Relating the QCD resummation procedures for transverse-momentum distributions

— "b_*" vs. "complex b" —

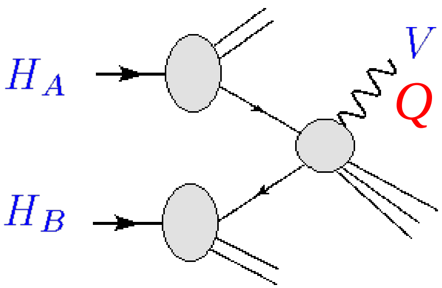
Kazuhiro Tanaka (Juntendo U)

Hadroproduction of vector boson

Drell, Yan ('70)

$$H_A + H_B \to V(Q, Q_T, y, \cdots) + X, \quad V = Z, W, \gamma^*$$

$$x_{A,B} = \frac{Q}{\sqrt{S}} e^{\pm y}$$



- at hadron colliders
- constraints for PDFs
- new physics search

$$d\sigma \propto \sum_{j=q,\bar{q}} e_j^2 f_{j/A}(x_A, Q^2) f_{\bar{j}/B}(x_B, Q^2) + \cdots$$

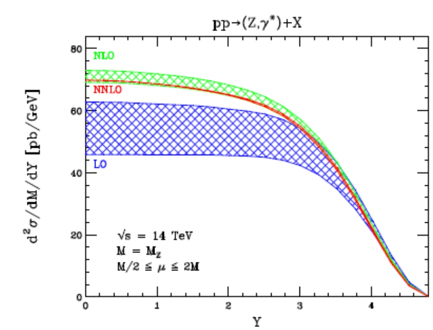
Perturbative QCD corrections（up to NNLO）:

- Total cross section

  Hamberg, Matsuura, van Neerven ('91), Harlander, Kilgore ('02)
- Rapidity distribution

  Anastasiou, Dixon, Melnikov, Kilgore ('02)
- Fully differential cross section

  Melnikov, petriello ('02), Catani, Cieri, Ferrera, de Florian, Grazzini ('09)



Anastasiou et al.

NLO EW corrections

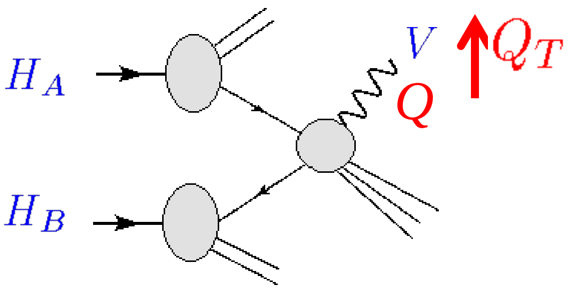
Dittmaier, Kramer ('02), Bauer, Wackeroth ('02), Caloni Calame et al. ('06)

Hadroproduction of vector boson

Drell, Yan ('70)

$$H_A + H_B \to V(Q, Q_T, y, \cdots) + X, \quad V = Z, W, \gamma^*$$

$$x_{A,B} = \frac{Q}{\sqrt{S}} e^{\pm y}$$

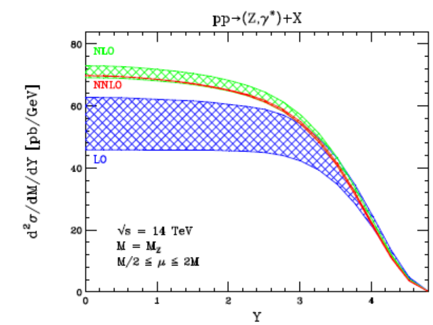


- at hadron colliders
- constraints for PDFs
- new physics search

$$d\sigma \propto \sum_{j=q,\bar{q}} e_j^2 f_{j/A}(x_A, Q^2) f_{\bar{j}/B}(x_B, Q^2) + \cdots$$

Perturbative QCD corrections（up to NNLO）:

- Total cross section
  Hamberg, Matsuura, van Neerven ('91), Harlander, Kilgore ('02)
- Rapidity distribution
  Anastasiou, Dixon, Melnikov, Kilgore ('02)
- Fully differential cross section
  Melnikov, petriello ('02), Catani, Cieri, Ferrera, de Florian, Grazzini ('09)



Anastasiou et al.

NLO EW corrections

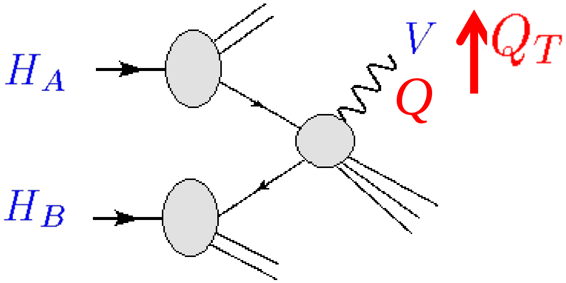
Dittmaier, Kramer ('02), Bauer, Wackeroth ('02), Caloni Calame et al. ('06)

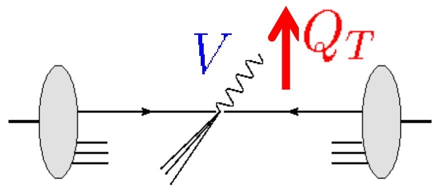
Hadroproduction of vector boson

Drell, Yan ('70)

$$H_A + H_B \to V(Q, Q_T, y, \cdots) + X, \quad V = Z, W, \gamma^*$$

$$x_{A,B} = \frac{Q}{\sqrt{S}} e^{\pm y}$$



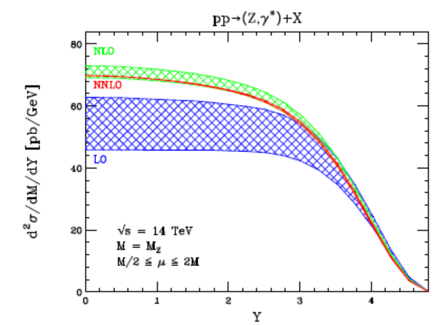


- at hadron colliders
- constraints for PDFs
- new physics search

$$d\sigma \propto \sum_{j=q,\bar{q}} e_j^2 f_{j/A}(x_A, Q^2) f_{\bar{j}/B}(x_B, Q^2) + \cdots$$

Perturbative QCD corrections（up to NNLO）:

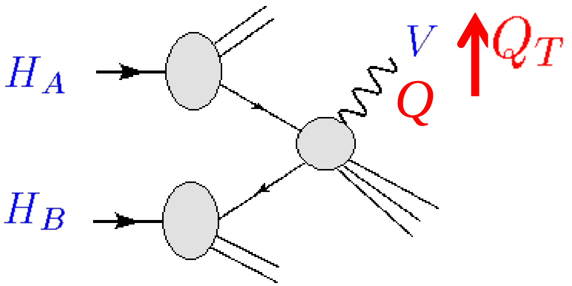
- Total cross section
  Hamberg, Matsuura, van Neerven ('91), Harlander, Kilgore ('02)
- Rapidity distribution
  Anastasiou, Dixon, Melnikov, Kilgore ('02)
- Fully differential cross section
  Melnikov, petriello ('02), Catani, Cieri, Ferrera, de Florian, Grazzini ('09)



Anastasiou et al.

NLO EW corrections

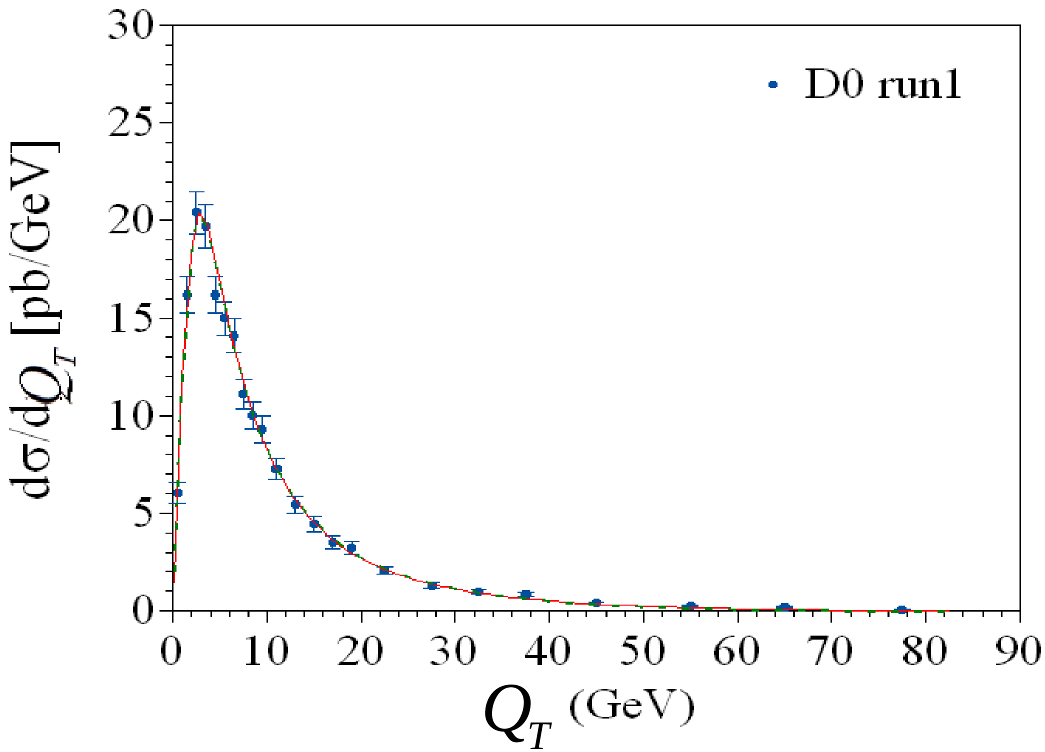
Dittmaier, Kramer ('02), Bauer, Wackeroth ('02), Caloni Calame et al. ('06)

