Modification of vector mesons in the nuclear medium

Tokyo Institute of Technology　　Keisuke Ohtani

原子核媒質中のハドロン研究 III, 10/20/2015

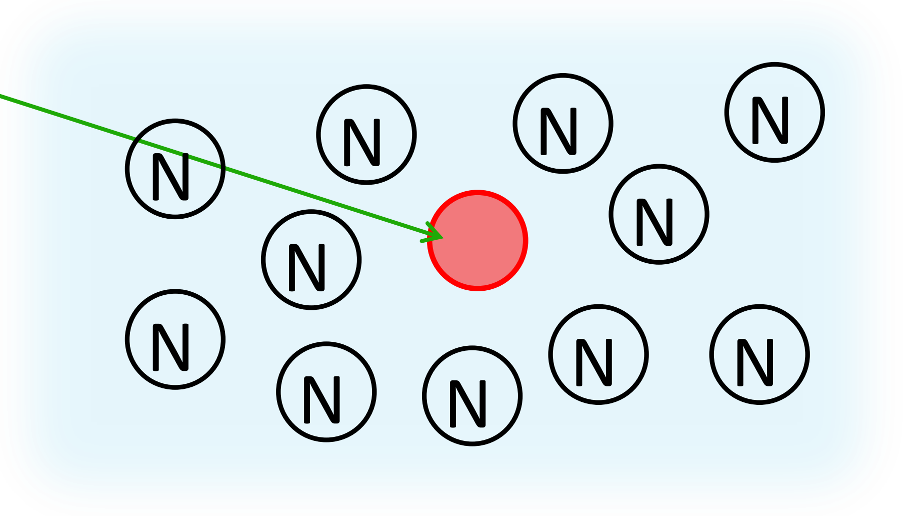
Outline

- Introduction
- ρ meson in the nuclear medium from QCD sum rules
- ϕ meson in the nuclear medium from QCD sum rules
- Summary

Introduction

▪Hadron properties in the nuclear medium

Probe hadron
(ρ, ω, ϕ, ･･)



Nuclear matter

The modification of hadron properties

- Mass shift
- Width broadening

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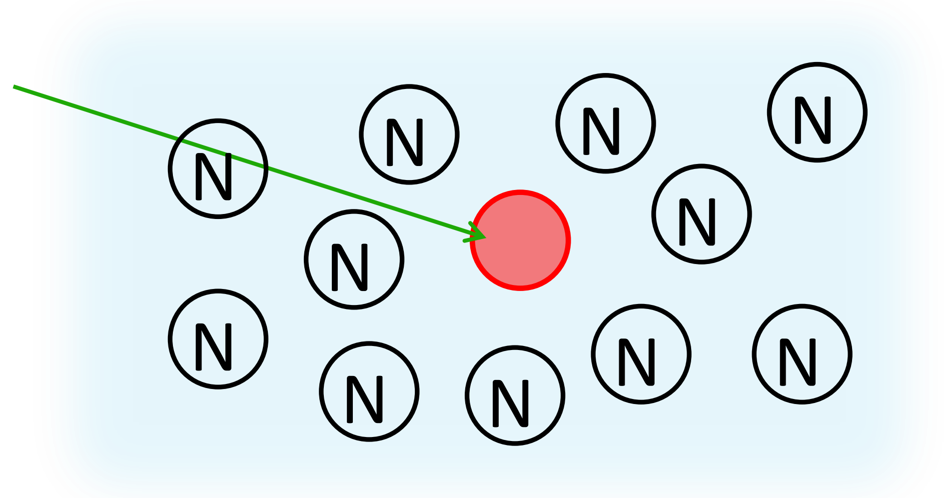
Partial restoration of the chiral symmetry

Interaction with the nucleons in the nuclear matter

Introduction

▪ Hadron properties in the nuclear medium

Probe hadron
(ρ, ω, ϕ, ・・)



Nuclear matter

The modification of hadron properties

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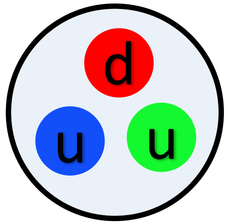
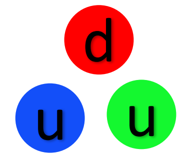
Partial restoration of the chiral symmetry

Interaction with the nucleons in the nuclear matter

Introduction

The relation between hadron mass and chiral symmetry



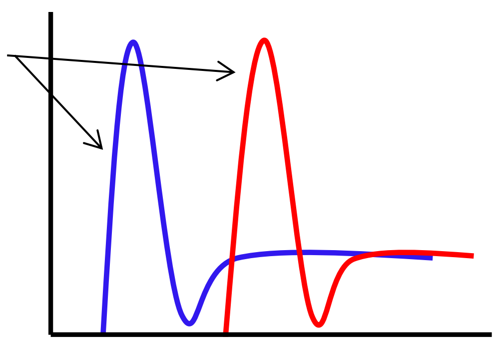


Nucleon mass $\approx$ 940MeV $>>$ three quark masses $\approx$ 20MeV

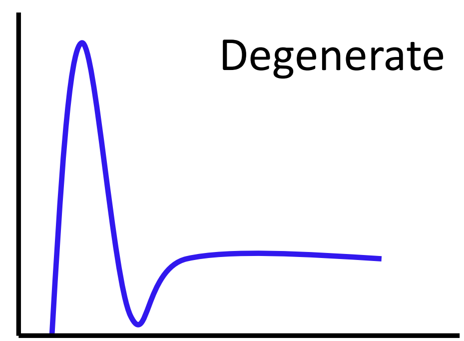
Spontaneous breaking of chiral symmetry ⇒ Origin of hadron mass

