

(γ, d) 反応による $\eta'(958)$ 中間子-原子核束縛状態の生成

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肥山詠美子氏(理化学研究所)

1. Introduction

Purpose

We like to know the possibility of formation of $\eta'(958)$ mesic nucleus by (γ, d) reaction

ϕ mesic nucleus by (γ, d)



by N. Ikeno et al., Phys. Rev. C 84, 054609(2011)

η' mesic nucleus by (γ, d)

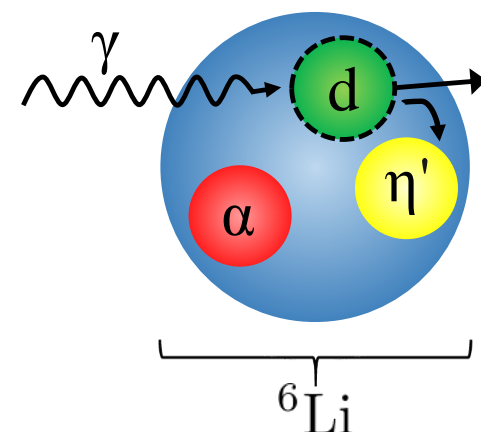
- In-medium η' properties
 \Rightarrow Information on $U_A(1)$ anomaly effect
- Possible at photon facilities ?
- Formation by (γ, p) and (p, d)

(Hideko Nagahiro, Satoru Hirenzaki, Phys. Rev. Lett. 94 (2005) 232503)

(Kenta Itahashi et al., Prog. Theor. Phys. 128 (2012) 601-613)

Improvements from N. Ikeno et al., Phys. Rev. C 84, 054609 (2011)

- Distortion effect
- Elementary cross section
- Realistic α density distribution
- Recoil effect

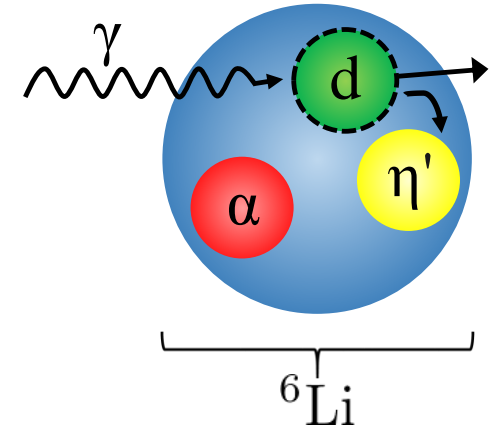


2. Two-nucleon pick-up reaction for ${}^6\text{Li}$ target by effective number (N_{eff}) approach

${}^6\text{Li}$ has well-developed cluster structure of $\alpha+d$

▣ formation cross section

$$\frac{d^2\sigma}{dEd\Omega} = \left(\frac{d\sigma}{d\Omega} \right)^{\text{ele}} \sum_f \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} \underbrace{N_{\text{eff}}}_{\uparrow}$$



N_{eff} : effective number of deuteron

$$N_{\text{eff}} = \sum_{JM} \left| \int \chi_d^*(\mathbf{r}) \left[\phi_{l_{\eta'}}^*(\mathbf{r}) \otimes \psi_{l_d}(\mathbf{r}) \right]_{JM} \chi_{\gamma}(\mathbf{r}) d\mathbf{r} \right|^2$$

χ_{γ}, χ_d : incident γ , emitted d wave function

$\phi_{l_{\eta'}}$: α - η' relative wave function

ψ_{l_d} : α - d relative wave function

$$\left(\frac{d\sigma}{d\Omega} \right)^{\text{ele}} : \text{Elementary cross section, } \left[\begin{array}{l} \Delta E = T_d - E_{\gamma} + S_d - B_{\eta'} + m_{\eta'} \\ \Gamma : \text{width of } \eta'\text{-meson bound states} \end{array} \right.$$

2-1. Initial state: α -d relative wave function (2s bound state)

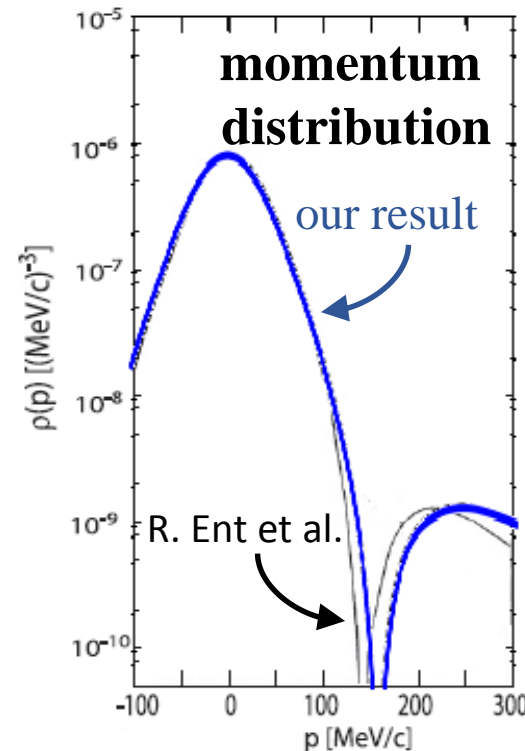
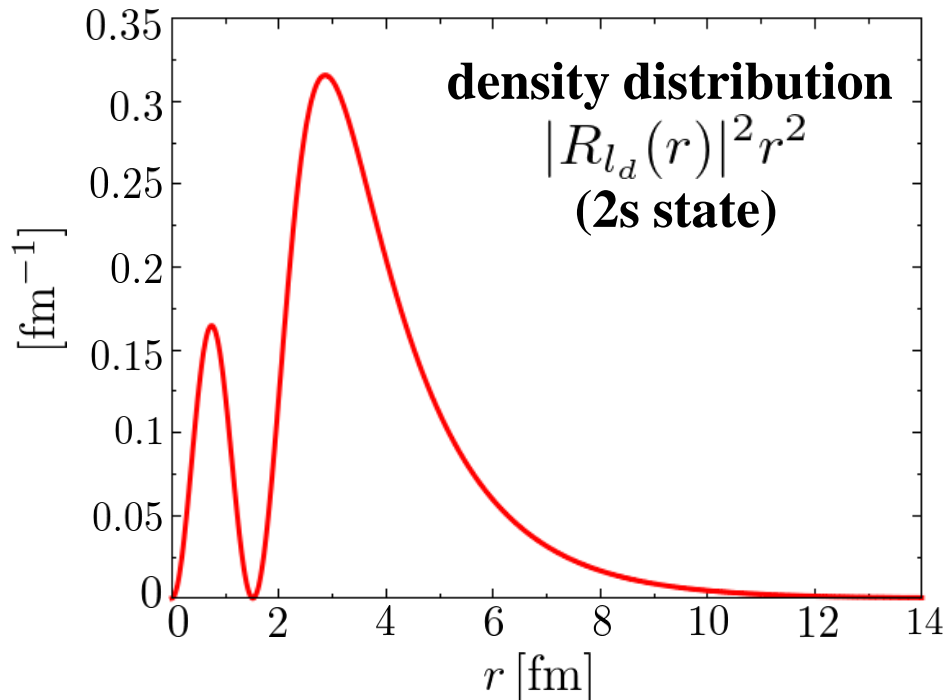
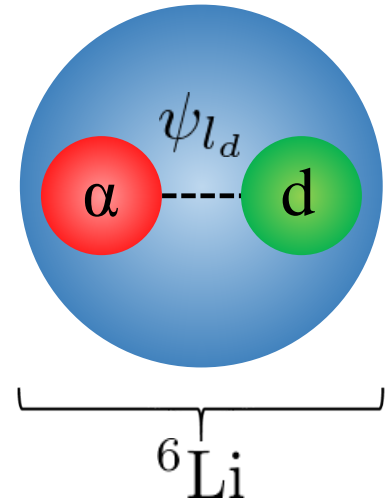
Probability of α +d cluster structure in ${}^6\text{Li}$ is reported to be 73%

(R. Ent et al., Phys. Rev. Lett. 57, 2367 (1986))

Schrödinger eq.

$$\left[-\frac{1}{2m} \nabla^2 + V(r) \right] \psi_{l_d}(\mathbf{r}) = E \psi_{l_d}(\mathbf{r})$$

$$V(r) = \frac{V_0}{1 + \exp((r - R)/a)} : \text{Woods-Saxon-type potential}$$



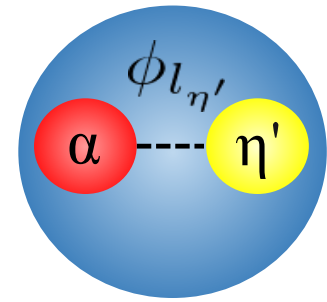
fix two parameters
to reproduce $\rho(p)$
in R. Ent et al.