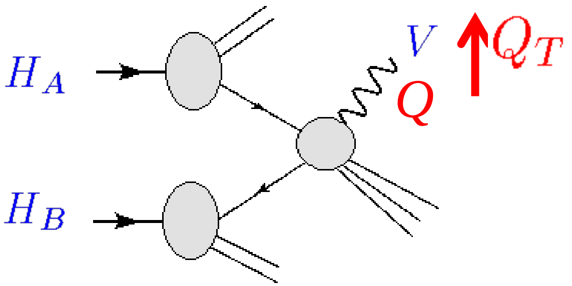


$$V = Z, W, \gamma^*$$

mostly produced at small Q_T

peak at 4-6 GeV for Z production

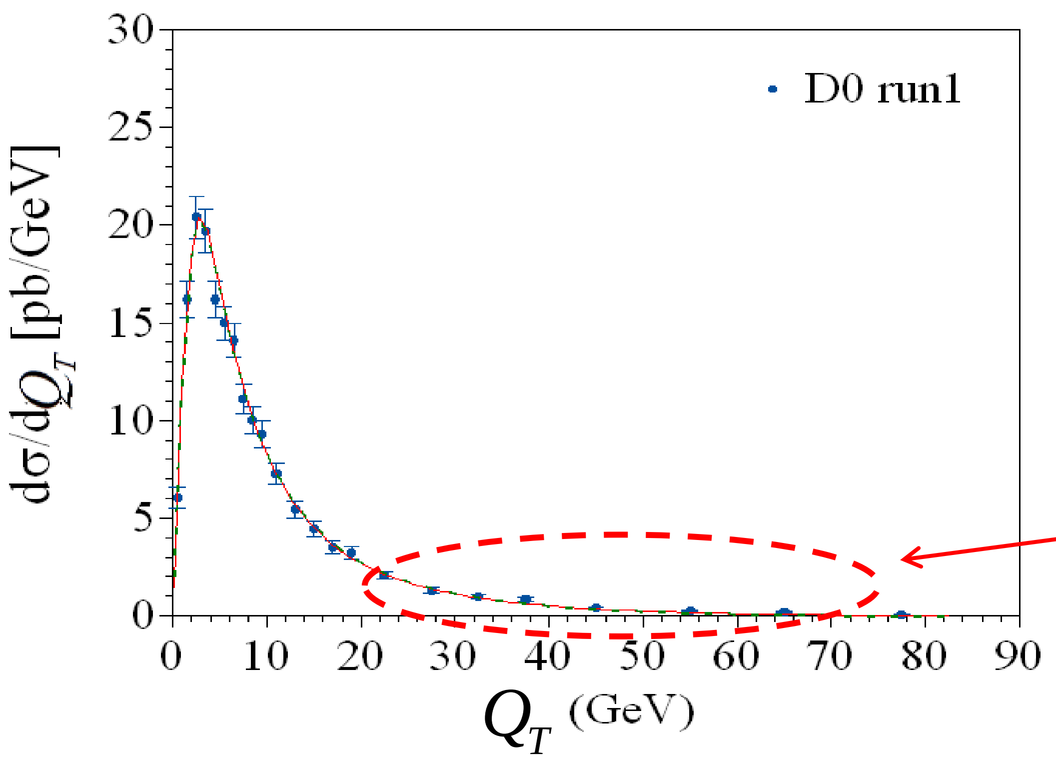


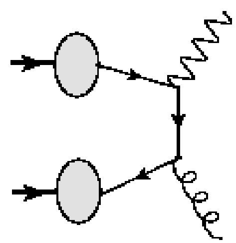


$V = Z, W, \gamma^*$

mostly produced at small Q_T

peak at 4-6 GeV for Z production

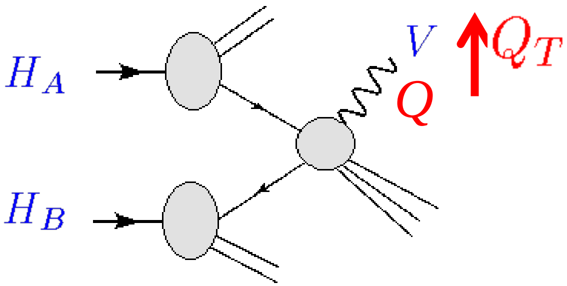




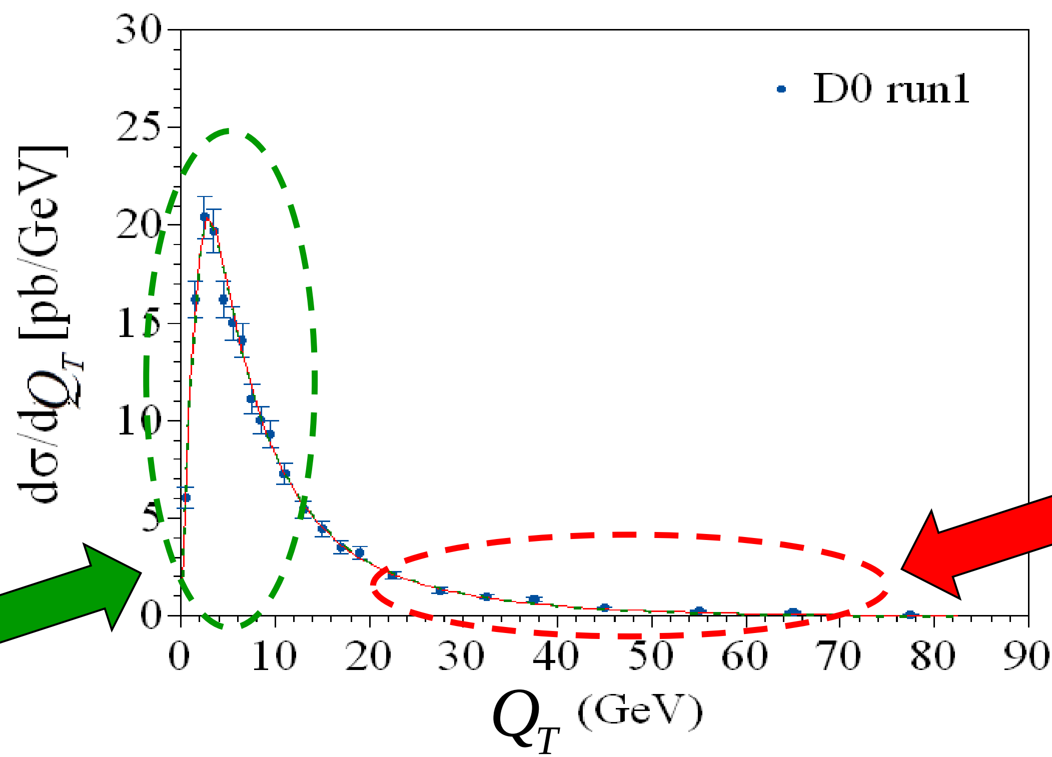
hard gluon emission

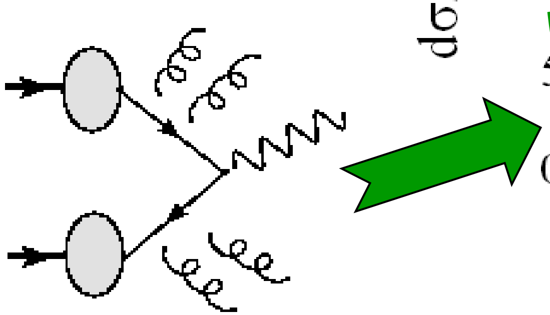
$V = Z, W, \gamma^*$

mostly produced at small Q_T

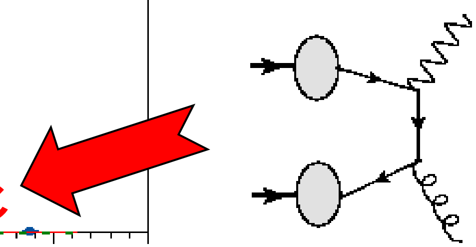


peak at 4-6 GeV for Z production

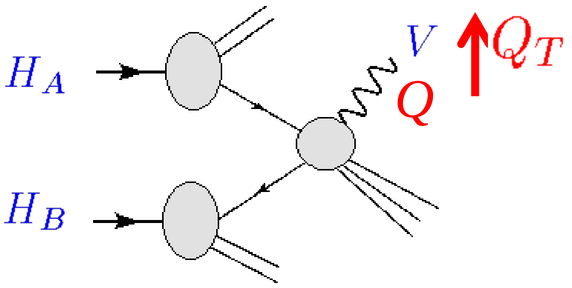




soft gluon emission



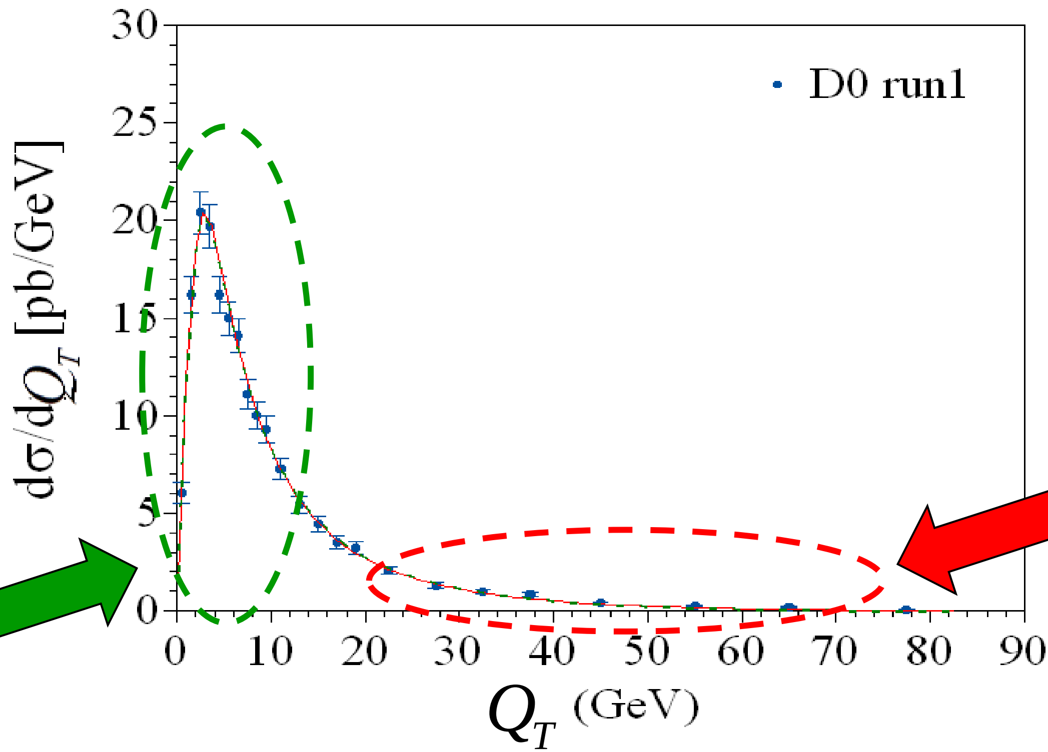
hard gluon emission

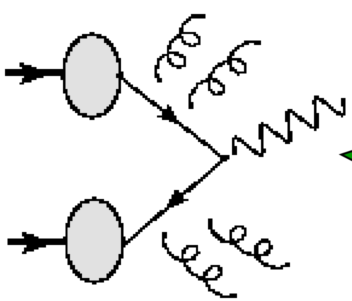


$V = Z, W, \gamma^*$

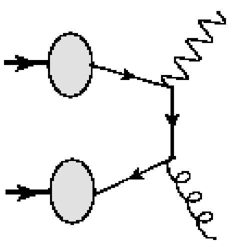
mostly produced at small Q_T

peak at 4-6 GeV for Z production



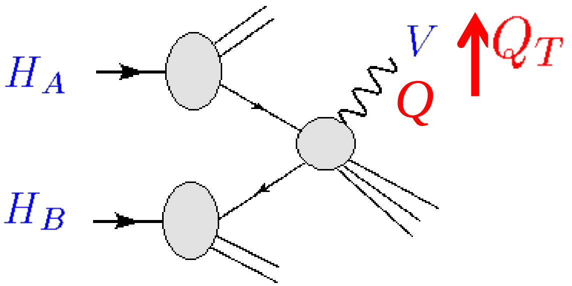


soft gluon emission



hard gluon emission

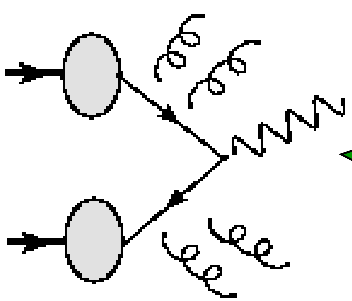
$$\alpha_s \ln \frac{Q}{Q_T}, \quad \alpha_s \ln^2 \frac{Q}{Q_T}$$

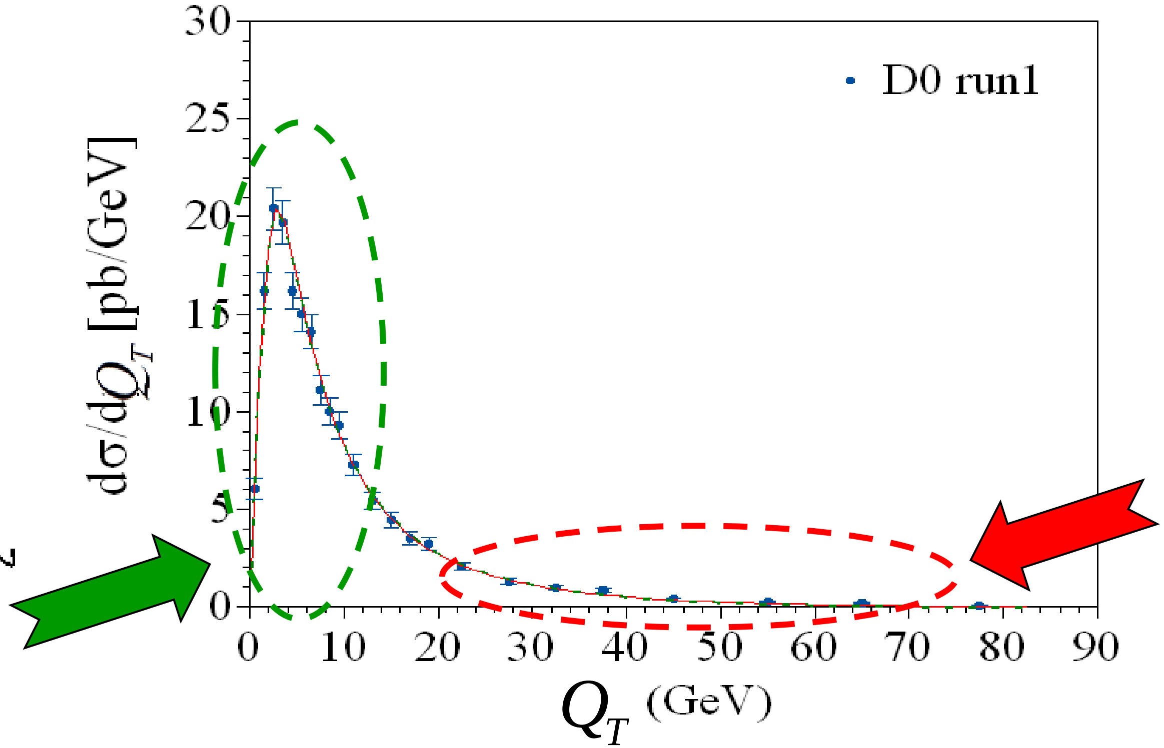


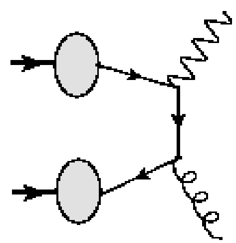
$$V = Z, W, \gamma^*$$

mostly produced at small Q_T

peak at 4-6 GeV for Z production







hard gluon emission

soft gluon emission

resummation

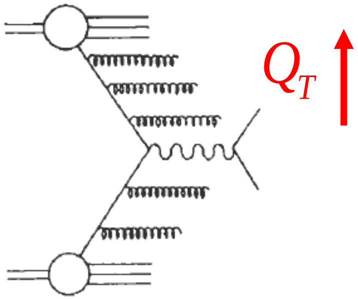
$$\alpha_s \ln \frac{Q}{Q_T}, \quad \alpha_s \ln^2 \frac{Q}{Q_T}$$

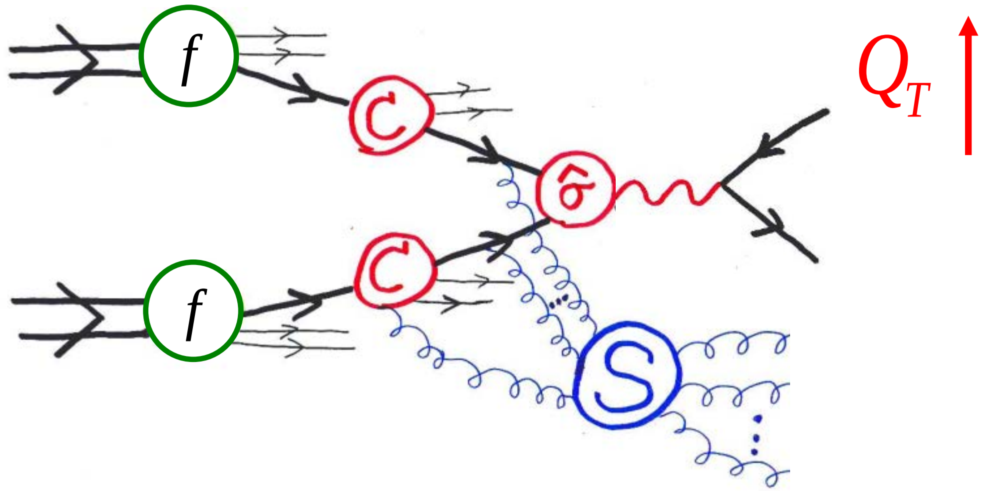
fixed-order

perturbation theory

Altarelli, Ellis, Greco, Maltinelli ('84)
Collins, Soper, Sterman ('85)
Catani, de Florian, Grazzini ('01)

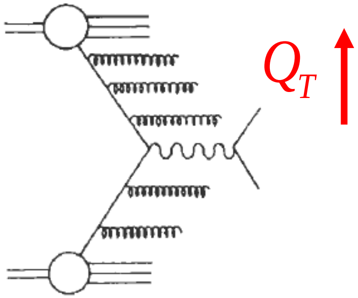
all-orders resummation

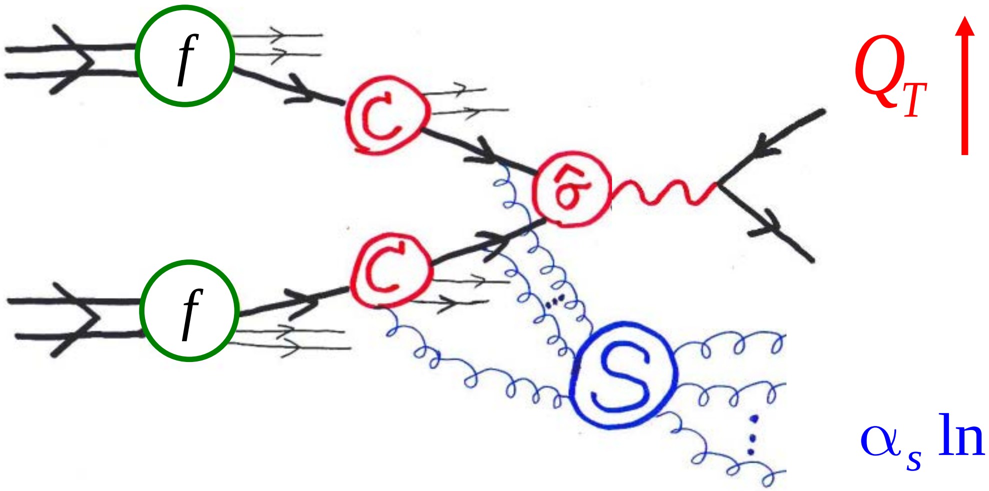




all-orders resummation

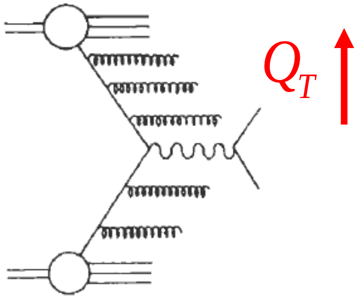
Altarelli, Ellis, Greco, Maltinelli ('84)
Collins, Soper, Sterman ('85)
Catani, de Florian, Grazzini ('01)