Chiral symmetry is broken

Spectral functions of the chiral partners

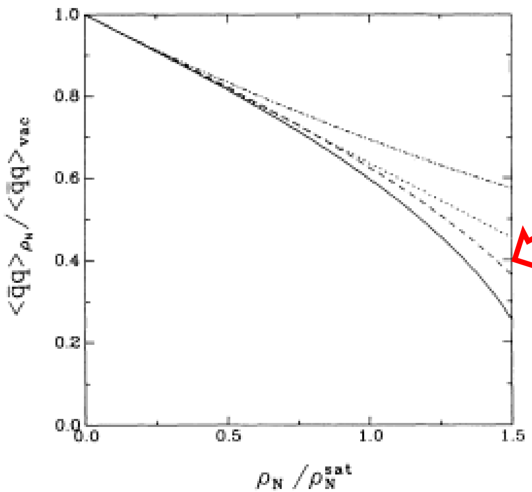


Chiral symmetry is restored



Introduction

An order parameter of chiral symmetry: Chiral condensate $\langle \bar{q}q \rangle$



Thomas D. Cohen, R.J. Furnstahl, and David K. Griegel.
Phys. Rev. C45 1881 (1992)

Linear density approximation

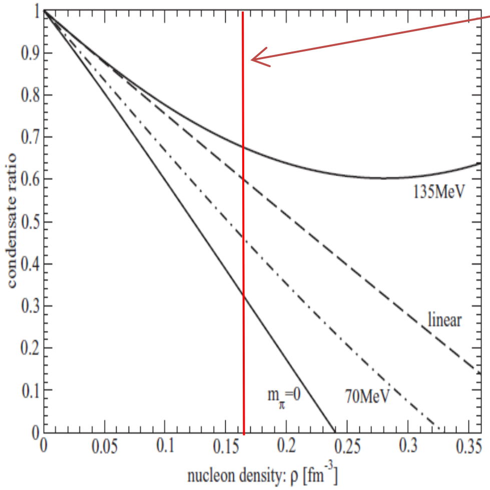
$$\frac{\langle \bar{q}q \rangle_{\rho N}}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{m_\pi^2 f_\pi^2}(m_q \frac{d\epsilon}{dm_q})$$
$$= 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho_N + \mathcal{O}(\rho_N^2)$$

Linear density approximation
(Free gas approximation)

The linear density approximation is valid up to the normal nuclear matter density.

Introduction

An order parameter of chiral symmetry: Chiral condensate $\langle \bar{q}q \rangle$



Normal nuclear matter density

$$\frac{\langle \bar{q}q \rangle_{\rho N}}{\langle \bar{q}q \rangle_0} = 1 - \frac{1}{m_\pi^2 f_\pi^2}\left(m_q \frac{d\epsilon}{dm_q}\right)$$
$$= 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho_N + \mathcal{O}(\rho_N^2)$$

Linear density approximation
(Free gas approximation)

N. Kaiser, P. de Homont, and W. Weise. Phys. Rev. C**77** 025204 (2008).

The linear density approximation is valid up to the normal nuclear matter density.

In-medium chiral condensate is about 35% smaller than the vacuum value.

⇨ Partial restoration of the chiral symmetry.

Introduction

QCD sum rules:

$$\Pi(q^2) = i\int e^{iqx}\langle 0|T[J(x)J(0)]|0\rangle d^4x$$
$$= \frac{1}{\pi}\int_0^\infty \frac{\Pi(s)}{s-q^2}ds = \int_0^\infty \frac{\rho(s)}{s-q^2}ds$$

Calculated by Operator product expansion (OPE):

$$\Pi(q^2) = C_0(q^2) + \sum_i C_i(q^2)\langle \mathcal{O}_i\rangle_0 \qquad \langle \mathcal{O}_i\rangle_0 = \langle \bar{q}q\rangle_0,\ \langle \frac{\alpha_s}{\pi}G^2\rangle_0,\ \langle \bar{q}q\bar{q}q\rangle_0 \cdots$$

Non perturbative contributions are expressed by some condensates.

Medium modification can be characterized by condensates

$$\Rightarrow \langle \mathcal{O}_i\rangle_{\rho_N} = \langle \bar{q}q\rangle_{\rho_N},\ \langle \frac{\alpha_s}{\pi}G^2\rangle_{\rho_N},\ \langle \bar{q}q\bar{q}q\rangle_{\rho_N},\ \langle \bar{q}\gamma_\mu D_\nu q\rangle_{\rho_N}, \cdots$$

A_2 (for $\langle \bar{q}\gamma_\mu D_\nu q\rangle_{\rho_N}$)

After the Borel transformation: $$G_{OPE}(M^2) = \int_0^\infty \exp\left(-\frac{s}{M^2}\right)\rho(s)ds$$

ρ meson in the nuclear medium

QCD sum rule for ρ meson channel

$$
\begin{aligned}
G_{OPE}(M^2) =& \frac{1}{4\pi^2}(1+\frac{\alpha_s}{\pi}) - \frac{1}{M^2}\frac{6m_q^2}{4\pi^2} + \frac{1}{M^4}(2m_q\langle\overline{q}q\rangle_{\rho_N} \\
&+ \frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle_{\rho_N}) - \frac{1}{M^6}\frac{112\pi}{81}\alpha_s\langle\overline{q}q\overline{q}q\rangle_{\rho_N} \\
&+ \frac{1}{M^4}A_2 M_N \rho - \frac{1}{M^6}\frac{5}{3}A_4 M_N^3 \rho + \cdots
\end{aligned}
$$

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$$

The medium effect strongly related to the four quark condensate $\langle\overline{q}q\overline{q}q\rangle_{\rho_N}$

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The medium effect strongly related to the four quark condensate $\langle\overline{q}q\overline{q}q\rangle_{\rho_N}$

