A Lattice Calculation of Charmed Baryon Electromagnetic Form Factors

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based on JHEP05(2014)125

Workshop on Progress on J-PARC hadron physics in 2014, Nov. 30 - Dec. 2
Ibaraki Quantum Beam Research Center, Tokai

MOTIVATION

- *Probe the hadron structure*
  - *size, charge radii, magnetic moment*
  - *Effect of heavy quarks*
    - *Previous work: heavy quark shrinks the mesons*

kuc, G. Erkol, M. Oka, A. Ozpineci, T.T. Takahashi PLB 719

$$\langle r^2 \rangle_D = 0.138\ fm^2$$
$$\langle r^2 \rangle_{D^*} = 0.185\ fm^2$$
$$\langle r^2 \rangle_\pi = 0.452\ fm^2$$

- *New data anticipated, theory should keep up*

OUTLINE

- *Lattice QCD*
- *Electromagnetic (EM) form factors*
  - *Parameterisation*
  - *Lattice Formulation*
- *Simulation Details*
- *Results*
- *Summary and outlook*

LATTICE QCD

Two key equations:

$$\lim_{T\to\infty} \langle \hat{O}_2(t)\hat{O}_1(0)\rangle_T = \sum_h \langle 0|\hat{O}_2|h\rangle\langle h|\hat{O}_1|0\rangle e^{-E_h t}$$

sum over Hamiltonian eigenstates (hadrons)

$$\left\langle \hat{O}_2(t)\hat{O}_1(0) \right\rangle = \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} O_2[\Psi(\vec{x},t)] O_1[\Psi(\vec{x},0)]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}}$$

path integral (quark d.o.f.)

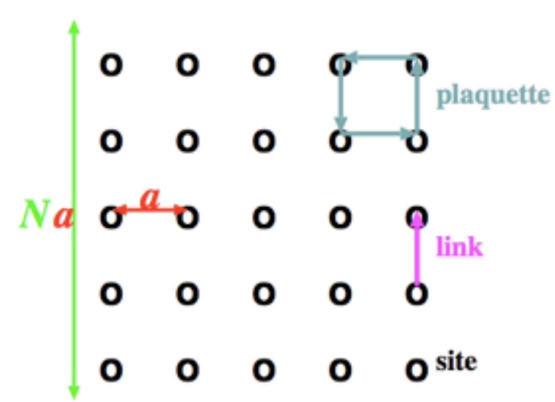
Tools of the stat. physics:

Importance Sampling

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]} \mathcal{O}[\Psi]}{\int \mathcal{D}[\Psi] e^{-S_E[\Psi]}} = \lim_{N\to\infty} \sum_{n=1}^{N} \mathcal{O}[U_n]$$

- e^{-S} acts as the Boltzmann factor
- Euclidean action to tame the oscillation
  - Wick rotation to imaginary time

- Discretize the space-time continuum
- Non-perturbatively regularizes theory
- Solvable by computers



continuum	→	lattice
x	→	$n = x/a \in \Lambda_E$
$\phi(x)$	→	ϕ_n
$\partial_\mu\phi(x)$	→	$\frac{1}{2}(\nabla_\mu + \nabla^*_\mu)\phi_n$
	$\nabla_\mu\phi_n$	$= (\phi_{n+\hat{\mu}} - \phi_n)/a$
	$\nabla^*_\mu\phi_n$	$= (\phi_n - \phi_{n-\hat{\mu}})/a$
$\Box\phi(x)$	→	$\sum_\mu \nabla^*_\mu\nabla_\mu\phi_n$
$\int d^4x$	→	$a^4\sum_n$

EM FORM FACTORS

$$\langle \mathcal{B}(p)|V_\mu|\mathcal{B}(p')\rangle = \bar{u}(p)\left[\gamma_\mu F_{1,\mathcal{B}}(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2m_\mathcal{B}} F_{2,\mathcal{B}}(q^2)\right] u(p)$$

Sachs FFs

$$G_{E,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + \frac{q^2}{4m_\mathcal{B}^2} F_{2,\mathcal{B}}(q^2)$$

$$G_{M,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + F_{2,\mathcal{B}}(q^2)$$

Charge Radii & Magnetic Moment

$$\langle r^2_{E,M}\rangle = -\frac{6}{G_{\mathrm{E,M}}(0)}\frac{d}{dQ^2}G_{E,M}(Q^2)\Big|_{Q^2=0}$$

$$G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{(1+Q^2/\Lambda^2_{E,M})^2}$$

$\Longrightarrow$

$$\langle r^2_{E,M}\rangle = \frac{12}{\Lambda^2_{E,M}}$$

$$\mu_B = G_M(0)\left(\frac{m_N}{2m_B}\right)\mu_N$$

EM FORM FACTORS

Lattice Formulation

$$\langle C^{\mathcal{B}}(t;\mathbf{p};\Gamma_4)\rangle = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}\Gamma_4^{\alpha\alpha'}\langle \text{vac}|T[\eta_{\mathcal{B}}^{\alpha}(x)\bar{\eta}_{\mathcal{B}}^{\alpha'}(0)]|\text{vac}\rangle$$

$$\langle C^{\mathcal{B}\mathcal{V}_\mu\mathcal{B}'}(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma)\rangle = -i\sum_{\mathbf{x_2},\mathbf{x_1}} e^{-i\mathbf{p}\cdot\mathbf{x_2}}e^{i\mathbf{q}\cdot\mathbf{x_1}}\Gamma^{\alpha\alpha'}\langle \text{vac}|T[\eta_{\mathcal{B}}^{\alpha}(x_2)V_\mu(x_1)\bar{\eta}_{\mathcal{B}'}^{\alpha'}(0)]|\text{vac}\rangle$$

$$R(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma};\mu) = \frac{\langle C^{\mathcal{B}\mathcal{V}_\mu\mathcal{B}'}(t_2,t_1;\mathbf{p}',\mathbf{p};\mathbf{\Gamma})\rangle}{\langle C^{\mathcal{B}\mathcal{B}}(t_2;\mathbf{p}';\Gamma_4)\rangle}$$

$$\times\left[\frac{\langle C^{\mathcal{B}\mathcal{B}}(t_2-t_1;\mathbf{p};\Gamma_4)\rangle\langle C^{\mathcal{B}\mathcal{B}}(t_1;\mathbf{p}';\Gamma_4)\rangle\langle C^{\mathcal{B}\mathcal{B}}(t_2;\mathbf{p}';\Gamma_4)\rangle}{\langle C^{\mathcal{B}\mathcal{B}}(t_2-t_1;\mathbf{p}';\Gamma_4)\rangle\langle C^{\mathcal{B}\mathcal{B}}(t_1;\mathbf{p};\Gamma_4)\rangle\langle C^{\mathcal{B}\mathcal{B}}(t_2;\mathbf{p};\Gamma_4)\rangle}\right]^{1/2}$$

$$\eta_{\Xi_{cc}}(x) = \epsilon^{ijk}[c^{Ti}(x)C\gamma_5\ell^j(x)]c^k(x)$$

$$\eta_{\Sigma_c}(x) = \epsilon^{ijk}[\ell^{Ti}(x)C\gamma_5 c^j(x)]\ell^k(x)$$

$$\eta_{\Omega_{cc}}(x) = \epsilon^{ijk}[c^{Ti}(x)C\gamma_5 s^j(x)]c^k(x)$$

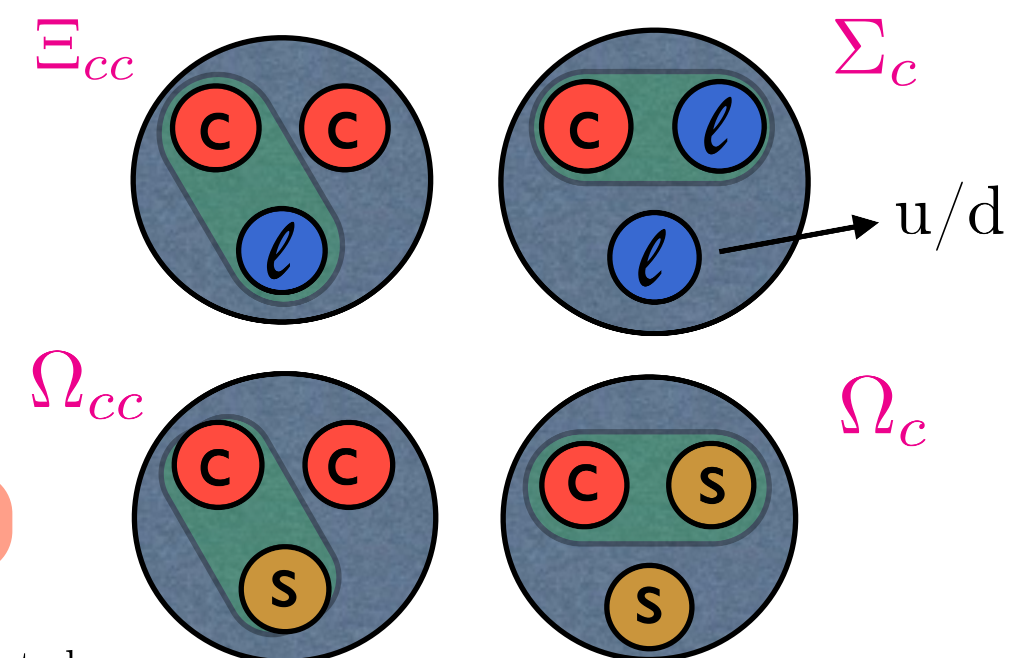
$$\eta_{\Omega_c}(x) = \epsilon^{ijk}[s^{Ti}(x)C\gamma_5 c^j(x)]s^k(x)$$

