# Approximate NNLO Corrections to Hadronic Jet Production

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# Outline

1. Single Inclusive Jet Production
2. Approximate NNLO Results   de Florian, Hinderer, Mukherjee, Ringer, Vogelsang PRL '13
3. Extension to $A_{LL}$
4. Conlusions

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# Single Inclusive Jet Production

$$H_1 + H_2 \to \mathrm{Jet}(p_T, \eta) + X$$

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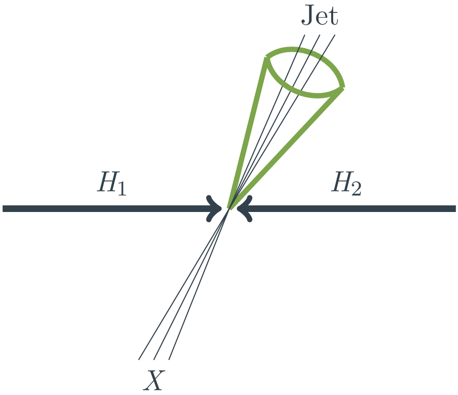
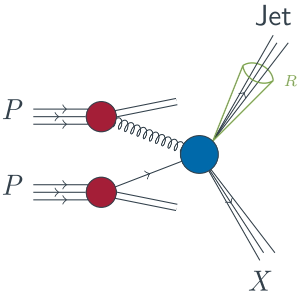


# Single Inclusive Jet Production - Cross Section



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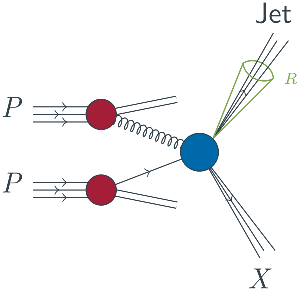


*anti*-$k_T$ algorithm $\quad p = -1$

$$d_{jk} \equiv \min(k_{T_j}^{2p}, k_{T_k}^{2p}) \frac{R_{jk}^2}{R^2}, \quad d_{jB} \equiv k_{T_j}^{2p}$$

$$R_{jk} = (\eta_j - \eta_k)^2 + (\Phi_j - \Phi_k)^2$$

(Cacciari, Salam, Soyez 2008)

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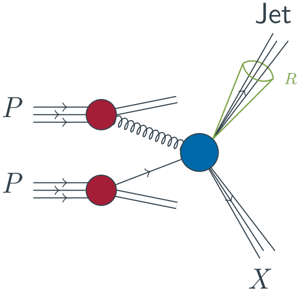


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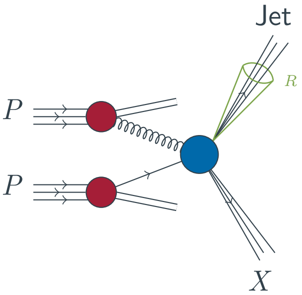


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$$v = \frac{u}{t+u}, \quad z = \frac{s_4}{s}$$

$$\frac{p_T^2 d^2\sigma}{dp_T^2 d\eta} = \int_0^{z_{\max}} dz \int_{v_{\min}}^{v_{\max}} dz \; x_a x_b \, f_a(x_a, \mu_F) f_b(x_b, \mu_F) \, \frac{d\hat{\sigma}_{ab}}{dv\, dz}(s, v, z, \mu_F, \mu_R; R)$$

$$z_{\max} = V(1-W), \quad v_{\min} = \frac{VW}{1-z}, \quad v_{\max} = 1 - \frac{V-z}{1-z},$$

$$V = 1 - \frac{p_T}{\sqrt{S}} e^{-\eta}, \quad VW = \frac{p_T}{\sqrt{S}} e^{-\eta}$$

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# The Cross Section

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## Perturbative expansion of the hard part:

$$s\frac{d\hat{\sigma}^2_{ab}}{dv\,dz} = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{\omega_{ab}^{(0)} + \left(\frac{\alpha_s}{\pi}\right)\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}\left(\alpha_s^3\right)\right\}$$

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# Analytical Results at NLO

Full analytical results at NLO in "Narrow Jet Approximation"

Jäger, Stratmann, Vogelsang 2004
Mukherjee, Vogelsang 2013
Kaufmann, Mukherjee, Vogelsang 2015

