Compositeness of near-threshold unstable states

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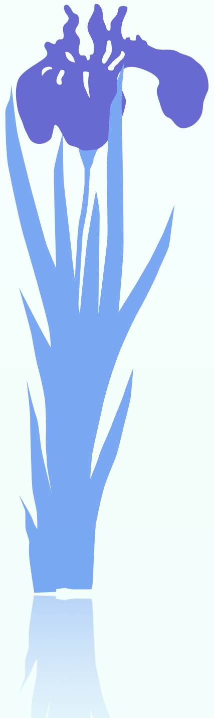
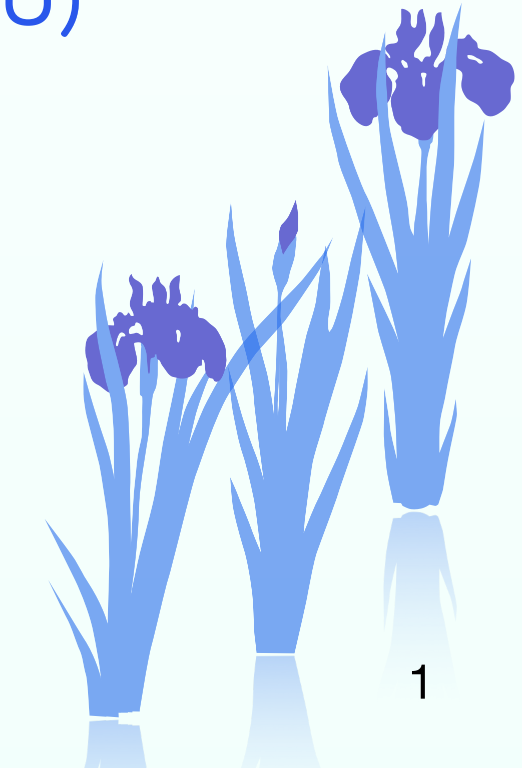




Contents

- Introduction ~compositeness of bound state~
- Extension to the quasi-bound state
- Applications to exotic hadrons ~ $\Lambda(1405), a_0(980), f_0(980)$ ~
- Conclusions

Introduction ~exotic hadrons~

- Exotic hadrons

Hadrons which do not coincide with the predictions of the quark model.

→ More complicated internal structure can be expected.

- tetra quark, penta quark
- hadron molecule ⋯

→ It is important to reveal the internal structure of exotics.

e.g. ; Λ(1405)

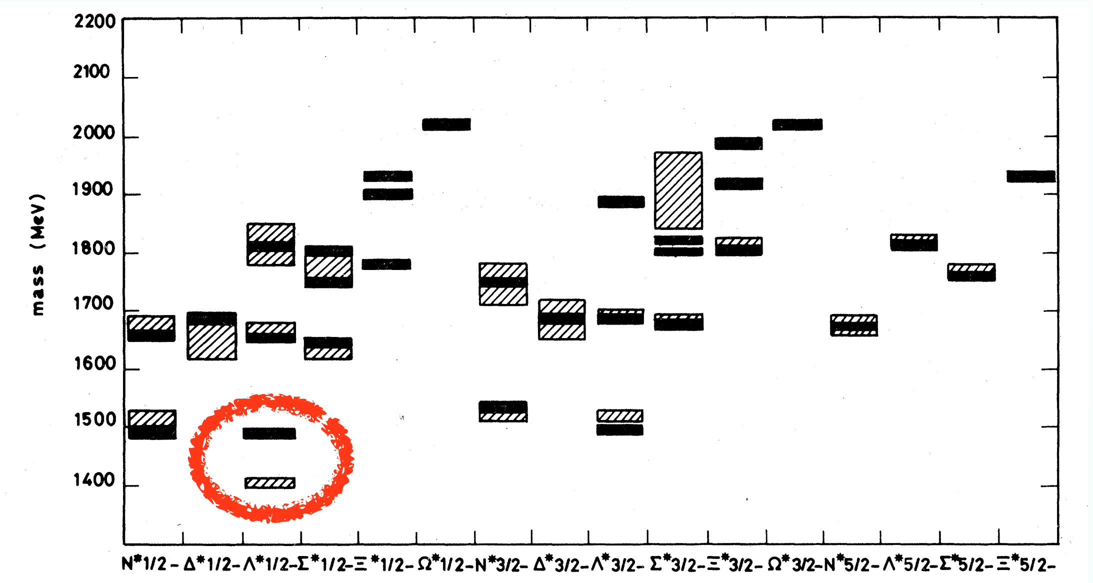
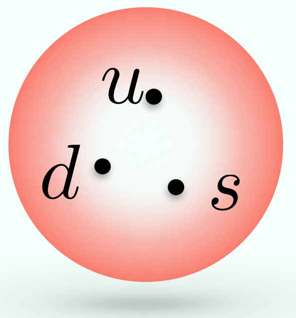