$$R(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma;\mu) \xrightarrow[t_2-t_1\gg a]{t_1\gg a} \Pi(\mathbf{p}',\mathbf{p};\Gamma;\mu)$$

$$\Pi(\mathbf{0},-\mathbf{q};\Gamma_4;\mu=4) = \left[\frac{(E_{\mathcal{B}}+m_{\mathcal{B}})}{2E_{\mathcal{B}}}\right]^{1/2} G_{E,\mathcal{B}}(q^2)$$

$$\Pi(\mathbf{0},-\mathbf{q};\Gamma_j;\mu=i) = \left[\frac{1}{2E_{\mathcal{B}}(E_{\mathcal{B}}+m_{\mathcal{B}})}\right]^{1/2} \epsilon_{ijk}q_k G_{M,\mathcal{B}}(q^2)$$

G(0) should be extrapolated from higher momenta



SIMULATION DETAILS

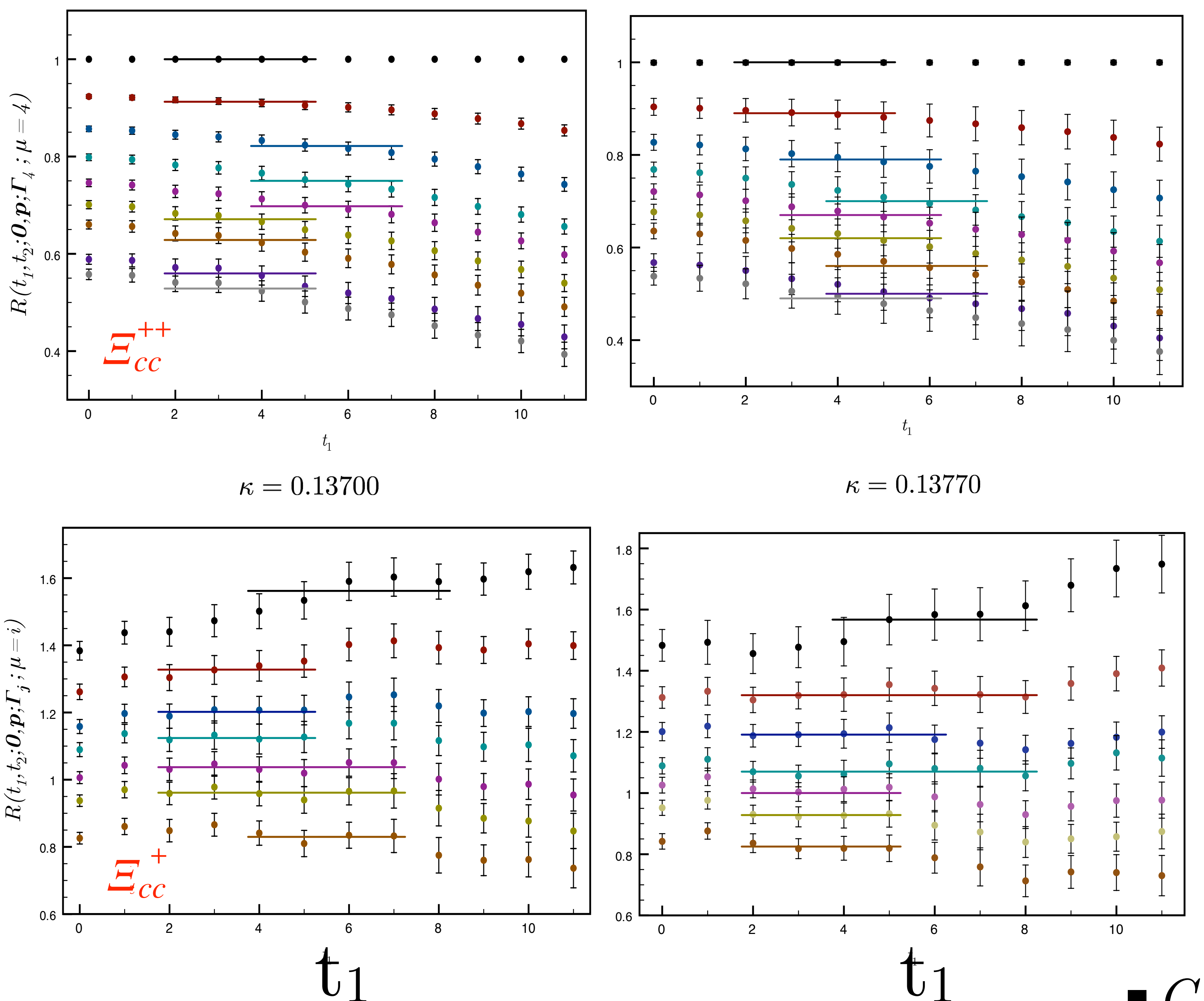
1. PACS-CS generated 32^3x64, β=1.9, 2+1 flavor (u/d,s) lattices Phys. Rev. D79 (034503)
   I. Gauge action: *Iwasaki*, Fermion action: *Clover*
   II. a = 0.0907(13) fm, a^{-1} = 2.176(31) GeV
   III. Box Size: $(2.9\ \mathrm{fm})^3$ x 5.8 fm
   IV. $\kappa_{ud} = 0.13700, 0.13727, 0.13754, 0.13770$,
      i. $m_\pi \sim 700, 570, 410, 300$ MeV
   V. $\kappa_s = 0.13640$, $\kappa_c = 0.1246$
2. Clover action for all valance quarks
   I. $c_E = c_B = 1/(u_0)^3$ (FermiLAB method)
   II. κ_c tuned to 1S $M_{\eta\text{-}J/\psi}$, $M_{D\text{-}D^*}$, $M_{D_s\text{-}D_s{}^*}$
3. Point-split (conserved) vector current: renormalisation not necessary
4. Connected diagrams only
5. Multiple Shell source - Wall sink pairs
   I. t =12 a separation
   II. Smearing: <r_l> ~ 0.5 fm, <r_c> ~ <r_l>/3
   III. Wall sinks: no need for sequential inversions, *caveat: increased noise!*
      i. Coulomb gauge fix: wall smearing is gauge dependent
6. Stat. errors single-elimination Jackknife analysis

RESULTS

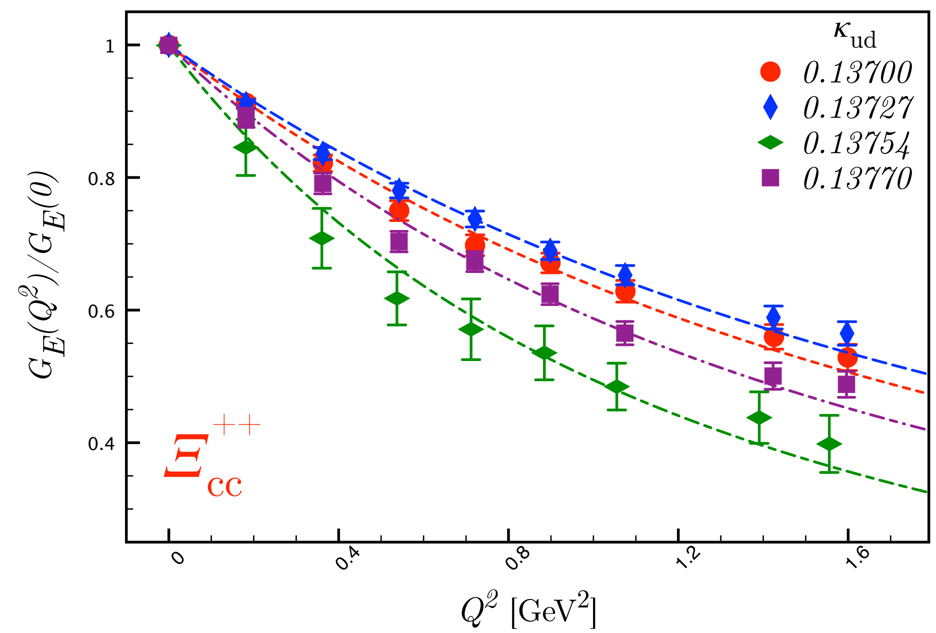
- $\kappa_{\mathrm{ud}} = 0.13700/70$
- p-value criteria
- 9 (7) 4-mom insertions for electric (magnetic) form factor