$$V_0 = -75 [\text{MeV}]$$

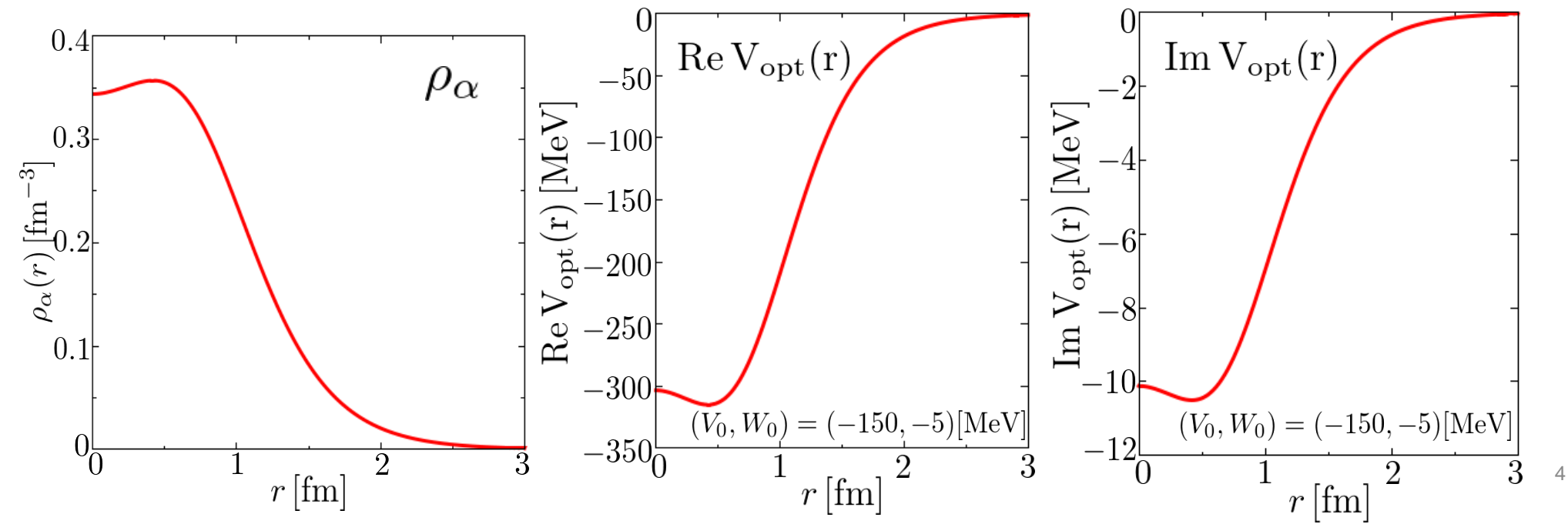
$$R = 2.0 [\text{fm}]$$

2-2. Final state: α - η' relative wave function

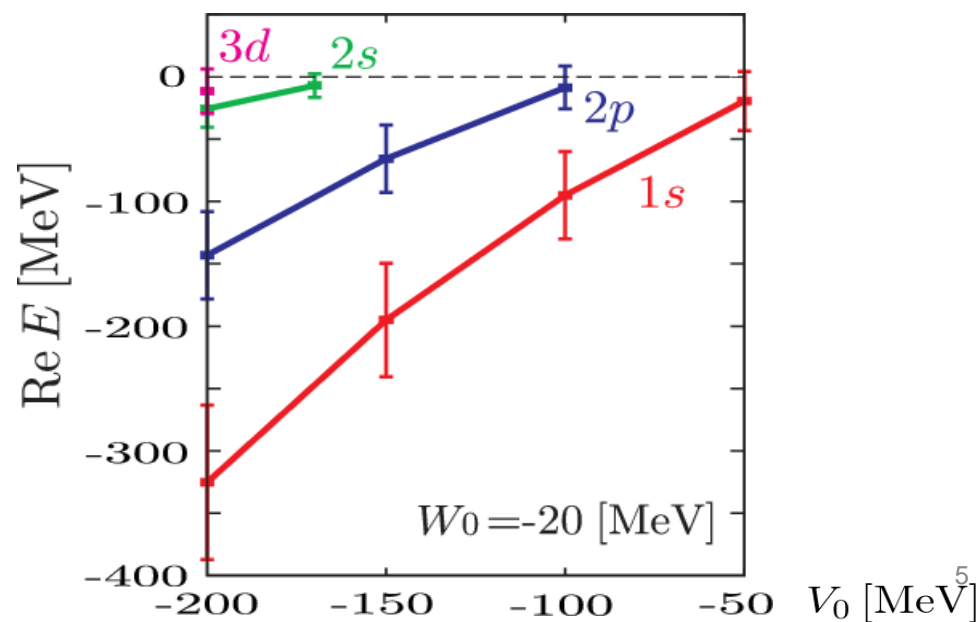
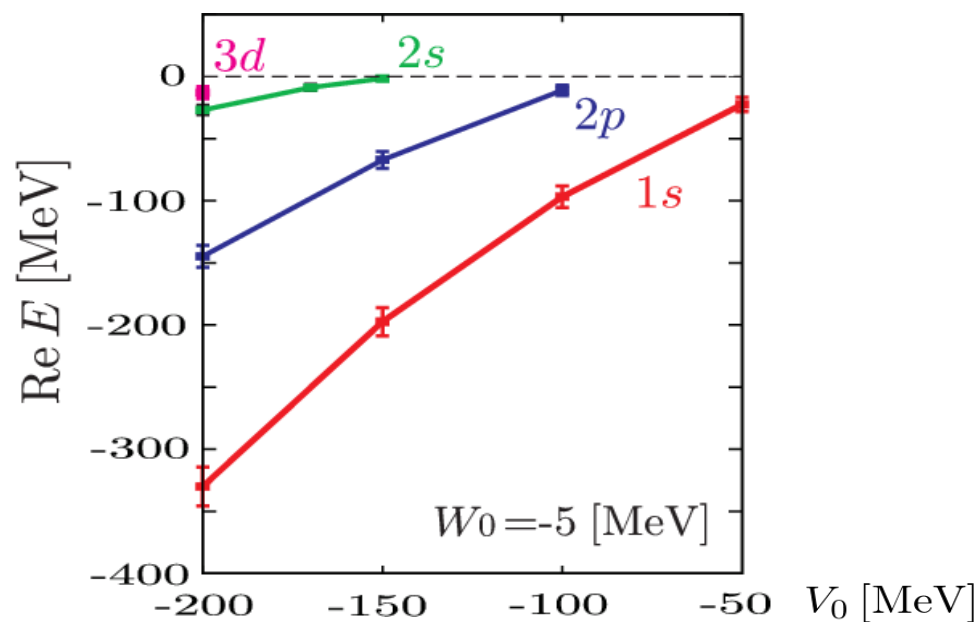
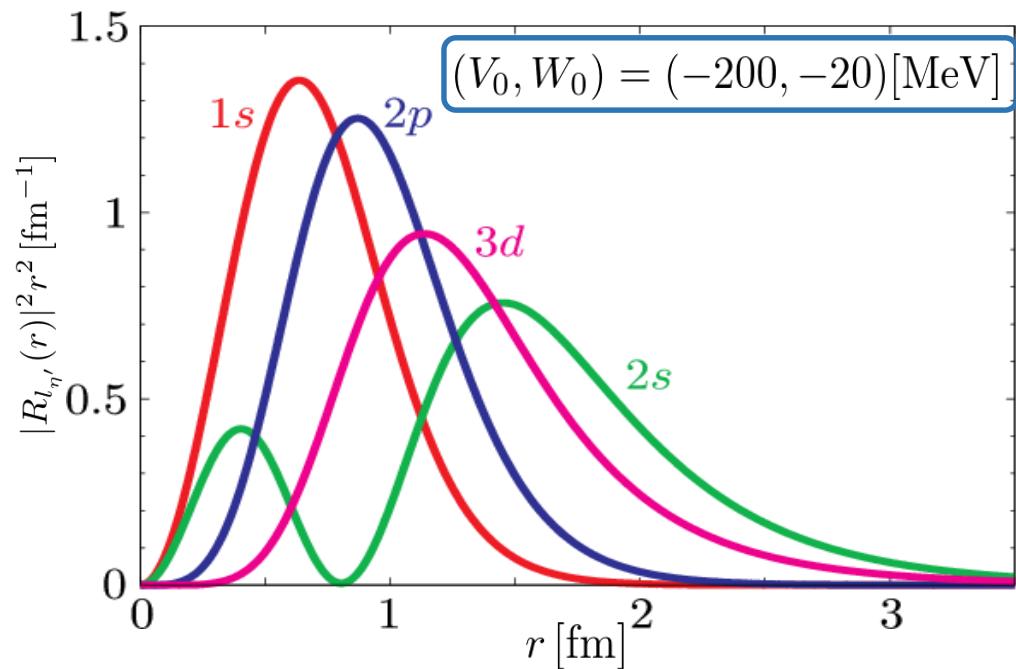
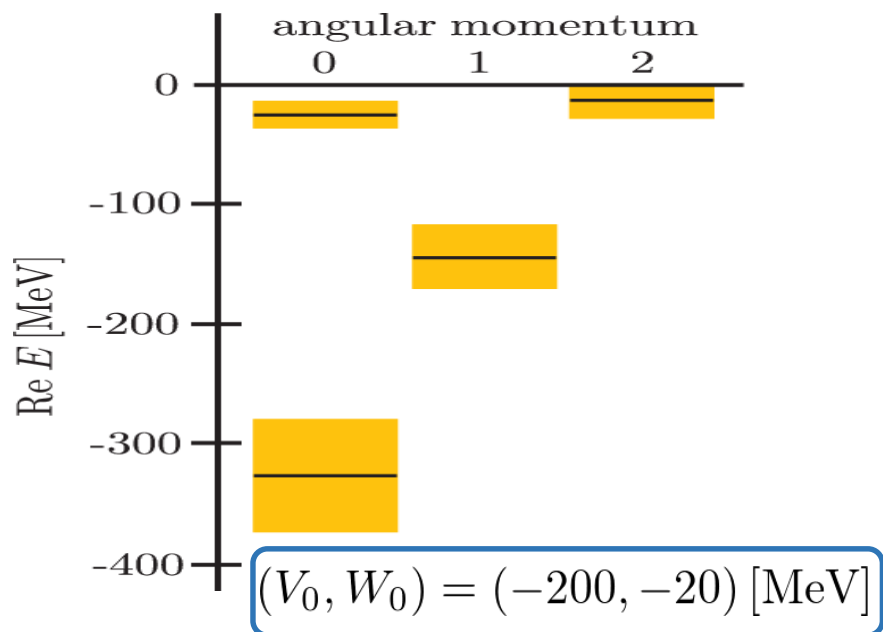
$$[-\nabla^2 + \mu^2 + 2\mu V_{\text{opt}}(r)] \phi_{l_{\eta'}}(\mathbf{r}) = E_{\text{KG}}^2 \phi_{l_{\eta'}}(\mathbf{r})$$



$$\left[\begin{array}{l} V_{\text{opt}}(r) = \frac{(V_0 + iW_0) \rho_{\alpha}(r)}{\rho_0} \\ V_0 = -50, -100, -150, -200 \text{ [MeV]}, \quad \rho_0 = 0.17 \text{ [fm}^{-3}\text{]} \\ W_0 = -5, -20 \text{ [MeV]} \quad (\text{H. Nagahiro et al., Phys. Rev. C 87, 045201 (2013)}) \\ \rho_{\alpha} : \text{Realistic } \alpha \text{ density distribution} \\ \text{Gaussian expansion method by Emiko Hiyama} \end{array} \right.$$



2-2. Final state: α - η' relative wave function



2-3. Scattering waves χ_γ and χ_d

$$N_{\text{eff}} = \sum_{JM} \left| \int \underline{\chi_d^*(\mathbf{r})} \left[\phi_{l_{\eta'}}^*(\mathbf{r}) \otimes \psi_{l_d}(\mathbf{r}) \right]_{JM} \underline{\chi_\gamma(\mathbf{r})} d\mathbf{r} \right|^2$$

1. Distortion effect (DWIA)

$$\chi_d^*(\mathbf{r}) \chi_\gamma(\mathbf{r}) = e^{i\mathbf{q} \cdot \mathbf{r}} \rightarrow e^{i\mathbf{q} \cdot \mathbf{r}} D(\mathbf{b}, z)$$

$$D(\mathbf{b}, z) = \exp \left[-\frac{\sigma_{\gamma N}}{2} \int_{-\infty}^z \rho_{^6\text{Li}}(\mathbf{b}, z') dz' - \frac{\sigma_{dN}}{2} \int_z^{+\infty} \rho_\alpha(\mathbf{b}, z') dz' \right]$$

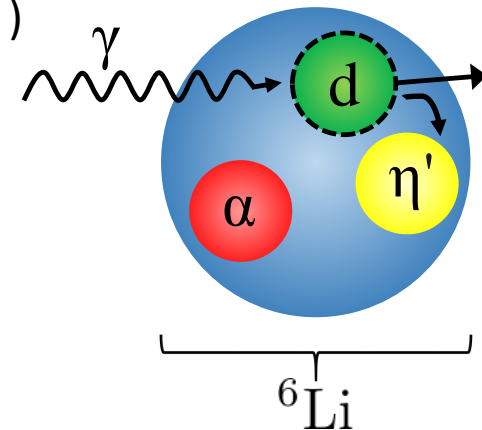
$$\begin{cases} \sigma_{\gamma N} = 0 \text{ [mb]} \\ \sigma_{dN} = 60 \text{ [mb]} \end{cases} \quad (\text{taken from } \sigma_{pd} \text{ in PDG (2012)})$$

2. Recoil effect

$$\vec{r} \rightarrow \boxed{\frac{M_\alpha}{m_{\eta'} + M_\alpha}} \vec{r}$$

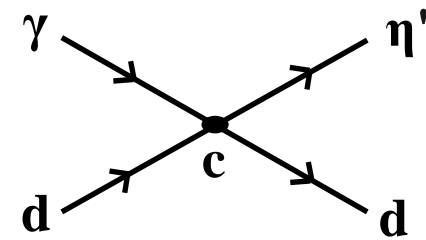
correction factor

(the same prescription as in T. Koike, T. Harada, Nucl. Phys. A 804 (2008) 231-273)

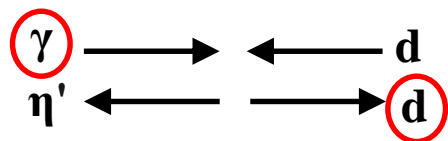


2-4. Elementary cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}}^{\text{ele}} = \frac{1}{2} \frac{|c|^2}{4\pi} \frac{p_\gamma p_{d'}}{\pi} \frac{M_d^2}{\lambda^{\frac{1}{2}}(s, M_d^2, 0)} \frac{1}{p_\gamma} \frac{1}{E_{d'} + \omega_{\eta'}} |F_d(\mathbf{q})|^2$$



scattering angle
in CM frame



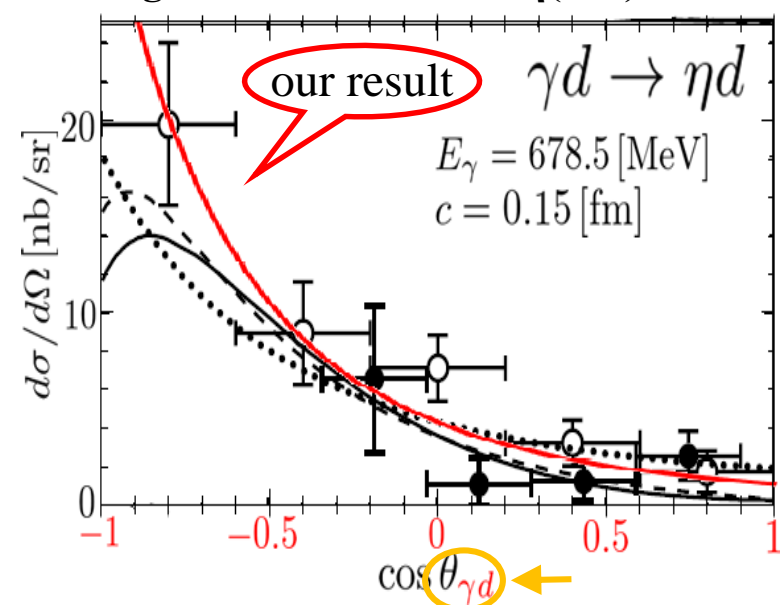
$$F_d(\mathbf{q}) = \int \psi_d^*(\mathbf{r}) e^{i\mathbf{q} \cdot \frac{\mathbf{r}}{2}} \psi_d(\mathbf{r}) d\mathbf{r} : \text{Form factor}$$