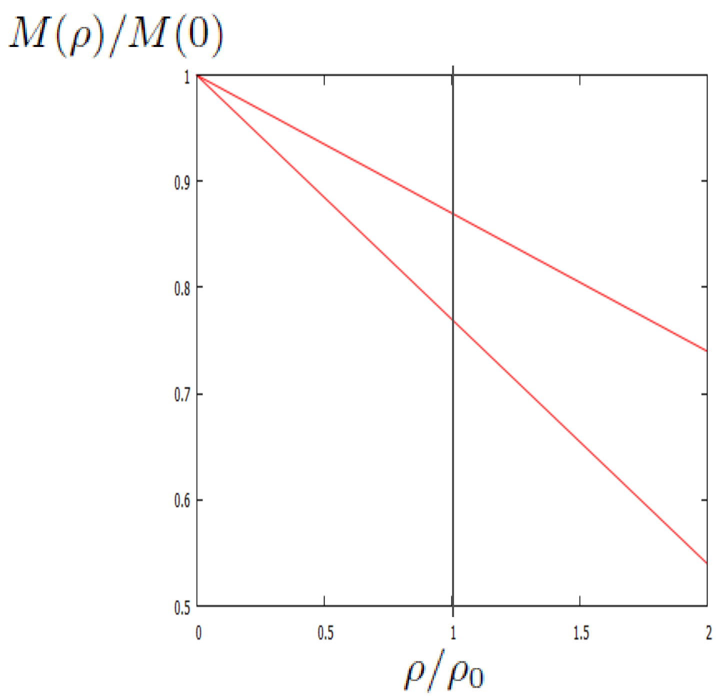
Four quark condensate: $\langle\overline{q}q\overline{q}q\rangle_{\rho_N} = \langle\overline{q}q\rangle_{\rho_N}^2$

ρ meson peak is represented by the delta function in the analysis.

$$M(\rho)/M(0) \simeq 1 - C(\rho/\rho_0)$$

$$C = 0.18 \pm 0.054$$

Mass modification



T. Hatsuda and S.H. Lee, Phys. Rev. C 46, R34 (1992).

ρ meson in the nuclear medium

QCD sum rule for ρ meson channel

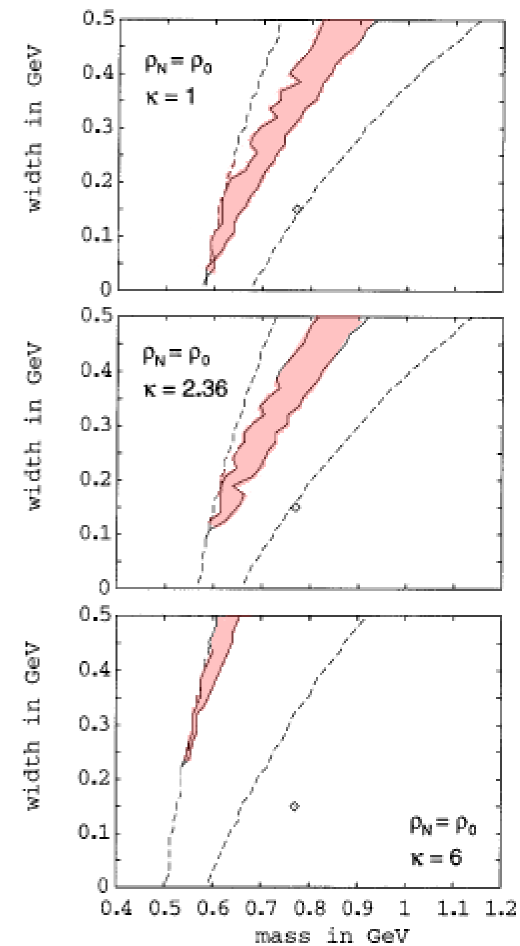
$$G_{OPE}(M^2) = \frac{1}{4\pi^2}(1+\frac{\alpha_s}{\pi}) - \frac{1}{M^2}\frac{6m_q^2}{4\pi^2} + \frac{1}{M^4}(2m_q\langle\overline{q}q\rangle_{\rho_N} + \frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle_{\rho_N}) - \frac{1}{M^6}\frac{112\pi}{81}\alpha_s\langle\overline{q}q\overline{q}q\rangle_{\rho_N} + \frac{1}{M^4}A_2 M_N\rho - \frac{1}{M^6}\frac{5}{3}A_4 M_N^3\rho + \cdots$$

The medium effect strongly related to the four quark condensate

Four quark condensate: $\langle\overline{q}q\overline{q}q\rangle_{\rho_N} = \kappa\langle\overline{q}q\rangle_{\rho_N}^2$

ρ meson peak is represented by the Breit-Wigner type function.

QCD sum rules provide weak constraint on the mass and the width



S. Leupold, W. Peters, and U. Mosel, Nucl. Phys. A628, 311 (1998).

ρ meson in the nuclear medium

QCD sum rule for ρ meson channel

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The medium effect strongly related to the four quark condensate $\langle\overline{q}q\overline{q}q\rangle_{\rho_N}$

Four quark condensate:

Factorization hypothesis in vacuum:

$$\langle\overline{q}q\overline{q}q\rangle_0 = \kappa\langle\overline{q}q\rangle_0^2$$

Large uncertainty $1 \leq \kappa \leq 10$

Factorization hypothesis in nuclear medium:

$$\langle\overline{q}q\overline{q}q\rangle_{\rho_N} = \langle\overline{q}q\rangle_{\rho_N}^2 = (\langle\overline{q}q\rangle_0 + \frac{\sigma_{\pi N}}{2m_q}\rho_N)^2$$

Is it valid?

Four quark condensate may also be related to the chiral symmetry.

