# Approximate degeneracy of heavy-light mesons with the same $L$

Takayuki Matsuki
in collaboration with Y. Dong, F.-Q. Lü, and T. Morii

Tokyo Kasei Univ.

March 3, 2016

# Index

# Motivation



- Careful observation of experimental spectra ;
  degeneracy among states with the same $L$ for $D/D_s/B/B_s$

# Motivation



- Careful observation of experimental spectra ; degeneracy among states with the same $L$ for $D/D_s/B/B_s$
- Relativistic potential model respecting heavy-quark symmetry does not conserve $L$

# Motivation



- Careful observation of experimental spectra ; degeneracy among states with the same $L$ for $D/D_s/B/B_s$
- Relativistic potential model respecting heavy-quark symmetry does not conserve $L$
- Godfrey-Isgur model respecting $L$
- Why do these two give the similar results?

# $D$ meson masses for different quark models

TABLE I: The $D$ meson masses in MeV from different quark models and experimental data.

| State | GI[1–3] | ZVR[6] | DE[7] | EFG[8] | MMS[9] | LS[10, 11] | EXP[13–16, 18] | Average | Gap |
|---|---|---|---|---|---|---|---|---|---|
| $D(1^1S_0)$ | 1874 | 1850 | 1868 | 1871 | 1869 | 1867 | 1867 | 1938 | |
| $D(1^3S_1)$ | 2038 | 2020 | 2005 | 2010 | 2011 | 2010 | 2009 | | |
| $D(1^3P_0)$ | 2398 | 2270 | 2377 | 2406 | 2283 | 2252 | 2361 | 2394 | 456 |
| $D_1(1P)$ | 2455 | 2400 | 2417 | 2426 | 2421 | 2402 | 2427 | | |
| $D_1'(1P)$ | 2467 | 2410 | 2490 | 2469 | 2425 | 2417 | 2422 | 2443 | 49 |
| $D(1^3P_2)$ | 2501 | 2460 | 2460 | 2460 | 2468 | 2466 | 2463 | | |
| $D(1^3D_1)$ | 2816 | 2710 | 2795 | 2788 | 2762 | 2740 | 2781 | 2763 | 330 |
| $D_2(1D)$ | 2816 | 2740 | 2775 | 2806 | 2800 | 2693 | 2745 | | |
| $D_2'(1D)$ | 2845 | 2760 | 2833 | 2850 | – | 2789 | 2745 | 2763 | 0 |
| $D(1^3D_3)$ | 2833 | 2780 | 2799 | 2863 | – | 2719 | 2800/2762 | | |
| $D(1^3F_2)$ | 3132 | 3000 | 3101 | 3090 | – | – | – | | |
| $D_3(1F)$ | 3109 | 3010 | 3074 | 3129 | – | – | – | | |
| $D_3'(1F)$ | 3144 | 3030 | 3123 | 3145 | – | – | – | | |
| $D(1^3F_4)$ | 3113 | 3030 | 3091 | 3187 | – | – | – | | |

# Careful observation of experimental spectra



- nonrelativistic and classical classification ${}^{2S+1}L_J$

# Careful observation of experimental spectra



- nonrelativistic and classical classification $^{2S+1}L_J$
- Is $L$ conserved? GI does but heavy-quark symmetry doesn't

# Careful observation of experimental spectra



- nonrelativistic and classical classification ${}^{2S+1}L_J$
- Is $L$ conserved? GI does but heavy-quark symmetry doesn't
- GI, ZVR, LS : Godfrey-Isgur model and its associates

# Careful observation of experimental spectra



- nonrelativistic and classical classification $^{2S+1}L_J$
- Is $L$ conserved? GI does but heavy-quark symmetry doesn't

- GI, ZVR, LS : Godfrey-Isgur model and its associates
- DE, MMS (EFG) : relativistic potential model respecting heavy-quark symmetry

# Careful observation of experimental spectra



- nonrelativistic and classical classification $^{2S+1}L_J$
- Is $L$ conserved? GI does but heavy-quark symmetry doesn't

- GI, ZVR, LS : Godfrey-Isgur model and its associates
- DE, MMS (EFG) : relativistic potential model respecting heavy-quark symmetry
- EXP : experimental data

# Careful observation of experimental spectra



- nonrelativistic and classical classification $^{2S+1}L_J$
- Is $L$ conserved? GI does but heavy-quark symmetry doesn't

- GI, ZVR, LS : Godfrey-Isgur model and its associates
- DE, MMS (EFG) : relativistic potential model respecting heavy-quark symmetry
- EXP : experimental data
- Average : average of a spin multiplet