- $\omega_{ab}^{(1)} \sim A + B\log(R) + \mathcal{O}(R^2)$
- different algorithms
  - anti - $k_T$ algorithm
  - cone algorithm
  - maximized jet function
- accuracy better than 2-3 % for $R \leq 0.7$

# The Partonic Cross Section

Perturbative expansion of the hard part:

$$\frac{sd\hat{\sigma}_{ab}^{2}}{dv\ dz} = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{\omega_{ab}^{(0)} + \left(\frac{\alpha_s}{\pi}\right)\omega_{ab}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega_{ab}^{(2)} + \mathcal{O}\left(\alpha_s^3\right)\right\}$$

# The Partonic Cross Section

Perturbative expansion of the hard part:

$$\frac{s d\hat{\sigma}^2_{ab}}{dv\ dz} = \left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \omega^{(0)}_{ab} + \left(\frac{\alpha_s}{\pi}\right) \omega^{(1)}_{ab} + \left(\frac{\alpha_s}{\pi}\right)^2 \omega^{(2)}_{ab} + \mathcal{O}\left(\alpha_s^3\right) \right\}$$

- threshold logarithms $\alpha_s^k \omega_{ab}^{(k)} \sim \alpha_s^k \left(\frac{\log(z)^m}{z}\right)_+$, with $0 \leq m \leq 2k-1$
- threshold logarithms dominate at partonic threshold: $z = \frac{s_4}{s} \rightarrow 0$

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Threshold resummation determine all order structure of logarithms

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# Mellin Transformation

mellin transformation

$$\tilde{\omega}_{ab}(v,N) = \int_0^1 dz\,(1-z)^{N-1}\, s\frac{d\sigma^2}{dv\,dz}$$

threshold logarithms

$$\left(\frac{\ln^{2k-1}(z)}{z}\right)_+ \rightarrow \ln^{2k} N + ....$$

threshold limit

$$N \rightarrow \infty$$

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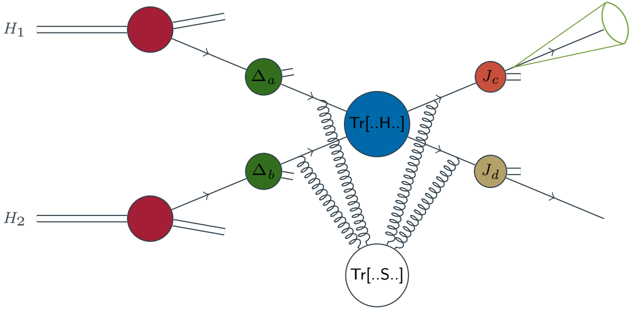
# Accuracy of Resummation

Fixed Order →

Resummation ↓

| | | | | | | |
|---|---|---|---|---|---|---|
| LO | $1$ | | | | | |
| NLO | $\alpha_s L^2$ | $\alpha_s L$ | $\alpha_s$ | | | |
| NNLO | $\alpha_s^2 L^4$ | $\alpha_s^2 L^3$ | $\alpha_s^2 L^2$ | $\alpha_s^2 L$ | $\alpha_s^2$ | |
| ... | ... | $\dots$ | ... | ... | ... | |
| $N^k LO$ | $\alpha_s^k L^{2k}$ | $\alpha_s^k L^{2k-1}$ | $\alpha_s^k L^{2k-2}$ | $\alpha_s^k L^{2k-3}$ | $\alpha_s^k L^{2k-4}$ | ... |
| | ↓ LL | | ↓ NLL | | ↓ NNLL | |

# Resummed Cross Section




$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \Delta_a(N_a)\, \Delta_b(N_b)\, J_c^{(\text{Jet})}(N)\, J_d^{(\text{recoil})}(N)\, \Delta_{ab\to cd}^{\text{int}}(N, v)\, \Delta_c^{(\text{ng})}(N)$$

Kidonakis, Oderda, Sterman 1998

$$\Delta_{ab\to cd}^{\text{int}}(N, v) = \text{Tr}(..S..H)$$

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# Resummed Cross Section

$$\Omega_{ab}^{\rm res}(v,N) = \sum_{cd} \Delta_a(N_a)\,\Delta_b(N_b)\,J_c^{\rm (Jet)}(N)\,J_d^{\rm (recoil)}(N)\,\Delta_{ab\to cd}^{\rm int}(N,v)\Delta_c^{\rm (ng)}(N)$$