ϕ meson in the nuclear medium

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$$
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The medium effect mainly come from the $m_s\langle\bar{s}s\rangle_{\rho_N}$

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The medium effect mainly come from the $m_s\langle\bar{s}s\rangle_{\rho_N}$

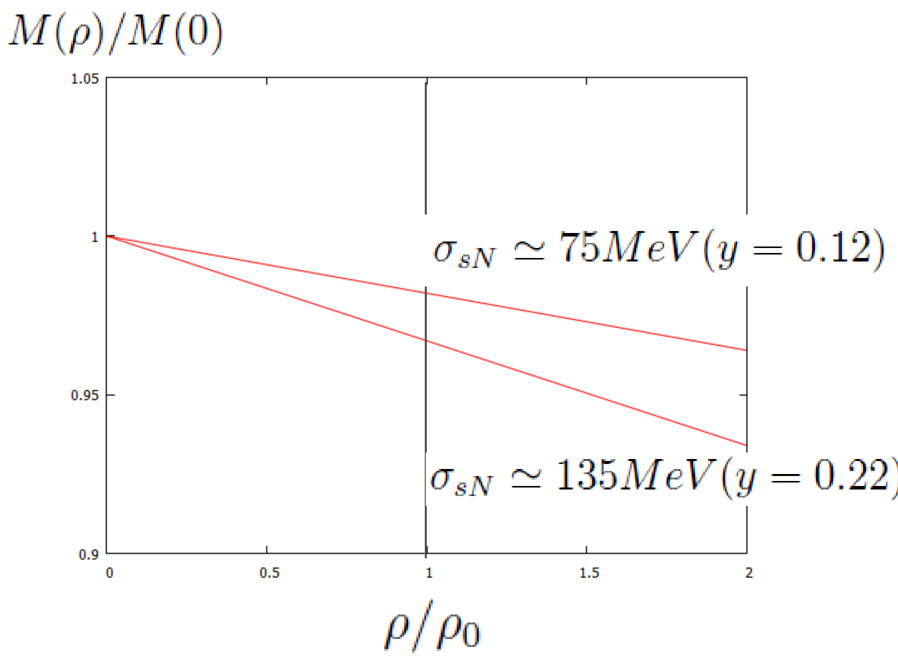
$$\langle\bar{s}s\rangle_{\rho_N} = \langle\bar{s}s\rangle_0 + \frac{\sigma_{sN}}{m_s}\rho_N$$

(In the linear density approximation)

$$\sigma_{sN} = m_s\langle N|\bar{s}s|N\rangle$$

$$M(\rho)/M(0) \simeq 1 - C(\rho/\rho_0)$$

$$C = (0.15 \pm 0.045)y$$



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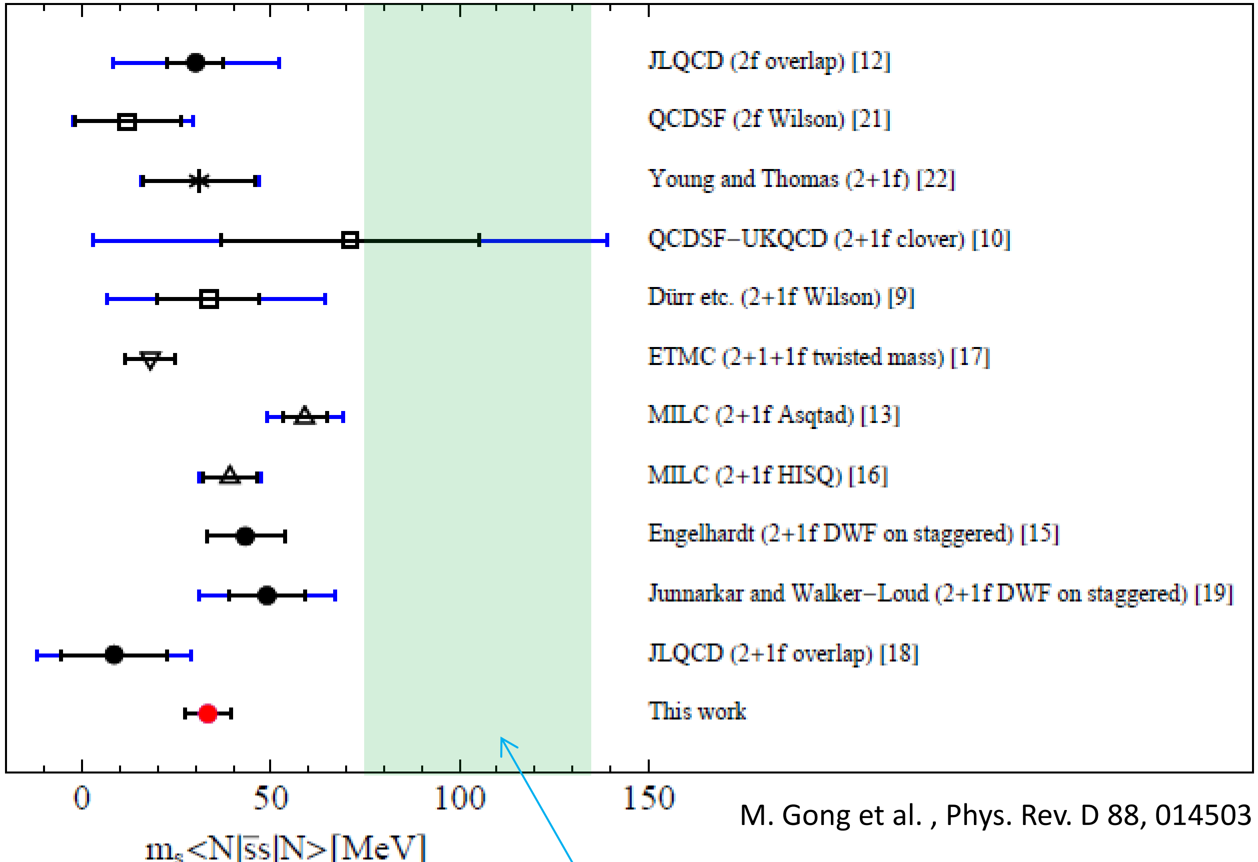
ϕ meson in the nuclear medium