plateaus and form factor fits: Ξ_{cc}

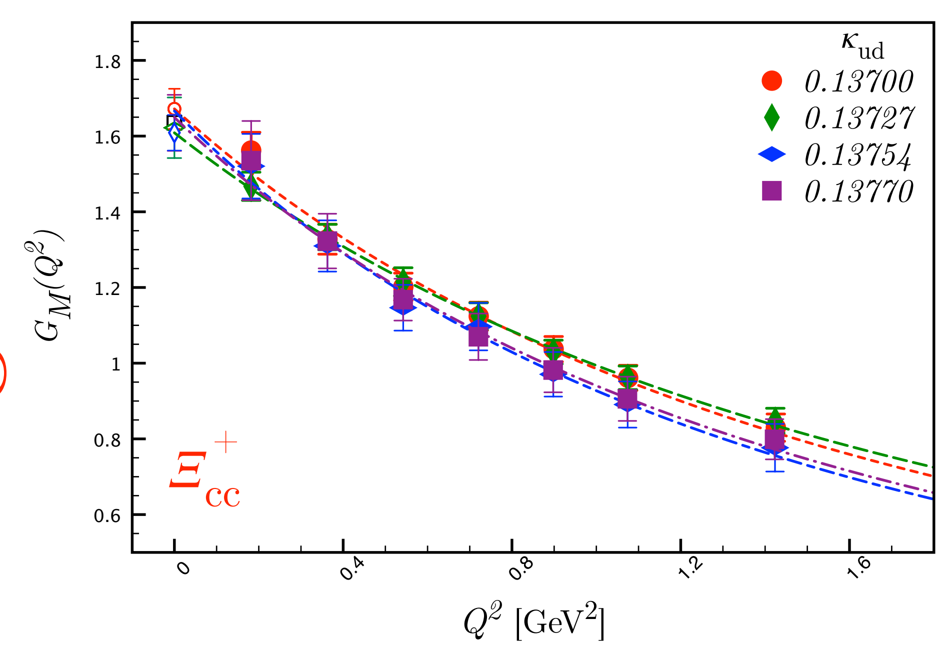
$m_\pi =$ 700 MeV   300 MeV



Electric



Magnetic



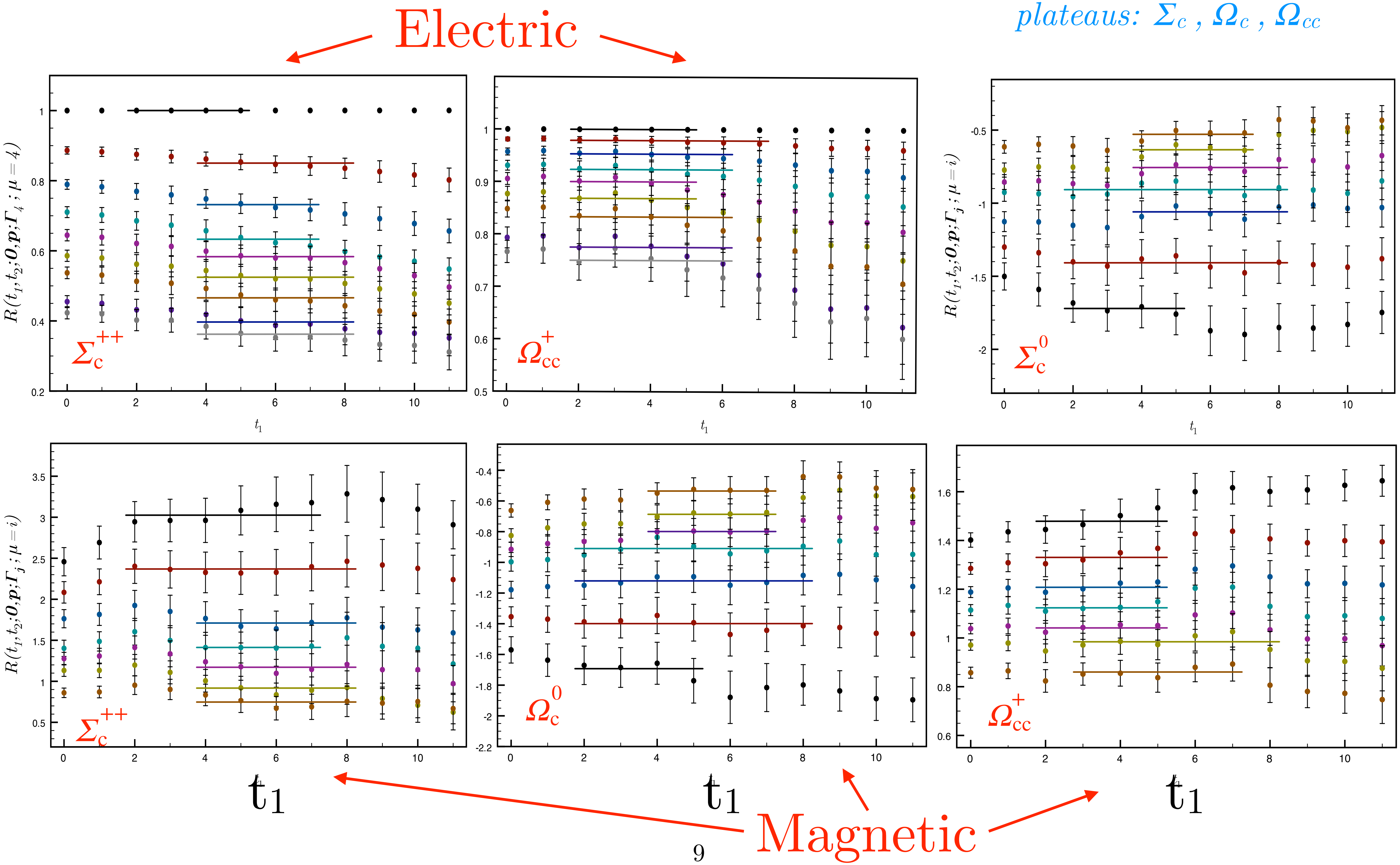
- *Good fits to the dipole form*
- *EFFs are normalised to unit charge*