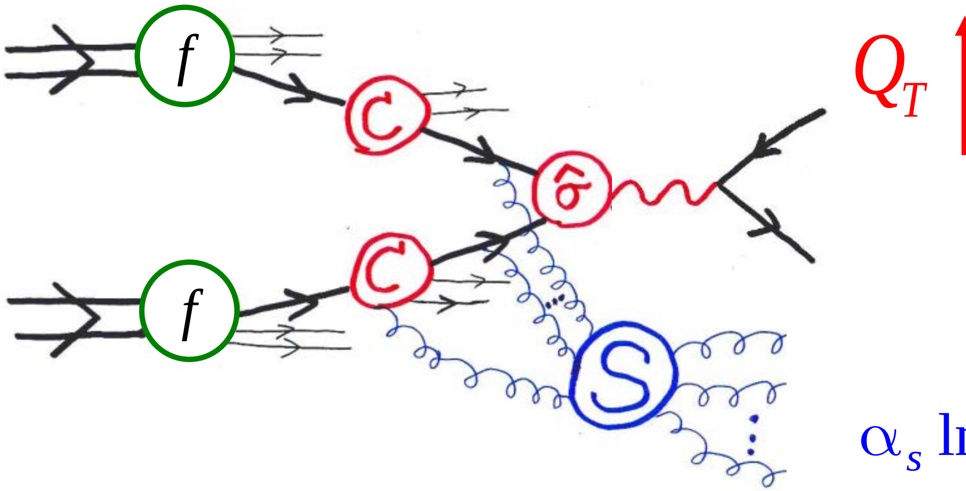


$$\alpha_s \ln^2 \frac{Q}{Q_T}, \quad \alpha_s \ln \frac{Q}{Q_T}$$

Altarelli, Ellis, Greco, Maltinelli ('84)
Collins, Soper, Sterman ('85)
Catani, de Florian, Grazzini ('01)



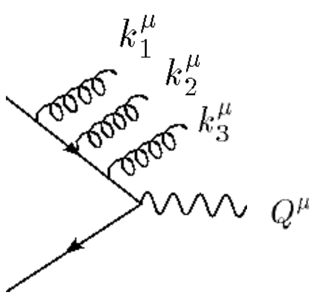
all-orders resummation

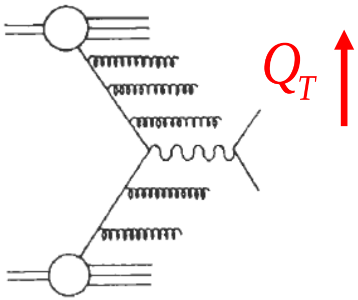


$$\alpha_s \ln^2 \frac{Q}{Q_T}, \quad \alpha_s \ln \frac{Q}{Q_T}$$

Impact parameter b space

$$\delta^{(2)}(Q_T - k_{1T} - k_{2T} - \cdots - k_{nT}) \to \int d^2 b e^{ibQ_T} \prod_n e^{-ib\cdot k_T}$$



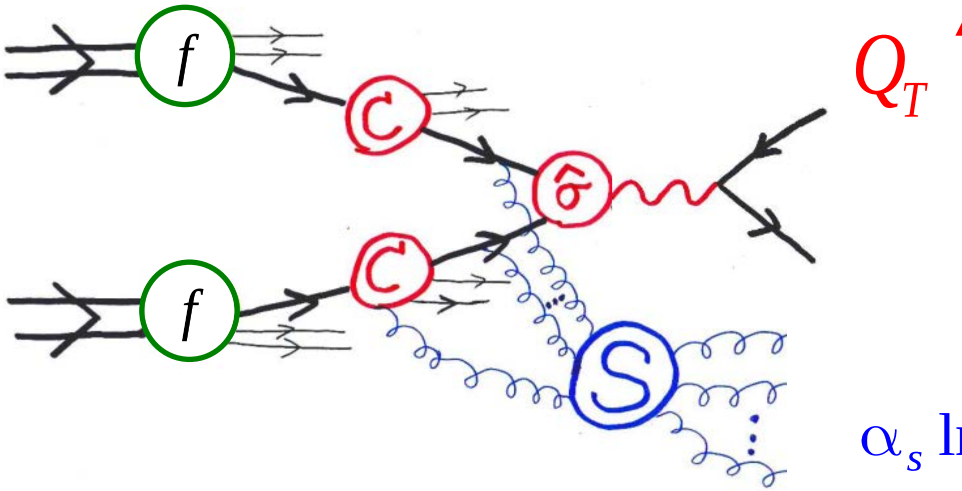


all-orders resummation

Altarelli, Ellis, Greco, Maltinelli ('84)

Collins, Soper, Sterman ('85)

Catani, de Florian, Grazzini ('01)

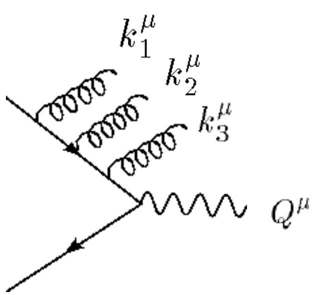


$$\alpha_s \ln^2 \frac{Q}{Q_T}, \quad \alpha_s \ln \frac{Q}{Q_T}$$

Impact parameter b space

$$\delta^{(2)}(Q_T - k_{1T} - k_{2T} - \cdots - k_{nT}) \to \int d^2 b e^{ibQ_T} \prod_n e^{-ib\cdot k_T}$$

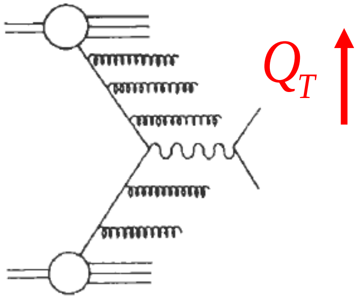
$$\alpha_s \ln \frac{Q^2 b^2}{b_0^2} \equiv \alpha_s L$$

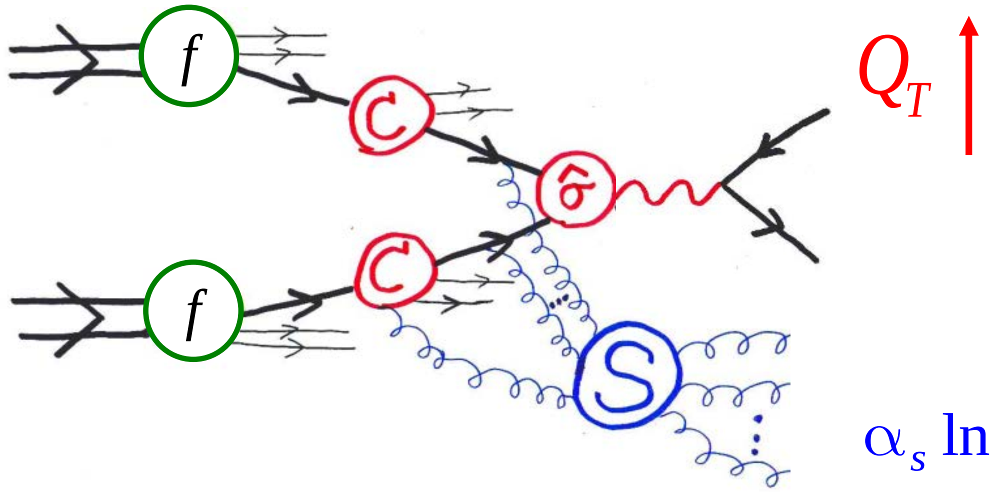


$$b_0 = 2e^{-\gamma_E}$$

all-orders resummation

Altarelli, Ellis, Greco, Maltinelli ('84)
Collins, Soper, Sterman ('85)
Catani, de Florian, Grazzini ('01)





$$\alpha_s \ln^2 \frac{Q}{Q_T}, \quad \alpha_s \ln \frac{Q}{Q_T}$$

$$d\sigma \propto \sum_{j=q,\bar{q}} e_j^2 \, f_{j/A}(x_A, Q^2)\, f_{\bar{j}/B}(x_B, Q^2) + \cdots$$

$$\Rightarrow \int d^2 b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_{j,i,k} \hat{\sigma}_{j\bar{j}}(Q^2)\, e^{S_j(b,Q)} (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right) (C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right) + \cdots$$

$$\left(C_{ji} \otimes f_{i/A}\right)(x, \mu^2) = \int_x^1 \frac{dz}{z} C_{ji}(z, \alpha_s(\mu^2))\, f_{i/A}(x/z, \mu^2)$$

$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}$$

$$A_q\left(\alpha_s\right) = C_F\frac{\alpha_s}{\pi} + \frac{1}{2}C_F\left\{\left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_G - \frac{5}{9}N_f\right\}\left(\frac{\alpha_s}{\pi}\right)^2 + \cdots$$

$$B_q\left(\alpha_s\right) = -\frac{3}{2}C_F\frac{\alpha_s}{\pi} + \cdots$$

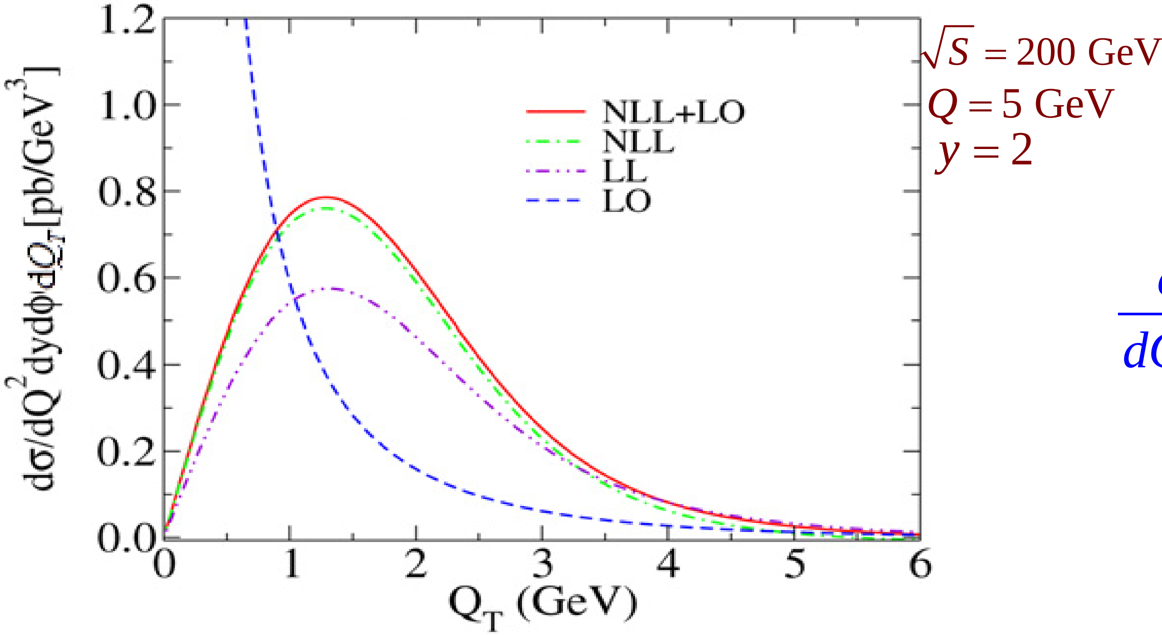
$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}$$

$$A_q(\alpha_s) = C_F\frac{\alpha_s}{\pi} + \frac{1}{2}C_F\left\{\left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_G - \frac{5}{9}N_f\right\}\left(\frac{\alpha_s}{\pi}\right)^2 + \cdots$$

$$B_q(\alpha_s) = -\frac{3}{2}C_F\frac{\alpha_s}{\pi} + \cdots$$

fully differential Drell-Yan

H. Kawamura, J. Kodaira, KT, NPB777 ('07) 203



$$\frac{d\sigma^{\text{fixed-order}}}{dQ^2dyd\phi dQ_T^2} \sim \sum_n \alpha_s^n\left[c_n\delta(Q_T^2) + \sum_{k=0}^{2n-1} d_{nk}\frac{\ln^k\left(Q^2/Q_T^2\right)}{Q_T^2}\right]$$

General Structure of Large Logs

LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s	
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	+…
N³LO	$\alpha_s^3 L^6$	$\alpha_s^3 L^5$	$\alpha_s^3 L^4$	+…
NᵏLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	+…
	LL	**NLL**	**NNLL**	

$$\alpha_s \ln \frac{Q^2 b^2}{b_0^2} \equiv \alpha_s L$$

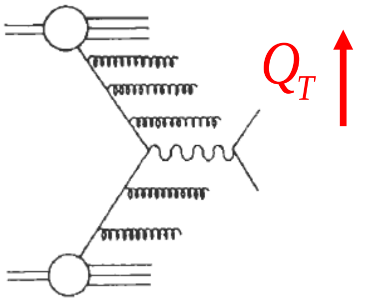
Collins, Soper ('81,82) , Collins, Soper, Sterman ('85)

J.Collins,"Foundations of perturbative QCD" (Cambridge, '11)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

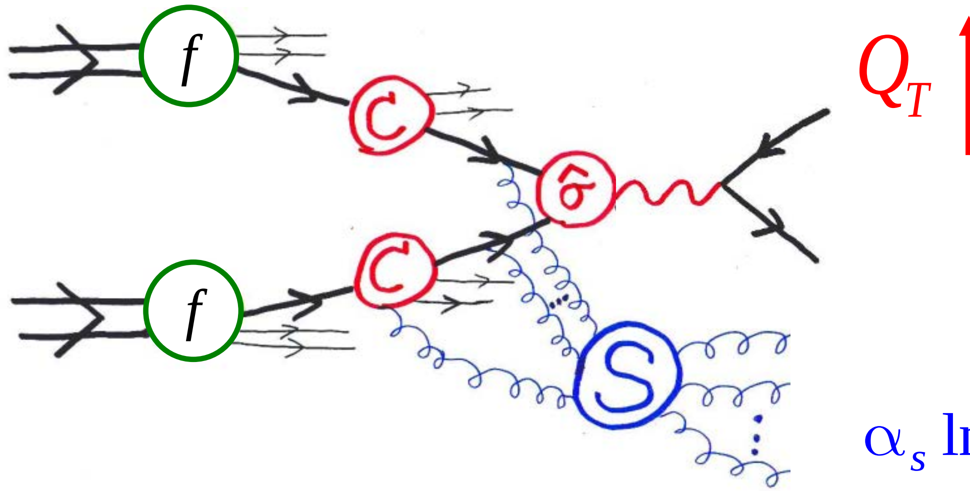
$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2 b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} W(b;Q,x_A,x_B) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b;Q,x_A,x_B)$$

$$W(b;Q,x_A,x_B) = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) W_j(b;Q,x_A,x_B),$$



all-orders resummation

Altarelli, Ellis, Greco, Maltinelli ('84)
Collins, Soper, Sterman ('85)
Catani, de Florian, Grazzini ('01)



$$\alpha_s \ln^2 \frac{Q}{Q_T}, \quad \alpha_s \ln \frac{Q}{Q_T}$$

$$d\sigma \propto \sum_{j=q,\bar{q}} e_j^2 \, f_{j/A}(x_A, Q^2)\, f_{\bar{j}/B}(x_B, Q^2) + \cdots$$

$$\Rightarrow \int d^2 b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_{j,i,k} \hat{\sigma}_{j\bar{j}}(Q^2)\, e^{S_j(b,Q)} \left(C_{ji} \otimes f_{i/A}\right)\left(x_A, \frac{b_0^2}{b^2}\right) \left(C_{\bar{j}k} \otimes f_{k/B}\right)\left(x_B, \frac{b_0^2}{b^2}\right) + \cdots$$

$$\left(C_{ji} \otimes f_{i/A}\right)(x, \mu^2) = \int_x^1 \frac{dz}{z} C_{ji}(z, \alpha_s(\mu^2))\, f_{i/A}(x/z, \mu^2)$$