M. Gong et al. , Phys. Rev. D 88, 014503 (2013)

$$\sigma_{sN} = m_s \langle N|\bar{s}s|N\rangle$$

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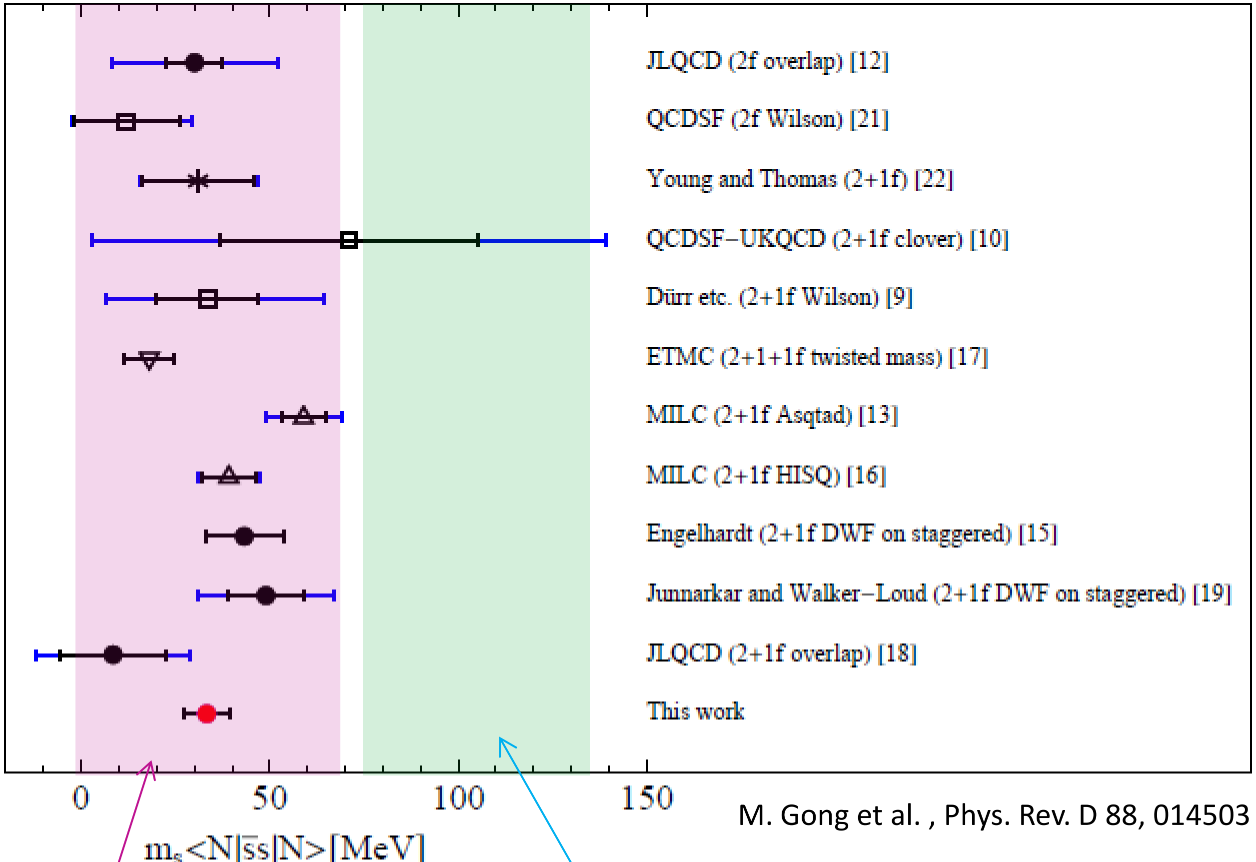
Recent study show that σ_{sN} is smaller than the previous values.

ϕ meson in the nuclear medium



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Recent study show that σ_{sN} is smaller than the previous values.

The analyses in this region are needed.

ϕ meson in the nuclear medium

QCD sum rule for ϕ meson channel

$$G_{OPE}(M^2) = c_0(\rho) + \frac{c_2(\rho)}{M^2} + \frac{c_4(\rho)}{M^4} + \frac{c_6(\rho)}{M^6} + \ldots$$

We take into account the higher order corrections

P. Gubler and K. Ohtani, Phys. Rev. D 90 094002 (2014).

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We take into account the higher order corrections

In the case of $c_4(\rho)$,

$$c_4(0) = \frac{1}{12}\left(1+\frac{7}{6}\frac{\alpha_s}{\pi}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + 2m_s\left(1+\frac{1}{3}\frac{\alpha_s}{\pi}\right)\langle\bar{s}s\rangle + \frac{3}{4\pi^2}m_s^4\left[1+4\log\left(\frac{M}{\mu}\right)-2\gamma_E\right]$$
$$-\frac{1}{6\pi^2}m_s^4\frac{\alpha_s}{\pi}\left[35-3\pi^2-24\zeta(3)+3\left(2\log\left(\frac{M}{\mu}\right)-\gamma_E\right)+18\left(2\log\left(\frac{M}{\mu}\right)-\gamma_E\right)^2\right]$$