- $\kappa_{ud} = 0.13700$
- p-value criteria

- 9 (7) 4-mom insertions for electric (magnetic) form factor

RESULTS

plateaus: Σ_c , Ω_c , Ω_{cc}

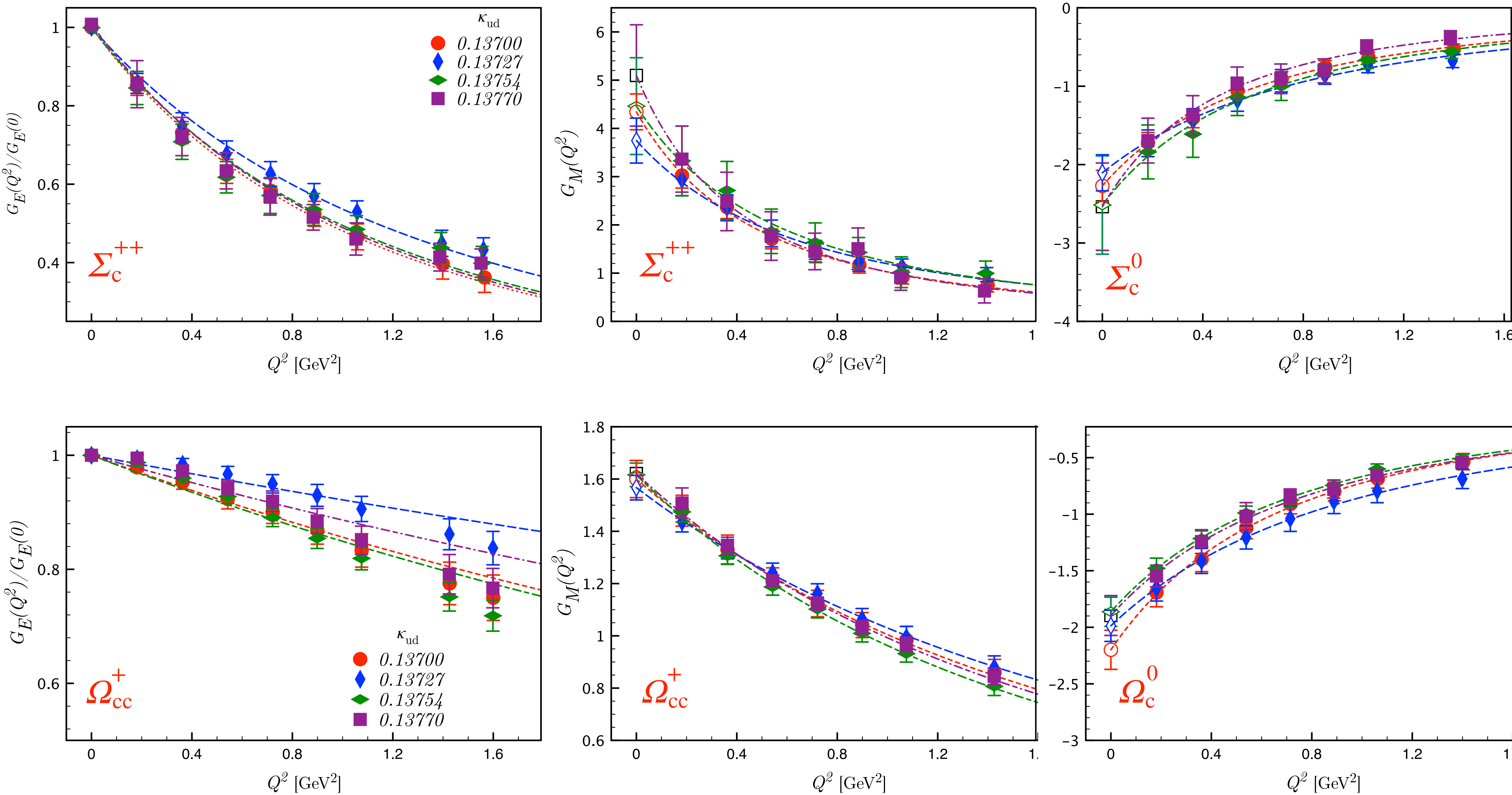


- $\kappa_{\text{ud}} = \text{All}$
- Dipole Form

- 9 (7) 4-mom insertions for electric (magnetic) form factor

RESULTS

form factors: Σ_c , Ω_c , Ω_{cc}



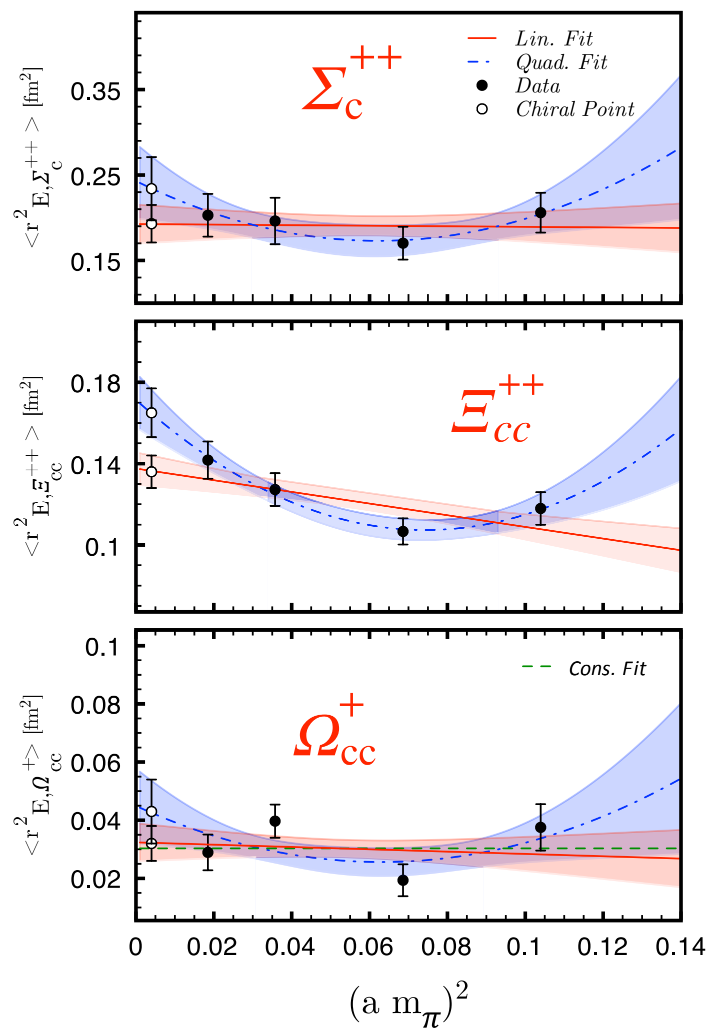
Electric

Magnetic

κ_{ud}(137-)		70	54	27	00
stat.	(Σ_c, Ξ_{cc})	170	150	100	100
	(Ω_c, Ω_{cc})	130	100	100	100

CHIRAL FITS

$\langle r^2_E \rangle$

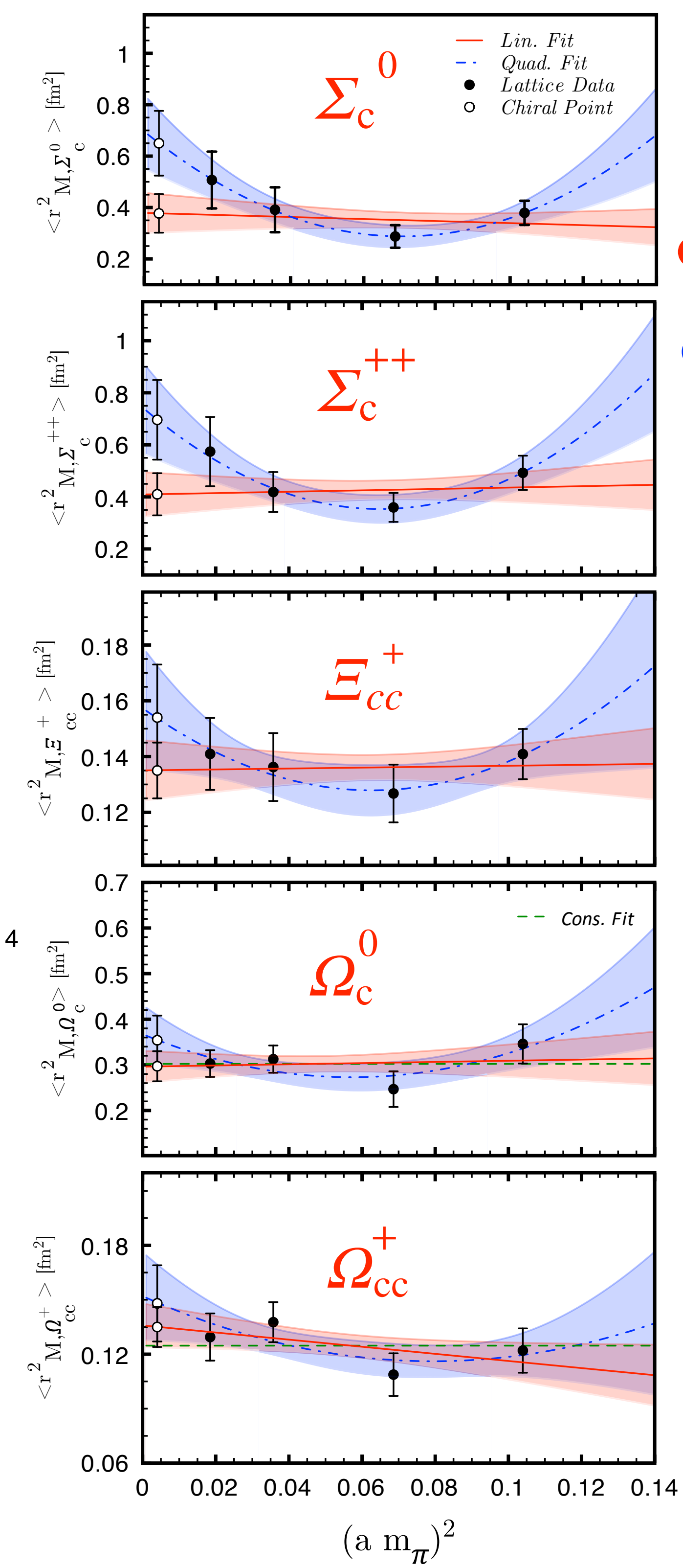


m_π ~ 300 410 570 700 MeV

Fit Form	Quad. Fit
uuc $\langle r^2_{E,\Sigma_c^{++}} \rangle$	0.234(37)
ucc $\langle r^2_{E,\Xi_{cc}^{++}} \rangle$	0.165(12)
scc $\langle r^2_{E,\Omega_{cc}^{+}} \rangle$	0.043(11)
dcc $\langle r^2_{E,\Xi_{cc}^{+}} \rangle = 0.042(9)$	

