The $\eta \to 3\pi$ decay in the nuclear medium as a possible probe for chiral restoration

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Introduction -- η→3π decay in free space

✓ Isospin symmetry breaking in QCD

$m_u \neq m_d$

Small contribution from QED

D.G. Sutherland, Phys. Lett. 23(1968)384.

Physical state ↘ Eigenstate of G parity ↙

$$\eta \neq \eta_8$$

(mixture of the G parity even and odd states)

➡ **Failure of the quantitative description…**

J.S.Bell,D.G.Sutherland,NPB4(1968)315., S.Weinberg,RPD11(1975)3583.

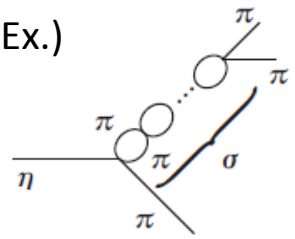
✓ Large effect of final state interaction (FSI)

2π resonances
- σ mode (s channel)
- ρ mode (t channel)

➡ Non-perturbative effect of FSI is important

cf.) C. Roiesnel and T. Truong, NPB187(1981)293.,
B. Borasoy and R. Nissler, EPJA26(2005)383.,
A. Abdel-Rehim, *et al.*, PRD67(2003)054001.,
S. Lanz, PoS CD12:007,2013.

Ex.)



$\eta \rightarrow 3\pi$ decay in nuclear medium

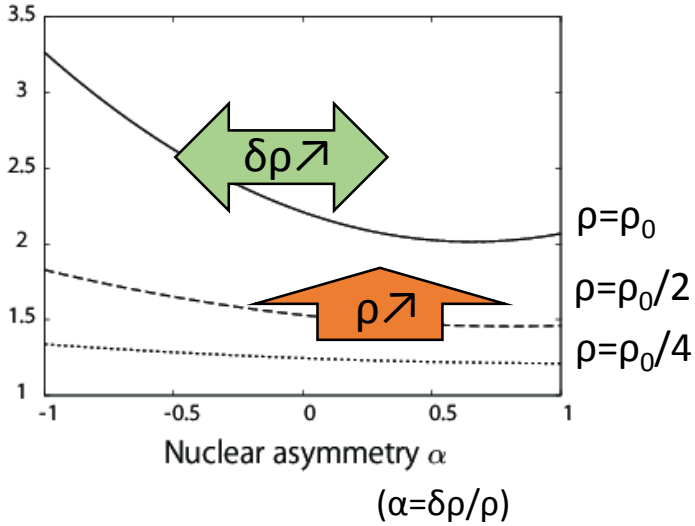
Previous study

The analysis of the nuclear medium effect on the $\eta \rightarrow 3\pi$ decay width with the non-linear σ model

Enhancement of the $\eta \rightarrow 3\pi$ decay width by $\rho=\rho_n+\rho_p$ and $\delta\rho=\rho_n-\rho_p$

S.S. and T. Kunihiro, PTEP (2015) 013D03, ibid., 089201.

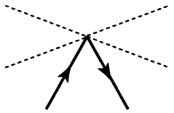
$\eta \rightarrow \pi^+\pi^-\pi^0$ decay width in asymmetric nuclear medium



(α=δρ/ρ)

✓ The enhancement by baryon number density ρ is large (factor 2~3 compared with the value @ρ=0)

※ Significant effect of the 4-meson NN vertex



: Similarity to the enhancement of the ππ cross section in nuclear medium
D. Jido, T. Hatsuda, T. Kunihiro, PRD63(2000)011901.

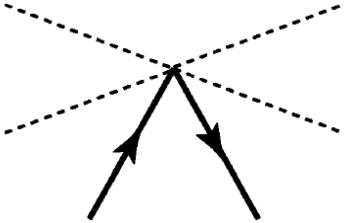
Chiral restoration is important!

Some relationship with the chiral restoration in nuclear medium?