Collins, Soper ('81,82) , Collins, Soper, Sterman ('85)

J.Collins,"Foundations of perturbative QCD" (Cambridge, '11)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2 b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} W(b;Q,x_A,x_B) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b;Q,x_A,x_B)$$

$$W(b;Q,x_A,x_B) = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) W_j(b;Q,x_A,x_B)$$

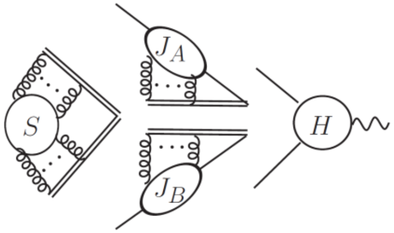
Collins, Soper ('81,82) , Collins, Soper, Sterman ('85)

J.Collins,"Foundations of perturbative QCD" (Cambridge, '11)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2 b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} W(b;Q,x_A,x_B) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b;Q,x_A,x_B)$$

$$W(b;Q,x_A,x_B) = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) W_j(b;Q,x_A,x_B)$$



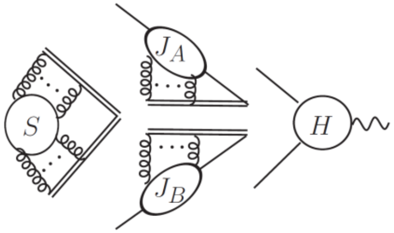
$$\frac{\partial}{\partial \ln Q^2} W_j(b;Q,x_A,x_B) = -\left[\int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} A_j(\alpha_s(\kappa^2)) + B_j(\alpha_s(Q^2))\right] W_j(b;Q,x_A,x_B)$$

Collins, Soper ('81,82) , Collins, Soper, Sterman ('85)
J.Collins,"Foundations of perturbative QCD" (Cambridge, '11)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} W(b;Q,x_A,x_B) = \int_0^\infty db\frac{b}{2}J_0(bQ_T)W(b;Q,x_A,x_B)$$

$$W(b;Q,x_A,x_B) = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) W_j(b;Q,x_A,x_B)$$



$$\frac{\partial}{\partial \ln Q^2} W_j(b;Q,x_A,x_B) = -\left[\int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} A_j(\alpha_s(\kappa^2)) + B_j(\alpha_s(Q^2))\right] W_j(b;Q,x_A,x_B)$$

$$W_j(b;Q,x_A,x_B) = e^{S_j(b,Q)}\, W_j(b;\frac{b_0}{b},x_A,x_B)$$

$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left\{ \left(\ln \frac{Q^2}{\mu^2} \right) A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right) \right\}$$

$$A_q(\alpha_s) = C_F \frac{\alpha_s}{\pi} + \frac{1}{2} C_F \left\{ \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_G - \frac{5}{9} N_f \right\} \left(\frac{\alpha_s}{\pi} \right)^2 + \cdots$$

$$B_q(\alpha_s) = -\frac{3}{2} C_F \frac{\alpha_s}{\pi} + \cdots$$

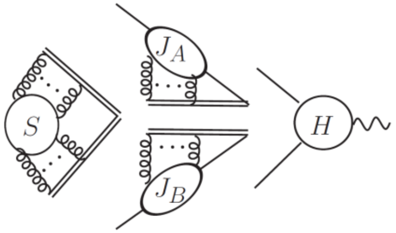
Collins, Soper ('81,82) , Collins, Soper, Sterman ('85)

J.Collins,"Foundations of perturbative QCD" (Cambridge, '11)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} W(b;Q,x_A,x_B) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b;Q,x_A,x_B)$$

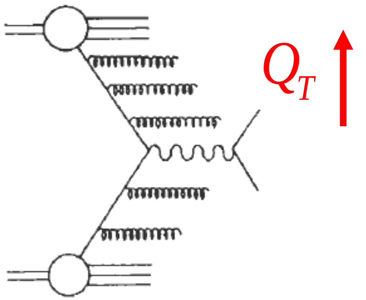
$$W(b;Q,x_A,x_B) = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) W_j(b;Q,x_A,x_B)$$



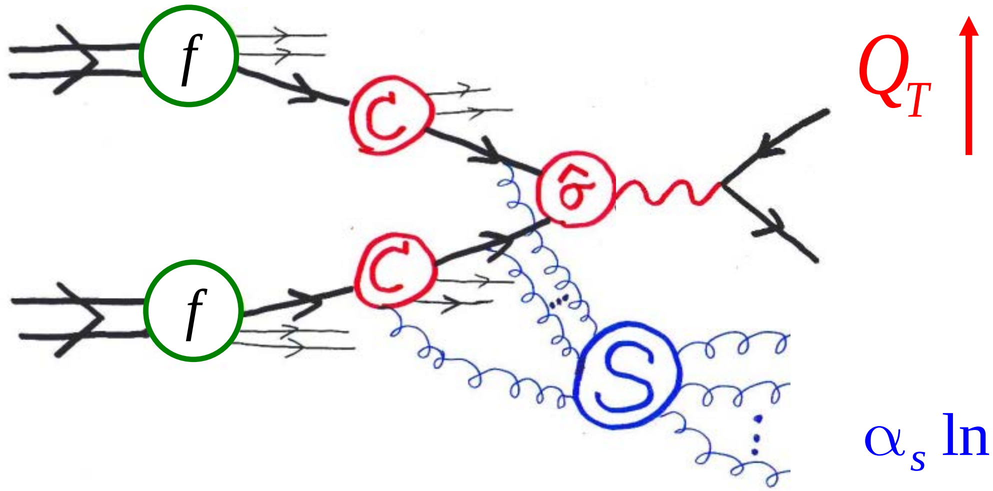
$$\frac{\partial}{\partial \ln Q^2} W_j(b;Q,x_A,x_B) = -\left[\int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} A_j(\alpha_s(\kappa^2)) + B_j(\alpha_s(Q^2))\right] W_j(b;Q,x_A,x_B)$$

$$W_j(b;Q,x_A,x_B) = e^{S_j(b,Q)} W_j\left(b;\frac{b_0}{b},x_A,x_B\right)$$

Altarelli, Ellis, Greco, Maltinelli ('84)
Collins, Soper, Sterman ('85)
Catani, de Florian, Grazzini ('01)



all-orders resummation



$$\alpha_s \ln^2 \frac{Q}{Q_T}, \quad \alpha_s \ln \frac{Q}{Q_T}$$

$$d\sigma \propto \sum_{j=q,\bar{q}} e_j^2 \, f_{j/A}(x_A, Q^2) \, f_{\bar{j}/B}(x_B, Q^2) + \cdots$$

$$\Rightarrow \int d^2 b \, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_{j,i,k} \hat{\sigma}_{j\bar{j}}(Q^2) \, e^{S_j(b,Q)} (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right) (C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right) + \cdots$$

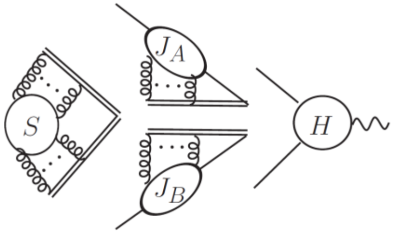
$$\left(C_{ji} \otimes f_{i/A}\right)(x, \mu^2) = \int_x^1 \frac{dz}{z} C_{ji}(z, \alpha_s(\mu^2)) \, f_{i/A}(x/z, \mu^2)$$

Collins, Soper ('81,82) , Collins, Soper, Sterman ('85)
J.Collins,"Foundations of perturbative QCD" (Cambridge, '11)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2 b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} W(b;Q,x_A,x_B) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b;Q,x_A,x_B)$$

$$W(b;Q,x_A,x_B) = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) W_j(b;Q,x_A,x_B)$$



$$\frac{\partial}{\partial \ln Q^2} W_j(b;Q,x_A,x_B) = -\left[\int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} A_j(\alpha_s(\kappa^2)) + B_j(\alpha_s(Q^2))\right] W_j(b;Q,x_A,x_B)$$

$$W_j(b;Q,x_A,x_B) = e^{S_j(b,Q)} W_j\left(b;\frac{b_0}{b},x_A,x_B\right)$$

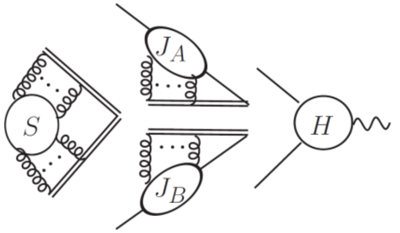
Collins, Soper ('81,82) , Collins, Soper, Sterman ('85)

J.Collins,"Foundations of perturbative QCD" (Cambridge, '11)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2 b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} W(b;Q,x_A,x_B) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) W(b;Q,x_A,x_B)$$

$$W(b;Q,x_A,x_B) = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) W_j(b;Q,x_A,x_B)$$



$$\frac{\partial}{\partial \ln Q^2} W_j(b;Q,x_A,x_B) = -\left[\int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} A_j(\alpha_s(\kappa^2)) + B_j(\alpha_s(Q^2))\right] W_j(b;Q,x_A,x_B)$$

$$W_j(b;Q,x_A,x_B) = e^{S_j(b,Q)} W_j(b;\frac{b_0}{b},x_A,x_B)$$

$$b \ll 1/\Lambda_{\mathrm{QCD}} \quad \left(Q_T \gg \Lambda_{\mathrm{QCD}}\right)$$

$$W_j(b;\frac{b_0}{b},x_A,x_B) \sim (C_{ji} \otimes f_{i/A})\left(x_A,\frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B,\frac{b_0^2}{b^2}\right) \equiv W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b},x_A,x_B)$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \xrightarrow{\; b \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \;} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) = (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \xrightarrow{\; b \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \;} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) = (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$= \bar{P}_{j/A}(x_A, b; b_0)\bar{P}_{\bar{j}/B}(x_B, b; b_0)$$

TMD

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \xrightarrow{\; b \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \;} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) = (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$W_j(b;\frac{b_0}{b}) = \bar{P}_{j/A}(x_A, b; b_0)\bar{P}_{\bar{j}/B}(x_B, b; b_0)$$

TMD

$$W_j(b;\frac{b_0}{b}) = W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \frac{W_j(b;\frac{b_0}{b})}{W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})} \equiv W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \xrightarrow{\; b \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \;} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) = (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$W_j(b;\frac{b_0}{b}) \equiv \overline{P}_{j/A}(x_A, b; b_0)\overline{P}_{\bar{j}/B}(x_B, b; b_0)$$

TMD

"primordial k_T"

$$F^{NP}(b) = e^{-g_{\mathrm{NP}} b^2}$$

$$W_j(b;\frac{b_0}{b}) = W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \frac{W_j(b;\frac{b_0}{b})}{W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})} \equiv W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}$$

$$A_q(\alpha_s) = C_F\frac{\alpha_s}{\pi} + \frac{1}{2}C_F\left\{\left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_G - \frac{5}{9}N_f\right\}\left(\frac{\alpha_s}{\pi}\right)^2 + \cdots$$

$$B_q(\alpha_s) = -\frac{3}{2}C_F\frac{\alpha_s}{\pi} + \cdots$$

$$\frac{d\sigma^{(\text{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \quad \xrightarrow{b \ll \frac{1}{\Lambda_{\text{QCD}}}} \quad W_j^{(\text{OPE})}(b;\frac{b_0}{b}) = (C_{ji}\otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k}\otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$= \bar{P}_{j/A}(x_A, b; b_0)\bar{P}_{\bar{j}/B}(x_B, b; b_0)$$

TMD

$$W_j(b;\frac{b_0}{b}) = W_j^{(\text{OPE})}(b;\frac{b_0}{b}) \frac{W_j(b;\frac{b_0}{b})}{W_j^{(\text{OPE})}(b;\frac{b_0}{b})} \equiv W_j^{(\text{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

"primordial k_T"

$$F^{NP}(b) = e^{-g_{\text{NP}} b^2}$$

$$g_{\text{NP}} = g_1 + g_2 \ln(Q/2Q_0)$$

$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}$$

$$A_q\left(\alpha_s\right) = C_F\frac{\alpha_s}{\pi} + \frac{1}{2}C_F\left\{\left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_G - \frac{5}{9}N_f\right\}\left(\frac{\alpha_s}{\pi}\right)^2 + \cdots$$

$$B_q\left(\alpha_s\right) = -\frac{3}{2}C_F\frac{\alpha_s}{\pi} + \cdots$$

$$= \frac{1}{\alpha_s(\mu_R^2)}h^{(0)}(\lambda) + h^{(1)}(\lambda) + \sum_{n=2}^{\infty}\left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^{n-1}h^{(n)}(\lambda)$$

$$h^{(0)}(\lambda) = \frac{A^{(1)}}{2\pi\beta_0^2}[\lambda + \ln(1-\lambda)]$$

$$\lambda = \beta_0\alpha_s(\mu_R^2)\ln\frac{Q^2b^2}{b_0^2} \equiv \beta_0\alpha_s(\mu_R^2)L$$

$$h^{(1)}(\lambda) = \frac{A^{(1)}\beta_1}{2\pi\beta_0^3}\left[\frac{1}{2}\ln^2(1-\lambda) + \frac{\lambda + \ln(1-\lambda)}{1-\lambda}\right] + \frac{B^{(1)}}{2\pi\beta_0}\ln(1-\lambda) - \frac{1}{4\pi^2\beta_0^2}\left[A^{(2)} - 2\pi\beta_0A^{(1)}\ln\frac{Q^2}{\mu_R^2}\right]\left[\frac{\lambda}{1-\lambda} + \ln(1-\lambda)\right]$$