*$\langle r^2_{E,p} \rangle = 0.770\ fm^2$

$\langle r^2_M \rangle$

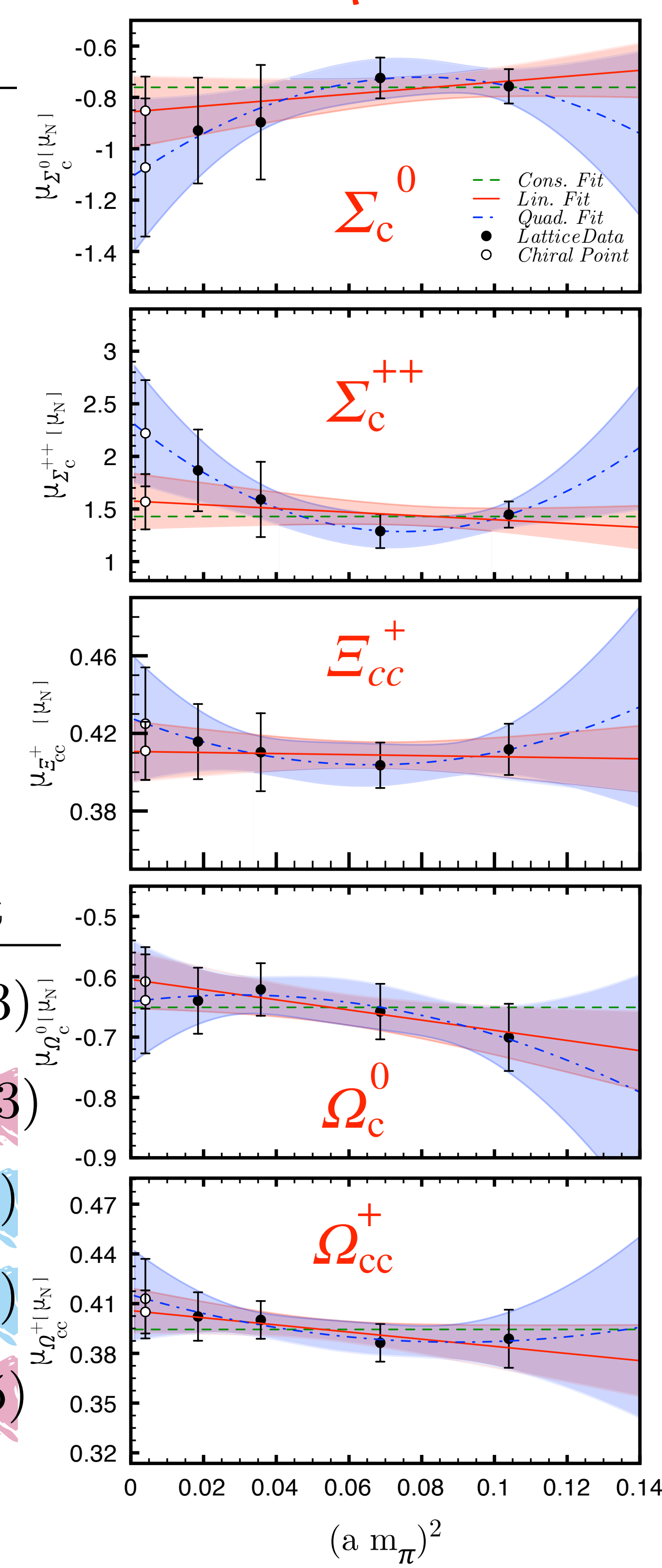


Fit Form	Quad. Fit
uuc $\langle r^2_{M,\Sigma_c^{++}} \rangle$	0.696(153)
ddc $\langle r^2_{M,\Sigma_c^{0}} \rangle$	0.650(126)
dcc $\langle r^2_{M,\Xi_{cc}^{+}} \rangle$	0.154(19)
scc $\langle r^2_{M,\Omega_{cc}^{+}} \rangle$	0.148(21)
ssc $\langle r^2_{M,\Omega_c^{0}} \rangle$	0.354(54)

*$\langle r^2_{M,p} \rangle = 0.604\ fm^2$

$\langle r^2_{M,n} \rangle = 0.862\ fm^2$

μ



Fit Form	Lin. Fit
uuc $\mu_{\Sigma_c^{++}}$	1.569(253)
ddc $\mu_{\Sigma_c^{0}}$	-0.852(133)
dcc $\mu_{\Xi_{cc}^{+}}$	0.411(15)
scc $\mu_{\Omega_{cc}^{+}}$	0.405(13)
ssc $\mu_{\Omega_c^{0}}$	-0.608(45)

*$\mu_p = 2.793\ \mu_N$

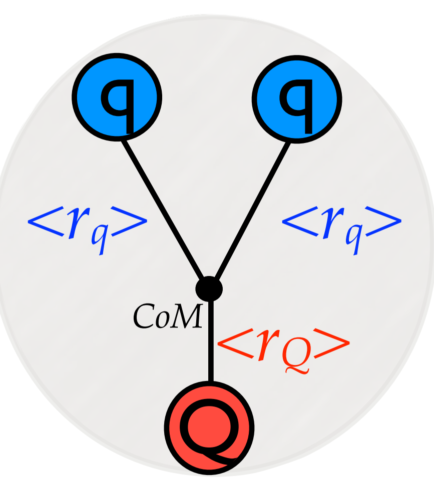
$\mu_n = -1.913\ \mu_N$

* PDG values

kuc, G. Erkol, B. Isildak, M. Oka, T.T. Takahashi, JHEP05(2014)125

QUARK CONTRIBUTIONS

Baryon	Fit Form	$\langle r_E^2\rangle_q$	$\langle r_E^2\rangle_Q$	$\langle r_M^2\rangle_q$	$\langle r_M^2\rangle_Q$	μ_q	μ_Q
		[fm^2]	[fm^2]	[fm^2]	[fm^2]	[μ_N]	[μ_N]
$\Sigma_c^{0,++}$	Lin. Fit	0.347(49)	0.032(18)	0.403(67)	0.098(80)	2.369(362)	-0.099(21)
	Quad. Fit	0.390(86)	0.066(32)	0.604(118)	0.236(183)	2.943(732)	-0.059(36)
$\Xi_{cc}^{+,++}$	Lin. Fit	0.386(33)	0.068(5)	0.426(60)	0.082(6)	-0.410(51)	0.430(8)
	Quad. Fit	0.410(46)	0.095(9)	0.612(115)	0.089(11)	-0.516(117)	0.433(16)
Ω_c^0	Lin. Fit	0.330(32)	0.064(10)	0.398(44)	0.056(19)	1.710(150)	-0.099(14)
	Quad. Fit	0.398(52)	0.069(22)	0.484(70)	0.054(38)	1.915(279)	-0.083(28)
Ω_{cc}^+	Lin. Fit	0.287(31)	0.078(7)	0.350(44)	0.095(9)	-0.370(26)	0.441(12)
	Quad. Fit	0.422(51)	0.104(13)	0.534(72)	0.101(16)	-0.428(58)	0.453(22)