Chiral restoration and softening of the σ mode

Significant effect of the 4π-NN vertex on the ππ scattering in nuclear medium

D. Jido, T. Hatsuda, T. Kunihiro, PRD63(2000)011901.

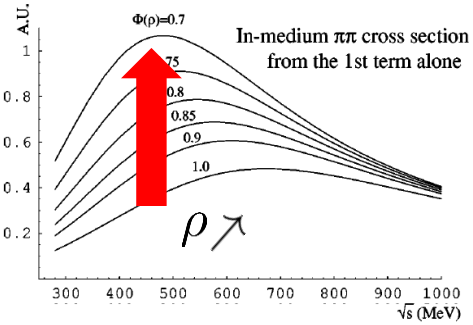


Chiral restoration in nuclear medium

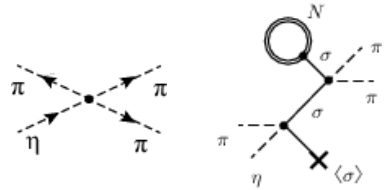
$$\langle\bar{q}q\rangle_\rho = \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)\langle\bar{q}q\rangle_{\rho=0} + O(\rho^{n>1})$$

E.G.Durkarev, E.M.Levin, NPA511(1990)679., T.D.Cohen,et al.,PRC45(1992)1881.

(Reduction of the order parameter of the spontaneous breaking of chiral symmetry)



✓ ππ scattering amplitude from linear σ model

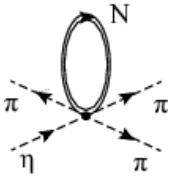


$$\mathcal{M}_{\pi\pi\to\pi\pi} \sim \frac{s - m_\pi^2}{s - m_\sigma^2}$$

Z. Aouissat, et al., PRC61(2000)012202.,
D. Davesne, et al., PRC62(2000)024604.

Large m_σ

✓ ππ scattering amplitude from non-linear σ model



$$\mathcal{M}_{\pi\pi\to\pi\pi} \sim \frac{1}{f_\pi^2}(1 + \alpha\rho)$$

(Dynamics other than NG bosons are contained in LECs)

V.Thorsson, A. Wirzba, NPA589(1995)633.

Experimental attempt in ππ system

(R.S.Hayano,T.Hatsuda, Rev.Mod.Phys.82(2010)2949.)

Chiral restoration in nuclear medium ➔ Enhancement of the cross section near $2m_\pi$

(Reflect the softening of the σ mode)

Purpose of this study

Investigate the significance of the role of the σ meson
and chiral restoration in the η→3π decay in nuclear medium

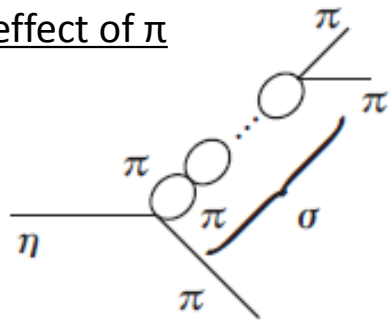
New possible probe for chiral restoration

Analysis with the linear σ model (explicit σ meson degree of freedom)

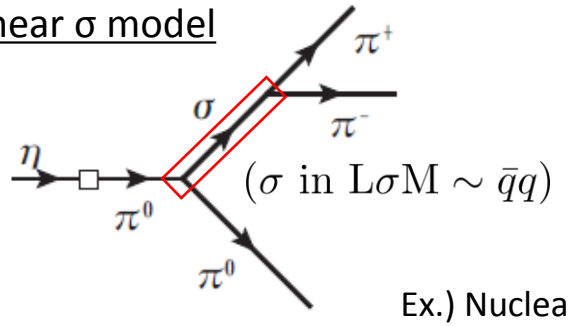
Advantages of linear σ model

- Non-perturbative inclusion of a part of the final state interaction

FSI effect of π



Linear σ model



Ex.) Nuclear force in intermediate range

- Effect of the chiral restoration through the softening of the σ mode

◎Remarkable effect in the case of the in-medium ππ scattering in linear σ model

Z. Aouissat, et al., PRC61(2000)012202., D. Davesne, et al., PRC62(2000)024604.

※The ρ-meson contribution is ignored for simplicity

Set up

- Linear sigma model with 3 flavor
  - Chiral SU(3) symmetry is respected
  - Explicit σ meson degree of freedom is included
  - Isospin symmetry breaking by non-degenerate u and d quarks