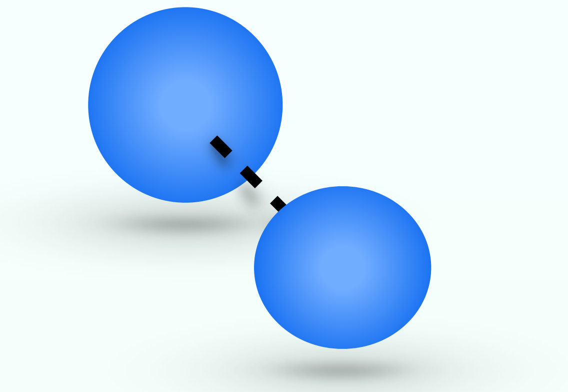
FIG. 1. Comparison of the predicted and observed spectrum of negative-parity baryons. The shaded regions corre-

N. Isgur, and G. Karl, Phys. Rev. D18, 4187 (1978)

exited Λ state(uds)



$\bar{K}N$ bound state



Compositeness of bound state

Previous work

Introduced to study deuteron by Weinberg.

S. Weinberg, Phys. Rev. 137, B672 (1965)

Composite Elementary

$X = 1$ $\longleftrightarrow$ $X = 0$

$Z = 0$ $Z = 1$

Condition

- weakly bound
- stable state
- s-wave

Output

- X ;weight of composite state $(0 < X < 1)$
- Z ;wave function renormalization $(0 < Z < 1)$
- a_0 ;scattering length
- B ;binding energy

$R = \frac{1}{\sqrt{2\mu B}}$

(μ;reduced mass of scat. state)

$$\boxed{a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(R_{\text{typ}}/R\right)\right\}}$$

typical length scale

We can extract the information of the internal structure using experimental observables.

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Extension to the quasi-bound state.

System

Two channel scattering

- scattering channel $|\boldsymbol{p}\rangle$
- decay channel $|\boldsymbol{p}'\rangle$

$|\boldsymbol{p}\rangle$ can decay to $|\boldsymbol{p}'\rangle$.

Unstable quasi-bound state $|QB\rangle$ exists near $|\boldsymbol{p}\rangle$ threshold.

The interaction has a typical length scale $R_{\rm typ}$.

Effective field theory

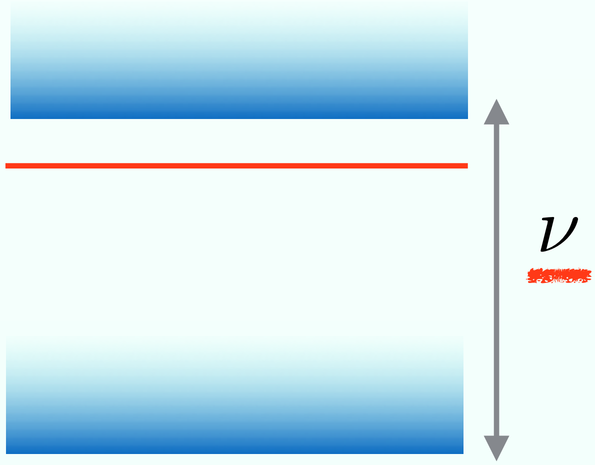
To discuss the near-threshold physics, we use following non-relativistic EFT.

