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# ***POLARIZED QUARK HADRONIZATION***

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*Collaborators:*

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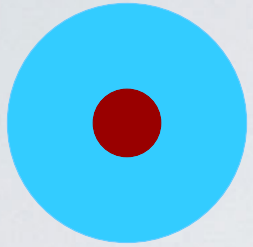


# Outlook

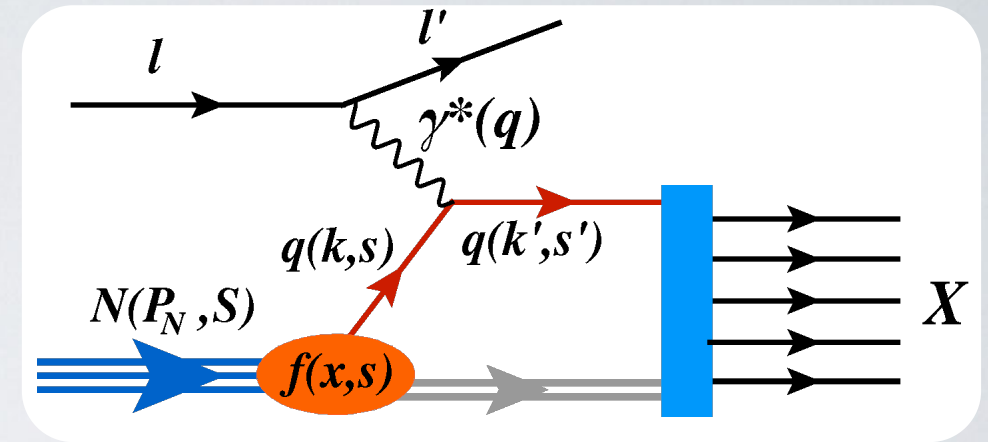
- ❖ *Introduction and Motivation.*
- ❖ *Brief overview of NJL-jet and the MC implementation.*
- ❖ *The recent progress on modelling polarised quark hadronisation.*
- ❖ *Conclusions.*

# NUCLEON PARTON DISTRIBUTION FUNCTIONS

- *Unpolarized* quark in *Unpolarized* nucleon.



$$f_1^q(x, Q^2)$$



- **The momentum and the spin of the partons are correlated with the polarization of the nucleon!**

- *Longitudinally* polarized quark in *Longitudinally* polarized nucleon.

$$g_{1L}^q(x, Q^2)$$

- *Transversely* polarized quark in *Transversely* polarized nucleon.

$$h_{1T}^q(x, Q^2)$$

Only for quarks (leading twist)!

Chiral-odd: Suppressed in Inclusive DIS: not well known!

# PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

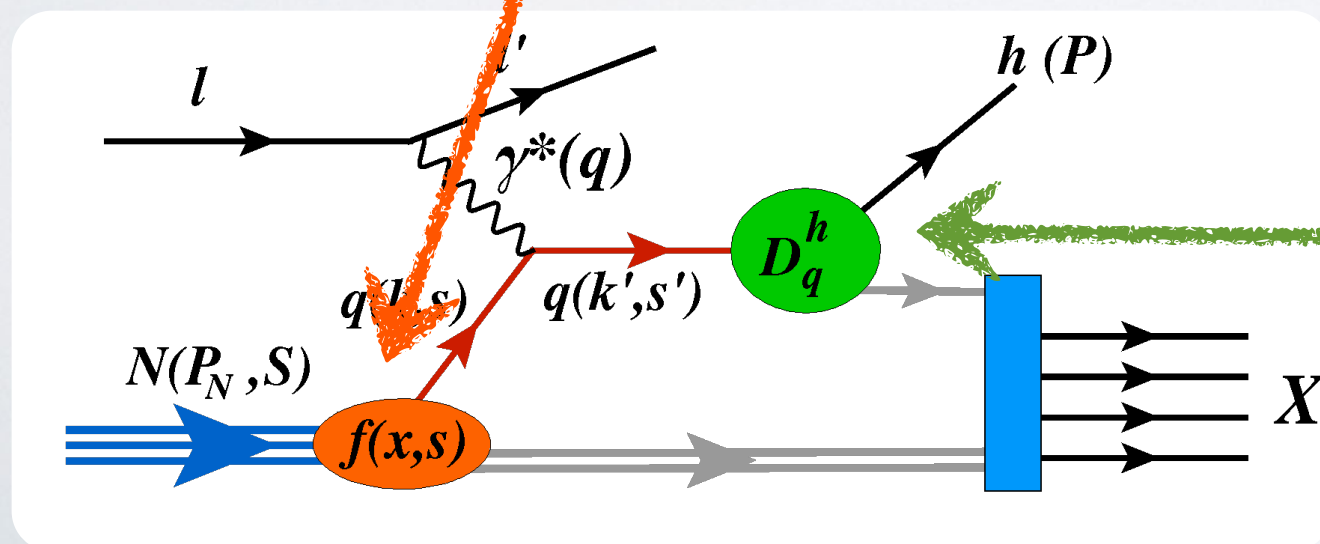
- **The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon:**
- **TMD PDFs**