Lagrangian of linear σ model

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}(\partial_\mu M \partial^\mu M^\dagger) - \frac{\mu^2}{2}\mathrm{tr}MM^\dagger - \frac{\lambda}{4}\mathrm{tr}(MM^\dagger)^2 - \frac{\lambda'}{4}\left(\mathrm{tr}MM^\dagger\right)^2 + \frac{B}{2}(\det M + \det M^\dagger) + \frac{A}{2}\mathrm{tr}(\chi M^\dagger + M\chi^\dagger) + \bar{N}\left(i\partial\!\!\!/ - gM_5\right)N$$

M : meson field

N : nucleon field

$$\chi = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$: Isospin symmetry breaking ($m_u \neq m_d \neq m_s$)

$$m_d - m_u = m_{K^\pm}^2 - m_{K^0}^2 - m_{\pi^\pm}^2 + m_{\pi^0}^2$$

$$M = M_{\mathrm{scalar}} + M_{\mathrm{pseudo\ scalar}} \qquad M_5 = M_{\mathrm{scalar}} + i\gamma_5 M_{\mathrm{pseudo\ scalar}}$$

$$M_{\mathrm{scalar}} = \sum_{a=0}^{8} \frac{\sigma_a \lambda_a}{\sqrt{2}} \qquad M_{\mathrm{pseudo\ scalar}} = \sum_{a=0}^{8} \frac{\pi_a \lambda_a}{\sqrt{2}}$$

※The 30% reduction of $\langle q^{bar}q \rangle$ is assumed.

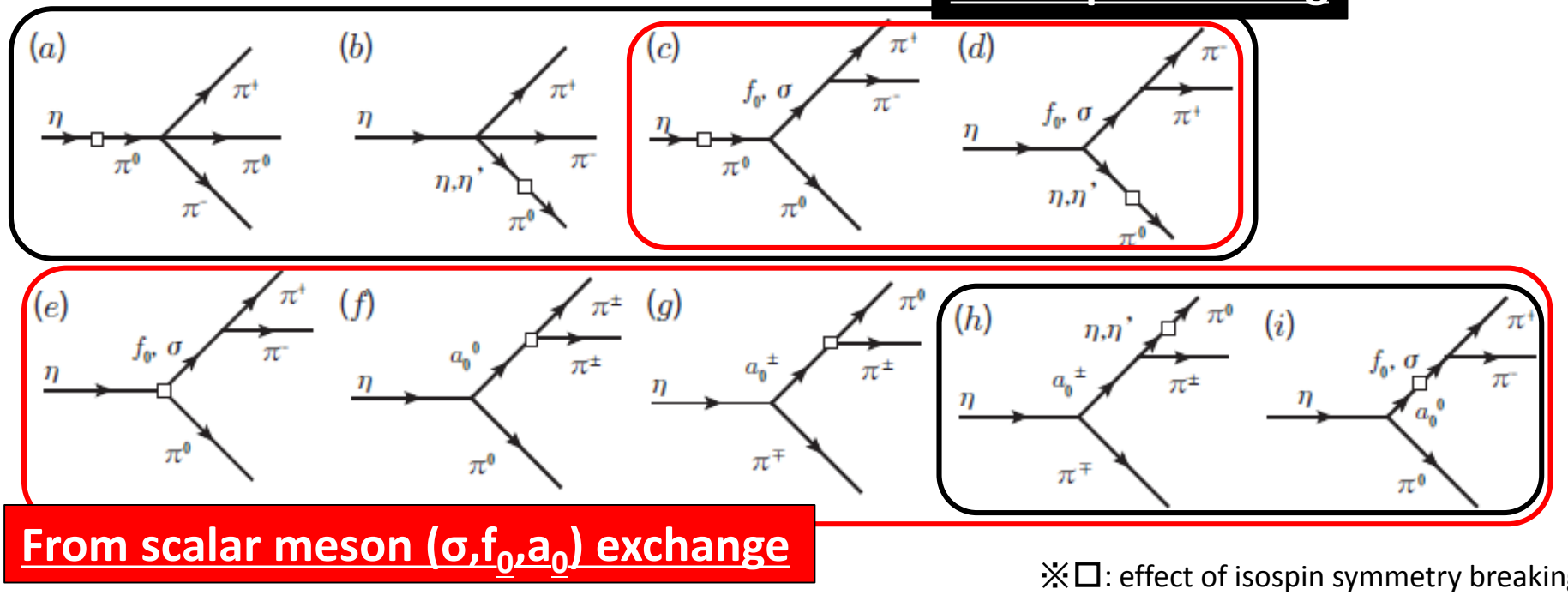
(K.Suzuki, et.al. Phys.Rev.Lett.92(2004)072302.)