Free field $H_{\rm free}$ eigenstate

$|\boldsymbol{p}\rangle$ scattering channel

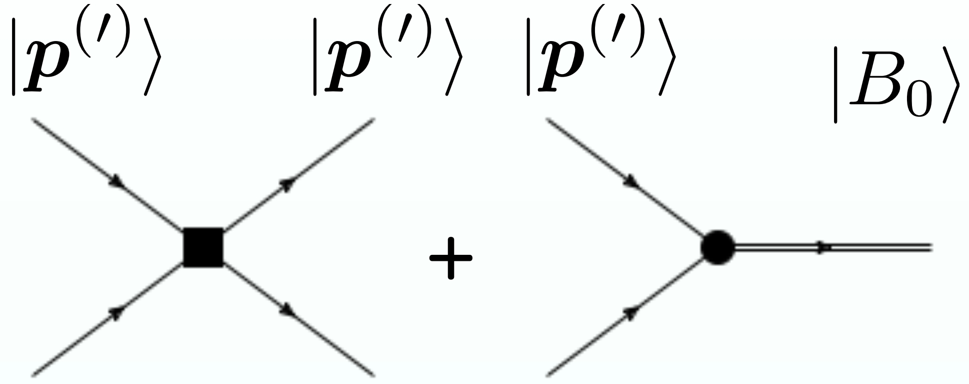
$|B_0\rangle$ discrete channel

$|\boldsymbol{p}'\rangle$ decay channel



Interaction

$\mathcal{H}_{\rm int} =$



point interaction

Eigenstate

$$H = H_{\rm free} + H_{\rm int}$$

$$H|QB\rangle = E_{QB}|QB\rangle$$

$$E_{QB} = -B - \underline{i\Gamma/2}$$; complex

We consider the compositeness of $|\boldsymbol{p}\rangle$channel ;$\boldsymbol{X}$.

Extension to the quasi-bound state.

Definition of compositeness

Bound state

Bound state $|B\rangle$ is normalized with $\langle B|B\rangle = 1$

- $X + Z = 1$
- $0 < X, Z < 1$

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \langle B|\boldsymbol{p}\rangle\langle\boldsymbol{p}|B\rangle$$
$$= \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} |\langle\boldsymbol{p}|B\rangle|^2$$

The probabilistic interpretation is guaranteed for X and Z.

Quasi-bound state

To normalize unstable state, we introduce Gamow state $|\overline{QB}\rangle$.

Normalization condition becomes

$$\langle\overline{QB}|QB\rangle = \langle QB^*|QB\rangle = 1.$$

T. Berggren, Nucl. Phys. A 109 (1968)

The expectation value of the any operator becomes complex number.

- $X + Z = 1$
- $0 \cancel{< X, Z < 1}\ X, Z \in C$

$$X \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \langle\overline{QB}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|QB\rangle$$

The probabilistic interpretation is not guaranteed!

Extension to the quasi-bound state.

Y. Kamiya and T. Hyodo, arXiv:1509.00146 [hep-ph].
accepted in Phys. Rev. C

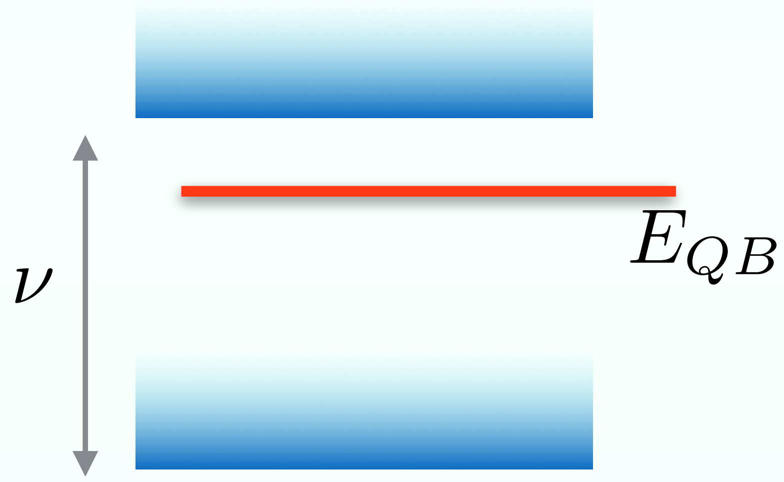
- Assuming $|E_{QB}|$ is small, we expand a_0 with respect to $1/R$.

$$a_0 = R\left[\underbrace{\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\rm typ}}{R}\right|\right)}_{\text{original}} + \underbrace{\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}_{\text{new}}\right]$$

$$R = \frac{1}{\sqrt{-2\mu E_{QB}}}$$

$$l = \frac{1}{\sqrt{2\mu\nu}}$$

If $\underline{|R_{\rm typ}/R|}$ and $\underline{|l/R|^3}$ are sufficiently smaller than 1, we can extract X from a_0 and E_{QB} .