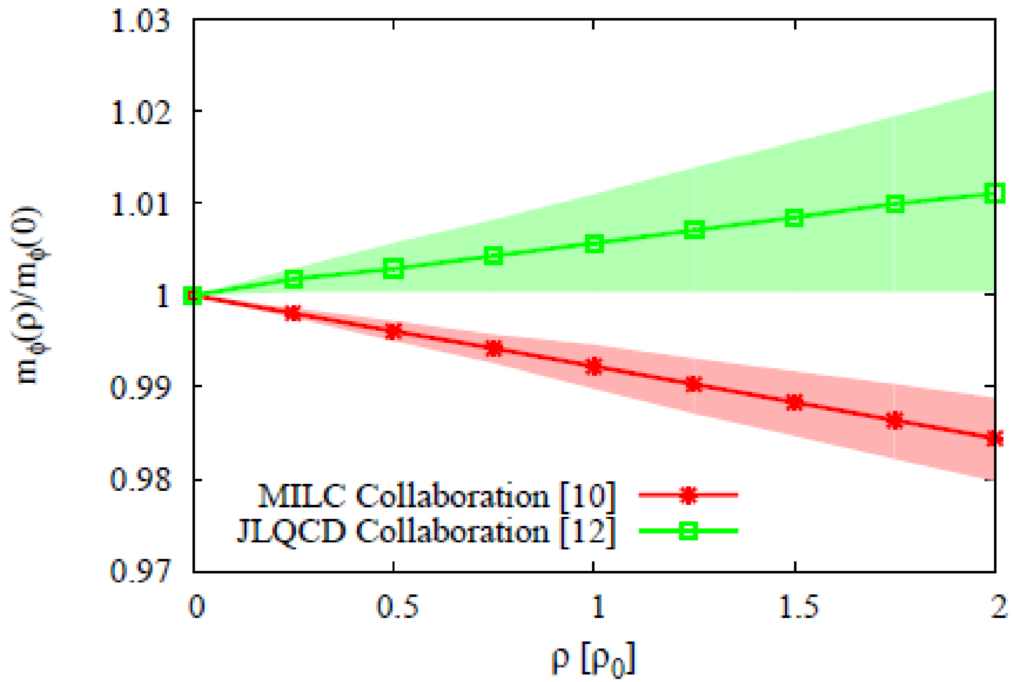
$$c_4(\rho) = c_4(0) + \rho\left[-\frac{2}{27}\left(1+\frac{7}{6}\frac{\alpha_s}{\pi}\right)M_N + \frac{56}{27}m_s\left(1+\frac{61}{168}\frac{\alpha_s}{\pi}\right)\langle N|\bar{s}s|N\rangle\right.$$
$$\left.+\frac{4}{27}m_q\left(1+\frac{7}{6}\frac{\alpha_s}{\pi}\right)\langle N|\bar{q}q|N\rangle + \left(1-\frac{5}{9}\frac{\alpha_s}{\pi}\right)A_2^s M_N - \underline{\frac{7}{12}\frac{\alpha_s}{\pi}A_2^g M_N}\right]$$

The contribution does not neglected.

P. Gubler and K. Ohtani, Phys. Rev. D 90 094002 (2014).

ϕ meson in the nuclear medium

Peak position of ϕ meson as a function of the density



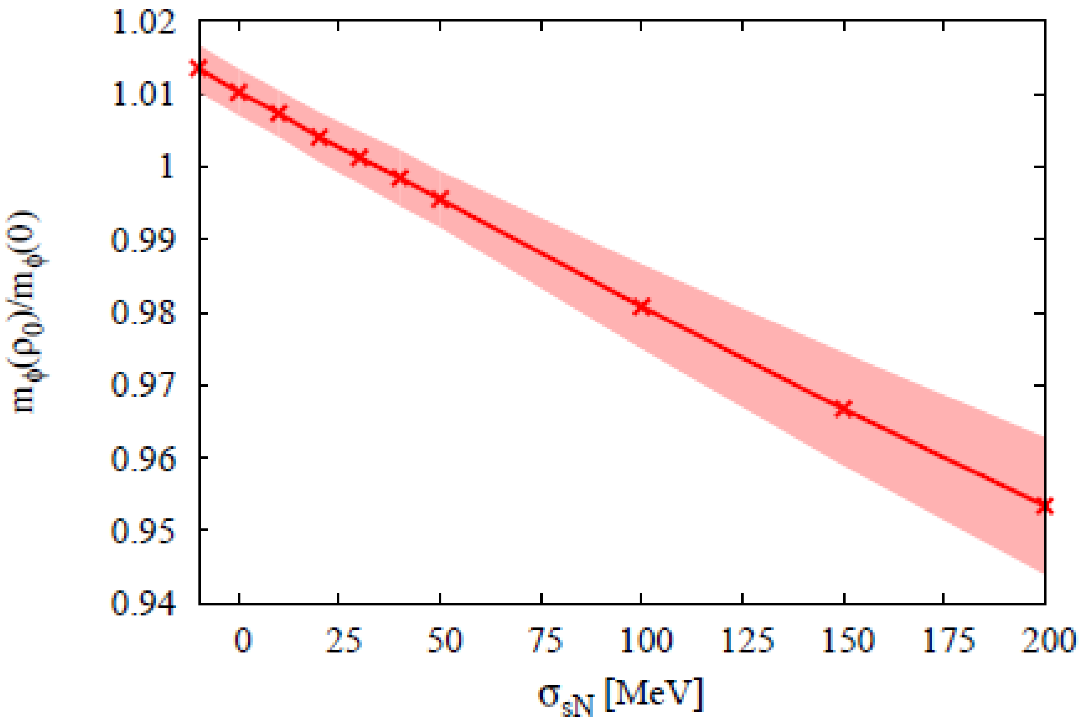
$\sigma_{sN} = 61 \pm 9\text{MeV}$

$\sigma_{sN} = 8 \pm 21\text{MeV}$

The ϕ meson mass shift in nuclear matter provide the constraint on the strangeness content of the nucleon.