$\psi_d(\mathbf{r})$: proton-neutron relative wave function in deuteron
by Bonn potential

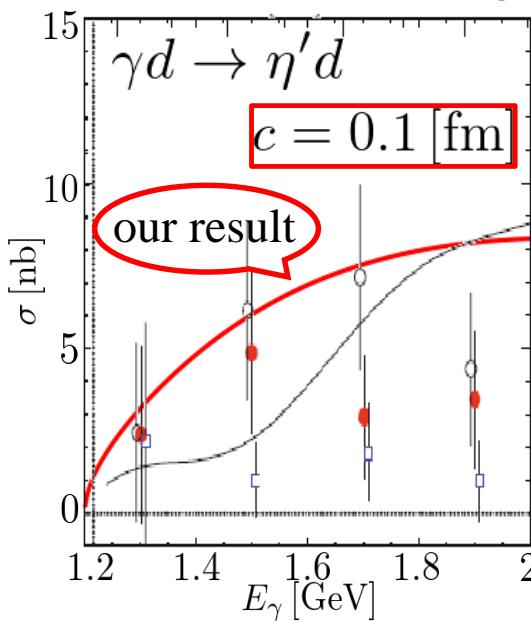
(R. Machleidt et al., Phys. Rep. 149, No.1 (1987) 1-89)

\mathbf{q} : momentum transfer in deuteron rest frame

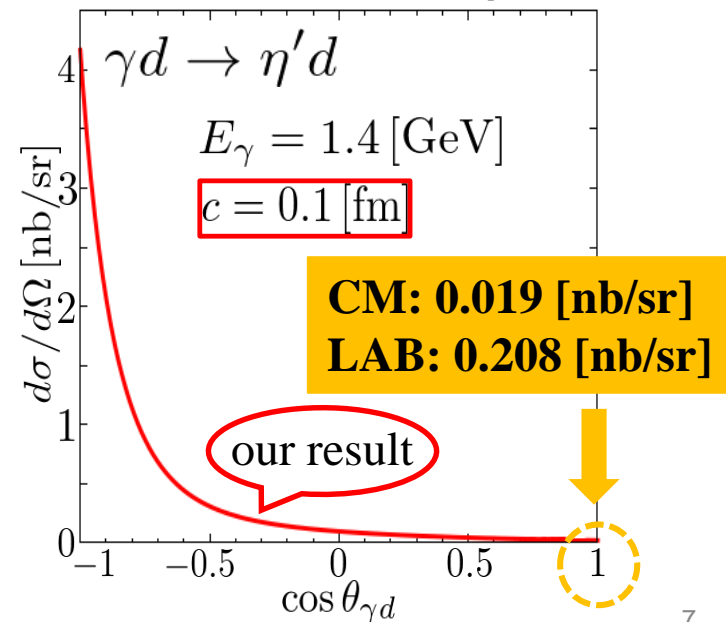
angular distribution of $\eta(548)$ in CM



total cross section of η'

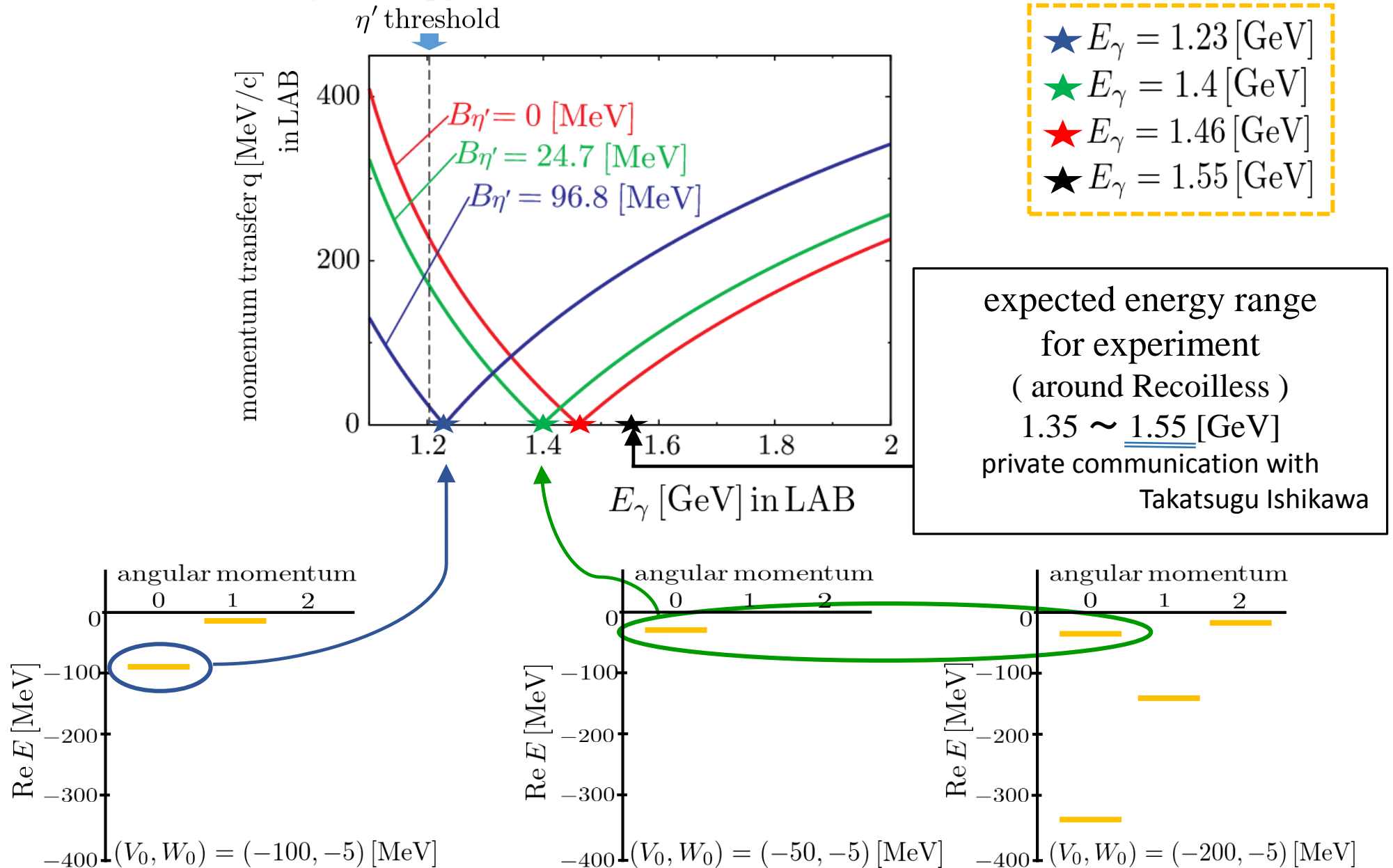


angular distribution of η' in CM

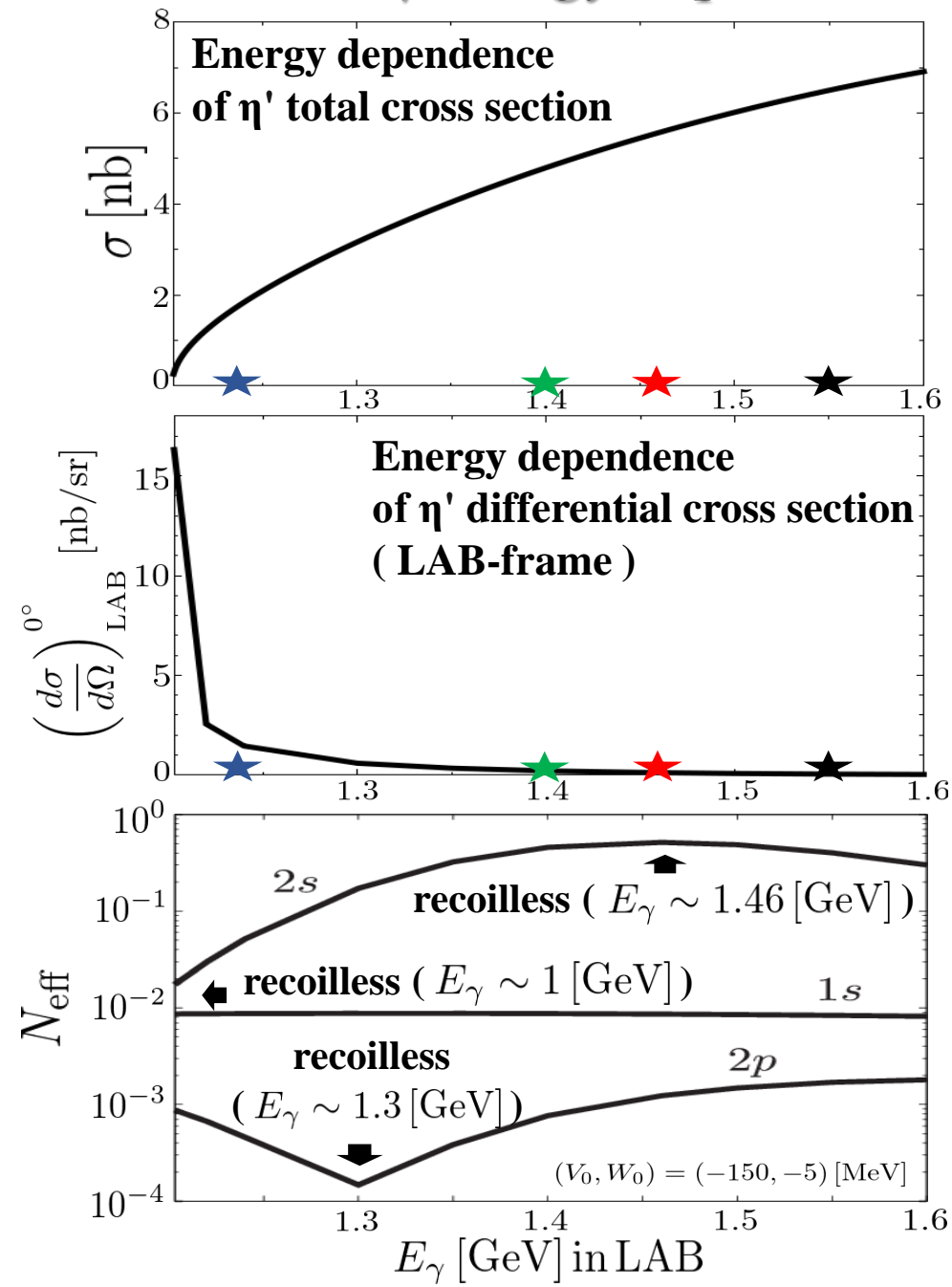


3. Result of η' mesic nucleus formation reaction

3-1. Incident γ energy and momentum transfer

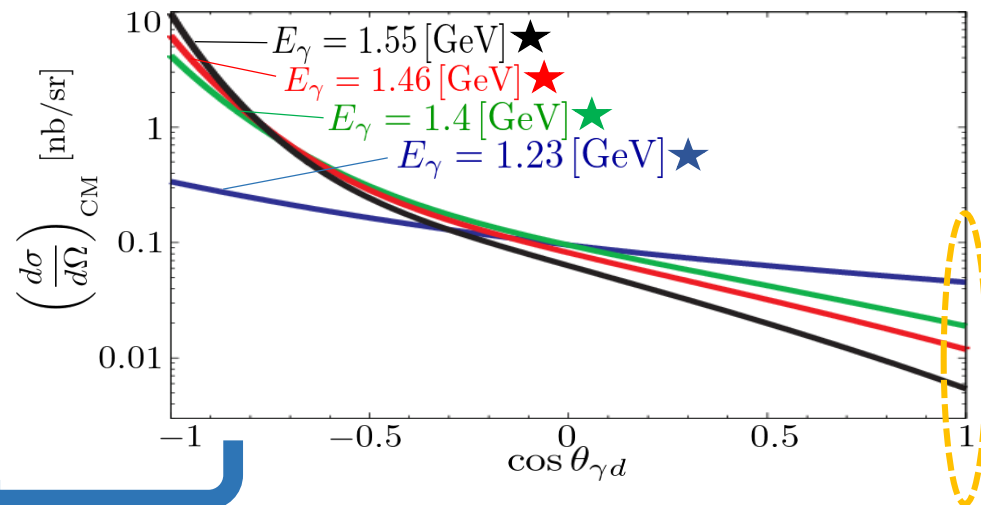


3-2. Incident γ energy dependence



Formation cross section

$$\frac{d^2\sigma}{dEd\Omega} = \left(\frac{d\sigma}{d\Omega} \right)^{\text{ele}} \sum_f \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} N_{\text{eff}}$$