Landau pole $\quad b_{LP} \sim \frac{1}{Q}e^{\frac{1}{2\beta_0\alpha_s(Q^2)}}$

$$\frac{d\sigma^{(\text{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \quad \xrightarrow{b \ll \frac{1}{\Lambda_{\text{QCD}}}} \quad W_j^{(\text{OPE})}(b;\frac{b_0}{b}) = (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$= \bar{P}_{j/A}(x_A, b; b_0)\bar{P}_{\bar{j}/B}(x_B, b; b_0) \quad \text{TMD}$$

"primordial k_T"

$$W_j(b;\frac{b_0}{b}) = W_j^{(\text{OPE})}(b;\frac{b_0}{b}) \frac{W_j(b;\frac{b_0}{b})}{W_j^{(\text{OPE})}(b;\frac{b_0}{b})} \equiv W_j^{(\text{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$F^{NP}(b) = e^{-g_{\text{NP}} b^2}$$

$$g_{\text{NP}} = g_1 + g_2 \ln(Q/2Q_0)$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \xrightarrow{b \ll \frac{1}{\Lambda_{\mathrm{QCD}}}} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) = (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$= \overline{P}_{j/A}(x_A, b; b_0)\overline{P}_{\bar{j}/B}(x_B, b; b_0)$$ TMD

"primordial k_T"

$$W_j(b;\frac{b_0}{b}) = W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \frac{W_j(b;\frac{b_0}{b})}{W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})} \equiv W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$F^{NP}(b) = e^{-g_{\mathrm{NP}} b^2}$$

$$g_{\mathrm{NP}} = g_1 + g_2 \ln(Q/2Q_0)$$

1. $e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \to e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})$ $\qquad b_* = \frac{b}{\sqrt{1+b^2/b_{\max}^2}}$

J. Collins, D. Soper, G. Sterman ('85)

C. Balazs, C. Yuan ('00)

J. Qiu, X. Zhang ('00)

A. Kulesza, W. Stirling ('02)

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\, e^{S_j(b,Q)}\, W_j\!\left(b;\frac{b_0}{b}\right) = \int_0^\infty db\,\frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\, e^{S_j(b,Q)}\, W_j\!\left(b;\frac{b_0}{b}\right)$$

$$W_j\!\left(b;\frac{b_0}{b}\right) \xrightarrow{\; b \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \;} W_j^{(\mathrm{OPE})}\!\left(b;\frac{b_0}{b}\right) = \left(C_{ji}\otimes f_{i/A}\right)\!\left(x_A,\frac{b_0^2}{b^2}\right)\left(C_{\bar{j}k}\otimes f_{k/B}\right)\!\left(x_B,\frac{b_0^2}{b^2}\right)$$

$$\| \quad \bar{P}_{j/A}(x_A,b;b_0)\,\bar{P}_{\bar{j}/B}(x_B,b;b_0)$$

TMD

$$W_j\!\left(b;\frac{b_0}{b}\right) = W_j^{(\mathrm{OPE})}\!\left(b;\frac{b_0}{b}\right)\frac{W_j\!\left(b;\frac{b_0}{b}\right)}{W_j^{(\mathrm{OPE})}\!\left(b;\frac{b_0}{b}\right)} \equiv W_j^{(\mathrm{OPE})}\!\left(b;\frac{b_0}{b}\right) F^{NP}(b)$$

"primordial k_T"

$$F^{NP}(b) = e^{-g_{\mathrm{NP}} b^2}$$

$$g_{\mathrm{NP}} = g_1 + g_2 \ln(Q/2Q_0)$$

1. $e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}\!\left(b;\frac{b_0}{b}\right) \to e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}\!\left(b_*;\frac{b_0}{b_*}\right)$ $\qquad b_* = \dfrac{b}{\sqrt{1+b^2/b_{\max}^2}}$

J. Collins, D. Soper, G. Sterman ('85)

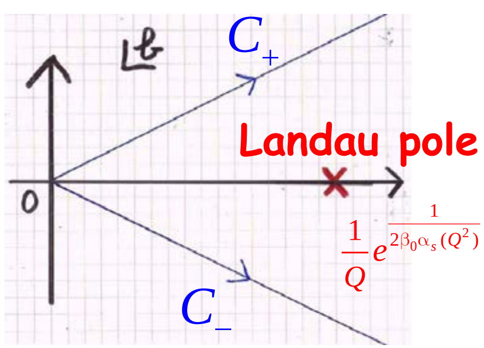
C. Balazs, C. Yuan ('00)

J. Qiu, X. Zhang ('00)

A. Kulesza, W. Stirling ('02)

2. $\int_0^\infty db \longrightarrow \int_{C_+ + C_-} db$

$$\left(J_0(bQ_T) = \frac{H_0^{(1)}(bQ_T) + H_0^{(2)}(bQ_T)}{2} \right)$$



E. Laenen, G. Sterman, W. Vogelsang ('00)

A. Kulesza, G. Sterman, W. Vogelsang ('02)

G. Bozzi, S. Catani, D. de Florian, M. Grazzini ('02)

H. Kawamura, J. Kodaira, KT ('07)

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2dQ_T^2dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T}\sum_j \hat{\sigma}_{j\bar{j}}(Q^2)e^{S_j(b,Q)}W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})F^{NP}(b)$$

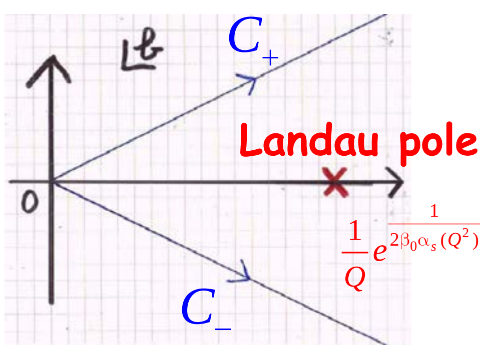
$$= \begin{cases} \displaystyle\sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\int_0^\infty db\frac{b}{2}J_0(bQ_T)e^{S_j(b_*,Q)}W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})F^{NP}(b) \\[3em] \displaystyle\sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T)+\int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right)e^{S_j(b,Q)}W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})\hat{F}^{NP}(b) \end{cases}$$

1. $e^{S_j(b,Q)}W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \to e^{S_j(b_*,Q)}W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})$ $\qquad b_* = \frac{b}{\sqrt{1+b^2/b_{\max}^2}}$

J. Collins, D. Soper, G. Sterman ('85)
C. Balazs, C. Yuan ('00)
J. Qiu, X. Zhang ('00)
A. Kulesza, W. Stirling ('02)

2. $\int_0^\infty db \to \int_{C_+ + C_-} db$

$$\left(J_0(bQ_T) = \frac{H_0^{(1)}(bQ_T)+H_0^{(2)}(bQ_T)}{2}\right)$$



E. Laenen, G. Sterman, W. Vogelsang ('00)
A. Kulesza, G. Sterman, W. Vogelsang ('02)
G. Bozzi, S. Catani, D. de Florian, M. Grazzini ('02)
H. Kawamura, J. Kodaira, KT ('07)

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

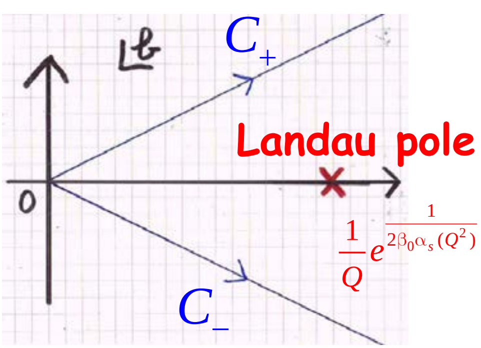
$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*}) F^{NP}(b) \\ \quad \to \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

1. $e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \to e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})$ $\qquad b_* = \frac{b}{\sqrt{1+b^2/b_{\max}^2}}$

J. Collins, D. Soper, G. Sterman ('85)
C. Balazs, C. Yuan ('00)
J. Qiu, X. Zhang ('00)
A. Kulesza, W. Stirling ('02)

2. $\int_0^\infty db \to \int_{C_+ + C_-} db$

$$\left(J_0(bQ_T) = \frac{H_0^{(1)}(bQ_T) + H_0^{(2)}(bQ_T)}{2} \right)$$



E. Laenen, G. Sterman, W. Vogelsang ('00)
A. Kulesza, G. Sterman, W. Vogelsang ('02)
G. Bozzi, S. Catani, D. de Florian, M. Grazzini ('02)
H. Kawamura, J. Kodaira, KT ('07)

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2 b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*}) F^{NP}(b) \\ \quad \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*}) F^{NP}(b) = e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \hat{F}^{NP}(b)$$

$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left\{ \left(\ln \frac{Q^2}{\mu^2} \right) A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right) \right\}$$

$$A_q(\alpha_s) = C_F \frac{\alpha_s}{\pi} + \frac{1}{2} C_F \left\{ \left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_G - \frac{5}{9} N_f \right\} \left(\frac{\alpha_s}{\pi} \right)^2 + \cdots$$

$$B_q(\alpha_s) = -\frac{3}{2} C_F \frac{\alpha_s}{\pi} + \cdots$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2\,dQ_T^2\,dy} = \frac{1}{4\pi}\int d^2b\; e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\, e^{S_j(b,Q)}\, W_j^{(\mathrm{OPE})}\left(b;\frac{b_0}{b}\right) F^{NP}(b)$$

$$= \begin{cases} \displaystyle\sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db\,\frac{b}{2} J_0(bQ_T)\, e^{S_j(b_*,Q)}\, W_j^{(\mathrm{OPE})}\left(b_*;\frac{b_0}{b_*}\right) F^{NP}(b) \\[2ex] \quad \rightarrow \left(\displaystyle\int_{C_+} db\,\frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db\,\frac{b}{4} H_0^{(2)}(bQ_T) \right) \\[2ex] \displaystyle\sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db\,\frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db\,\frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)}\, W_j^{(\mathrm{OPE})}\left(b;\frac{b_0}{b}\right) \hat{F}^{NP}(b) \end{cases}$$

$$\hat{F}^{NP}(b) = F^{NP}(b)\, e^{-\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right) A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}}\; \frac{W_j^{(\mathrm{OPE})}\left(b_*;\frac{b_0}{b_*}\right)}{W_j^{(\mathrm{OPE})}\left(b;\frac{b_0}{b}\right)}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*}) F^{NP}(b) \\ \quad \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$\hat{F}^{NP}(b) = F^{NP}(b) e^{-\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right) A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}} \frac{W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})}{W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})}$$

accurate around $|b| \sim b_{\max} \sim 1\ \mathrm{GeV}^{-1}$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b}) = \int_0^\infty db \frac{b}{2} J_0(bQ_T) \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j(b;\frac{b_0}{b})$$

$$W_j(b;\frac{b_0}{b}) \xrightarrow{\; b \ll \frac{1}{\Lambda_{\mathrm{QCD}}} \;} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) = (C_{ji} \otimes f_{i/A})\left(x_A, \frac{b_0^2}{b^2}\right)(C_{\bar{j}k} \otimes f_{k/B})\left(x_B, \frac{b_0^2}{b^2}\right)$$

$$= \bar{P}_{j/A}(x_A, b; b_0)\bar{P}_{\bar{j}/B}(x_B, b; b_0)$$

TMD

"primordial k_T"

$$F^{NP}(b) = e^{-g_{\mathrm{NP}} b^2}$$

$$g_{\mathrm{NP}} = g_1 + g_2 \ln(Q/2Q_0)$$

$$W_j(b;\frac{b_0}{b}) = W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \frac{W_j(b;\frac{b_0}{b})}{W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})} \equiv W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

1. $e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \to e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})$ $\quad b_* = \dfrac{b}{\sqrt{1+b^2/b_{\max}^2}}$

J. Collins, D. Soper, G. Sterman ('85)

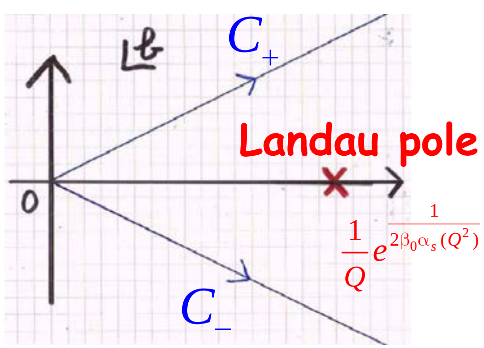
C. Balazs, C. Yuan ('00)

J. Qiu, X. Zhang ('00)

A. Kulesza, W. Stirling ('02)

2. $\int_0^\infty db \longrightarrow \int_{C_+ + C_-} db$

$$\left(J_0(bQ_T) = \frac{H_0^{(1)}(bQ_T) + H_0^{(2)}(bQ_T)}{2} \right)$$



E. Laenen, G. Sterman, W. Vogelsang ('00)

A. Kulesza, G. Sterman, W. Vogelsang ('02)

G. Bozzi, S. Catani, D. de Florian, M. Grazzini ('02)

H. Kawamura, J. Kodaira, KT ('07)

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$e^{-\left[g_1+g_2\ln(Q/2Q_0)\right]b^2}$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\int_0^\infty db\frac{b}{2}J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*}) F^{NP}(b) \\ \quad \left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T) + \int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T) + \int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1+\hat{g}_2\ln(Q/2Q_0)\right]b^2}$$

$$\hat{F}^{NP}(b) = F^{NP}(b)\, e^{-\int_{b_0^2/b_*^2}^{b_0^2/b^2}\frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right)+B_j\left(\alpha_s(\mu^2)\right)\right\}} \frac{W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})}{W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})}$$

$$e^{-\left[g_1+g_2\ln(Q/2Q_0)\right]b^2}$$

$$e^{-\left[\hat{g}_1+\hat{g}_2\ln(Q/2Q_0)\right]b^2}$$

accurate around $|b| \sim b_{\max} \sim 1\ \mathrm{GeV}^{-1}$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$e^{-[g_1+g_2\ln(Q/2Q_0)]b^2}$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\int_0^\infty db\frac{b}{2}J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*}) F^{NP}(b) \\ \quad \left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T) + \int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T) + \int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-[\hat{g}_1+\hat{g}_2\ln(Q/2Q_0)]b^2}$$

$$\hat{F}^{NP}(b) = F^{NP}(b) e^{-\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right)+B_j\left(\alpha_s(\mu^2)\right)\right\}} \frac{W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*})}{W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})}$$

$$e^{-[\hat{g}_1+\hat{g}_2\ln(Q/2Q_0)]b^2} \qquad e^{-[g_1+g_2\ln(Q/2Q_0)]b^2}$$

accurate around $|b| \sim b_{\max} \sim 1\ \mathrm{GeV}^{-1}$

$$W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) = (C_{ji}\otimes f_{i/A})\left(x_A,\frac{b_0^2}{b^2}\right)(C_{\bar{j}k}\otimes f_{k/B})\left(x_B,\frac{b_0^2}{b^2}\right)$$

$$\frac{d\sigma^{(\text{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\text{OPE})}(b; \frac{b_0}{b}) F^{NP}(b)$$

$$e^{-[g_1+g_2\ln(Q/2Q_0)]b^2}$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\text{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \qquad \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\text{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-[\hat{g}_1+\hat{g}_2\ln(Q/2Q_0)]b^2}$$