ϕ meson in the nuclear medium

Peak positions of the ϕ meson at nuclear matter density as a function of σ_{sN}

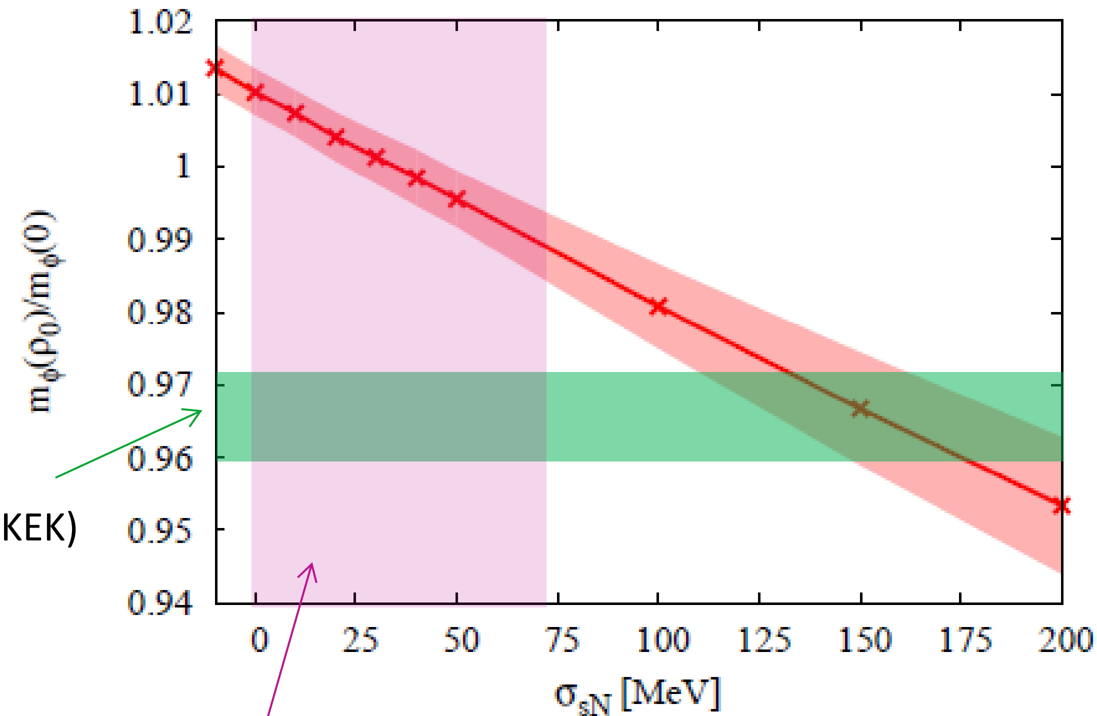


$$\frac{m_\phi(\rho)}{m_\phi(0)} - 1 = \left[b_0 - b_1\left(\frac{\sigma_{sN}}{1\,\mathrm{MeV}}\right)\right]\frac{\rho}{\rho_0}$$

$b_0 = 1.0 \cdot 10^{-2}$ $b_1 = 2.86 \cdot 10^{-4}$

ϕ meson in the nuclear medium

Peak positions of the ϕ meson at nuclear matter density as a function of σ_{sN}



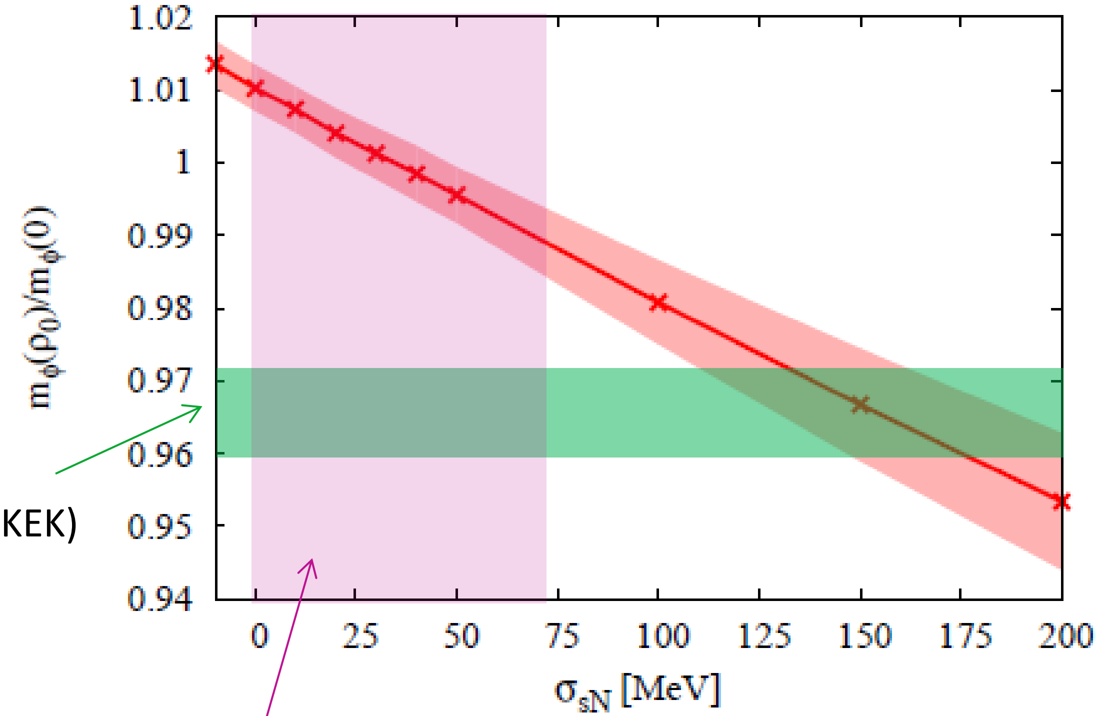
Δm_ϕ=35±7 MeV
(E325 experiment at KEK)

Recent studies give the values of σ_{sN} in this region

Δm_ϕ=35±7 MeV ⇨ σ_{sN} >100 MeV , which seems to be in contradiction with recent lattice result.