Calculation in free space

J. Schechter and Y. Ueda, PRD4(1971)733., W. Hudnall and J. Schechter, PRD9(1974)2111., S.Raby, PRD13(1976)2594.

Diagrams

From $\eta^{(\prime)}\pi^0$ mixing



From scalar meson (σ,f_0,a_0) exchange

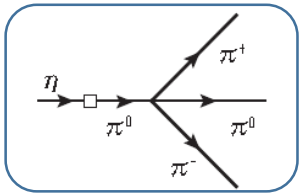
※□: effect of isospin symmetry breaking

Matrix element of $\eta \to 3\pi$ decay with linear σ model

$$\mathcal{M}^{\mathrm{L}\sigma\mathrm{M}}_{\eta\to\pi^+\pi^-\pi^0} = \mathcal{M}^{\mathrm{contact}}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{M}^{\mathrm{isoscalar}}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{M}^{\mathrm{isovector}}_{\eta\to\pi^+\pi^-\pi^0}$$

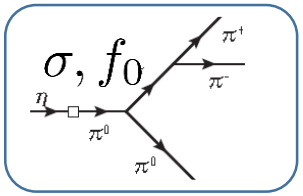
・Contribution purely from meson 4pt. vertex

$$\mathcal{M}^{\mathrm{contact}}_{\eta\to\pi^+\pi^-\pi^0} = 2(-\sin\theta_{\eta\pi^0}) g_{\pi_3\pi_3\pi^+\pi^-} + 2\sin\theta_{\eta\pi^0} g_{\eta\eta\pi^+\pi^-} + \sin\theta_{\eta'\pi^0} g_{\eta\eta'\pi^+\pi^-}$$



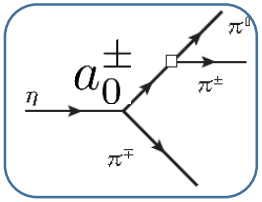
・Contribution from scalar meson (σ, f_0) exchange

$$\mathcal{M}^{\mathrm{isoscalar}}_{\eta\to\pi^+\pi^-\pi^0} = -g_{\sigma\eta\pi^0}\frac{1}{s-m_\sigma^2}g_{\sigma\pi^+\pi^-} - g_{f_0\eta\pi^0}\frac{1}{s-m_{f_0}^2}g_{f_0\pi^+\pi^-}$$



・Contribution from isovector meson (a_0) exchange

$$\mathcal{M}^{\mathrm{isovector}}_{\eta\to\pi^+\pi^-\pi^0} = -g_{\eta\pi^- a_0^0}\frac{1}{s-m_{a_0^0}^2}g_{a_0^0\pi^0\pi^0} - g_{\eta\pi^- a_0^+}\frac{1}{t-m_{a_0^+}^2}g_{a_0^+\pi^-\pi^0} - g_{\eta\pi^- a_0^+}\frac{1}{u-m_{a_0^-}^2}g_{a_0^-\pi^+\pi^0}$$



The η→3π decay: ~200eV

※1. About 70eV w/o scalar meson contribution
← Large effect of scalar meson exchange

※2. A certain dependence on the σ meson mass (180~250eV)
(The σ meson mass is an input.)