$$\hat{g}_1 = g_1 + \frac{1}{b^2} \int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2} \left\{ \left(\ln \frac{4Q_0^2}{\mu^2} \right) A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right) \right\} + \ln\left[W_j^{(\text{OPE})}(b) \big/ W_j^{(\text{OPE})}(b_*) \right]$$

$$\hat{g}_2 = g_2 + \frac{2}{b^2} \int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2} A_j\left(\alpha_s(\mu^2)\right)$$

accurate around $|b| \sim b_{\max} \sim 1\,\text{GeV}^{-1}$

$$b_* = \frac{b}{\sqrt{1+b^2/b_{\max}^2}}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) F^{NP}(b)$$

$$e^{-\left[g_1+g_2\ln(Q/2Q_0)\right]b^2}$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db\frac{b}{2}J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*;\frac{b_0}{b_*}) F^{NP}(b) \\ \quad \to \left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T) + \int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T) + \int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1+\hat{g}_2\ln(Q/2Q_0)\right]b^2}$$

$$\hat{g}_1 = g_1 + \frac{1}{b^2}\int_{b_0^2/b_*^2}^{b_0^2/b^2}\frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{4Q_0^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\} + \ln\left[W_j^{(\mathrm{OPE})}(b)/W_j^{(\mathrm{OPE})}(b_*)\right]$$

$$\simeq g_1 - \frac{1}{b_{\max}^2}\left\{2A_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)\ln\frac{2Q_0 b_{\max}}{b_0} + B_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)\right\} + \cdots$$

$$\hat{g}_2 = g_2 + \frac{2}{b^2}\int_{b_0^2/b_*^2}^{b_0^2/b^2}\frac{d\mu^2}{\mu^2}A_j\left(\alpha_s(\mu^2)\right) \simeq g_2 - \frac{2}{b_{\max}^2}A_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)$$

accurate around $|b| \sim b_{\max} \sim 1\,\mathrm{GeV}^{-1}$

$$b_* = \frac{b}{\sqrt{1+b^2/b_{\max}^2}}$$

$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}$$

$$A_j(\alpha_s) = A_j^{(1)}\frac{\alpha_s}{\pi} + A_j^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + A_j^{(3)}\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots$$

$$\simeq 1.33\frac{\alpha_s}{\pi} + 2.67\left(\frac{\alpha_s}{\pi}\right)^2 + 6.71\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots$$

$$\hat{g}_2 \simeq g_2 - \frac{2}{b_{\max}^2}A_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)$$

$$\simeq g_2 - 0.5\ \mathrm{GeV}^2 \qquad \left(b_{\max} \sim 1\ \mathrm{GeV}^{-1}\right)$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b}) F^{NP}(b)$$

$$e^{-[g_1+g_2\ln(Q/2Q_0)]b^2}$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-[\hat{g}_1+\hat{g}_2\ln(Q/2Q_0)]b^2}$$

$$\hat{g}_1 = g_1 + \frac{1}{b^2}\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{4Q_0^2}{\mu^2}\right) A_j(\alpha_s(\mu^2)) + B_j(\alpha_s(\mu^2))\right\} + \ln\left[W_j^{(\mathrm{OPE})}(b)/W_j^{(\mathrm{OPE})}(b_*)\right]$$

$$\simeq g_1 - \frac{1}{b_{\max}^2}\left\{2A_j(\alpha_s(b_0^2/b_{\max}^2))\ln\frac{2Q_0 b_{\max}}{b_0} + B_j(\alpha_s(b_0^2/b_{\max}^2))\right\} + \cdots$$

$$\hat{g}_2 = g_2 + \frac{2}{b^2}\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2} A_j(\alpha_s(\mu^2)) \simeq g_2 - \frac{2}{b_{\max}^2} A_j(\alpha_s(b_0^2/b_{\max}^2))$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \frac{1}{4\pi}\int d^2b\, e^{i\boldsymbol{b}\cdot\boldsymbol{Q}_T} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}\left(b; \frac{b_0}{b}\right) F^{NP}(b)$$

$$e^{-\left[g_1 + g_2 \ln(Q/2Q_0)\right]b^2}$$

$$= \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}\left(b_*; \frac{b_0}{b_*}\right) F^{NP}(b) \\ \quad \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T)\right) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T)\right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}\left(b; \frac{b_0}{b}\right) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right]b^2}$$

$$\hat{g}_1 = g_1 + \frac{1}{b^2}\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{4Q_0^2}{\mu^2}\right) A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\} + \ln\left[W_j^{(\mathrm{OPE})}(b) / W_j^{(\mathrm{OPE})}(b_*)\right]$$

$$\simeq g_1 - \frac{1}{b_{\max}^2}\left\{2A_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)\ln\frac{2Q_0 b_{\max}}{b_0} + B_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)\right\} + \cdots$$

$$\hat{g}_2 = g_2 + \frac{2}{b^2}\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2} A_j\left(\alpha_s(\mu^2)\right) \simeq g_2 - \frac{2}{b_{\max}^2} A_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)$$

$$\hat{g}_2 < g_2 \quad !!$$

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$e^{-\left[g_1 + g_2 \ln(Q/2Q_0)\right] b^2}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right] b^2}$$

1. $b_* = b \Big/ \sqrt{1 + b^2 / b_{\max}^2}$

Global fit

$\left(b_{\max} = 0.5\ \mathrm{GeV}^{-1}\right)$

$g_1 = -0.08\ \mathrm{GeV}^2,\ g_2 = 0.67\ \mathrm{GeV}^2$ $\left(Q_0 = 1.6\ \mathrm{GeV}\right)$

A. Kulesza, W. Stirling ('03)

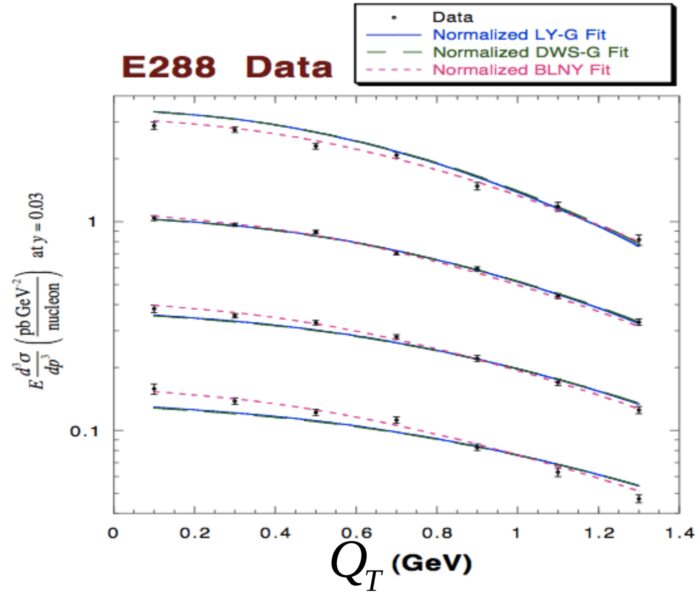
$g_1 = 0.016\ \mathrm{GeV}^2,\ g_2 = 0.54\ \mathrm{GeV}^2$ $\left(Q_0 = 1.6\ \mathrm{GeV}\right)$

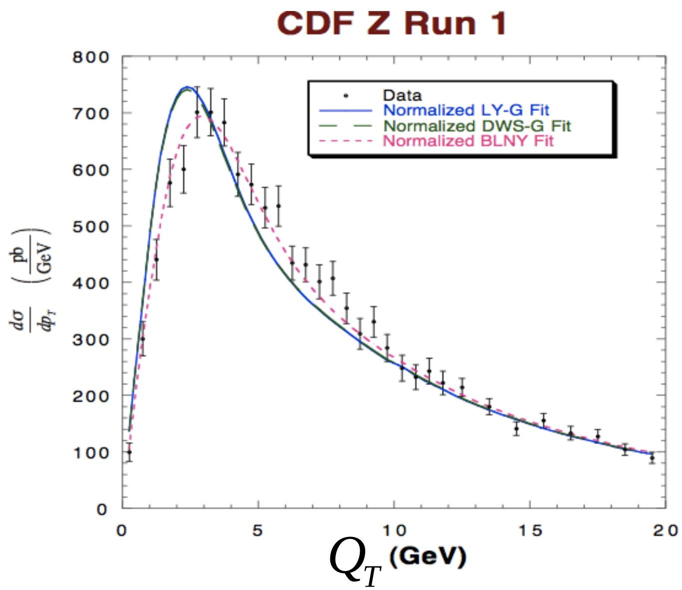
F. Landry, R. Brock, P. Nadolsky. C. Yuan ('03)

F. Landry, R. Brock, P. Nadolsky. C. Yuan, Phys. Rev. D **67**, 073016 (2003)

Fit to Drell-Yan and Z boson production

$$F^{NP}(b) = e^{-\left[g_1 + g_2 \ln(Q/2Q_0) + g_1 g_3 \ln(100 x_A x_B)\right] b^2}$$





g_1	0.21	0.016
g_2	0.68	0.54
g_3	-0.60	0.00

b_{max}=0.5GeV^{-1}

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\text{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\text{fin})}}{dQ^2 dQ_T^2 dy}$$

$$e^{-\left[g_1 + g_2 \ln(Q/2Q_0)\right]b^2}$$

$$\frac{d\sigma^{(\text{res})}}{dQ^2 dQ_T^2 dy} = \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\text{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\text{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right]b^2}$$

1. $b_* = b \Big/ \sqrt{1 + b^2 / b_{\max}^2}$

Global fit

$\left(b_{\max} = 0.5\ \text{GeV}^{-1}\right)$

$$g_1 = -0.08\ \text{GeV}^2,\ g_2 = 0.67\ \text{GeV}^2 \quad \left(Q_0 = 1.6\ \text{GeV}\right)$$

A. Kulesza, W. Stirling ('03)

$$g_1 = 0.016\ \text{GeV}^2,\ g_2 = 0.54\ \text{GeV}^2 \quad \left(Q_0 = 1.6\ \text{GeV}\right)$$

F. Landry, R. Brock, P. Nadolsky. C. Yuan ('03)

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$e^{-\left[g_1 + g_2 \ln(Q/2Q_0)\right] b^2}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right] b^2}$$

1. $b_* = b \Big/ \sqrt{1 + b^2 / b_{\max}^2}$

Global fit

$\left(b_{\max} = 0.5\ \mathrm{GeV}^{-1}\right)$

$g_1 = -0.08\ \mathrm{GeV}^2,\ g_2 = 0.67\ \mathrm{GeV}^2$ $\left(Q_0 = 1.6\ \mathrm{GeV}\right)$

A. Kulesza, W. Stirling ('03)

$g_1 = 0.016\ \mathrm{GeV}^2,\ g_2 = 0.54\ \mathrm{GeV}^2$ $\left(Q_0 = 1.6\ \mathrm{GeV}\right)$

F. Landry, R. Brock, P. Nadolsky. C. Yuan ('03)

2. $\int_0^\infty db \to \int_{C_+ + C_-} db$

$$\hat{g}_2 \simeq g_2 - \frac{2}{b_{\max}^2} A_j\left(\alpha_s(b_0^2 / b_{\max}^2)\right)$$

$$S_j(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left\{\left(\ln\frac{Q^2}{\mu^2}\right)A_j\left(\alpha_s(\mu^2)\right) + B_j\left(\alpha_s(\mu^2)\right)\right\}$$

$$A_j\left(\alpha_s\right) = A_j^{(1)}\frac{\alpha_s}{\pi} + A_j^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + A_j^{(3)}\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots$$

$$\simeq 1.33\frac{\alpha_s}{\pi} + 2.67\left(\frac{\alpha_s}{\pi}\right)^2 + 6.71\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots$$

$$\hat{g}_2 \simeq g_2 - \frac{2}{b_{\mathrm{max}}^2}A_j\left(\alpha_s(b_0^2/b_{\mathrm{max}}^2)\right)$$

$$\simeq g_2 - 0.5\ \mathrm{GeV}^2 \qquad \left(b_{\mathrm{max}} \sim 1\ \mathrm{GeV}^{-1}\right)$$

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$e^{-\left[g_1 + g_2 \ln(Q/2Q_0)\right] b^2}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right] b^2}$$

1. $b_* = b \Big/ \sqrt{1 + b^2 / b_{\max}^2}$

Global fit

$\left(b_{\max} = 0.5\ \mathrm{GeV}^{-1}\right)$

$g_1 = -0.08\ \mathrm{GeV}^2,\ g_2 = 0.67\ \mathrm{GeV}^2 \quad \left(Q_0 = 1.6\ \mathrm{GeV}\right)$

A. Kulesza, W. Stirling ('03)

$g_1 = 0.016\ \mathrm{GeV}^2,\ g_2 = 0.54\ \mathrm{GeV}^2 \quad \left(Q_0 = 1.6\ \mathrm{GeV}\right)$

F. Landry, R. Brock, P. Nadolsky. C. Yuan ('03)

2. $\int_0^\infty db \to \int_{C_+ + C_-} db$

$$\hat{g}_2 \simeq g_2 - \frac{2}{b_{\max}^2} A_j\left(\alpha_s(b_0^2 / b_{\max}^2)\right)$$

$$\frac{d\sigma}{dQ^2 dQ_T^2 dy} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2 dy}$$

$$e^{-\left[g_1 + g_2 \ln(Q/2Q_0)\right] b^2}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right] b^2}$$

1. $b_* = b \big/ \sqrt{1 + b^2 / b_{\max}^2}$

Global fit

$\left(b_{\max} = 0.5\ \mathrm{GeV}^{-1}\right)$

$g_1 = -0.08\ \mathrm{GeV}^2,\ g_2 = 0.67\ \mathrm{GeV}^2$ $\left(Q_0 = 1.6\ \mathrm{GeV}\right)$

A. Kulesza, W. Stirling ('03)

$g_1 = 0.016\ \mathrm{GeV}^2,\ g_2 = 0.54\ \mathrm{GeV}^2$ $\left(Q_0 = 1.6\ \mathrm{GeV}\right)$

F. Landry, R. Brock, P. Nadolsky. C. Yuan ('03)

" revised b_* "

$\left(b_{\max} = 1.5\ \mathrm{GeV}^{-1}\right)$

$g_2 \simeq 0.19\ \mathrm{GeV}^2$

A. Konychev, P. Nadolsky ('06)

2. $\int_0^\infty db \to \int_{C_+ + C_-} db$

$$\hat{g}_2 \simeq g_2 - \frac{2}{b_{\max}^2} A_j\left(\alpha_s(b_0^2 / b_{\max}^2)\right)$$