N/q	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1 h_{1T}^\perp$

♦ Survive after TM integration!

• Accessible in SIDIS Process

• NEED TMD Fragmentation Functions



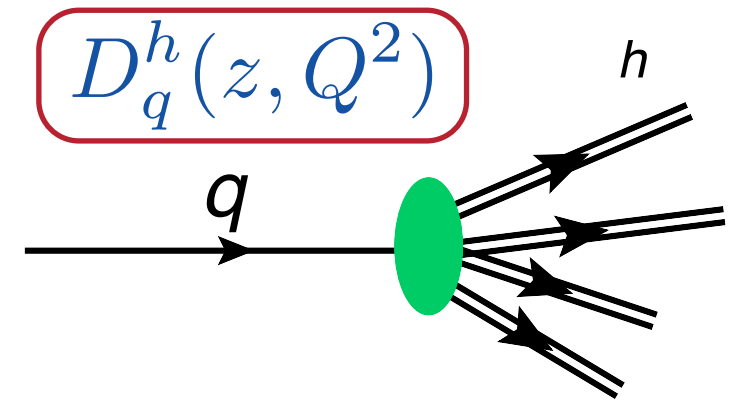
q/h	U
U	$D_1$
L	
T	$H_1^\perp$

\* unpol/spinless h!



# Collinear Fragmentation Functions (FFs)

- **Unpolarized FF** is the number density of hadron  $h$  with LC momentum fraction  $z$ , produced by quark  $q$ :



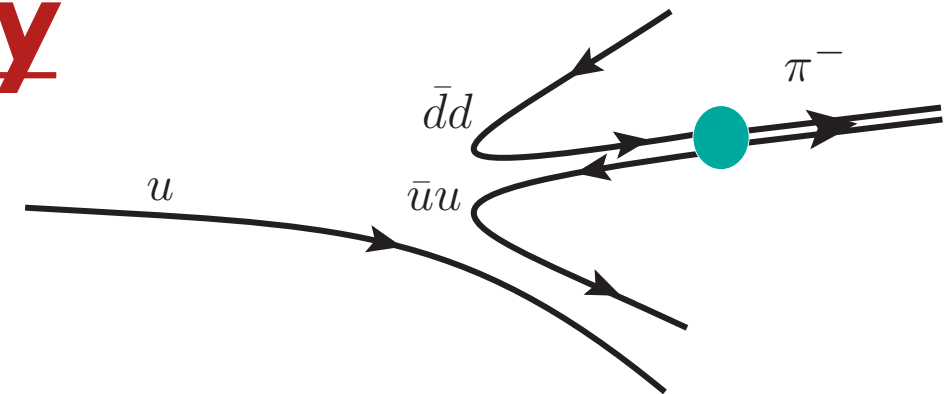
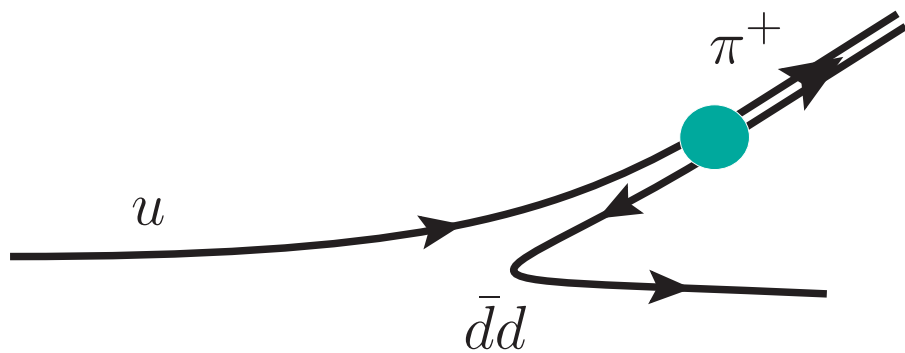
- **Favoured**: the produced hadron has a valence quark of the same flavour.