- Notice
  - a_0 ,E_{QB}, X are all complex numbers, then above relation is established among them.
  - If the the contribution of decaying mode is neglected, the compositeness relation is same to the one for bound state.
  - The same argument is valid for the case with $\mathrm{Re}\ E_h > 0$.

Interpretation of X

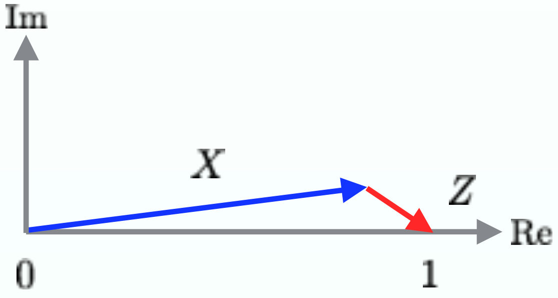
Interpretation of the complex compositeness

- There is no common interpretation of the complex X.

(1) close to bound state case

$$\begin{cases} X = 0.8 + 0.1i \\ Z = 0.2 - 0.1i \end{cases}$$

small cancellation in X+Z



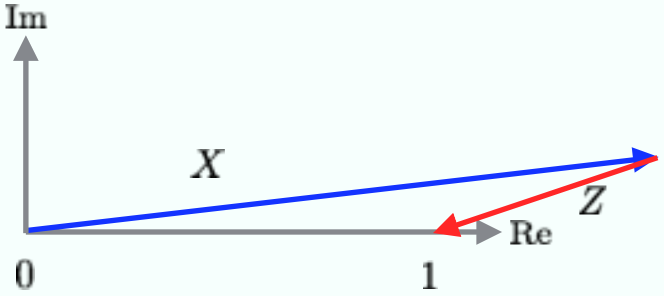
bound state case

$$\begin{cases} X = 0.8 \\ Z = 0.2 \end{cases}$$

probabilistic interpretation is available

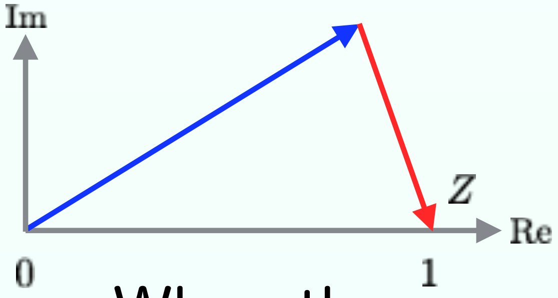
(2-a) When real part is not in [0,1]

$$\begin{cases} X = \underline{1.9} + 0.2i \\ Z = \underline{-0.9} - 0.2i \end{cases}$$



(2-b) When imaginary part is large.

$$\begin{cases} X = 0.9 \underline{+ 0.8i} \\ Z = 0.1 \underline{- 0.8i} \end{cases}$$



large cancellation in X+Z

When the cancelation is small,
we can interpret the complex compositeness.

Interpretation of X

Our proposal

c.f. T. Berggren, Phys. Lett. B 33 (1979) 8

For probabilistic interpretation we define the following real quantities.