$\langle r^2\rangle_{E,\Sigma_c^{++}}$ = 0.234(37) fm^2

$\langle r^2\rangle_{E,\Xi_{cc}^{+}}$ = 0.042(9) fm^2

$\langle r^2\rangle_{E,\Xi_{cc}^{++}}$ = 0.165(12) fm^2

$\langle r^2\rangle_{E,\Omega_{cc}^{+}}$ = 0.043(11) fm^2

$\langle r^2\rangle_{M,\Sigma_c^{0}}$ = 0.650(126) fm^2

$\langle r^2\rangle_{M,\Sigma_c^{++}}$ = 0.696(153) fm^2

$\langle r^2\rangle_{M,\Xi_{cc}^{+}}$ = 0.154(19) fm^2

$\langle r^2\rangle_{M,\Omega_c^{0}}$ = 0.354(54) fm^2

$\langle r^2\rangle_{M,\Omega_{cc}^{+}}$ = 0.148(21) fm^2

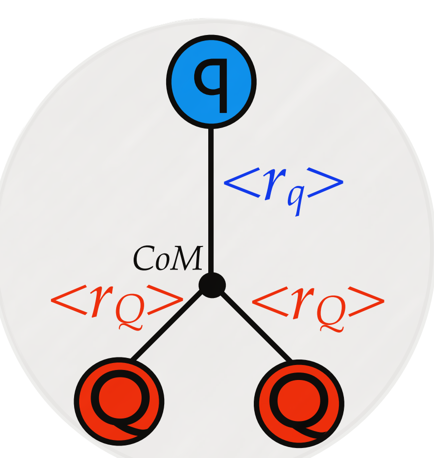
$\mu_{\Sigma_c^0}$ = - 0.852(133) μ_N

$\mu_{\Sigma_c^{++}}$ = 1.569(253) μ_N

$\mu_{\Xi_{cc}^{+}}$ = 0.411(15) μ_N

$\mu_{\Omega_c^0}$ = - 0.608(45) μ_N

$\mu_{\Omega_{cc}^{+}}$ = 0.405(13) μ_N

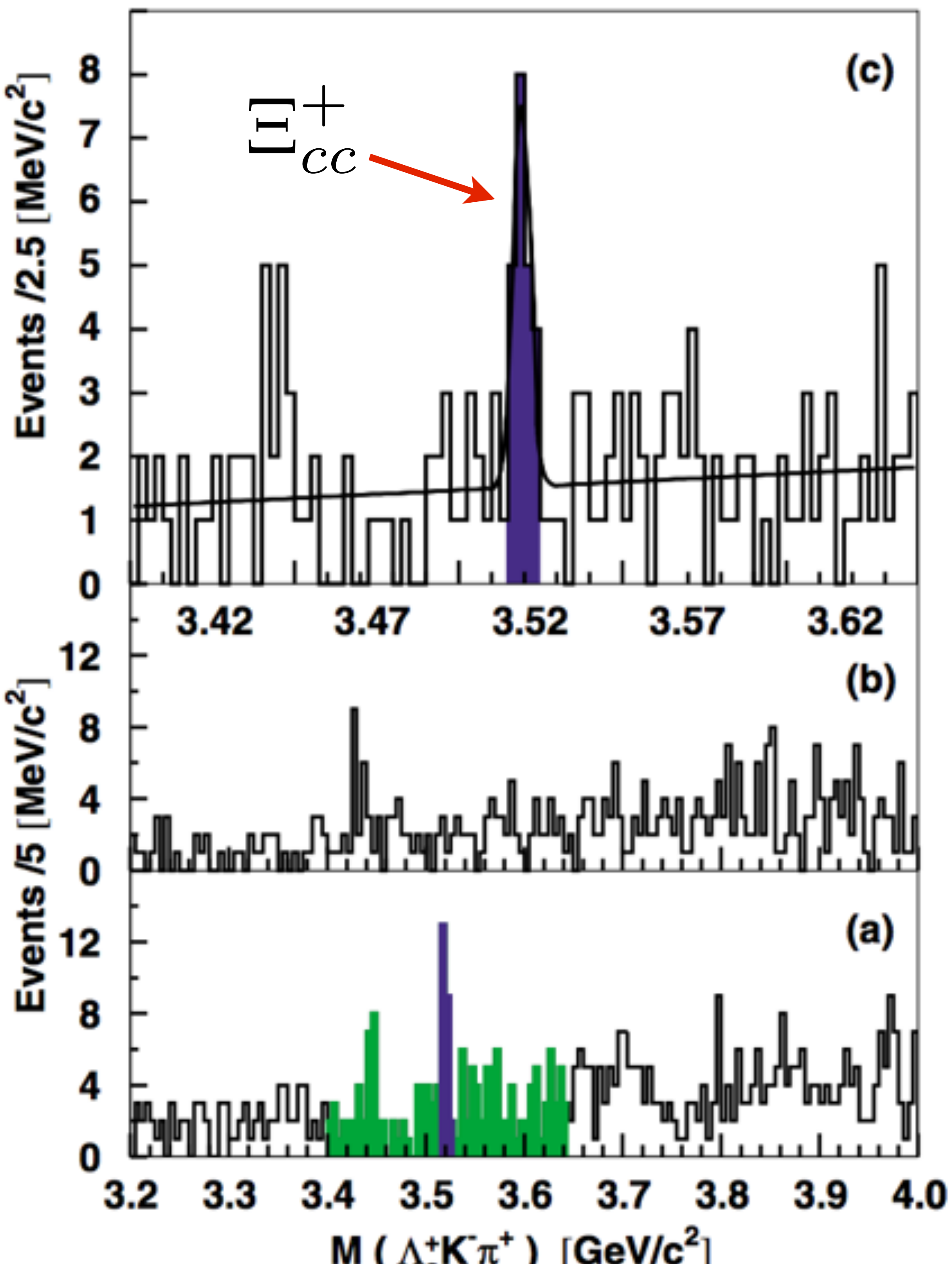


- Double-quark contribution dominates
- Heavy quarks shift centre of mass (CoM) closer to themselves

kuc, G. Erkol, B. Isildak, M. Oka, T.T. Takahashi, JHEP05(2014)125

DOUBLY CHARMED Ξ_{CC}

SELEX Collaboration (2002)



- Ξ_{cc}^{+} ***omitted from PDG summary table.***

$$\left.\begin{matrix}\Xi_{cc}^{+}(3443)\\ \Xi_{cc}^{+}(3520)\end{matrix}\right\} \to \Lambda_c^+ K^- \pi^+$$

$$\Xi_{cc}^{+}(3520) \to p^+ D^+ K^-$$

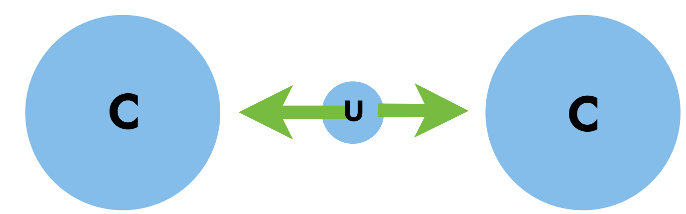
$$\left.\begin{matrix}\Xi_{cc}^{++}(3452)\\ \Xi_{cc}^{++}(3541)\end{matrix}\right\} \to \Lambda_c^+ K^- \pi^+ \pi^+$$

isospin splitting

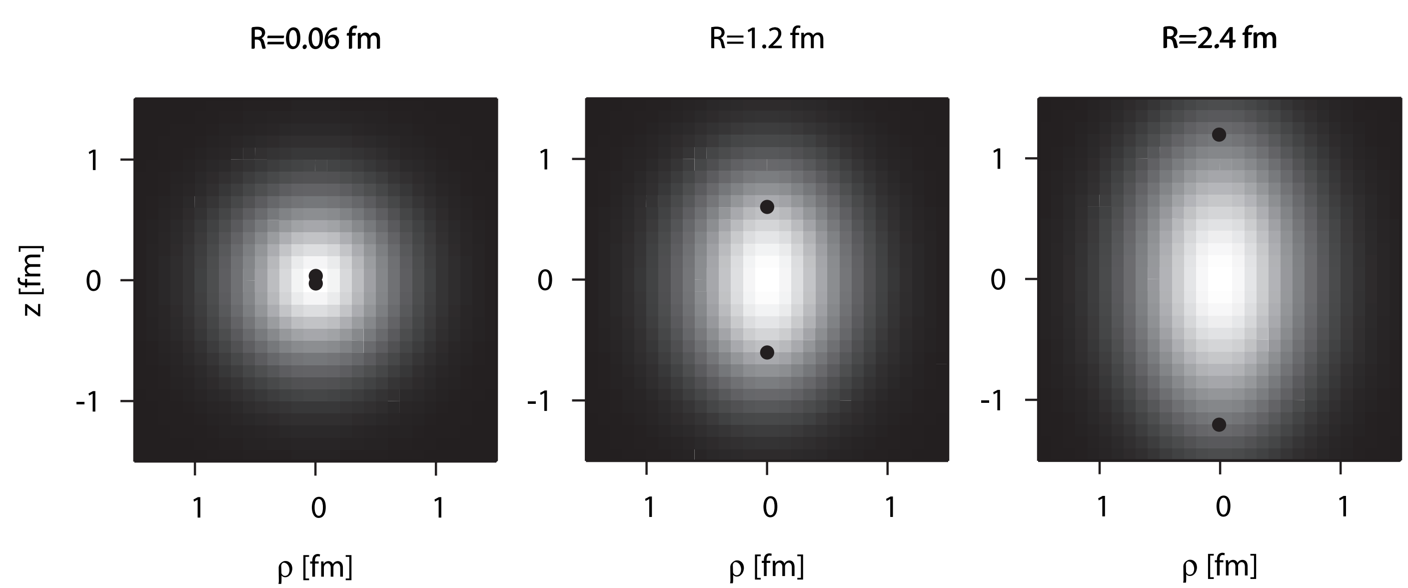
$\Xi_{cc}^{+}(3520) \leftrightarrow \Xi_{cc}^{++}(3541)$