$E_\gamma \rightarrow$ larger

✓ η' total cross section (σ) increases

✓ elementary cross section $\left(\frac{d\sigma}{d\Omega} \right)$

\rightarrow decreases at $\theta_{\gamma d} = 0^\circ$

✓ N_{eff} vary

according to the matching condition

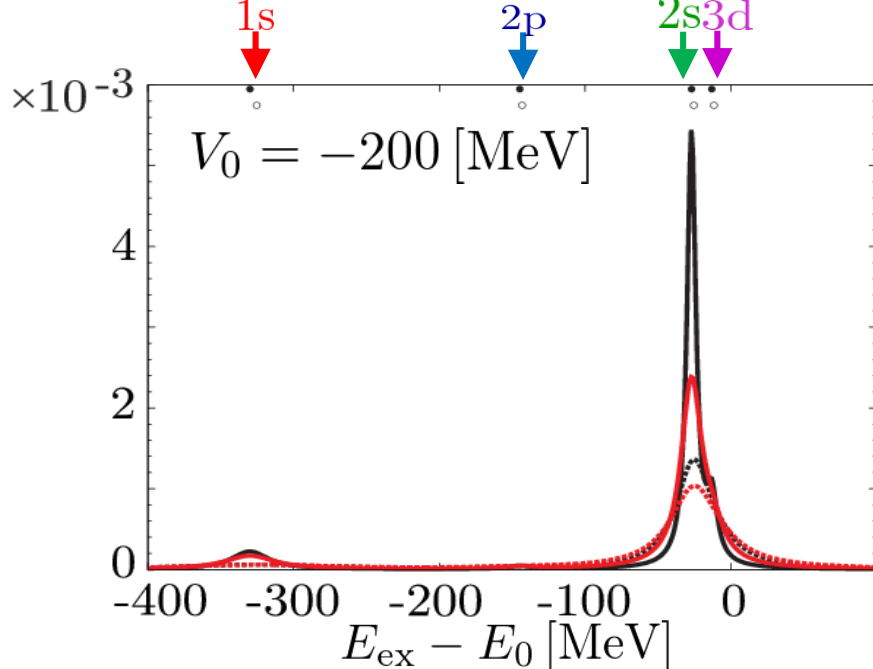
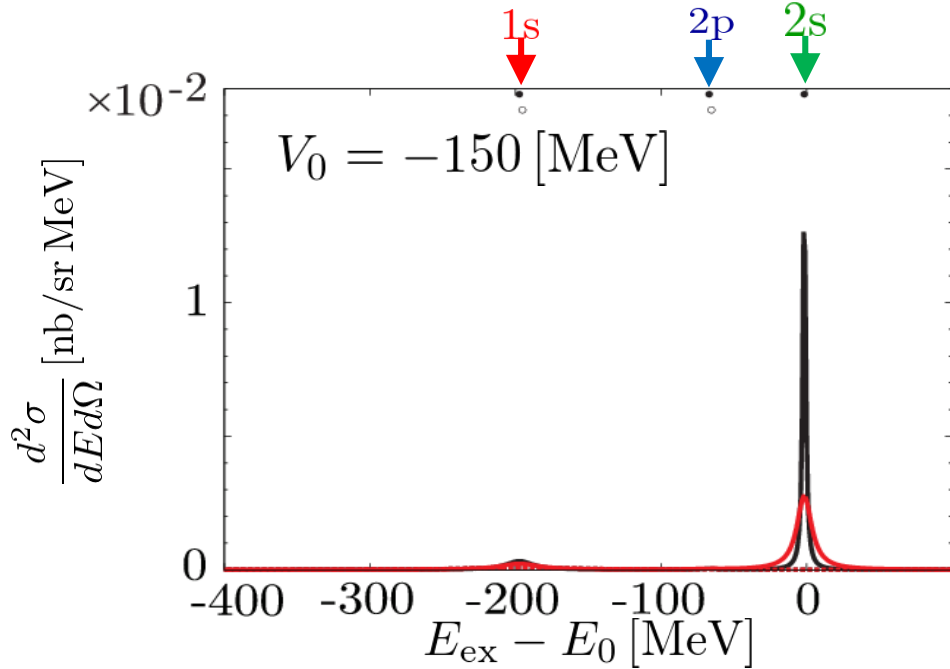
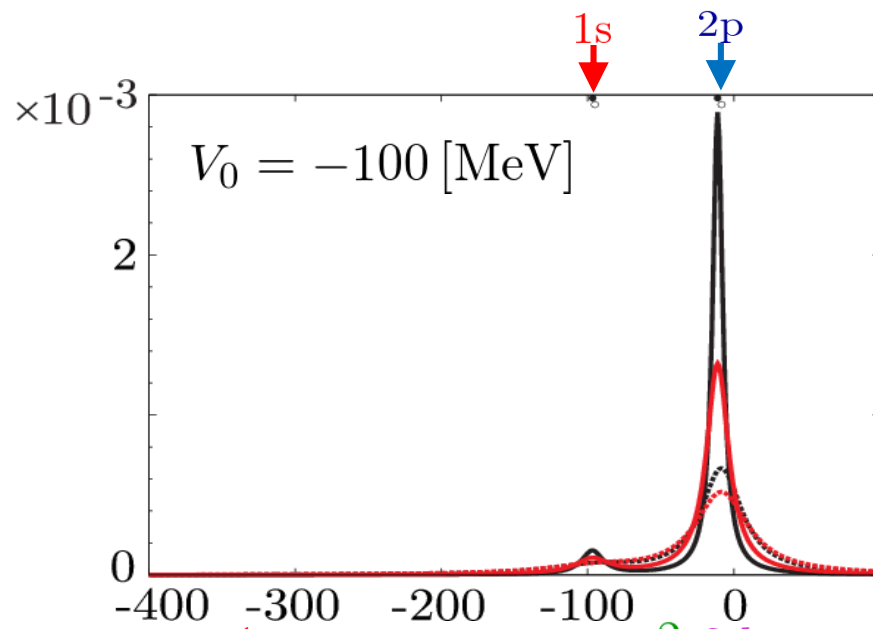
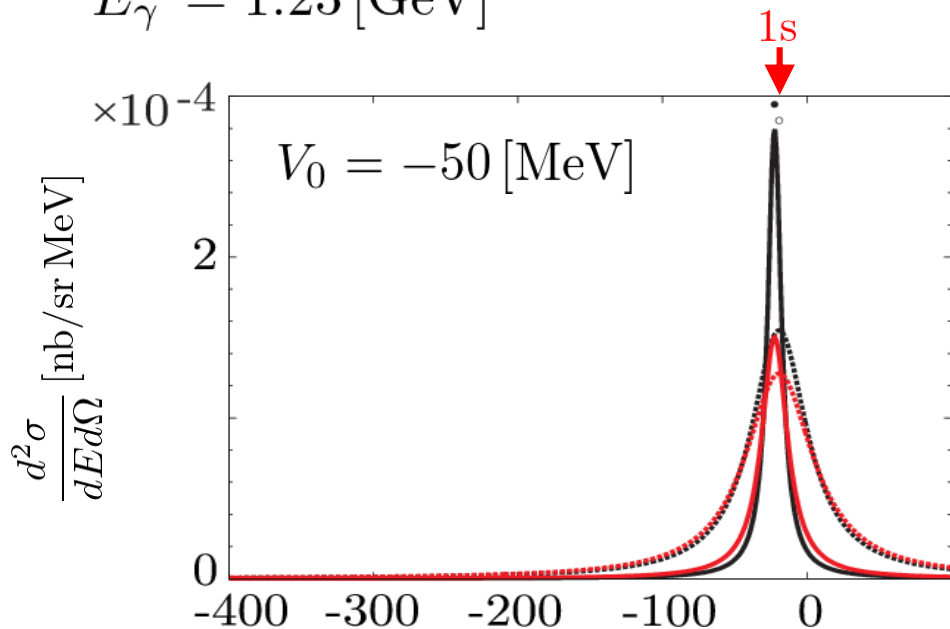
3-3. Formation cross section of η' bound state