※3. Experimental value: 300eV
from PDG (J.Beringer,et al.,PRD86(2012)01001.)
← ρ meson is ignored

Medium effect on $\eta \rightarrow 3\pi$ decay

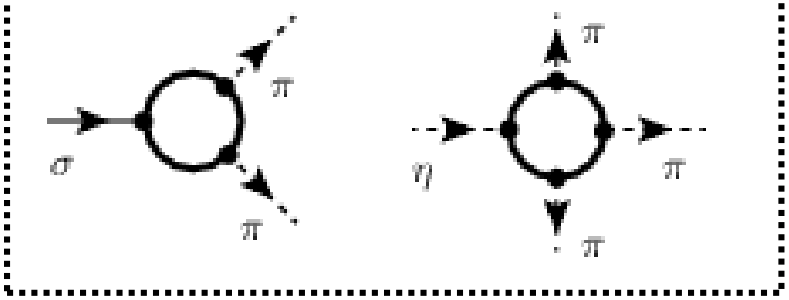
※1. Nucleon field (solid line) ← Mean field approximation
※2. Inclusion of nucleon 1-loop ← Fermi momentum k_f: small

Self energy

※ Mass modification of the ps mesons (π, η): relatively small
(30~40MeV enhancement @ $\rho=\rho_0$)

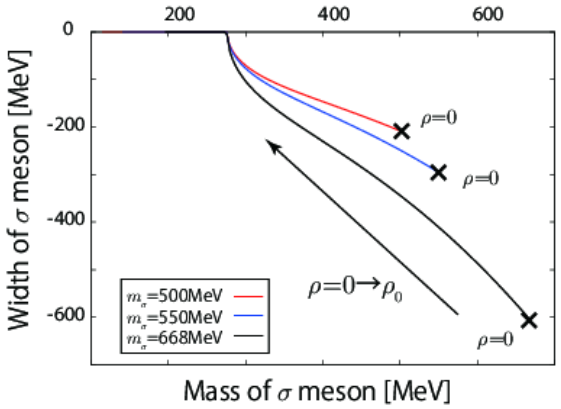
⇔ Large reduction of mass and width of σ meson
(several 100MeV reduction @$\rho=\rho_0$)

Correction of vertices



✓ Width of the σ meson: tree level

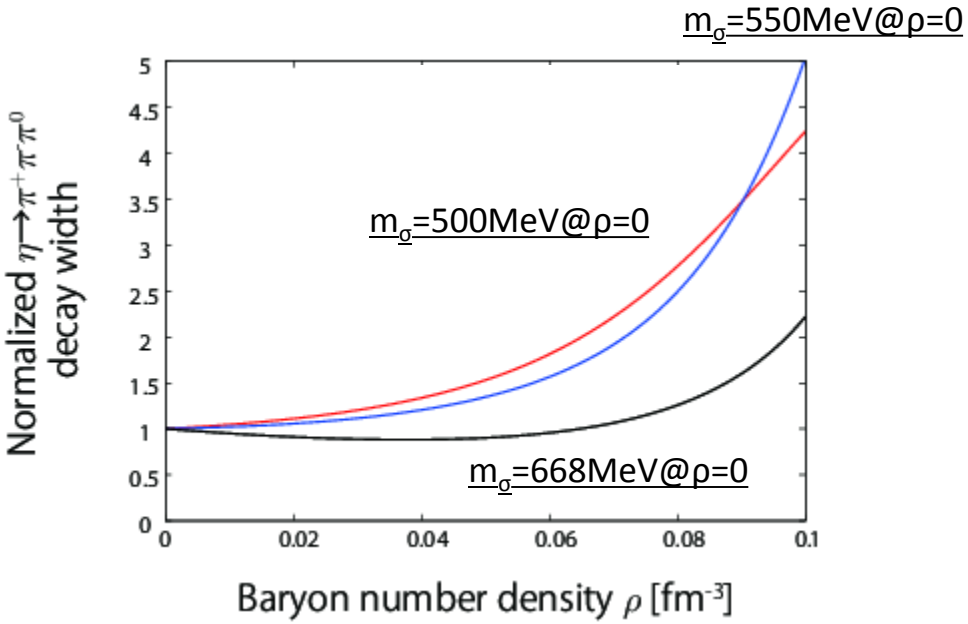
$$\Gamma_\sigma = \int d\Pi \left| \sigma \text{—} \overset{\pi}{\underset{\pi}{\frown}} \right|^2$$



