=-, Excited state is $\left|1_{13}1_{24}\right\rangle\left|P_{13}V_{24}\right\rangle$

A11=13.9, A22=2, A33 $\cong$ 0, A12=0

E=3770.5 MeV

This is the sum of two mesons($\eta_c+\rho$).



Summary

1. For $cq\bar{c}\bar{q}$, ground state is just the sum of two meson states in $(J/\psi + \rho)$ configuration .

2. Excited state in $(J/\psi + \rho)$ configuration corresponds to $\frac{1}{\sqrt{2}}(DD^* - D^*D)$ in $(D + D^*)$ configuration.

3. There is no tetraquark state near E=3872MeV or 3900MeV in this model.

4. If we could put the additional interaction term, then we might be able to obtain bounded tetraquark state in this nonrelativistic potential model.