Global fit at NLL using " complex b " approach

M. Hirai, H. Kawamura, KT

Experimental data sets

Group	Exp	$\sqrt{s}$ (GeV)	Target	Q_T range (GeV)	Q range (GeV)	# of data (pT < 22 GeV)
DY	R209	62	P-P	0.2 – 1.8	5.0 - 8.0	5
DY	R209	62	P-P	0.2 – 1.8	8.0 - 11.0	5
Z^0	CDF run-0	1800	P-Pbar	0.0 – 22.8	75 - 105	7
Z^0	CDF run-1	1800	P-Pbar	0.0 – 22.0	66 - 116	33
Z^0	D0 run-1	1800	P-Pbar	0.0 – 22.0	75 - 105	15

$$\frac{d\sigma}{dQ^2 dQ_T^2} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2} = \sum_j \hat{\sigma}_{jj}(Q^2)\left(\int_{C_+} db\,\frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db\,\frac{b}{4} H_0^{(2)}(bQ_T)\right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}\left(b;\frac{b_0}{b}\right)\hat{F}^{NP}(b)$$

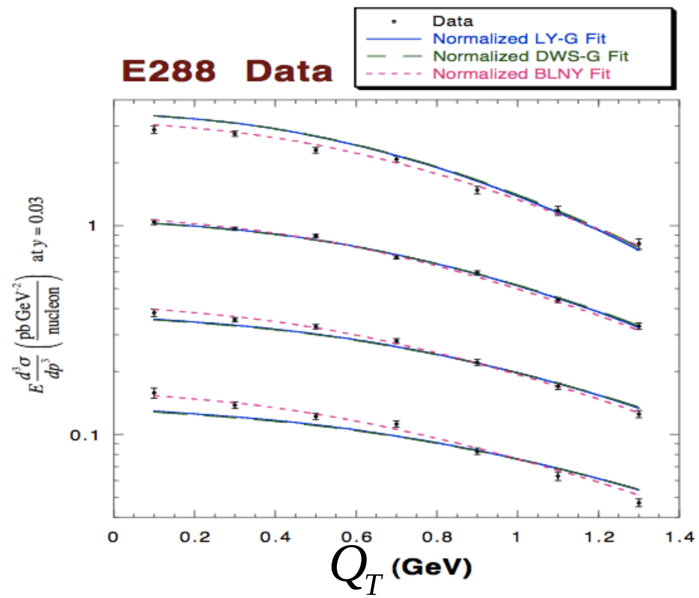
2-parameter fit of $\hat{F}^{NP}(b) = e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right] b^2}$

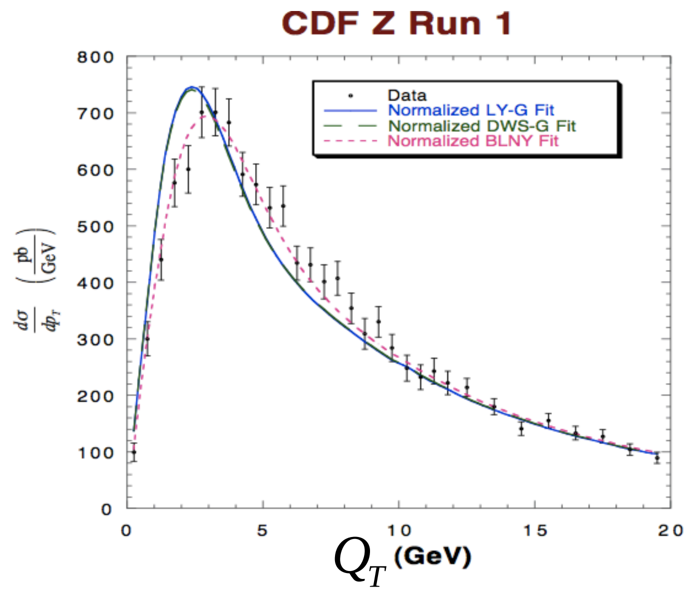
determine $\hat{g}_1, \; \hat{g}_2$

F. Landry, R. Brock, P. Nadolsky. C. Yuan, Phys. Rev. D **67**, 073016 (2003)

Fit to Drell-Yan and Z boson production

$$F^{NP}(b) = e^{-\left[g_1 + g_2 \ln(Q/2Q_0) + g_1 g_3 \ln(100 x_A x_B)\right] b^2}$$





g_1	0.21	0.016
g_2	0.68	0.54
g_3	-0.60	0.00

b_{max}=0.5GeV^{-1}

Global fit at NLL using " complex b " approach

M. Hirai, H. Kawamura, KT

Experimental data sets

	Exp	√s (GeV)	Target	Q_T range (GeV)	Q range (GeV)	# of data (pT < 22 GeV)
DY	R209	62	P-P	0.2 – 1.8	5.0 - 8.0	5
DY	R209	62	P-P	0.2 – 1.8	8.0 - 11.0	5
Z^0	CDF run-0	1800	P-Pbar	0.0 – 22.8	75 - 105	7
Z^0	CDF run-1	1800	P-Pbar	0.0 – 22.0	66 - 116	33
Z^0	D0 run-1	1800	P-Pbar	0.0 – 22.0	75 - 105	15

$$\frac{d\sigma}{dQ^2 dQ_T^2} = \frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2} + \frac{d\sigma^{(\mathrm{fin})}}{dQ^2 dQ_T^2}$$

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2} = \sum_j \hat{\sigma}_{jj}(Q^2)\left(\int_{C_+} db\frac{b}{4}H_0^{(1)}(bQ_T) + \int_{C_-} db\frac{b}{4}H_0^{(2)}(bQ_T)\right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b;\frac{b_0}{b})\hat{F}^{NP}(b)$$

2-parameter fit of $\hat{F}^{NP}(b) = e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right]b^2}$

➡ **determine** $\hat{g}_1, \ \hat{g}_2$

M. Hirai, H. Kawamura, KT

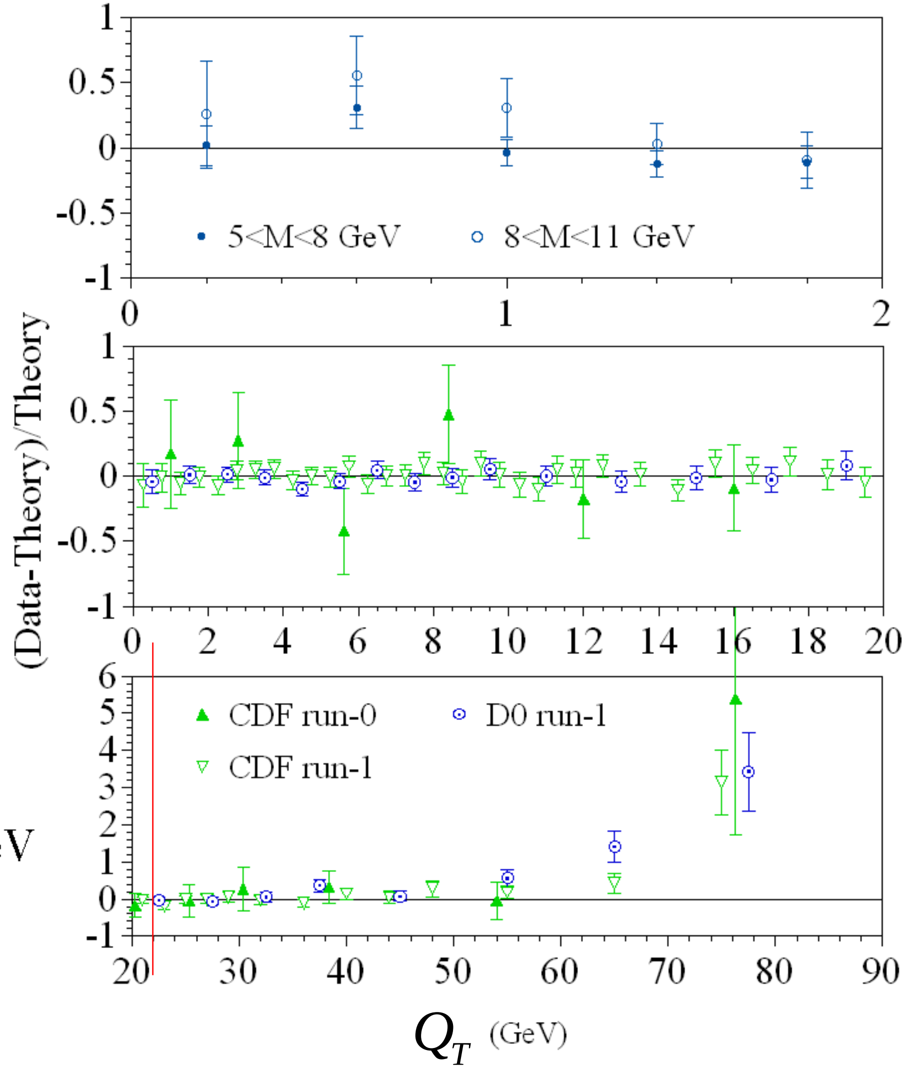
Data vs. Theory

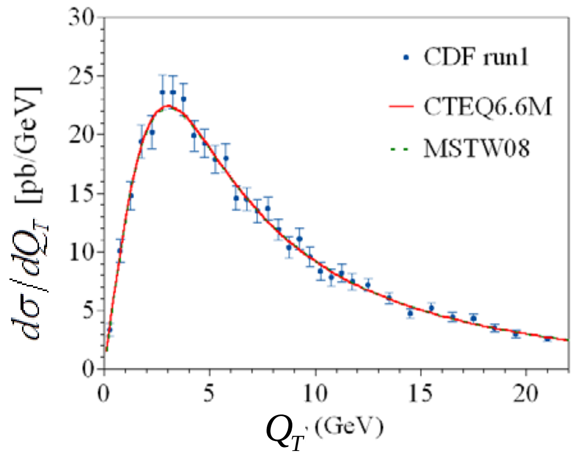
CTEQ6.6

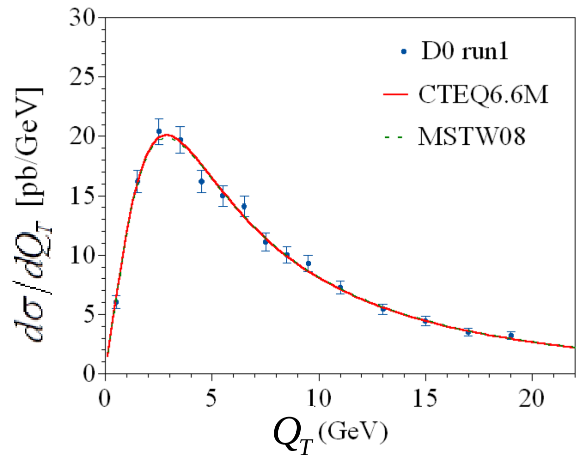
R209

D0, CDF

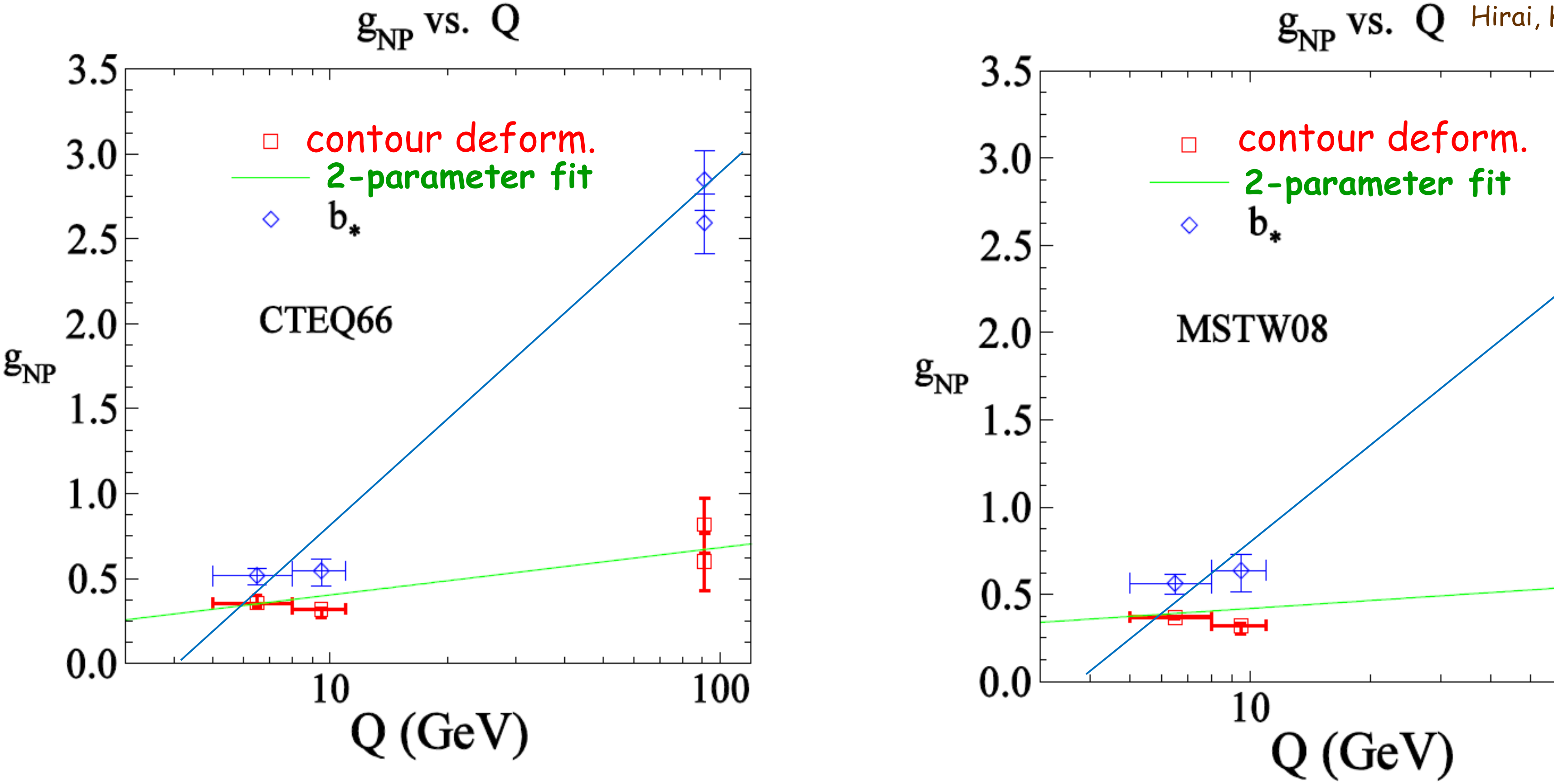
$Q_T < 22\text{GeV}$







Hirai, Kawamura, KT



$$\hat{g}_1 = 0.241^{+0.026}_{-0.028}\ \mathrm{GeV}^2 \qquad \hat{g}_1 = 0.330^{+0.024}_{-0.026}\ \mathrm{GeV}^2$$