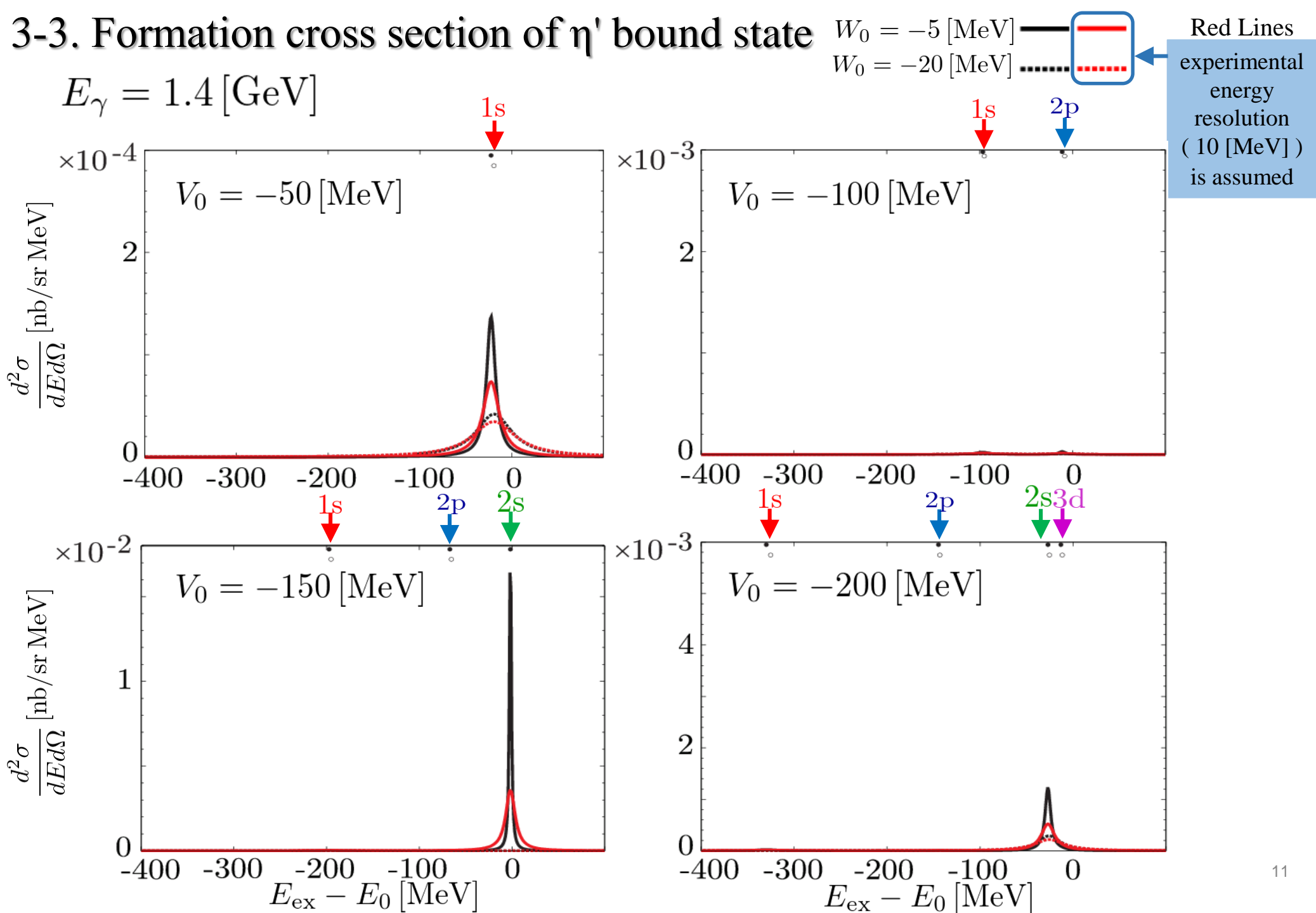
$$E_\gamma = 1.23 \text{ [GeV]}$$

$$W_0 = -5 \text{ [MeV]}$$

$$W_0 = -20 \text{ [MeV]}$$

Red Lines
experimental
energy
resolution
(10 [MeV])
is assumed





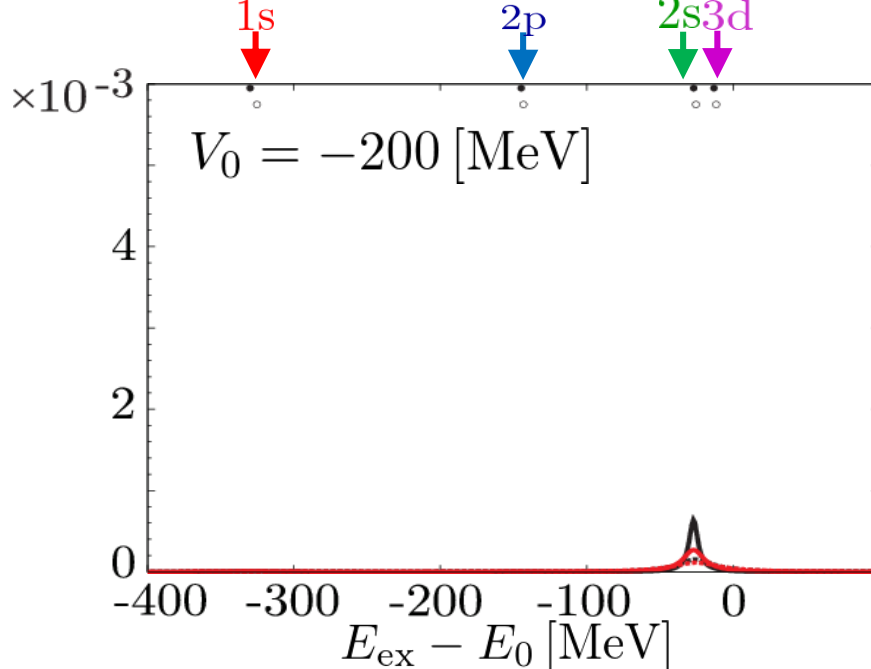
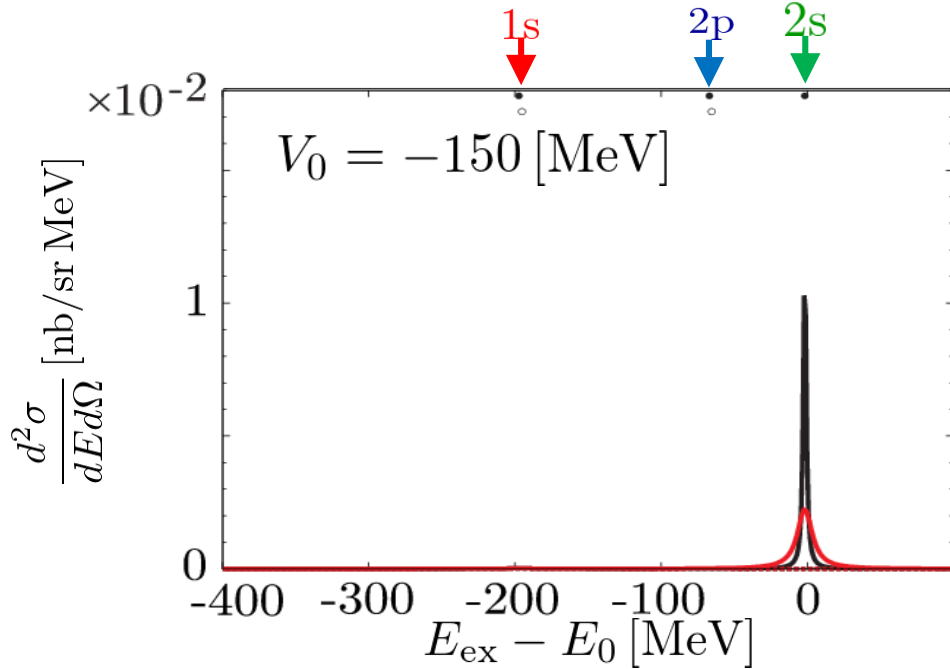
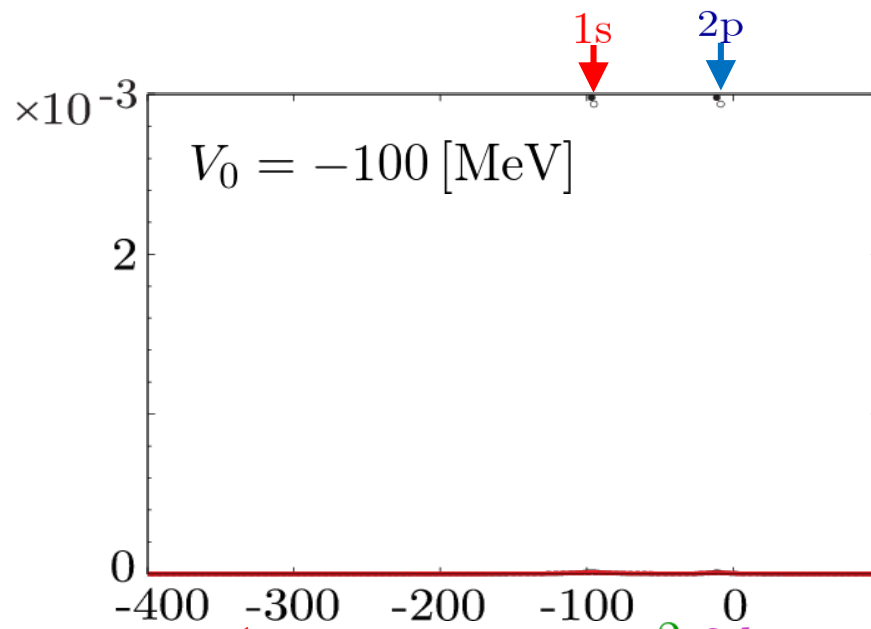
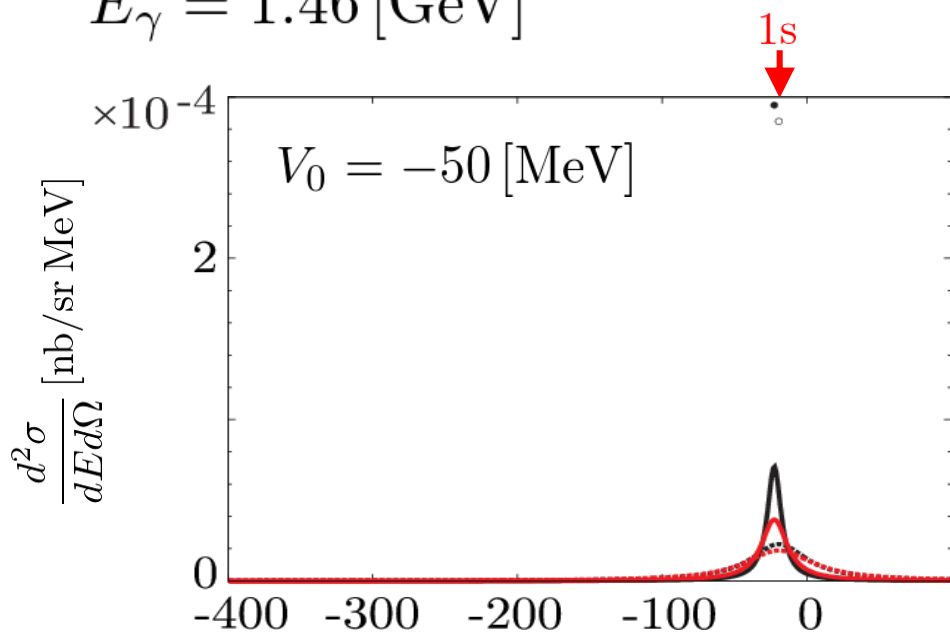
3-3. Formation cross section of η' bound state

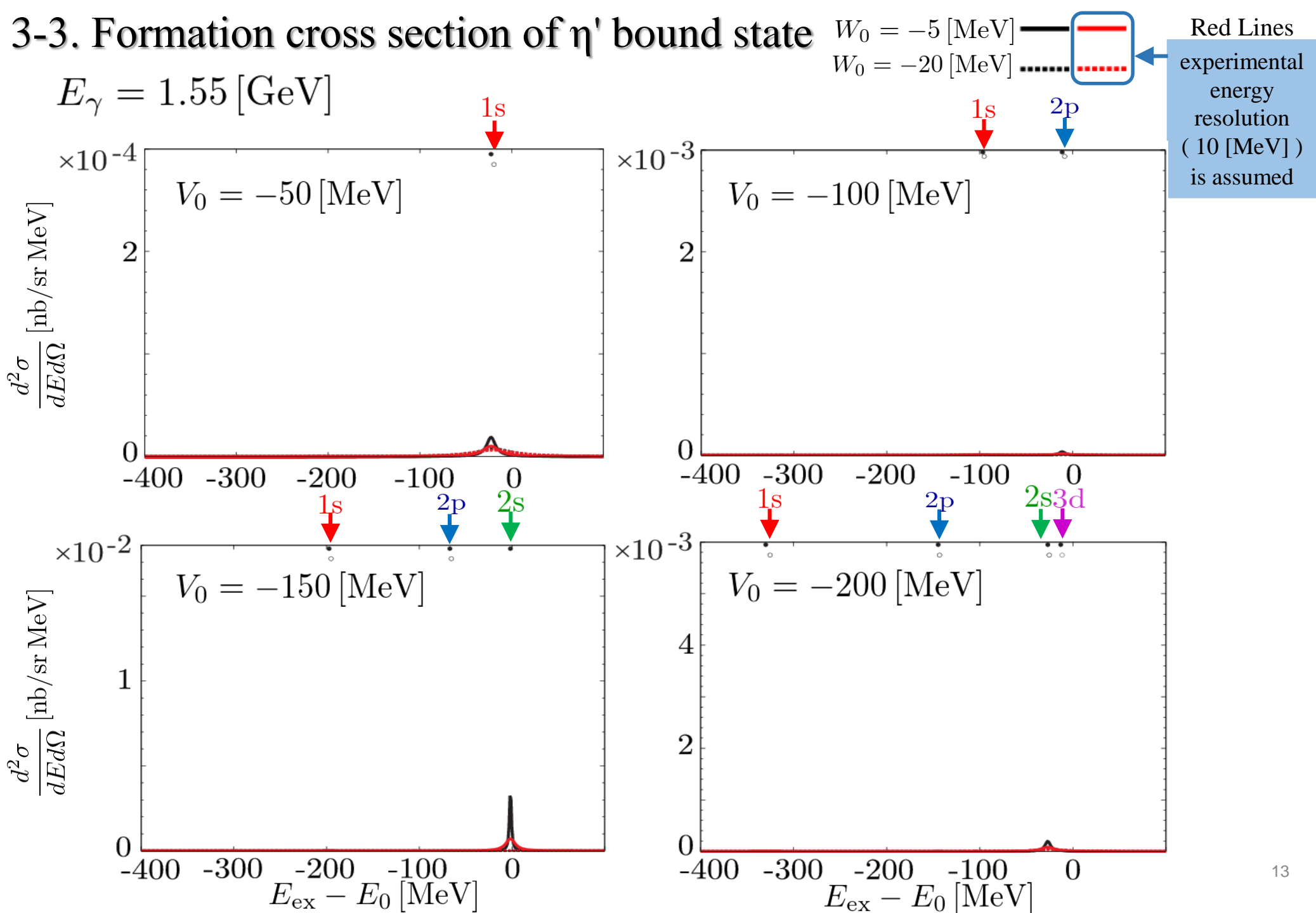
$$E_\gamma = 1.46 \text{ [GeV]}$$

$$W_0 = -5 \text{ [MeV]}$$

$$W_0 = -20 \text{ [MeV]}$$

Red Lines
experimental
energy
resolution
(10 [MeV])
is assumed





ここまでで分かったこと

- ✓ $E_\gamma \rightarrow$ 大 の時、 η' 中間子束縛状態の生成断面積 \rightarrow 小さくなる
 \rightarrow deuteronの形状因子が大きく影響している

より大きい生成断面積を得るために



${}^6\text{Li}$ 標的内での
deuteronのshrinkの効果



さらに低い
入射エネルギー E_γ での計算

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入射エネルギー E_γ での計算

3-4. Effects of shrinkage of quasi-deuteron in ${}^6\text{Li}$ target

Deuteron wave function

$$\psi_d(\mathbf{r}) = \underbrace{\psi_S(\mathbf{r})}_{\text{S-wave part}} + \underbrace{\psi_D(\mathbf{r})}_{\text{D-wave part}}$$