$\tilde{X}$; probability to find the scattering state in physical state

$\tilde{Z}$; probability to find the other states

U ; degree of uncertainty of the interpretation

$$\tilde{X} \equiv \frac{1-|Z|+|X|}{2}$$

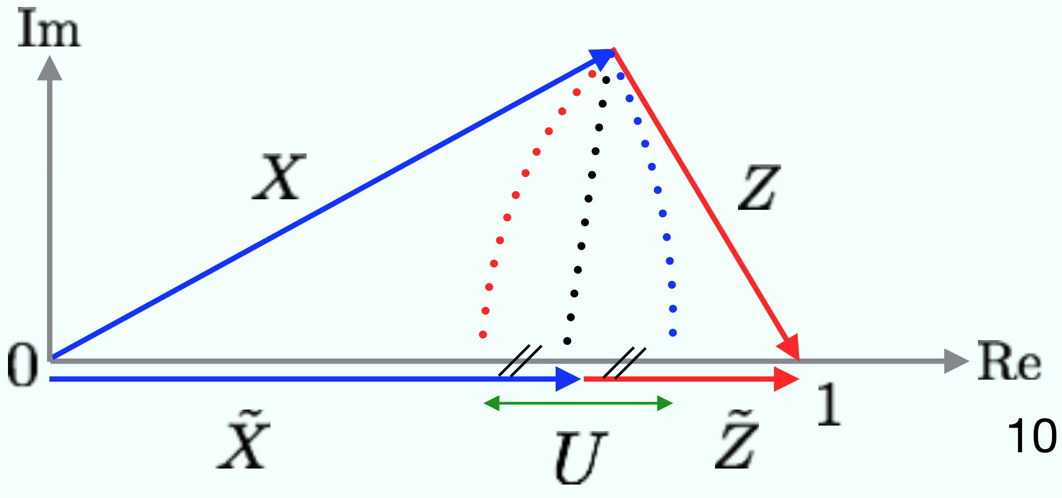
$$\tilde{Z} \equiv \frac{1-|X|+|Z|}{2}$$

$$U \equiv |Z|+|X|-1$$

conditions :

- $\tilde{X}+\tilde{Z}=1$
- $0 \leq \tilde{X}, \tilde{Z} \leq 1$
- When the cancellation is 0, $\tilde{X}=X, \tilde{Z}=Z, U=0$.
- U becomes large when the cancellation becomes large.

If U is small, we interpret $\tilde{X}$ as the probability.



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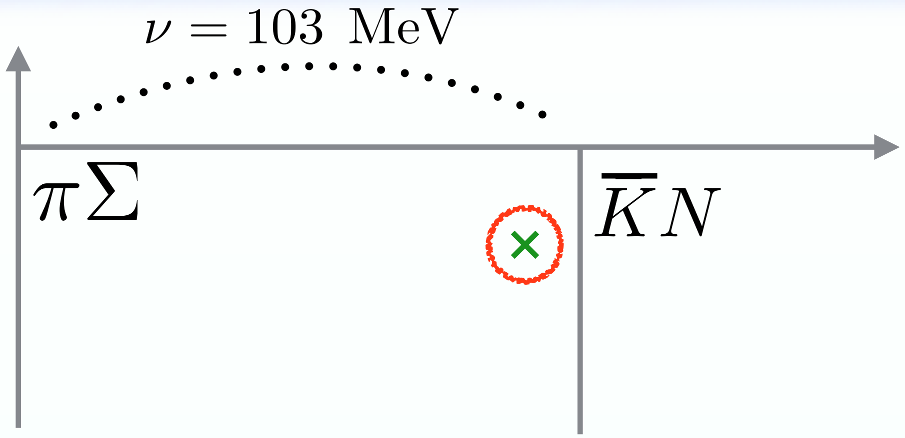
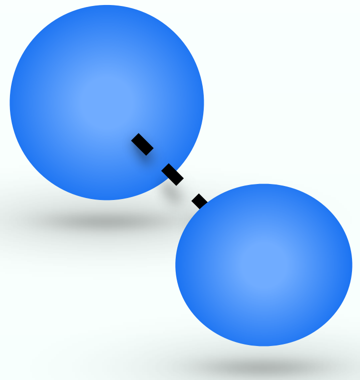
Compositeness of exotics

- $\Lambda(1405)$ ($I = 0$ $\bar{K}N$ scattering)

$J^P = \frac{1}{2}^-$



$\bar{K}N$ molecule?

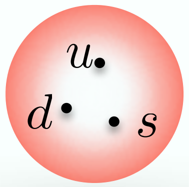


$\tilde{X} = 1$

or

other components?

$\tilde{X} = 0$



e.g.

- Λ excited states(uds)
- penta-quark state

…

R_{typ} is estimated from rho meson exchange int.