# Careful observation of experimental spectra

- nonrelativistic and classical classification $^{2S+1}L_J$
- Is $L$ conserved? GI does but heavy-quark symmetry doesn't

- GI, ZVR, LS : Godfrey-Isgur model and its associates
- DE, MMS (EFG) : relativistic potential model respecting heavy-quark symmetry
- EXP : experimental data
- Average : average of a spin multiplet
- Gap : gap between spin multiplets ($\sim \Lambda_Q = 300$ MeV)

Degeneracy
T. Matsuki
Motivation
Analytical analysis
Conclusions and summary

expanding system in $1/m_Q$

$$\left(H_{-1}+H_0+\cdots-m_Q\right)\left(\psi_\ell^0+\psi_\ell^1+\cdots\right)$$
$$=\left(E_\ell^0+E_\ell^1+\cdots-m_Q\right)\left(\psi_\ell^0+\psi_\ell^1+\cdots\right)$$

# Relativistic potential model respecting heavy-quark symmetry

expanding system in $1/m_Q$

$$\begin{aligned}&(H_{-1} + H_0 + \cdots - m_Q)\left(\psi_\ell^0 + \psi_\ell^1 + \cdots\right)\\ = &\left(E_\ell^0 + E_\ell^1 + \cdots - m_Q\right)\left(\psi_\ell^0 + \psi_\ell^1 + \cdots\right)\end{aligned}$$

Lowest order equation of motion

$$\left(\vec{\alpha}_q \cdot \vec{p} + \beta_q m_q\right)\Psi_\ell^+ = E_\ell^0 \Psi_\ell^+$$

One Dirac particle equation ($\Psi_\ell^+$ : $4\times4$ spinor)

# Godfrey-Isgur model



Equation of motion which conserves $L$, $J$, and $j_\ell$

$$H\Psi = (H_0 + V)\Psi = E\Psi,$$
$$H_0 = \sqrt{p^2 + m_1} + \sqrt{p^2 + m_2^2},$$
$$V = H^{conf} + H^{hyp} + H^{SO}$$

# Godfrey-Isgur model



Equation of motion which conserves $L$, $J$, and $j_\ell$

$$H\Psi = (H_0 + V)\Psi = E\Psi,$$
$$H_0 = \sqrt{p^2 + m_1} + \sqrt{p^2 + m_2^2},$$
$$V = H^{conf} + H^{hyp} + H^{SO}$$

$H^{SO}$ (spin-orbit term) breaks rotational symmetry

# Anylytical analysis (heavy-quark symmetry)



Lowest order Hamiltonian

$$\begin{aligned} H_0 &= \vec{\alpha}_q \cdot \vec{p} + m_q \beta_q, \\ \left[H_0, \vec{L}\right] &= -i\alpha_q \times \vec{p}, \left[H_0, \frac{1}{2}\vec{\Sigma}_q\right] = i\vec{\alpha}_q \times \vec{p} \end{aligned}$$

which conserves light quark degrees of freedom $\vec{j}_\ell = \vec{L} + 1/2\vec{\Sigma}_q$ and total angular momentum $\vec{j} = \vec{j}_\ell + 1/2\vec{\Sigma}_Q$

# Analytical analysis (E.V. of $[H_0, \vec{L}^2]$)



Calculate $[H_0, \vec{L}^2]$

$$\mathcal{M} = [H_0, \vec{L}^2] = i\alpha_{qj}\left(ip_j - r_j p^2 + (r \cdot p)p_j\right) \equiv \alpha_{qj} f_j(r, p)$$

# Analytical analysis (E.V. of $[H_0, \vec{L}^2]$)



Calculate $[H_0, \vec{L}^2]$

$$\mathcal{M} = [H_0, \vec{L}^2] = i\alpha_{qj}\left(ip_j - r_j p^2 + (r\cdot p)p_j\right) \equiv \alpha_{qj} f_j(r,p)$$

For any operator $\mathcal{O}$ and eigenfunction $\Psi_\ell$, we have

**Theorem 2.1**

$$\langle \Psi_\ell | \left[H_0, \mathcal{O}\right] \Psi_\ell \rangle = 0$$

# Analytical analysis (E.V. of $[H_0, \vec{L}^2]$)



Calculate $[H_0, \vec{L}^2]$

$$\mathcal{M} = [H_0, \vec{L}^2] = i\alpha_{qj}\left(ip_j - r_j p^2 + (r\cdot p)p_j\right) \equiv \alpha_{qj} f_j(r,p)$$