$$u \rightarrow \pi^+ (u\bar{d}) \quad u \rightarrow K^+ (u\bar{s})$$

- **Unfavoured** (disfavoured): NO valence quark of the same flavour.

$$u \rightarrow \pi^- (\bar{u}d) \quad u \rightarrow K^- (\bar{u}s)$$

**Naively**



## ► Symmetries:

♦ Charge Conjugation:  $q \Leftrightarrow \bar{q}$

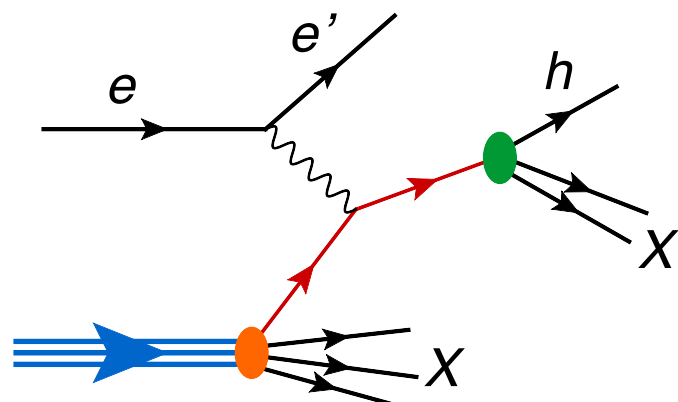
♦ Isospin:  $u \Leftrightarrow d$

$$\begin{aligned} D_u^{\pi^+} &= D_{\bar{u}}^{\pi^-} \\ D_s^{K^-} &= D_{\bar{s}}^{K^+} \end{aligned}$$

$$\begin{aligned} D_u^{\pi^+} &= D_d^{\pi^-} \\ D_u^{K^+} &= D_d^{K^0} \end{aligned}$$

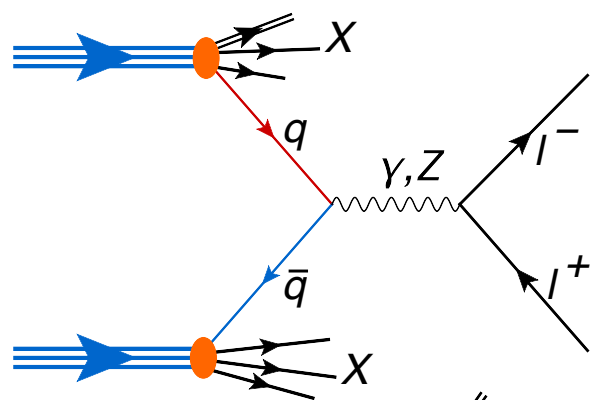


# FACTORIZATION AND UNIVERSALITY



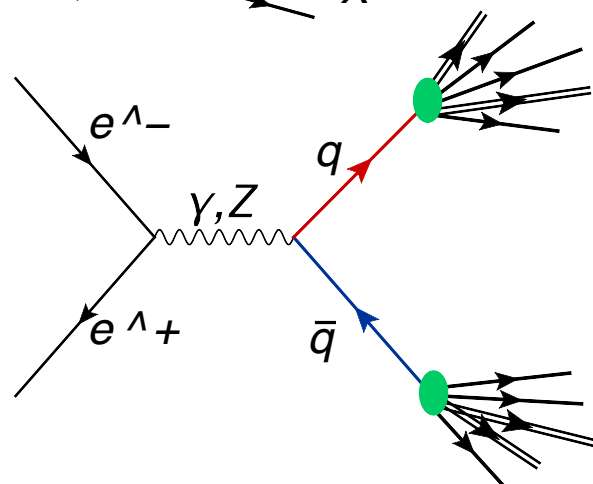
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP \rightarrow ehX} = \sum_q f_q^P \otimes \sigma^{eq \rightarrow eq} \otimes D_q^h$$