$\Xi_{cc}^{+}(3443) \leftrightarrow \Xi_{cc}^{++}(3452)$

- Large isospin splitting (9 and 21 MeV) indicates a compact baryon.
  Brodsky et al. PLB 2011



- A. Yamamoto, H.Suganuma, H. Iida shows light quark is situated in the bright region
  Phys. Rev. D 77, 014036 (2008)



RESULTS

comparison with other calculations

(magnetic moments)

	Our result		[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
	Lin. fit	Quad. fit									
$\mu_{\Sigma_c^0}$	-0.852(133)	-1.073(269)	-1.78	-1.04	-	-1.043	-1.60	-1.391	-1.17	-1.015	-1.6(2)
$\mu_{\Sigma_c^{++}}$	1.569(253)	2.220(505)	3.07	1.76	-	1.679	2.20	2.44	2.18	2.279	2.1(3)
$\mu_{\Xi_{cc}^+}$	0.411(15)	0.425(29)	0.94	0.72	$0.785^{+0.050}_{-0.030}$	0.722	0.84	0.774	0.77	-	-
$\mu_{\Omega_c^0}$	-0.608(45)	-0.639(88)	-0.90	-0.85	-	-0.774	-0.90	-0.85	-0.92	-0.960	-
$\mu_{\Omega_{cc}^+}$	0.405(13)	0.413(24)	0.74	0.67	$0.635^{+0.012}_{-0.015}$	0.668	0.697	0.639	0.70	0.785	-

- **Bottom line:** signs match but LQCD results underestimate the mag. moms (or other models overestimate)

[1] B. Julia Diaz et al., Rel. QM, hep-ph/0401096
[2] Faessler et al., Rel. 3QM, hep-ph/0602193
[3] C. Albertus et al., NRQM, hep-ph/0610030
[4] Bertonas et al., Bag Model, hep-ph/1209.2900
[5] N. Sharma et al., χCQM, hep-ph/1003.4338
[6] N. Barik et al., indep-QM, PRD 28 (1983)
[7] S. Kumar et al., eff. mass and screened charge, J.Phys G31(2005)
[8] B. Patel et al., hyper central model, hep-ph/0710.3828
[9] S.-L. Zhu et al., QCD spectral SR, hep-ph/9708411

SUMMARY & OUTLOOK

- Summary
  - Charm quark shrinks the baryon's size, they are compact!
    - Magnitude of the observables are systematically small compared to the that of i.e. proton's.
  - CoM is closer to Charm quark(s).
    - Ξ_{cc} is peculiar
  - Doubly represented quarks have the dominant contribution.
- Outlook
  - Almost physical point calculation on $\kappa_{ud} = 0.13781$ ($m_\pi \sim 150$ MeV) PAC-CS configurations.
    - Spin-1/2 states as well as spin-3/2 states.

THANK YOU

BACKUP SLIDES

1S STATIC MASSES

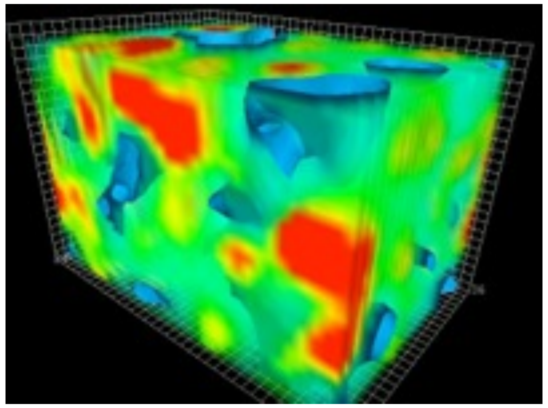
$\kappa_{val}^{u,d}$	m_{η_c}	$m_{J/\Psi}$	m_D	m_{D^*}	m_{D_s}	$m_{D_s^*}$
	[GeV]	[GeV]	[GeV]	[GeV]	[GeV]	[GeV]
Lin. Fit	2.979(2)	3.063(3)	1.895(6)	2.021(13)	2.018(4)	2.138(7)
Exp.	2.980	3.097	1.865	2.007	1.968	2.112
PACS-CS [17]	2.986(1)(13)	3.094(1)(14)	1.871(10)(8)	1.994(11)(9)	1.958(2)(9)	2.095(3)(10)

	$1S\ M_{\eta_c,\,J/\psi}$	$1S\ M_{D,\,D^*}$	$1S\ M_{D_s,\,D_s^*}$
This work	3.042(3)	1.990(42)	2.108(6)
Exp.	3.068	1.963	2.076
PACS-CS	3.067(15)	1.972(11)	2.061(12)

BARYON MASSES

$\kappa_{val}^{u,d}$	m_{Σ_c}	m_{Ω_c}	$m_{\Xi_{cc}}$	$m_{\Omega_{cc}}$
Lin. Fit	2.553(18)	2.740(24)	3.660(14)	3.755(18)
Exp.	2.455	2.695	3.519	-
PACS-CS [18]	2.467(39)(11)	2.673(5)(12)	3.603(15)(16)	3.704(5)(16)

Visualizations of QCD Project (bit.ly/vqcd)



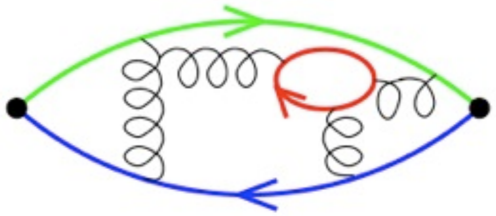
WORKFLOW

e^{-S_F} reduces to det[ ] terms

$$< \mathcal{O} > = -\frac{1}{\mathcal{Z}} \int \mathcal{D}[U] e^{-S_G[U]} \det[D_{q_1}] \det[D_{q_2}] \ldots Tr[\Gamma D_{q_1}^{-1}(n|m) \Gamma D_{q_2}^{-1}(n|m) \ldots]$$

$$\mathcal{Z} = \int \mathcal{D}[U] e^{-S_G[U]} \det[D_{q_1}] \det[D_{q_2}] \ldots$$

- Generate gauge configurations (lattices)
  - $\det[D_q]$ terms include sea-quark effects (unquenched)



- Compute the fermion propagators

Choose a Dirac Delta (Point) $S_0^{m_0,\alpha_0,a_0}(m)_a^\alpha = \delta(m - m_0)\delta_{\alpha\alpha_0}\delta_{aa_0}$

or Smeared source $S^{n_0,\alpha_0,a_0} = \sum_i^N F(U)$, N $\rightarrow$ # of iter.

$$F(U) = \begin{cases} e^{\sigma \nabla^2}, & \text{gaussian smearing} \\ \sigma \rightarrow 0, & \text{wall smearing} \end{cases}$$

$$\nabla^2 = \sum_{j=1}^{3} \left(U_j(\mathbf{n}, n_t)\delta(\mathbf{n} + \hat{j}, m) + U_j^\dagger(\mathbf{n} - \hat{j}, n_t)\delta(\mathbf{n} - \hat{j}, n_t) \right)$$