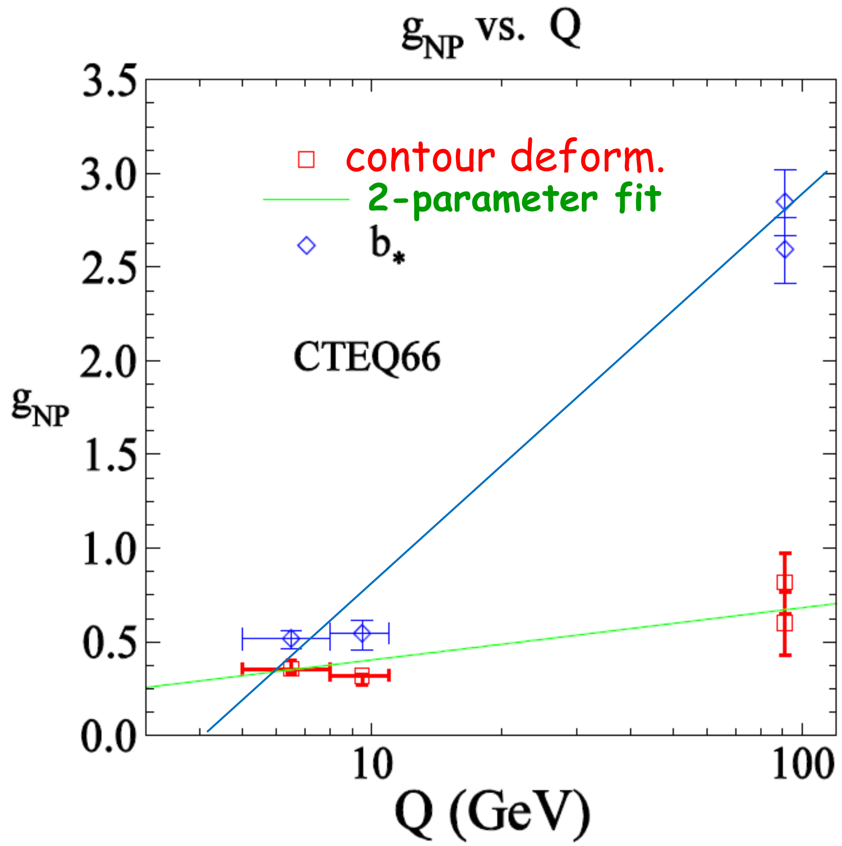
$$\hat{g}_2 = 0.121^{+0.041}_{-0.038}\ \mathrm{GeV}^2 \qquad \hat{g}_2 = 0.066^{+0.039}_{-0.037}\ \mathrm{GeV}^2$$

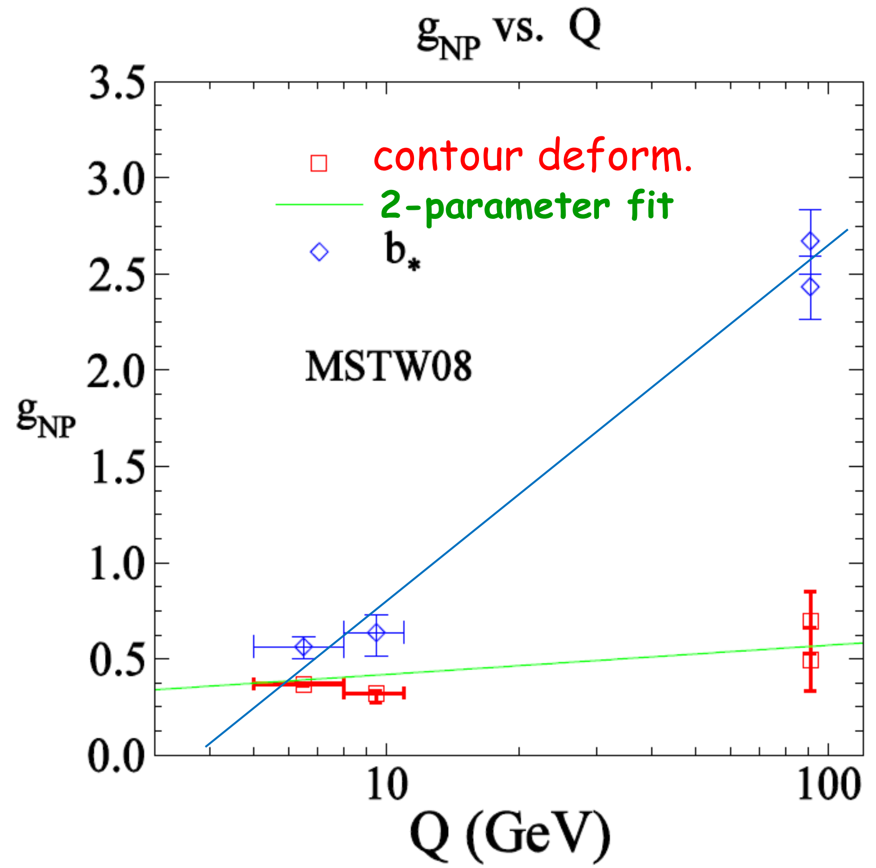
$$\hat{g}_{NP} = \hat{g}_1 + \hat{g}_2 \ln\left(Q/2Q_0\right)$$

$$\left(Q_0 = 1.3\ \mathrm{GeV}\right)$$

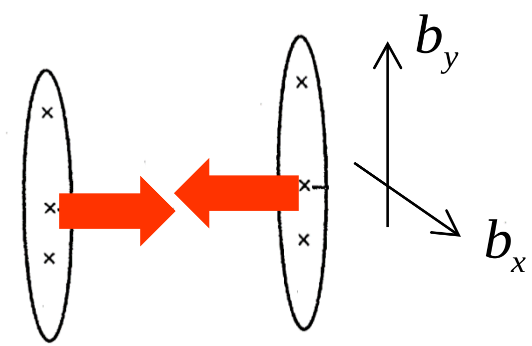
$$g_1 = 0.016\ \mathrm{GeV}^2,\ g_2 = 0.54\ \mathrm{GeV}^2$$

F. Landry, R. Brock, P. Nadolsky. C. Yuan ('03)





$$\frac{d\sigma^{(\text{res})}}{dQ^2\,dQ_T^2} = \sum_j \hat{\sigma}_{j\bar{j}}(Q^2)\left(\int_{C_+} db\,\frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db\,\frac{b}{4} H_0^{(2)}(bQ_T)\right) e^{S_j(b,Q)}\, W_j^{(\text{OPE})}\!\left(b;\frac{b_0}{b}\right)\hat{F}^{NP}(b)$$



$$\hat{F}^{NP}(b) = e^{-\hat{g}_{\text{NP}} b^2}$$

$\left(\hat{g}_{\text{NP}} = \hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right)$

$$e^{-\frac{k_T^2}{4\hat{g}_{\text{NP}}}}$$

$$\langle k_T^2 \rangle = 4\hat{g}_{\text{NP}} \lesssim 2\ \text{GeV}^2$$

$$\langle k_T^2 \rangle_{\text{1-proton}} \lesssim 1\ \text{GeV}^2$$

Summary: QCD resummation for Q_T distribution

"b_*" vs. "complex b"

$$\frac{d\sigma^{(\mathrm{res})}}{dQ^2 dQ_T^2 dy} = \begin{cases} \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S_j(b_*,Q)} W_j^{(\mathrm{OPE})}(b_*; \frac{b_0}{b_*}) F^{NP}(b) \\ \sum_j \hat{\sigma}_{j\bar{j}}(Q^2) \left(\int_{C_+} db \frac{b}{4} H_0^{(1)}(bQ_T) + \int_{C_-} db \frac{b}{4} H_0^{(2)}(bQ_T) \right) e^{S_j(b,Q)} W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b}) \hat{F}^{NP}(b) \end{cases}$$

$$\hat{F}^{NP}(b) = F^{NP}(b) e^{-\int_{b_0^2/b_*^2}^{b_0^2/b^2} \frac{d\mu^2}{\mu^2} \left\{ \left(\ln \frac{Q^2}{\mu^2} \right) A_j(\alpha_s(\mu^2)) + B_j(\alpha_s(\mu^2)) \right\}} \frac{W_j^{(\mathrm{OPE})}(b_*; \frac{b_0}{b_*})}{W_j^{(\mathrm{OPE})}(b; \frac{b_0}{b})}$$

$$\hat{F}^{NP}(b) = e^{-\left[\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right] b^2}$$

$$\hat{g}_2 \simeq g_2 - \frac{2}{b_{\max}^2} A_j\left(\alpha_s(b_0^2/b_{\max}^2)\right)$$

global fit to experimental data

$$\hat{g}_1 = 0.241^{+0.026}_{-0.028} \ \mathrm{GeV}^2 \quad \hat{g}_2 = 0.121^{+0.041}_{-0.038} \ \mathrm{GeV}^2 \quad \text{CTEQ6.6}$$

$$\hat{g}_1 = 0.330^{+0.024}_{-0.026} \ \mathrm{GeV}^2 \quad \hat{g}_2 = 0.066^{+0.039}_{-0.037} \ \mathrm{GeV}^2 \quad \text{MSTW08}$$

$\Leftrightarrow$

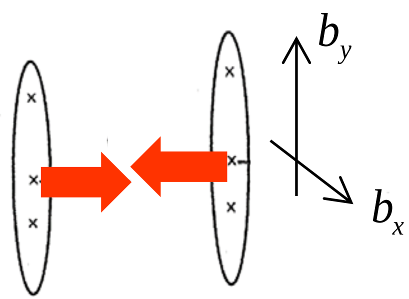
$g_1 = 0.016\ \mathrm{GeV}^2,\ g_2 = 0.54\ \mathrm{GeV}^2 \left(b_{\max} = 0.5\ \mathrm{GeV}^{-1}\right)$

F. Landry, R. Brock, P. Nadolsky. C. Yuan ('03)

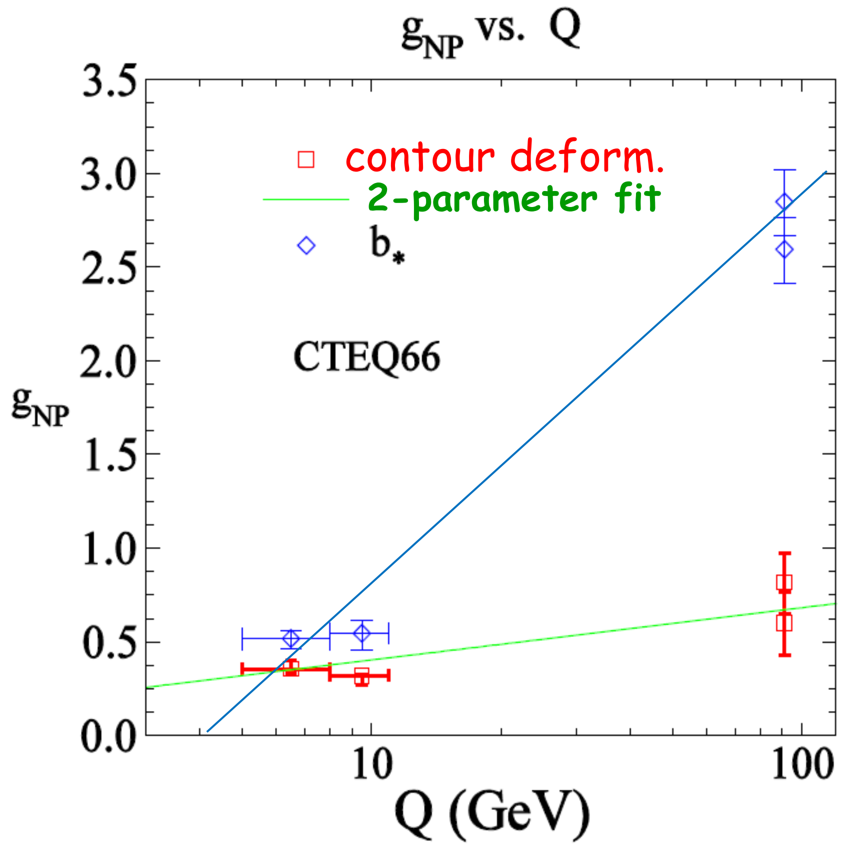
"revised b_*" $\left(b_{\max} = 1.5\ \mathrm{GeV}^{-1}\right)$ $g_2 \simeq 0.19\ \mathrm{GeV}^2$

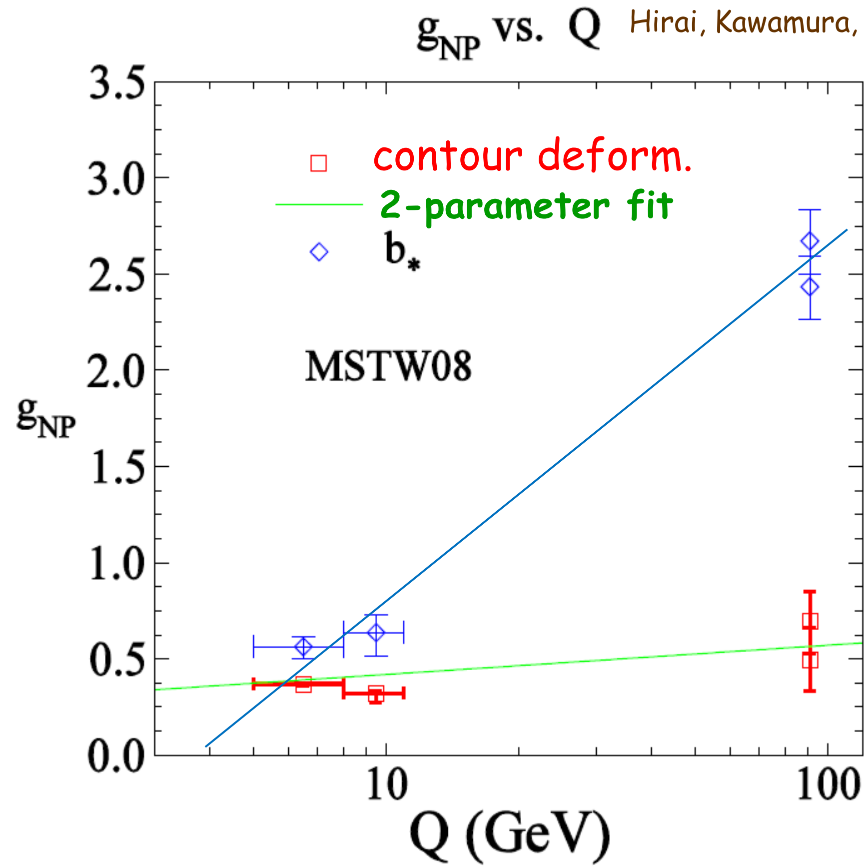
A. Konychev, P. Nadolsky ('06)

$$\langle k_T^2 \rangle_{\text{1-proton}} = 2\left\{\hat{g}_1 + \hat{g}_2 \ln(Q/2Q_0)\right\} \lesssim 1\ \mathrm{GeV}^2$$



Hirai, Kawamura, KT





Experimental data sets

	Exp	√s (GeV)	Target	Q range (GeV)	# of data (pT < 22 GeV)
DY	R209	62	P-P	5.0 - 8.0	5
DY	R209	62	P-P	8.0 - 11.0	5
Z^0	CDF run-0	1800	P-Pbar	75 - 105	7
Z^0	CDF run-1	1800	P-Pbar	66 - 116	33
Z^0	D0 run-1	1800	P-Pbar	75 - 105	15