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# Resumed Cross Section

$$\Omega_{ab}^{\text{res}}(v,N) = \sum_{cd} \boxed{\Delta_a(N_a)\,\Delta_b(N_b)}\, J_c^{(\text{Jet})}(N)\, J_d^{(\text{recoil})}(N)\, \Delta_{ab\to cd}^{\text{int}}(N,v)\Delta_c^{(\text{ng})}(N)$$

- initial state, soft-collinear gluon emission:

$$\Delta_i(N_i) = R_i\left(\alpha_s\left(\mu_r\right)\right)\, \exp\left\{\int_0^1 dz \frac{z^{N_i-1}-1}{1-z}\left[\int_{\mu_F^2}^{(1-z)^2 s} \frac{d\mu^2}{\mu^2} A_i\left(\alpha_s\left(\mu\right)\right) + D_i\left(\alpha_s((1-z)s)\right)\right]\right\}$$

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# Resummed Cross Section

$$\Omega_{ab}^{\mathrm{res}}(v,N) = \sum_{cd} \Delta_a(N_a)\,\Delta_b(N_b)\,J_c^{(\mathrm{Jet})}(N)\,\boxed{J_d^{(\mathrm{recoil})}(N)}\,\Delta_{ab\to cd}^{\mathrm{int}}(N,v)\Delta_c^{(\mathrm{ng})}(N)$$

- initial state, soft-collinear gluon emission:

$$\Delta_i(N_i) = R_i\left(\alpha_s\left(\mu_r\right)\right)\,\exp\left\{\int_0^1 dz\,\frac{z^{N_i-1}-1}{1-z}\left[\int_{\mu_F^2}^{(1-z)^2 s}\frac{d\mu^2}{\mu^2}A_i\left(\alpha_s\left(\mu\right)\right) + D_i\left(\alpha_s((1-z)s)\right)\right]\right\}$$

- unobserved final state, soft-collinear gluon emission:

$$J_i^{(\mathrm{recoil})} = R_i^J(\alpha_s\left(\mu_r\right))\exp\left\{\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left[\int_{(1-z)^2 s}^{(1-z)s}\frac{d\mu^2}{\mu^2}A_i\left(\alpha_s\left(\mu\right)\right) + \frac{1}{2}B_i\left(\alpha_s((1-z)s)\right)\right]\right\}$$

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# Resummed Cross Section

$$\Omega_{ab}^{\text{res}}(v, N) = \sum_{cd} \Delta_a(N_a)\,\Delta_b(N_b)\,J_c^{(\text{Jet})}(N)\,J_d^{(\text{recoil})}(N)\,\Delta_{ab\to cd}^{\text{int}}(N, v)\boxed{\Delta_c^{(\text{ng})}(N)}$$

- non global contribution $\Delta_c^{(NG)}$

  3rd tower effect starting at NNLO, arising from boundary of the jet

Dasgupta, Salam 2001; Banfi, Dasgupta 2004

# Resumed Cross Section

$$\Omega_{ab}^{\rm res}(v,N) = \sum_{cd} \Delta_a(N_a)\,\Delta_b(N_b)\;J_c^{(\rm Jet)}(N)\;J_d^{(\rm recoil)}(N)\;\boxed{\Delta_{ab\to cd}^{\rm int}(N,v)}\Delta_c^{(\rm ng)}(N)$$

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- trace part

$$\Delta_{ab\to cd}^{\rm int}(N,v) = {\rm Tr}\left\{H(\alpha_s(\mu_r^2)\mathcal{S}^\dagger S(\alpha_s(S/\bar{N}))\mathcal{S}\right\}$$

matrices in space of color exchange operators (2-,3-,8-dimensional)

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# Resummed Cross Section

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# Resumed Cross Section

$$\Delta^{\text{int}}_{ab\to cd}(N,v) = \text{Tr}\left\{H(\alpha_s(\mu_r^2)\mathcal{S}^\dagger \boxed{S(\alpha_s(S/\bar{N}))}\mathcal{S}\right\}$$