$(R_{\mathrm{typ}} \sim 0.25 \ \mathrm{fm})$

$$\left|\frac{R_{\mathrm{typ}}}{R}\right| \lesssim 0.17$$

$$R = \frac{1}{\sqrt{-2\mu E_{QB}}}$$

$$l = \frac{1}{\sqrt{2\mu\nu}}$$

l is estimated from difference of the threshold energy

$$\left|\frac{l}{R}\right|^3 \lesssim 0.14$$

$$a_0 = R\left[\frac{2X}{1+X} + \mathcal{O}\left(\left|\frac{R_{\mathrm{typ}}}{R}\right|\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right] \Rightarrow X = \frac{a_0}{2R - a_0} \Rightarrow \tilde{X}, U$$

can be neglected

Compositeness of exotics

- $\Lambda(1405)$ in $I=0$ $\bar{K}N$ scattering

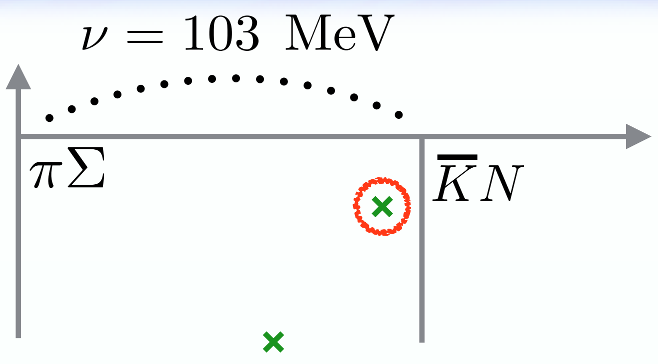
We use E_{QB} and a_0 in the following papers.

(1) Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A 881 98 (2012)

(2) M. Mai and U. G. Meissner, Nucl. Phys. A 900, 51 (2013)

(3) Z. H. Guo and J. A. Oller, Phys. Rev. C 87, 035202 (2013)

(4)M. Mai and U.-G. Meißner, Eur. Phys. J. A 51, 30 (2015).



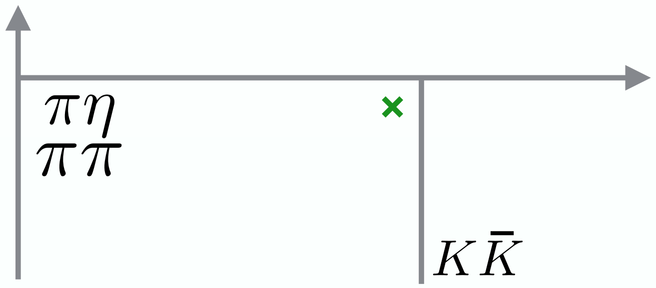
Ref.	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	-10-i26	1.39-i0.85	1.3+i0.1	1.0	0.5
(2)	-4-i8	1.81-i0.92	0.6+i0.1	0.6	0.0
(3)	-13-i20	1.30-i0.85	0.9-i0.2	0.9	0.1
(4)-1	2-i10	1.21-i1.47	0.6+i0.0	0.6	0.0
(4)-2	- 3-i12	1.52-i1.85	1.0+i0.5	0.8	0.6

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- $\tilde{X}$ is close to 1.

$\Lambda(1405)$: $\bar{K}N$ composite dominance

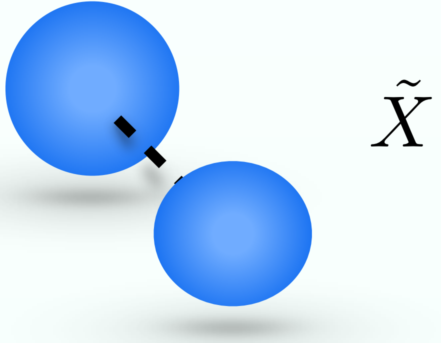
Compositeness of exotics

- $a_0(980)$ 、$f_0(980)$ ($K\bar{K}$ scattering)
  $(I=1)$ $(I=0)$
  $J^{PC}=0^{++}$



$K\bar{K}$ molecule ?

J. D. Weinstein and N. Isgur, PRD 41 (1990)



$\tilde{X}=1$

or

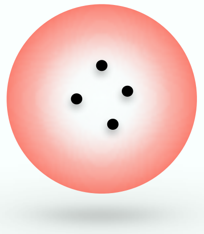
other components?

$\tilde{X}=0$

e.g.