For any operator $\mathcal{O}$ and eigenfunction $\Psi_\ell$, we have

**Theorem 2.1**

$$\langle \Psi_\ell | \, [H_0, \mathcal{O}] \, \Psi_\ell \rangle = 0$$

**Proof.**

$$\langle \Psi_\ell | \, [H_0, \mathcal{O}] \, \Psi_\ell \rangle = \langle \Psi_\ell | \left(E_0\mathcal{O} - \mathcal{O}E_0\right) |\Psi_\ell\rangle = 0$$

□

# Analytical analysis (actual wave function)



General actual wave function is given by

$$\psi_\ell = \Psi_\ell^+ + \sum_{\ell'} \left( c_+^{\ell,\ell'} \Psi_{\ell'}^+ + c_-^{\ell,\ell'} \Psi_{\ell'}^- \right),$$

where $\ell = \{k, j, m\}$ with a total angular momentum $j$ and its $z$-component $m$. $(k, j_\ell) = (-1, (1/2)^-), (+1, (1/2)^+), \cdots$ and

# Analytical analysis (actual wave function)



General actual wave function is given by

$$\psi_\ell = \Psi_\ell^+ + \sum_{\ell'} \left( c_+^{\ell,\ell'} \Psi_{\ell'}^+ + c_-^{\ell,\ell'} \Psi_{\ell'}^- \right),$$

where $\ell = \{k, j, m\}$ with a total angular momentum $j$ and its $z$-component $m$. $(k, j_\ell) = (-1, (1/2)^-), (+1, (1/2)^+), \cdots$ and

$$\Psi_\ell^+ = \begin{pmatrix} 0 \ \psi_{jm}^k \end{pmatrix}, \quad \Psi_\ell^- = \begin{pmatrix} \psi_{jm}^k \ 0 \end{pmatrix},$$

$$\psi_{jm}^k(r, \Omega) = \frac{1}{r} \begin{pmatrix} u_k(r) y_{jm}^k \\ i v_k(r) y_{jm}^{-k} \end{pmatrix}$$

# Analytical analysis (actual wave function)



General actual wave function is given by

$$\psi_\ell = \Psi_\ell^+ + \sum_{\ell'} \left( c_+^{\ell,\ell'} \Psi_{\ell'}^+ + c_-^{\ell,\ell'} \Psi_{\ell'}^- \right),$$

where $\ell = \{k, j, m\}$ with a total angular momentum $j$ and its $z$-component $m$. $(k, j_\ell) = (-1, (1/2)^-), (+1, (1/2)^+), \cdots$ and

$$\Psi_\ell^+ = \begin{pmatrix} 0 \ \psi_{jm}^k \end{pmatrix}, \quad \Psi_\ell^- = \begin{pmatrix} \psi_{jm}^k \ 0 \end{pmatrix},$$

$$\psi_{jm}^k(r, \Omega) = \frac{1}{r} \begin{pmatrix} u_k(r) y_{jm}^k \\ i v_k(r) y_{jm}^{-k} \end{pmatrix}$$

$\Psi_\ell^{+/-}$ : positive/negative component of $Q$

# Analytical analysis ($0^-$ & $1^-$ states)



$0^-$ & $1^-$ states are given by

$$
\begin{aligned}
\psi_\ell(0^-) &= \Psi^+_{-1} + c^{-1,1}_{1-}\Psi^-_1 + O\left(1/m_Q^2\right), \\
\psi_\ell(1^-) &= \Psi^+_{-1} + c^{-1,2}_{1+}\Psi^+_2 + c^{-1,-2}_{1-}\Psi^-_{-2} + O\left(1/m_Q^2\right),
\end{aligned}
$$

# Analytical analysis ($0^-$ & $1^-$ states)



$0^-$ & $1^-$ states are given by

$$\begin{aligned}
\psi_\ell(0^-) &= \Psi^+_{-1} + c_{1-}^{-1,1}\Psi^-_1 + O\left(1/m_Q^2\right), \\
\psi_\ell(1^-) &= \Psi^+_{-1} + c_{1+}^{-1,2}\Psi^+_2 + c_{1-}^{-1,-2}\Psi^-_{-2} + O\left(1/m_Q^2\right),
\end{aligned}$$

$c_{1\pm}^{k',k}$ are of the order of $1/m_Q$.