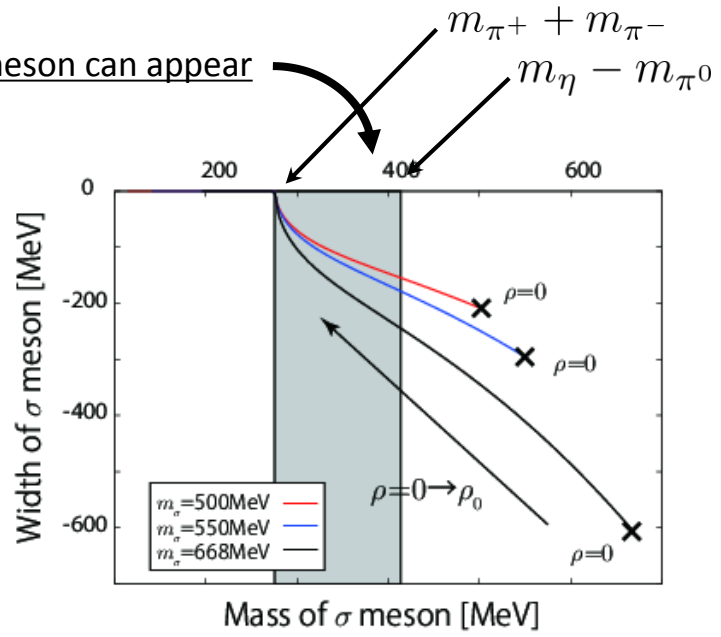
(The σ meson mass @ρ=0 is an input.)

Results

$\eta \to \pi^+\pi^-\pi^0$ decay width in nuclear medium
(width is normalized by the value @ ρ=0)



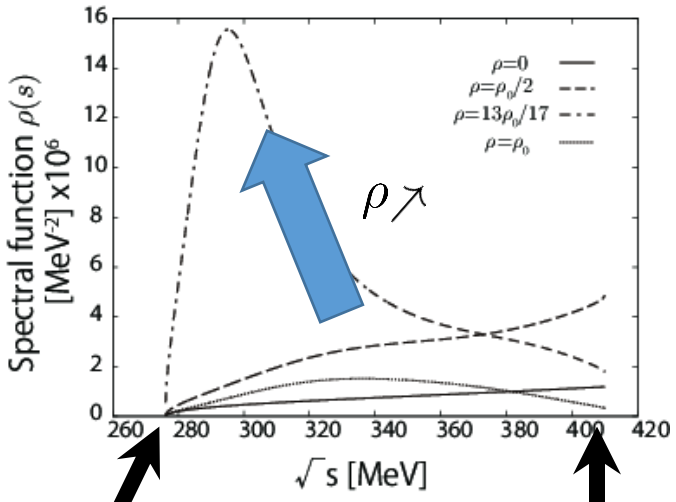
On-shell σ meson can appear

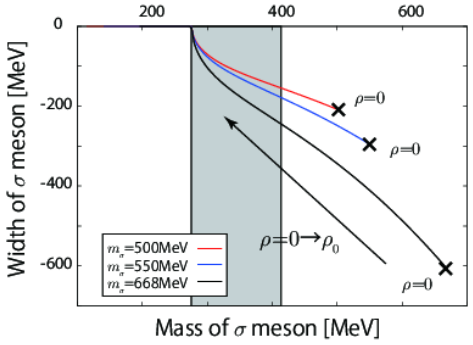


- $\eta \to \pi^+\pi^-\pi^0$ decay width is enhanced by the nuclear medium in the wide range of σ meson mass
- Large dependence of the enhancement on the σ meson mass @ρ=0
  - The enhancement is factor 2~5 ← m_σ=500~668MeV
  - The width is enhanced in the smaller ρ region with smaller m_σ @ρ=0