- soft matrix
  - soft, large angle gluon emission

$$S = S^{(0)} + \frac{\alpha_s}{\pi} S^{(1)}$$

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# Resumed Cross Section

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$$\mathcal{S} = \mathcal{P}\exp\left[\frac{1}{2}\int_s^{s/\bar{N}^2} \frac{d\mu^2}{\mu^2}\Gamma_S^{(f)}(\alpha_s(\mu^2))\right]$$

$$(\Gamma_S^{(f)})_{KL} = (\Gamma_{S'}^{(f)})_{KL} + \delta_{KL}\frac{\alpha_s}{\pi}\sum_{i=a,b,c,d} C_{f_i}\frac{1}{2}\left[-\ln(2\nu_i)+1-i\pi\right]$$

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- hard scattering matrix
  - contains far off shell part

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# Resummed Cross Section

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$$\left(\frac{\alpha_s}{\pi}\right)^k \frac{\sum_i A_i^{(1)}}{(k-1)!}\mathrm{Tr}\underbrace{\left\{H^{(1)}S^{(0)} + H^{(0)}S^{(1)}\right\}}_{\text{matching to NLO}\to\text{NLL accuracy}} \ln^{2k-2}\bar{N}$$

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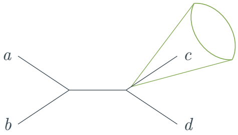
# Jet Mass at Threshold

LO: 


$\Rightarrow$ jet massless

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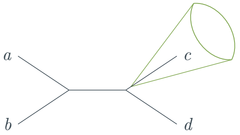
LO: 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- Scheme 1: keep jet massless at threshold

$$\exp\left[\frac{\alpha_s}{\pi}\ln^2\bar{N}\left(C_a + C_b - \frac{1}{2}C_c - \frac{1}{2}C_d\right)\right]$$

Kidonakis, Owens 2000
Moch, Kumar 2013

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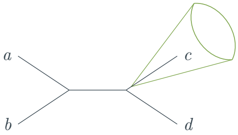


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no dependence on $R$

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# Threshold NLO Results

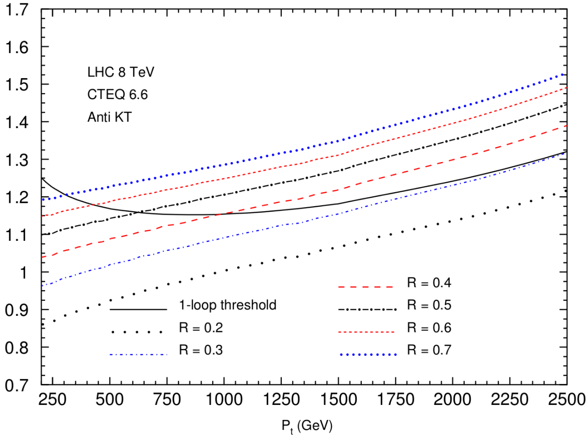


Figure : Moch/Kumar 13'

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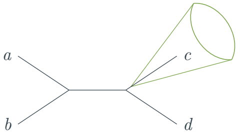


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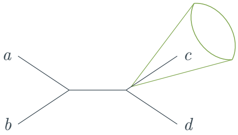


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Kidonakis, Owens 2000
Moch, Kumar 2013

- Scheme 2: jet allowed to be massive at threshold

$$\exp\left[\frac{\alpha_s}{\pi}\ln^2\bar{N}\left(C_a + C_b - \frac{1}{2}C_d\right) + \frac{\alpha_s}{\pi}\boxed{C_c\ln(R)\ln(\bar{N})}\right]$$

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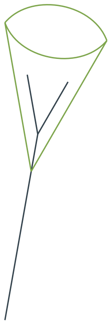
# Jet Mass At Threshold



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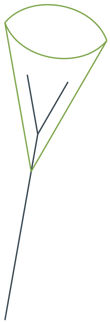
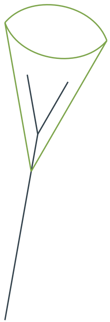


## Jet massless at threshold:

- Two partons in the jet, restrictions:
  - one must be arbitrarily soft or
  - they are exactly collinear

$\rightarrow$ no $R$-dependence in $J_c^{(\mathrm{Jet})}$

# Jet Mass At Threshold



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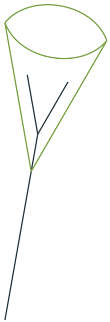