# Analytical analysis ($0^-$ & $1^-$ states)



$0^-$ & $1^-$ states are given by

$$\psi_\ell(0^-) = \Psi^+_{-1} + c^{-1,1}_{1-}\Psi^-_1 + O\left(1/m_Q^2\right),$$
$$\psi_\ell(1^-) = \Psi^+_{-1} + c^{-1,2}_{1+}\Psi^+_2 + c^{-1,-2}_{1-}\Psi^-_{-2} + O\left(1/m_Q^2\right),$$

$c^{k',k}_{1\pm}$ are of the order of $1/m_Q$.
Using these w.f.'s, matrix elements are given by

$$\left\langle\Psi^+_{\ell'}|\mathcal{M}|\Psi^-_\ell\right\rangle = \left\langle\Psi^-_{\ell'}|\mathcal{M}|\Psi^+_\ell\right\rangle = 0,$$
$$\left\langle\Psi^\pm_{\ell'}|\mathcal{M}|\Psi^\pm_\ell\right\rangle = i\int d^3r\frac{1}{r}\left[-v_{k'}(r)y^{-k'\dagger}_{j'm'}\sigma_n f_n(r,p)\right.$$
$$\left.\times\left(\frac{1}{r}u_k(r)y^k_{jm}\sigma_i\right) + u_{k'}(r)y^{k'\dagger}_{j'm'}\sigma_n f_n(r,p)\left(\frac{1}{r}v_k(r)y^{-k}_{jm}\sigma_i\right)\right]$$

# Conclusions and summary

- Matrix element (M.E.) starts from $(1/m_Q)^2$ for $0^-$

# Conclusions and summary

- Matrix element (M.E.) starts from $(1/m_Q)^2$ for $0^-$
- M.E. starts from $1/m_Q$ for $1^-$ (e.g., $\langle\Psi_{-1}^{+}|\mathcal{M}|\Psi_{2}^{+}\rangle$)

# Conclusions and summary



- Matrix element (M.E.) starts from $(1/m_Q)^2$ for $0^-$
- M.E. starts from $1/m_Q$ for $1^-$ (e.g., $\langle\Psi^+_{-1}|\mathcal{M}|\Psi^+_2\rangle$)
- general M.E. starts from $1/m_Q$ because of general expression for wave function

# Conclusions and summary



- Matrix element (M.E.) starts from $(1/m_Q)^2$ for $0^-$
- M.E. starts from $1/m_Q$ for $1^-$ (e.g., $\langle\Psi^+_{-1}|\mathcal{M}|\Psi^+_2\rangle$)
- general M.E. starts from $1/m_Q$ because of general expression for wave function
- Neglection of $v_k(r)$ leads to rotational symmetry
  $\vec{L}^2 y^k_{jm} = k(k+1)y^k_{jm} = L(L+1)y^k_{jm}$

# Conclusions and summary



- Matrix element (M.E.) starts from $(1/m_Q)^2$ for $0^-$
- M.E. starts from $1/m_Q$ for $1^-$ (e.g., $\langle\Psi^+_{-1}|\mathcal{M}|\Psi^+_2\rangle$)
- general M.E. starts from $1/m_Q$ because of general expression for wave function
- Neglection of $v_k(r)$ leads to rotational symmetry $\vec{L}^2 y^k_{jm} = k(k+1)y^k_{jm} = L(L+1)y^k_{jm}$
- This corresponds to nonrelativistic limit of heavy quark symmetry!!

# Conclusions and summary



- Matrix element (M.E.) starts from $(1/m_Q)^2$ for $0^-$
- M.E. starts from $1/m_Q$ for $1^-$ (e.g., $\langle\Psi^+_{-1}|\mathcal{M}|\Psi^+_2\rangle$)
- general M.E. starts from $1/m_Q$ because of general expression for wave function
- Neglection of $v_k(r)$ leads to rotational symmetry $\vec{L}^2 y^k_{jm} = k(k+1)y^k_{jm} = L(L+1)y^k_{jm}$
- This corresponds to nonrelativistic limit of heavy quark symmetry!!
- Application of our idea : $QQq$, $QQ\bar{Q}q$, heavy quarks + a brown mock of light quarks

# Conclusions and summary



- Matrix element (M.E.) starts from $(1/m_Q)^2$ for $0^-$
- M.E. starts from $1/m_Q$ for $1^-$ (e.g., $\langle \Psi^+_{-1}|\mathcal{M}|\Psi^+_2 \rangle$)
- general M.E. starts from $1/m_Q$ because of general expression for wave function
- Neglection of $v_k(r)$ leads to rotational symmetry $\vec{L}^2 y^k_{jm} = k(k+1)y^k_{jm} = L(L+1)y^k_{jm}$
- This corresponds to nonrelativistic limit of heavy quark symmetry!!
- Application of our idea : $QQq$, $QQ\bar{Q}q$, heavy quarks + a brown mock of light quarks
- LHCb and forthcoming BelleII are expected to test our observation

# Thanks for your attention

## Thanks for your attention!