Radial wave function

$$R_S(r) = \frac{u(r)}{r}, \quad R_D(r) = \frac{w(r)}{r}$$

$$1 = \int_0^\infty dr (u^2(r) + w^2(r)) : \text{Normalization}$$

Parameterization

$$\begin{cases} u(r) = \sum_j C_j \exp(-m_j r) \\ w(r) = \sum_j D_j \exp(-m_j r) \left(1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2} \right) \end{cases}$$

(R. Machleidt et al., Phys. Rep. 149, No.1 (1987) 1-89)

Form factor

$$F_d(\mathbf{q}) = \int \psi_d^*(\mathbf{r}) e^{i\mathbf{q} \cdot \frac{\mathbf{r}}{2}} \psi_d(\mathbf{r}) d\mathbf{r}$$

Elementary cross section

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{CM}}^{\text{ele}} = \frac{1}{2} \frac{|c|^2}{4\pi} \frac{p_\gamma p_{d'}}{\pi} \frac{M_d^2}{\lambda^{\frac{1}{2}}(s, M_d^2, 0)} \frac{1}{p_\gamma} \frac{1}{E_{d'} + \omega_{\eta'}} |F_d(\mathbf{q})|^2$$

Scale (Shrink) factor α

$$0 < \alpha < 1$$

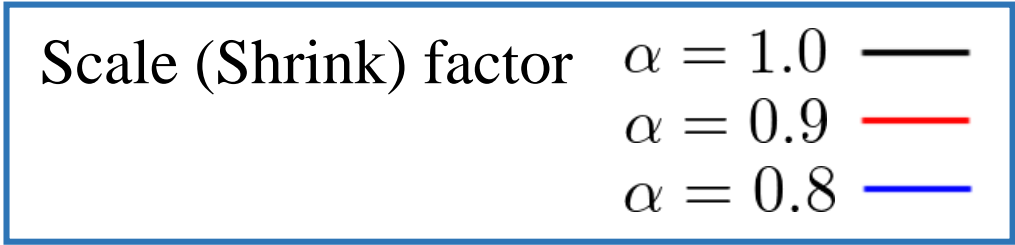
$$m_j \rightarrow \frac{1}{\alpha} m_j$$

$$(C_j, D_j) \rightarrow \frac{1}{\sqrt{\alpha}} (C_j, D_j)$$

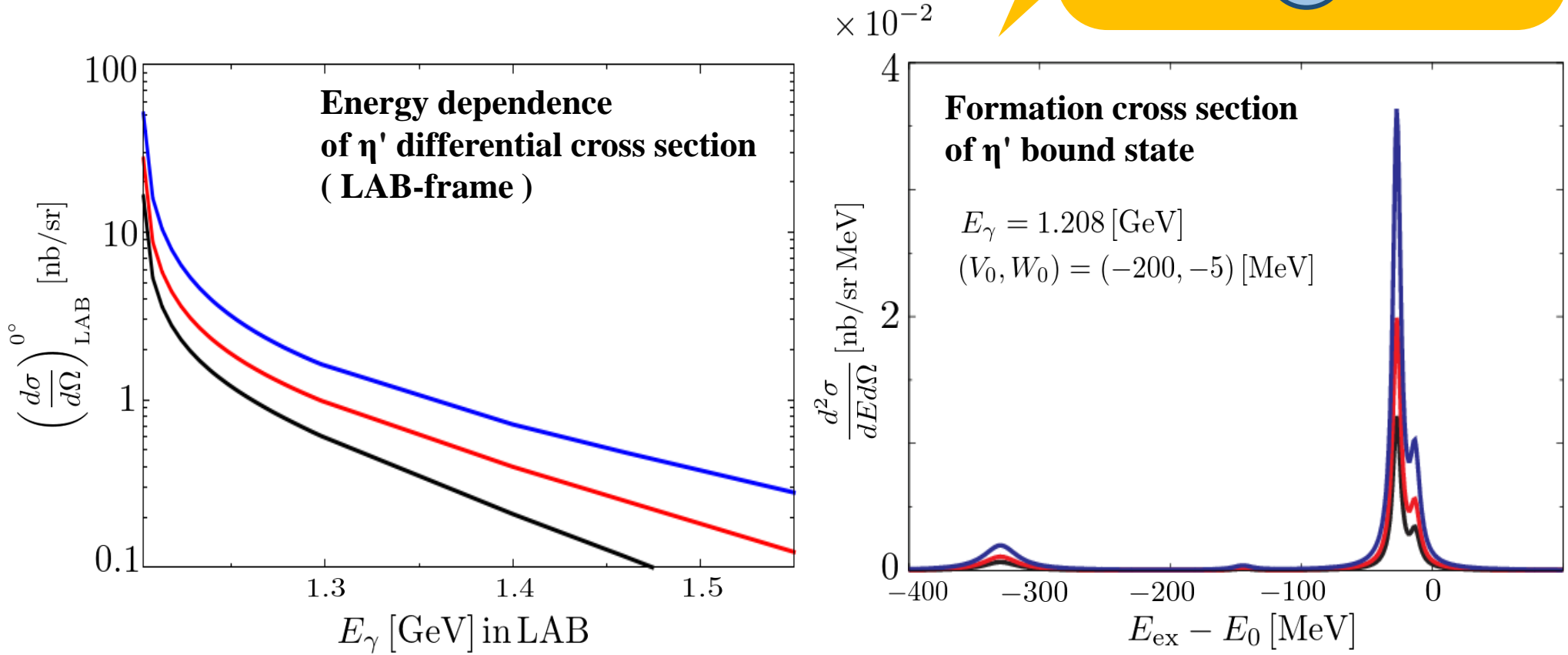
• Normalization: invariant

$$\bullet \sqrt{\langle r^2 \rangle} \rightarrow \alpha \sqrt{\langle r^2 \rangle}$$

3-4. Effects of shrinkage of quasi-deuteron



for $\alpha = 0.8$
 $\left(\sqrt{\langle r^2 \rangle} \rightarrow 0.8 \sqrt{\langle r^2 \rangle}\right)$
 $\sim \times \textcircled{3}$



3-5.

より大きい生成断面積を得るために



${}^6\text{Li}$ 標的内の
deuteronのshrinkの効果

さらに低い
入射エネルギー E_γ での計算

threshold
for nuclear target

threshold
for elementary process



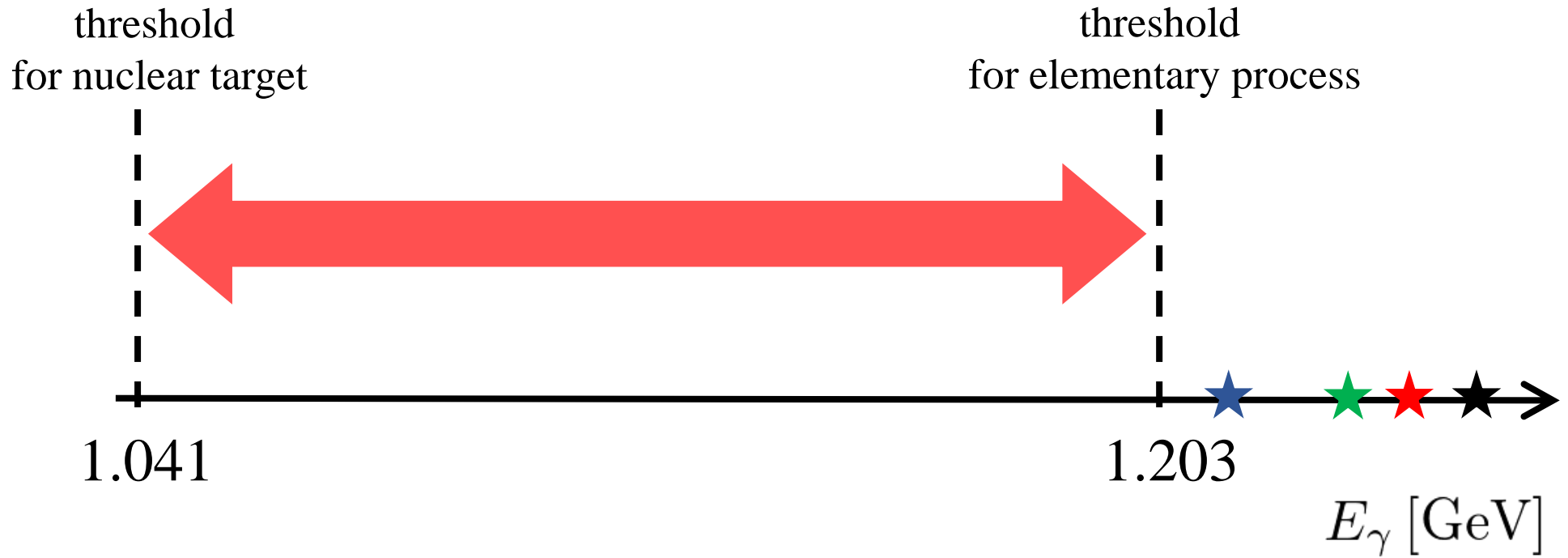
$$\frac{d^2\sigma}{dEd\Omega} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{ele}} \sum_f \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} N_{\text{eff}}$$

1.041

1.203



E_γ [GeV]

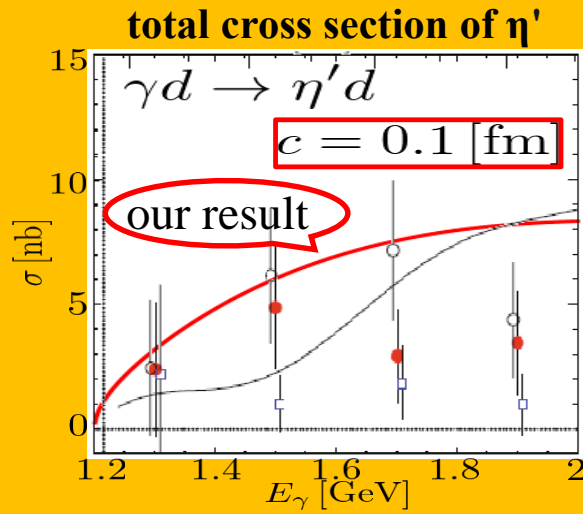


□ 生成断面積の新たな定式化

$$\frac{d^2\sigma}{dEd\Omega} = \sum_f \frac{|c|^2 M_d p_{d'}}{16\pi^2 m_{\eta'} p_\gamma} \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} N_{\text{eff}} |F_d(\mathbf{q})|^2$$

threshold
for nuclear target

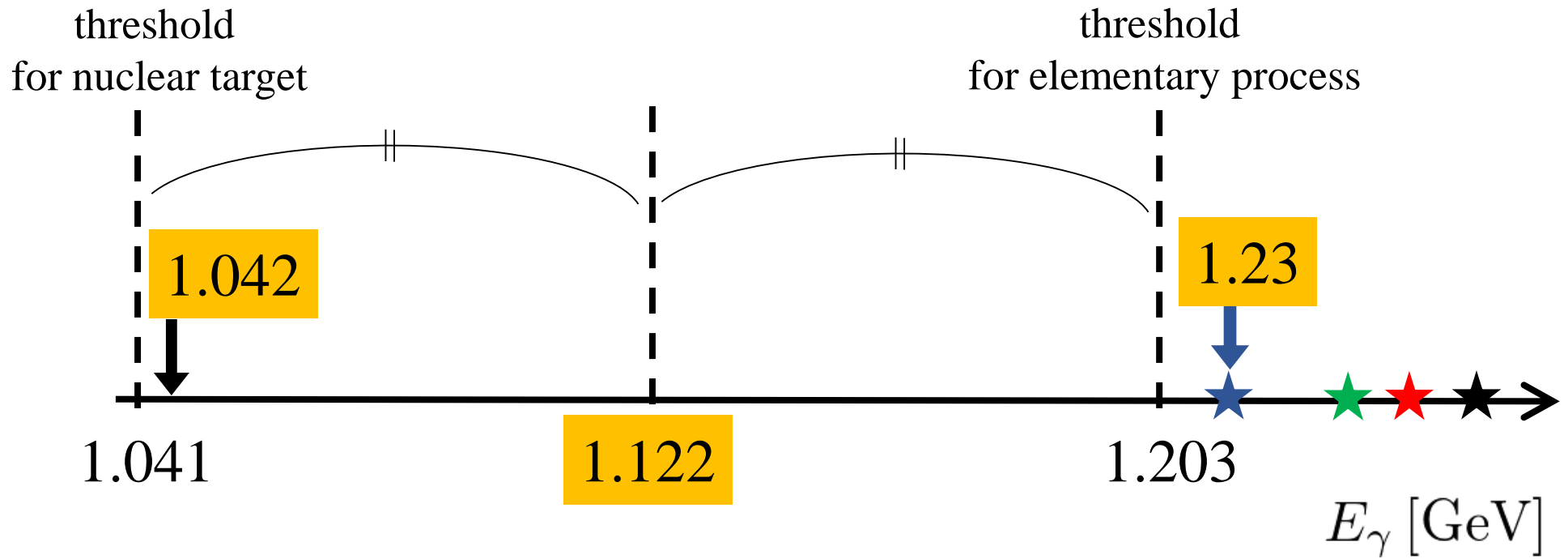
threshold
for elementary process



(I. Jaegle et al., Eur. Phys. J. A (2011) 47: 11)