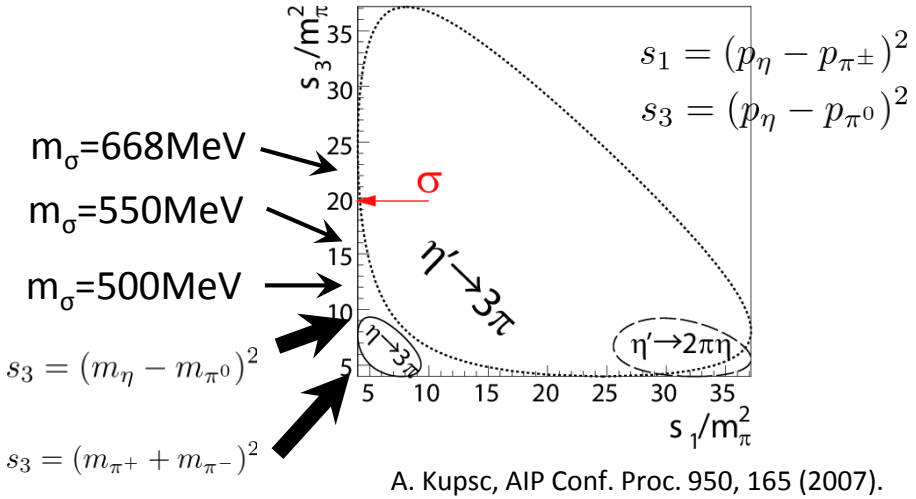
Spectral function of sigma meson in nuclear medium

m_σ=550MeV





$$\rho(s) = -\frac{1}{\pi}\mathrm{Im}G_\sigma(s)$$



A. Kupsc, AIP Conf. Proc. 950, 165 (2007).

※Similar discussion in ππ scattering

T. Hatsuda, T. Kunihiro, H. Shimizu, PRL82(1999)2840

(Spectral enhancement near 2π threshold: general phenomenon)

Summary

- The σ meson in linear σ model plays

  important role in $\eta \rightarrow 3\pi$ decay even in free space

  (about 200eV with the σ meson exchange)

- $\eta \rightarrow \pi^+\pi^-\pi^0$ decay width is enhanced

  by the effect of nuclear medium

- The enhancement largely depends on the σ meson mass @ρ=0

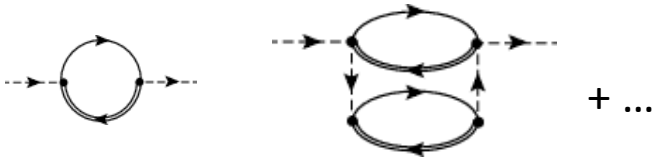
  (2~5 times larger than the value @ρ=0 from m_σ=500~668MeV)

- The enhancement reflects the softening of the σ mode

  (modification of spectral function of σ meson)

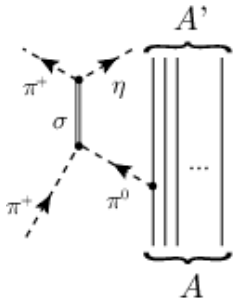
Future prospects

- Effect of the asymmetric nuclear medium ($\delta\rho \neq 0$)
- More reasonable treatment of the final state interaction
  - The $\pi\pi$ composite component of the σ mode
  - Contribution from the ρ meson
- Contribution of the excited baryons (N^*, Δ) in nuclear medium and nuclear absorption



+ …

- Influence on the $\pi A \rightarrow \pi \eta A'$ reaction in forward direction



※ Sensitivity to $\delta\rho = \rho_n - \rho_p$

Thank you for your attention!