- tetra quark state

R. L. Jaffe, PRD 15 (1977)



- $q\bar{q}$ meson state

…

$$\left|\frac{R_{\mathrm{typ}}}{R}\right| \lesssim 0.17 \qquad \left|\frac{l}{R}\right|^3 \lesssim 0.04$$

$$a_0 = R\left[\frac{2X}{1+X} + \underline{\mathcal{O}\left(\left|\frac{R_{\mathrm{typ}}}{R}\right|\right)} + \underline{\mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)}\right] \Rightarrow X = \frac{a_0}{2R-a_0} \Rightarrow \tilde{X}, U$$

can be neglected

Compositeness of exotics

- $a_0(980)$ in $K\overline{K}$ scattering

  We determine E_{QB} and a_0 from Flatte parameters which are obtained experimental analysis.

c. f. : V. Baru et al. Phys. Lett. B 586, 53 (2004)

T. Sekihara and S. Kumano, Phys. Rev. D 92, 034010 (2015)

(1) G. S. Adams et al. [CLEO Collaboration], Phys. Rev. D 84, 112009 (2011)

(2) F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 681, 5 (2009)

(3) D. V. Bugg, Phys. Rev. D 78, 074023 (2008)

(4) S. Teige et al. [E852 Collaboration], Phys. Rev. D 59, 012001 (1999)

Set	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	31-i70	-0.03-i0.53	0.2-i0.2	0.3	0.1
(2)	3-i25	0.17-i0.77	0.2-i0.2	0.2	0.1
(3)	9-i36	0.05–i0.63	0.2-i0.2	0.2	0.1
(4)	15-i29	-0.13-i0.52	0.1-i0.4	0.1	0.1

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- $\tilde{X}$ is close to 0.

⇨ $a_0(980)$: small $K\bar{K}$ fraction

Compositeness of exotics

c. f. T. Sekihara and S. Kumano, Phys. Rev. D 92, no. 3, 034010 (2015)

- $f_0(980)$ in $K\overline{K}$ scattering

  We determine E_{QB} and a_0 from Flatte parameters which are obtained experimental analysis.

(1) T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 84, 052012 (2011)

(2) F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 634, 148 (2006)

(3) A. Garmash et al. [Belle Collaboration], Phys. Rev. Lett. 96, 251803 (2006)

(4) M. Ablikim et al. [BES Collaboration], Phys. Lett. B 607, 243 (2005)

(5) J. M. Link et al. [FOCUS Collaboration], Phys. Lett. B 610, 225 (2005)

(6) M. N. Achasov et al., Phys. Lett. B 485, 349 (2000)

Ref.	E_{QB} (MeV)	a_0 (fm)	X	$\tilde{X}$	U
(1)	19-i30	0.02-i0.95	0.3-0.3	0.4	0.2
(2)	-6 -i10	0.84-i0.85	0.3-i0.1	0.3	0.0
(3)	-8 -i28	0.64-i0.83	0.4-i0.2	0.4	0.1
(4)	10-i18	0.51-i1.58	0.7-i0.3	0.6	0.1
(5)	-10-i29	0.49-i0.67	0.3-i0.1	0.3	0.0
(6)	10-i7	0.52-i2.41	0.9-i0.2	0.9	0.1

- U is small enough. —> $\tilde{X}$ can be considered as the probability.
- Values of $\tilde{X}$ are not consistent.

More precise analysis is needed.

Contents

Conclusions

Conclusions

Y. Kamiya and T. Hyodo, arXiv:1509.00146 [hep-ph].

accepted in Phys. Rev. C

- We extend the weak-binding relation to quasi-bound states.

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(|R_{\mathrm{typ}}/R|\right) + \mathcal{O}\left(|l/R|^3\right)\right\}$$

If the absolute value of the eigenenergy is small enough,

the compositeness is model-independently determined only from observables.

- We construct interpretation of complex X.

$$\tilde{X} \equiv \frac{1-|Z|+|X|}{2}, \qquad U \equiv |X|+|Z|-1$$

If the uncertainty U is small, we interpret $\tilde{X}$ as the probability.

- We apply the method to exotic hadrons and discuss the internal structures.

$\Lambda(1405)$:$\bar{K}N$ composite dominance

$a_0(980)$: not $K\bar{K}$ dominance