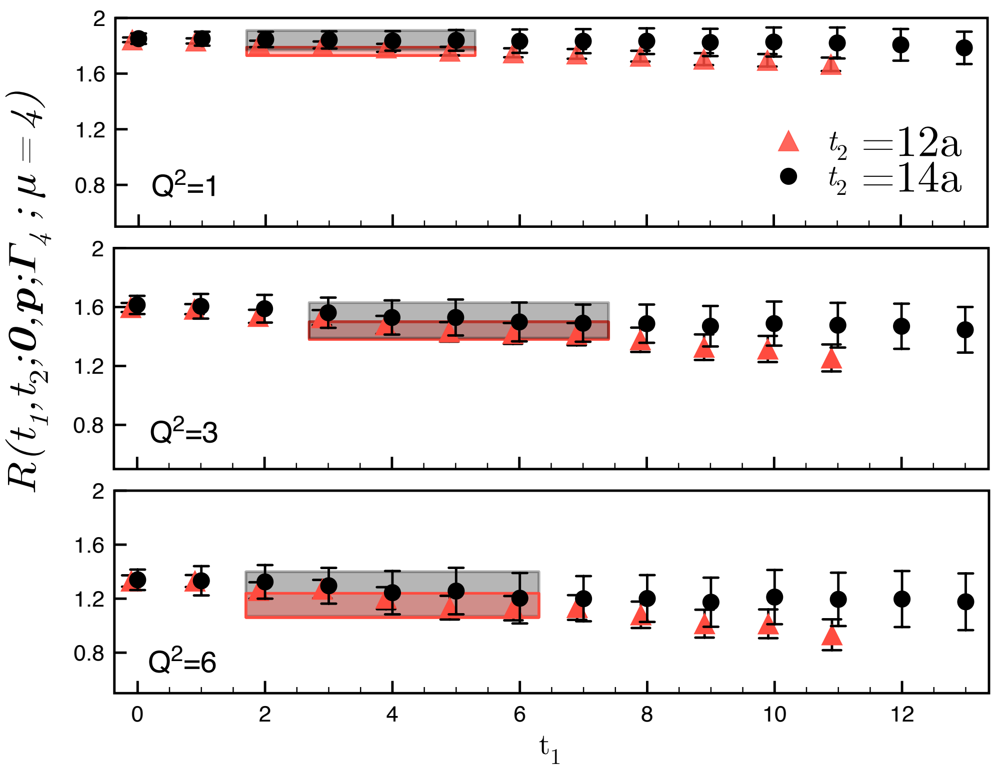
- Compute hadron properties
  - Contract props

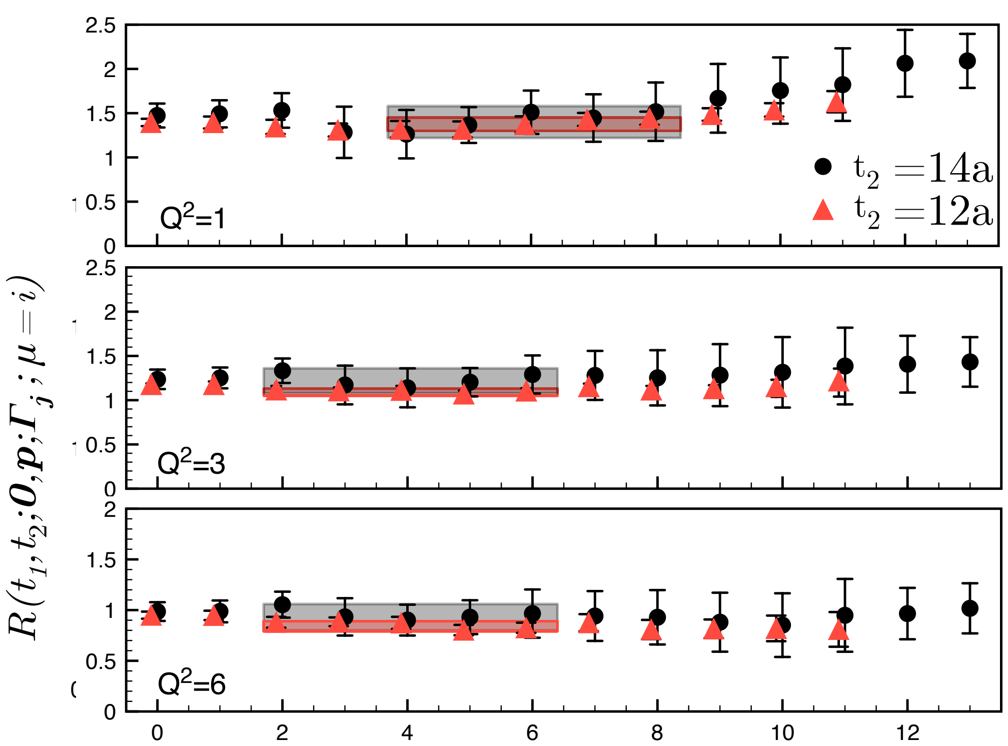
- Calculate on different lattices (Importance sampling)

$$\langle \mathcal{O} \rangle = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}[U_n]$$

SIMULATION DETAILS

Smallest time separation possible





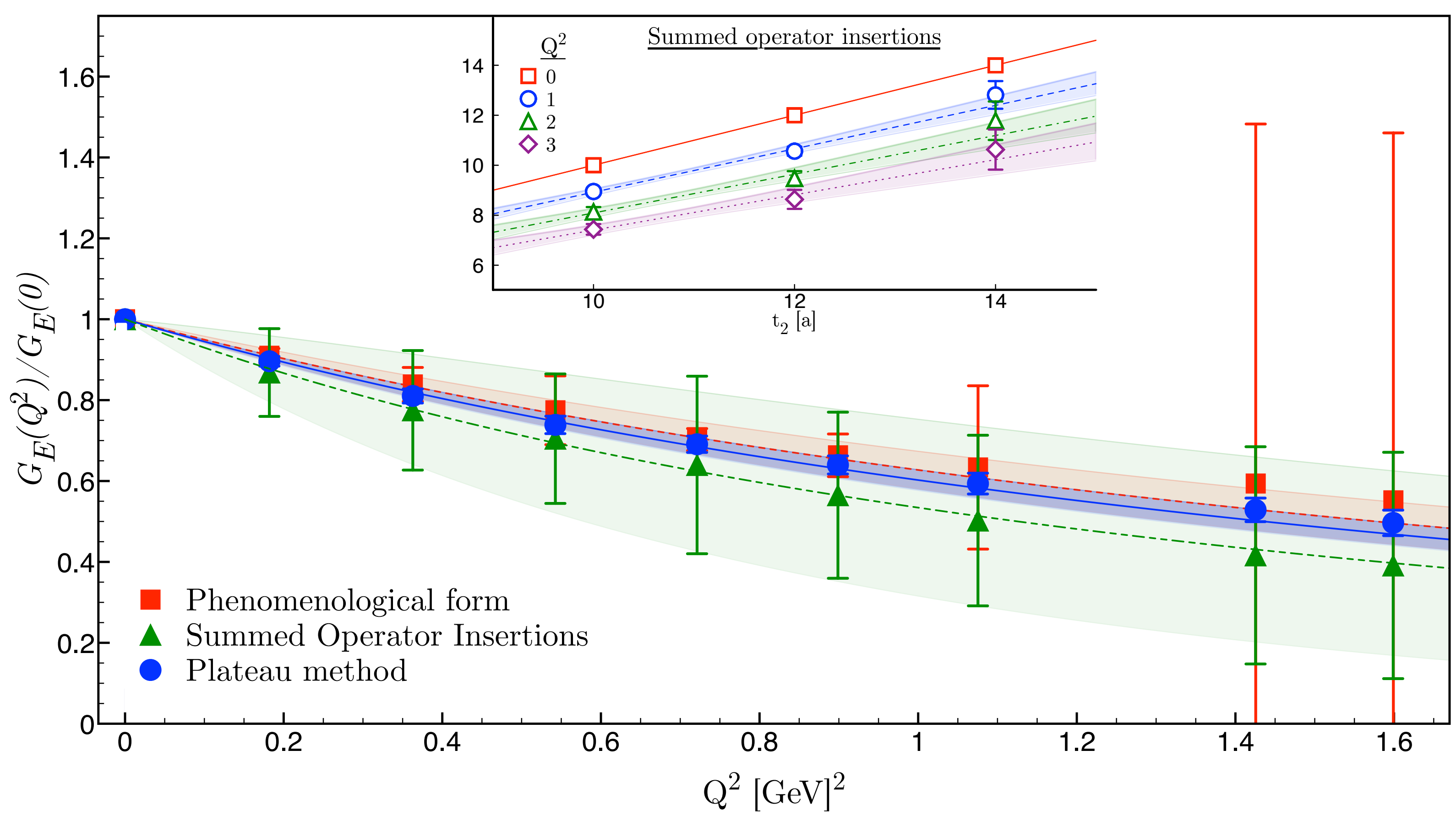
Excited State Contamination

- Phenomenological Form

$$R(t_2,t_1) = G_{E,M} + b_1\, e^{-\Delta t_1} + b_2\, e^{-\Delta(t_2-t_1)}$$

- Summed Operator Insertions

$$S(t_s) = \sum_{t=0}^{t_s} R(\vec{q},t,t_s) \to c(\Delta,\Delta') + t_s\left(G_{E,M} + \mathscr{O}\left(e^{-\Delta t_s}\right) + \mathscr{O}\left(e^{-\Delta' t_s}\right)\right)$$



Ξ_{cc} - κ_{ud}=0.13700 // Plateau fits = 100 confs // SOI = 30 conf