ϕ meson in the nuclear medium

Peak positions of the ϕ meson at nuclear matter density as a function of σ_{sN}



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Recent studies give the values of σ_{sN} in this region.

Higher order corrections: higher order m_s terms, higher twist terms
Momentum dependence of the ϕ meson mass shift

Summary

- The medium modifications of p meson were investigated QCD sum rule.

- The modifications are strongly related to the four quark condensate .

- We analyze the ϕ meson in the nuclear medium from QCD sum rules using the newest values of strangeness content of the nucleon.

- The ϕ meson mass shift in nuclear matter provide the constraint on the strangeness content of the nucleon.

QCD sum rule for ϕ meson channel

$$G_{OPE}(M^2) = c_0(\rho) + \frac{c_2(\rho)}{M^2} + \frac{c_4(\rho)}{M^4} + \frac{c_6(\rho)}{M^6} + \dots .$$

$$c_0(0) = \frac{1}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right) \qquad c_2(0) = \frac{m_s^2}{4\pi^2}\left[-6 - 4\frac{\alpha_s}{\pi}\left(4 - 6\log\left(\frac{M}{\mu}\right) + 3\gamma_E\right)\right]$$

$$c_6(0) = -\frac{112}{81}\pi\alpha_s\kappa_0\langle\bar{s}s\rangle^2 + \frac{1}{18}m_s^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle - \frac{4}{3}m_s^3\langle\bar{s}s\rangle$$

$$\begin{aligned} c_6(\rho) =& c_6(0) + \rho\Big[-\frac{224}{81}\pi\alpha_s\kappa_N\langle\bar{s}s\rangle\langle N|\bar{s}s|N\rangle - \frac{104}{81}m_s^3\langle N|\bar{s}s|N\rangle \\ & + \frac{8}{81}m_s^2 m_q\langle N|\bar{q}q|N\rangle - \frac{4}{81}m_s^2 M_N - \frac{3}{4}m_s^2 A_2^s M_N - \frac{5}{6}A_4^s M_N^3\Big] \end{aligned}$$

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$$c_4(0) = \frac{1}{12}\left(1+\frac{7}{6}\frac{\alpha_s}{\pi}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + 2m_s\left(1+\frac{1}{3}\frac{\alpha_s}{\pi}\right)\langle\bar{s}s\rangle + \frac{3}{4\pi^2}m_s^4\left[1+4\log\left(\frac{M}{\mu}\right)-2\gamma_E\right]$$
$$-\frac{1}{6\pi^2}m_s^4\frac{\alpha_s}{\pi}\left[35-3\pi^2-24\zeta(3)+3\left(2\log\left(\frac{M}{\mu}\right)-\gamma_E\right)+18\left(2\log\left(\frac{M}{\mu}\right)-\gamma_E\right)^2\right]$$

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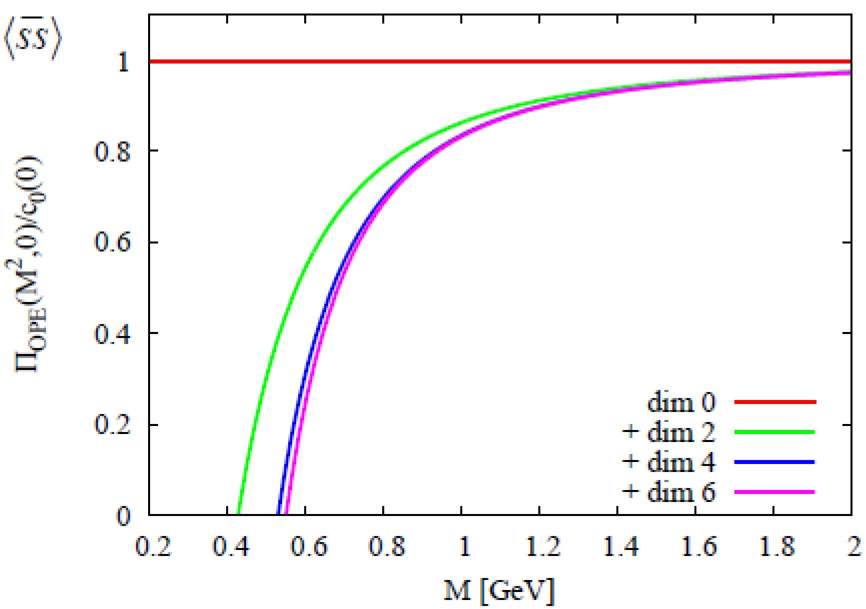

$$\langle \overline{q}q \rangle_{\rho_N} = \langle \overline{q}q \rangle_0 + \frac{\sigma_{\pi N}}{2m_q} \rho_N \qquad \sigma_{\pi N} = 2m_q \langle N | \overline{q}q | N \rangle$$

$$\langle \overline{s}s \rangle_{\rho_N} = \langle \overline{s}s \rangle_0 + \frac{\sigma_{sN}}{m_s} \rho_N \qquad \sigma_{sN} = m_s \langle N | \overline{s}s | N \rangle$$

(In the linear density approximation)

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_{\rho_N} = \langle \frac{\alpha_s}{\pi} G^2 \rangle_0 - \frac{8}{9} \Big[M_N - \sigma_N - \sigma_{sN} \Big] \rho_N + \cdots$$

$$A_n^q(\mu^2) = 2 \int_0^1 dx\, x^{n-1} [q(x,\mu^2) + (-1)^n \overline{q}(x,\mu^2)]$$

$$A_n^g(\mu^2) = \frac{1+(-1)^n}{2} \int_0^1 dx\, x^{n-1} g(x,\mu^2)$$