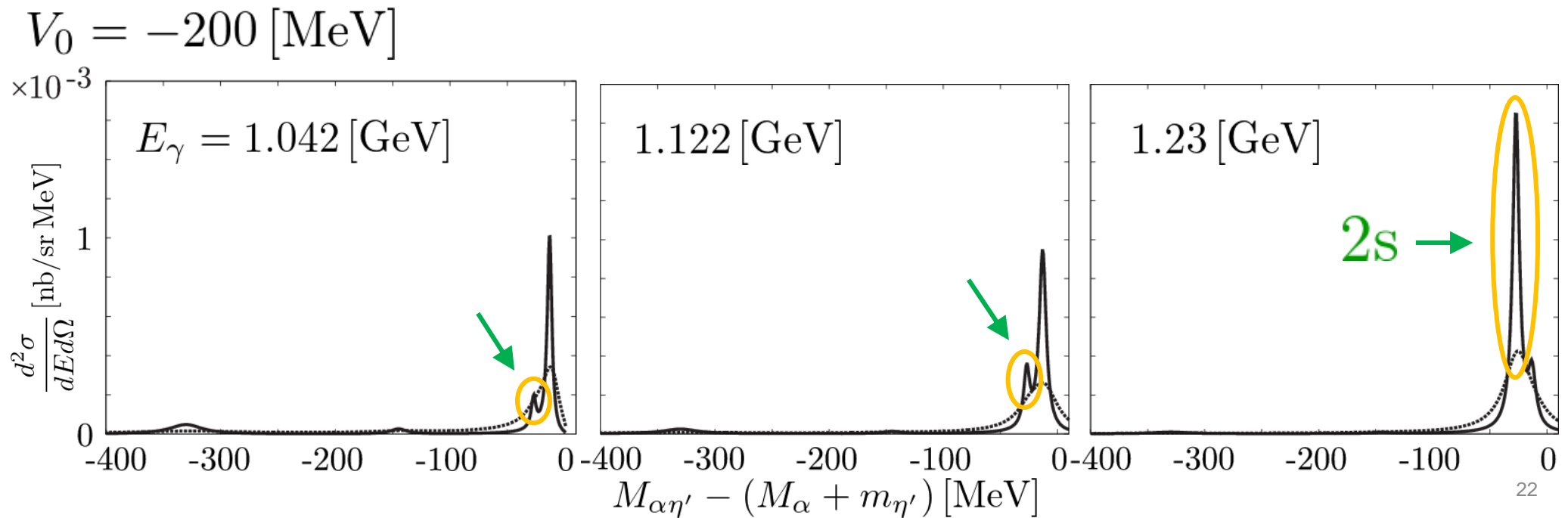
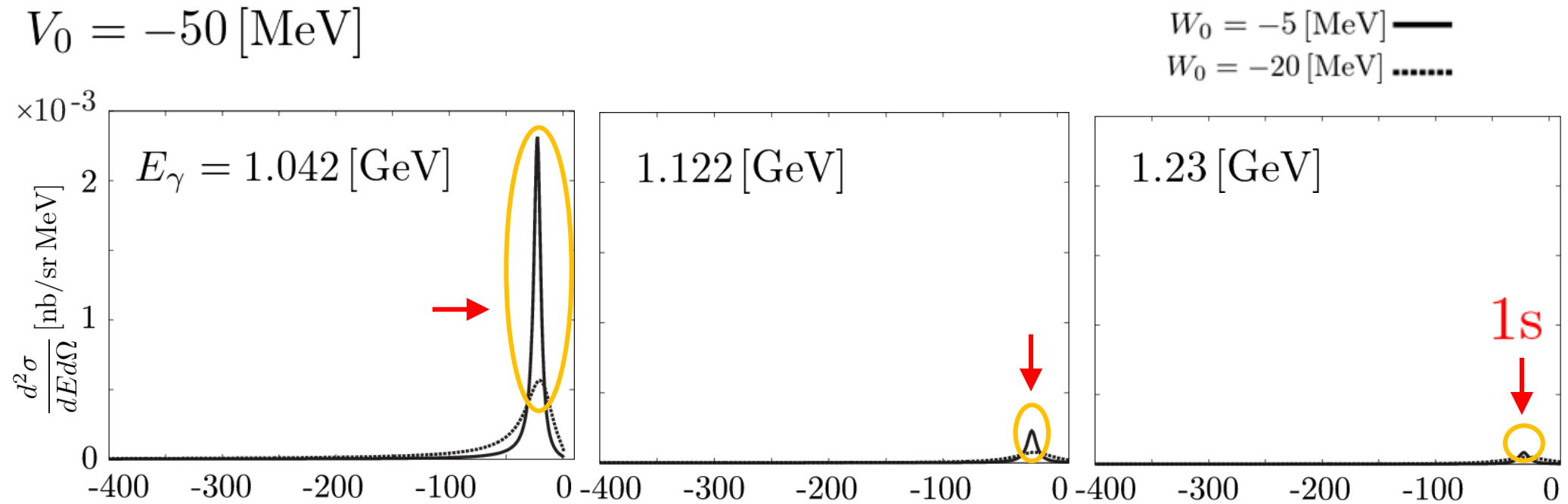
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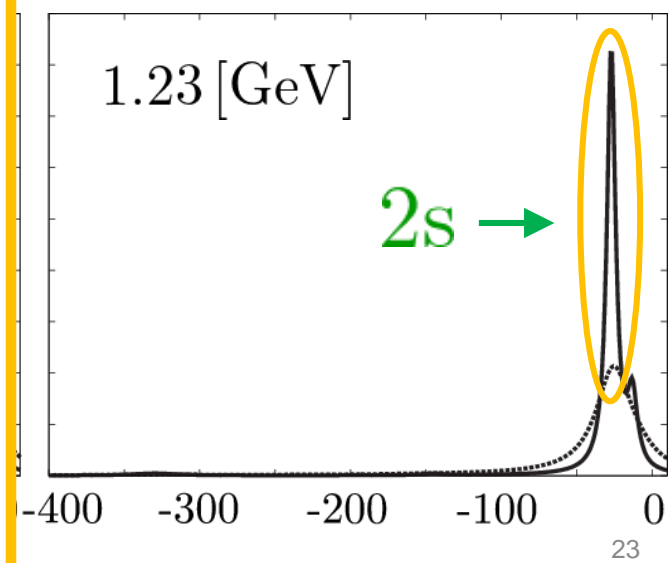
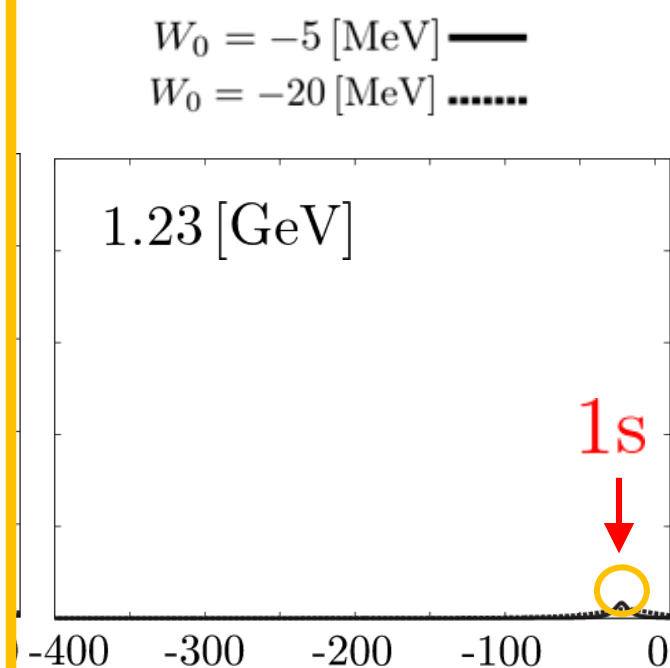
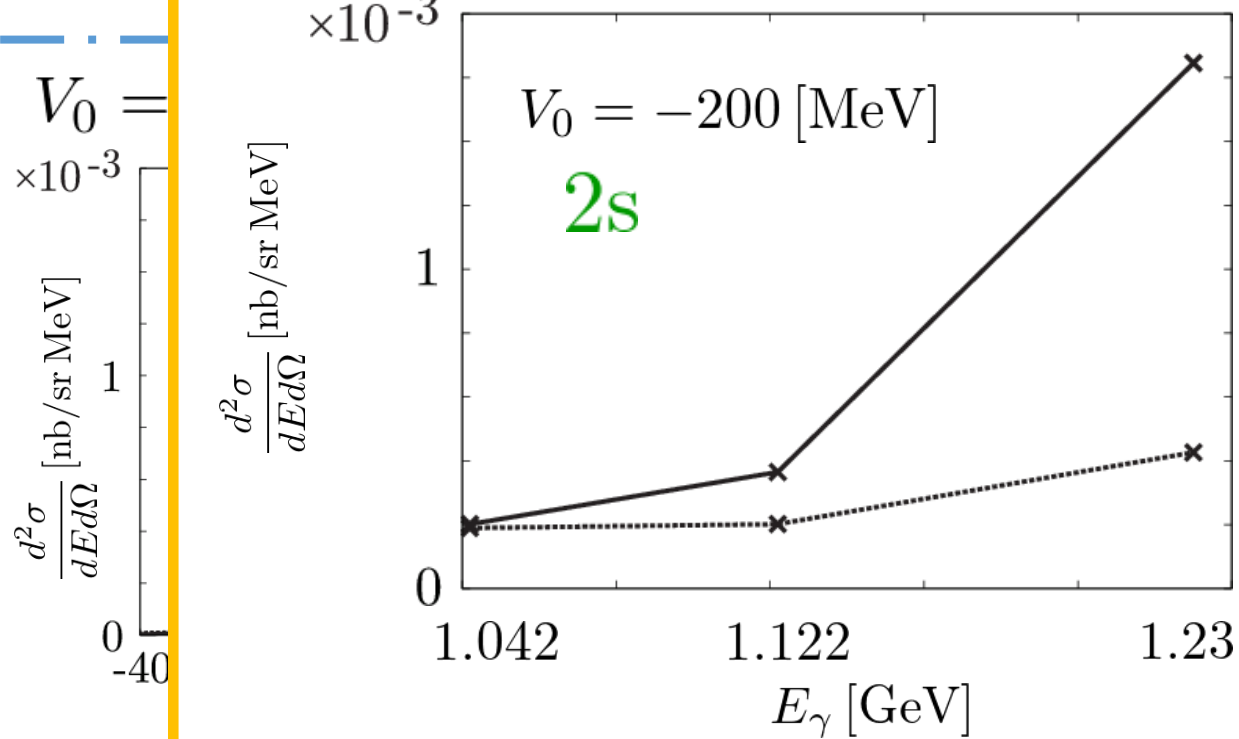
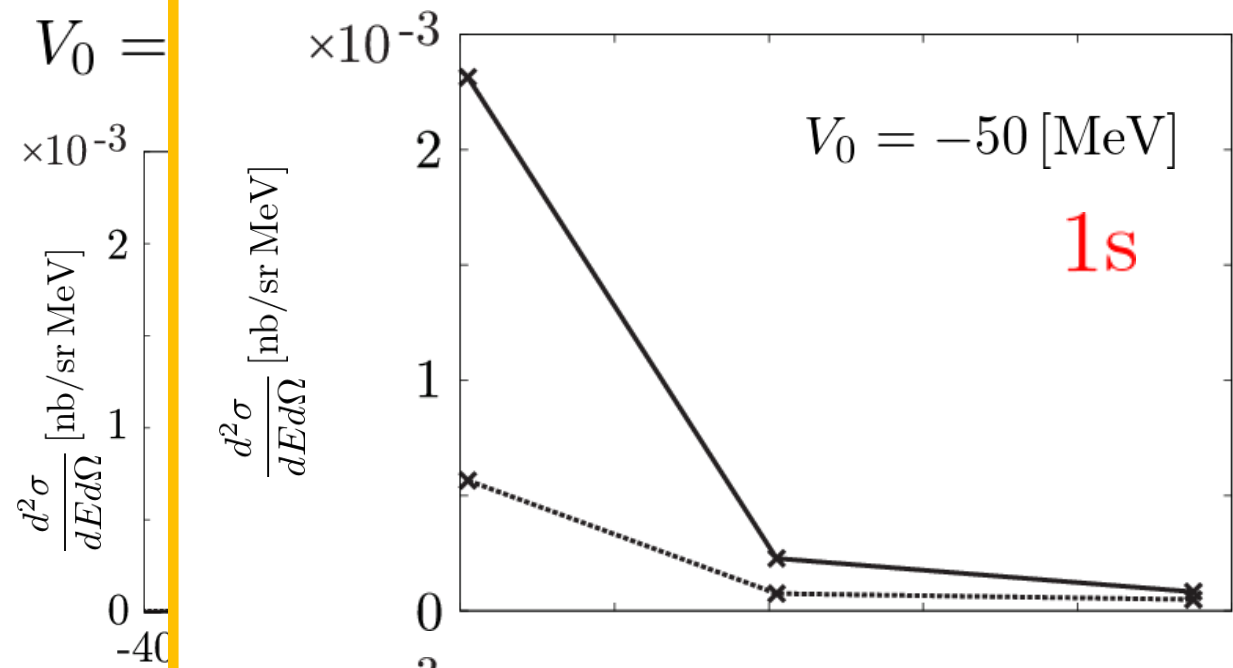
$$\frac{d^2\sigma}{dEd\Omega} = \sum_f \frac{|c|^2 M_d p_{d'}}{16\pi^2 m_{\eta'} p_\gamma} \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} N_{\text{eff}} |F_d(\mathbf{q})|^2$$