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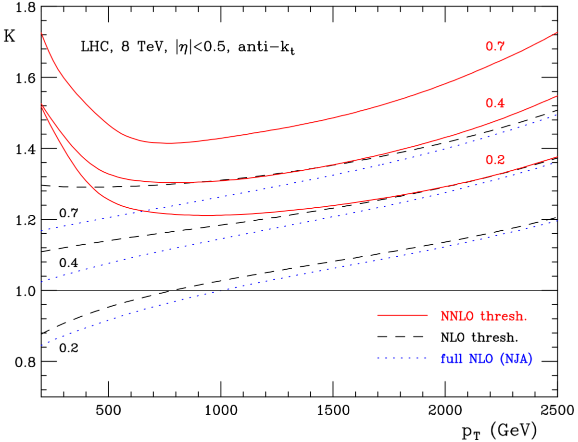
- more possible final states

$\rightarrow$ generates $R$-dependence in $J_c^{(\mathrm{Jet})}$

$$J_c^{(\mathrm{Jet})}(\bar{N}) = \exp\left\{ \int_s^{s/\bar{N}^2} \frac{dq^2}{q^2} \alpha_s(q^2) \left[ -\frac{C_c}{2\pi} \log\left( \frac{p_T^2 R^2}{s} \right) \right] \right\}$$

de Florian, Vogelsang 2013

# Approximate NNLO Results

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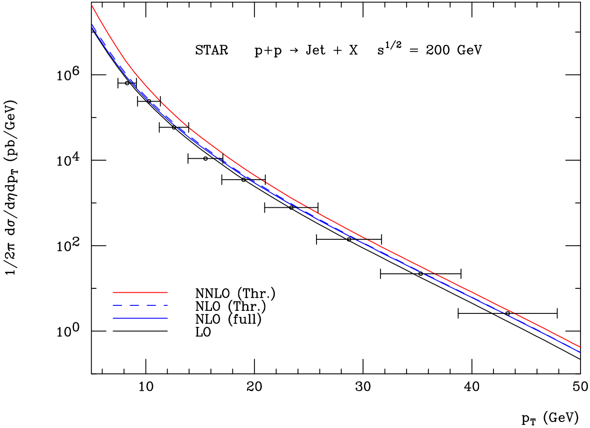


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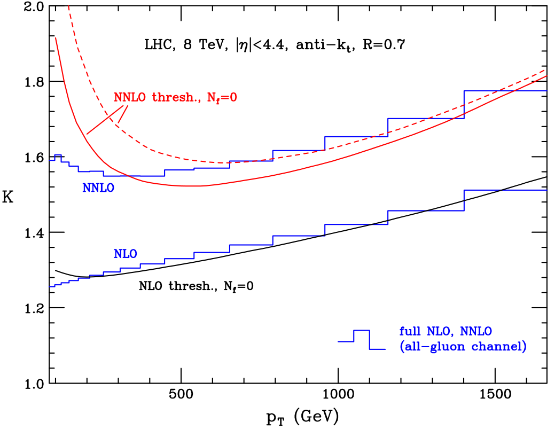
# K-factors in the gluon only channel

$$v = \frac{u}{t+u}$$

$$v' = \frac{t+s}{s}$$

scale choice

$$p_{T,1} \neq p_T$$




Currie, Gehrmann-De-Ritter,Glover, Pires 2013

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# Upcoming Work

Approximate NNLO results for $A_{LL}$

- measured at STAR

$$\Omega_{ab}^{\text{res}}\,(v,N) = \sum_{cd}\,\Delta_a(N_a)\,\Delta_b(N_b)\,J_c^{(\text{Jet})}(N)\,J_d^{(\text{recoil})}(N)\,\text{Tr}(..S..H)\,\Delta_c^{(\text{ng})}(N)$$

Soft gluon emission is spin independent

- $\Delta_i(N_i),\ J_c^{\text{Jet}}(N),\ J_d^{\text{recoil}},\ \Delta^{\text{NG}}(N),\ S$ remain unchanged
  see eg. de Florian, Vogelsang, Wagner 2008
- $H_{UU} \neq H_{LL}$, they are related to hard radiation

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# Conclusions

- Single inclusive jet production
- Full result are available up to NLO (NJA)
- Approximate NNLO
- Observed jet should be massive at threshold
- $A_{LL}$

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