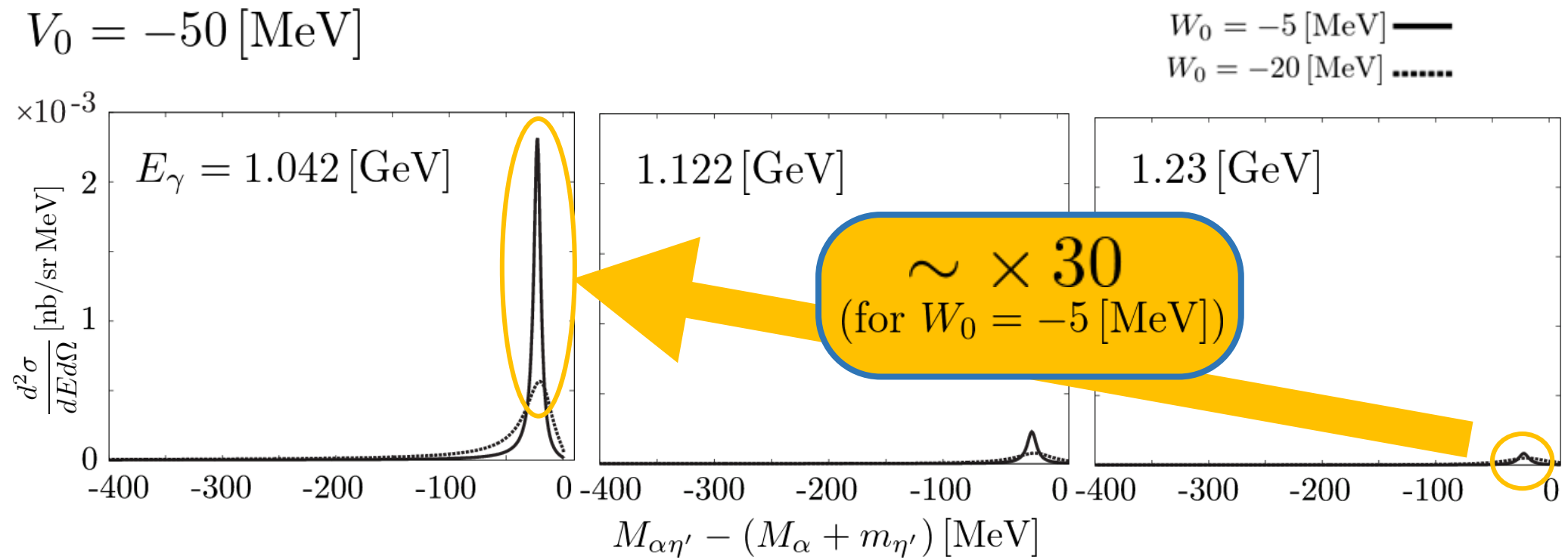


□ 生成断面積の新たな定式化

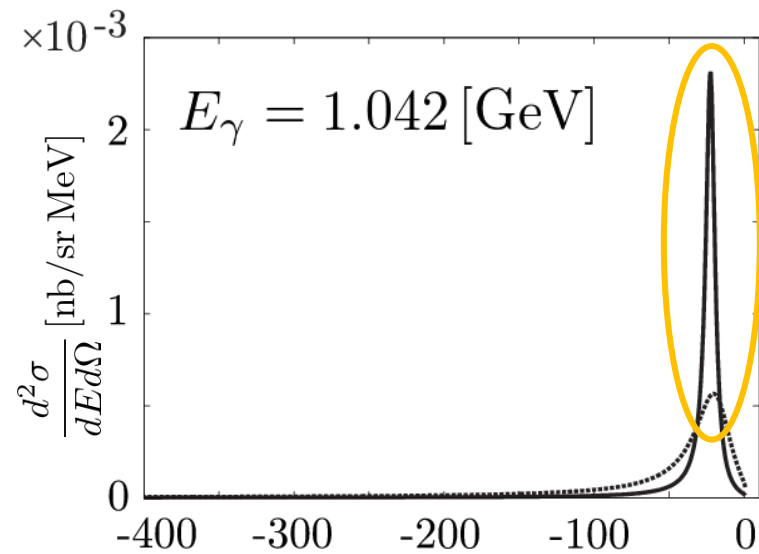
$$\frac{d^2\sigma}{dEd\Omega} = \sum_f \frac{|c|^2 M_d p_{d'}}{16\pi^2 m_{\eta'} p_\gamma} \frac{\Gamma}{2\pi} \frac{1}{\Delta E^2 + \Gamma^2/4} N_{\text{eff}} |F_d(\mathbf{q})|^2$$



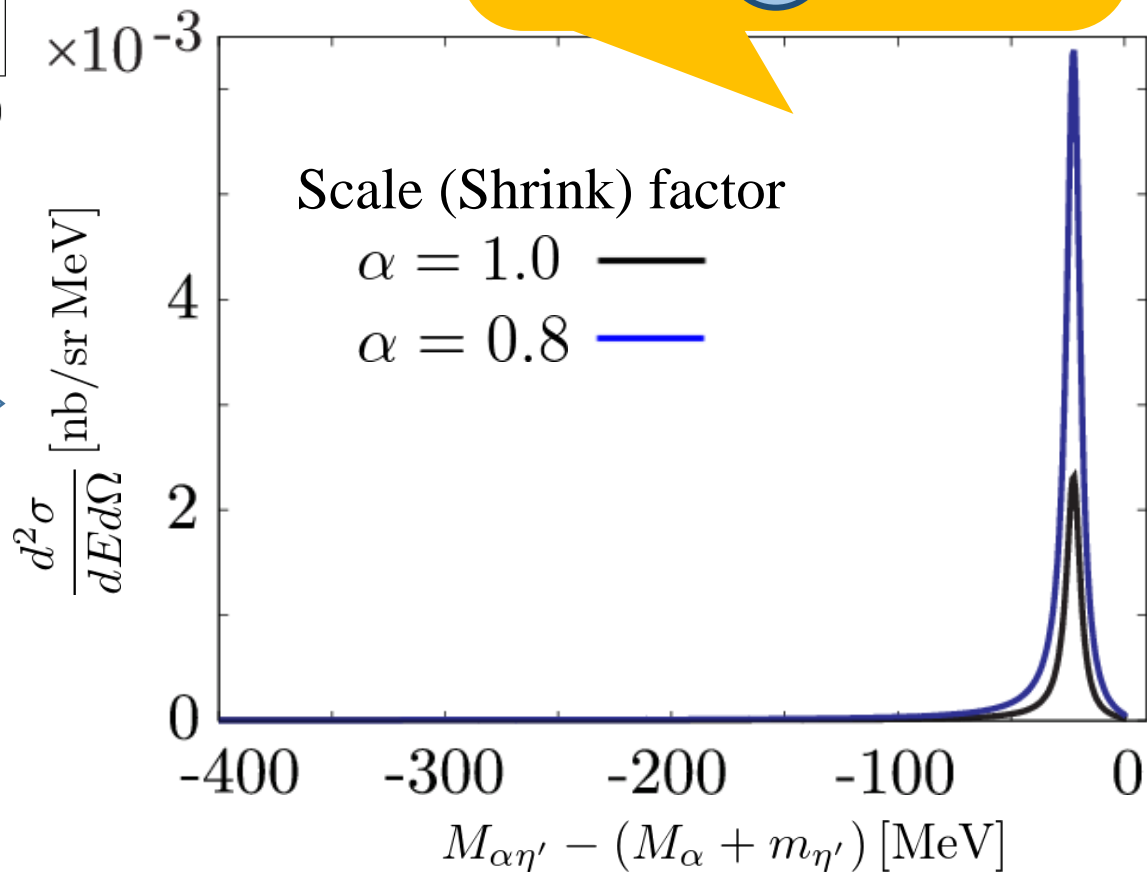




$$V_0 = -50 \text{ [MeV]}$$



Effects of
possible deuteron shrinkage



for $\alpha = 0.8$

$$\left(\sqrt{\langle r^2 \rangle} \rightarrow 0.8 \sqrt{\langle r^2 \rangle} \right)$$

$$\sim \times 3$$

4. Summary

Purpose of this work

To know the possibility of formation of $\eta'(958)$ mesic nucleus by (γ, d) reaction

Formalism

- Effective number approach
 - Improvements from N. Ikeno et al., Phys. Rev. C 84, 054609 (2011)
 - Distortion effect
 - Elementary cross section
 - Realistic α density distribution
 - Recoil effect
- ‘ Formation of ϕ mesic nucleus ’

Numerical results at $E_\gamma = 1.23 \sim 1.55$ [GeV] (above $\gamma + d \rightarrow \eta' + d$ threshold)

- Formation of the η' mesic nucleus in recoilless kinematics is possible
- Formation cross section
 - Peak height is smaller than 0.01 [nb/sr MeV] for almost all cases
 - Larger cross sections at $E_\gamma = 1.23$ [GeV] than other energies considered here
 - The bound η' states form well-separated peak structures in the spectra

Discussions

➤ Effects of the deuteron form factor

Large effects for the formation cross section of η' mesic nucleus

⇒ For larger E_γ , $\left(\frac{d\sigma}{d\Omega}\right)^{\text{ele}}$ decreases at $\theta_{\gamma d} = 0^\circ$


⇒ Experiments with photon energy
around η' production threshold ($E_\gamma \sim 1.2$ [GeV]) could be better



➤ To get larger cross section

- Possible deuteron shrinkage effects
⇒ For 0.8 times shorter $\sqrt{\langle r^2 \rangle}$, cross section becomes about 3 times larger
- Lower incident γ energy
(nuclear target threshold \Leftrightarrow elementary process threshold)

×100 Enhancement for 1s state for $(V_0, W_0) = (-50, -5)$ [MeV] case

8.3×10^{-5} [nb/sr MeV]		6.4×10^{-3} [nb/sr MeV]
$[E_\gamma = 1.23 \text{ [GeV]}]$	Lower incident γ energy Deuteron shrinkage effects	$[E_\gamma = 1.042 \text{ [GeV]}, \alpha = 0.8]$

Conclusion

${}^6\text{Li} (\gamma, d)$ reaction

pick-up quasi-deuteron in target

$$\blacktriangleright \frac{d^2\sigma}{dEd\Omega} \leq 0.01 \text{ [nb/sr MeV]}$$

for all cases considered here

including possible deuteron shrinkage in ${}^6\text{Li}$
and optimizing the incident γ energy