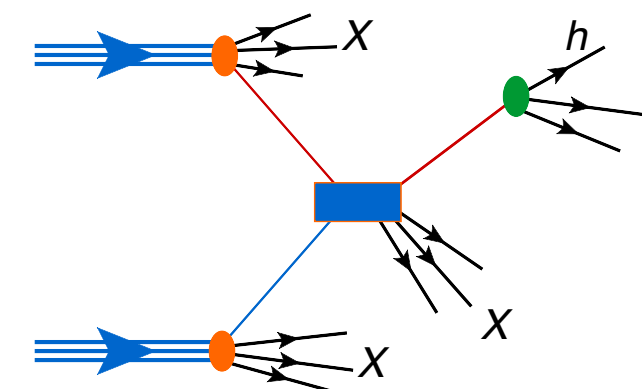
- DRELL-YAN (DY)

$$\sigma^{PP \rightarrow l^+ l^- X} = \sum_{q, q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q} \rightarrow l^+ l^-}$$



- $e^+ e^-$

$$\sigma^{e^+ e^- \rightarrow hX} = \sum_q \sigma^{e^+ e^- \rightarrow q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



- Hadron Production

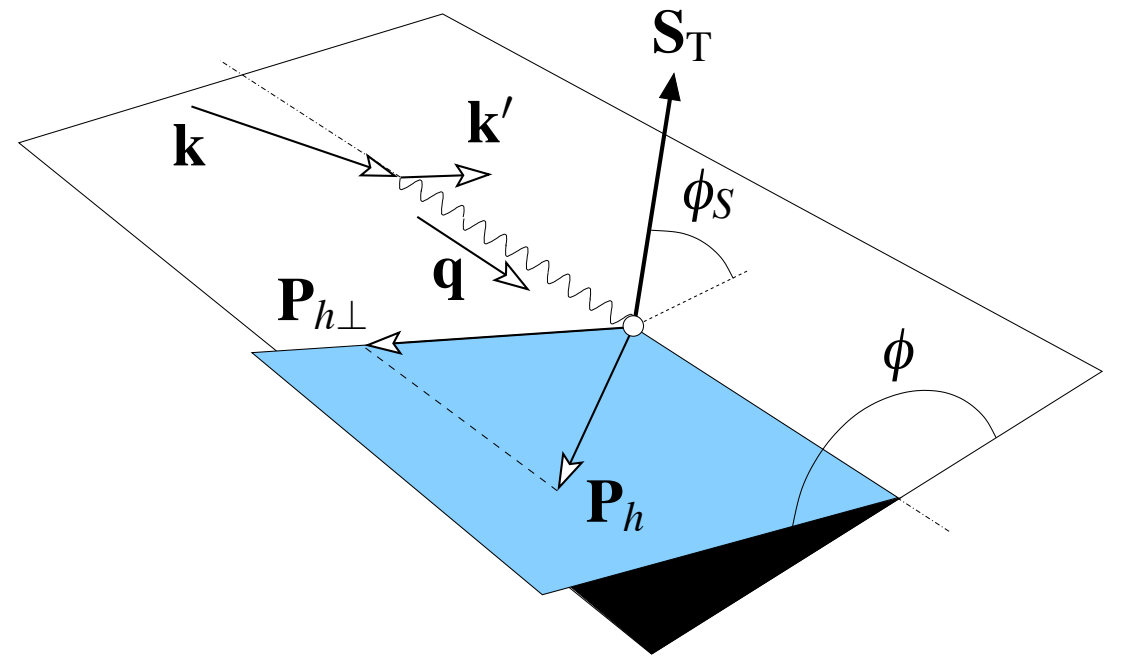
$$\sigma^{PP \rightarrow hX} = \sum_{q, q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq' \rightarrow qq'} \otimes D_q^h$$



# TMDs from SIDIS $e P \rightarrow e' h X$

A. Bacchetta et al., JHEP08 023 (2008).

- For polarized SIDIS cross-section there are **18 terms** in leading twist expansion:



$$\frac{d\sigma}{dx dy dz d\phi_S d\phi_h dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \dots$$

$$+ |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \dots \right]$$

**Collins term**

► Access the structure functions via **specific** modulations.

► LO Matching to **convolutions** of PDFs and FFs:  $P_T^2 \ll Q^2$

$$F_{UU,T} \sim \mathcal{C}[f_1 D_1] \quad F_{UT}^{\sin(\phi_h + \phi_S)} \sim \mathcal{C}[h_1 H_1^{\perp}]$$

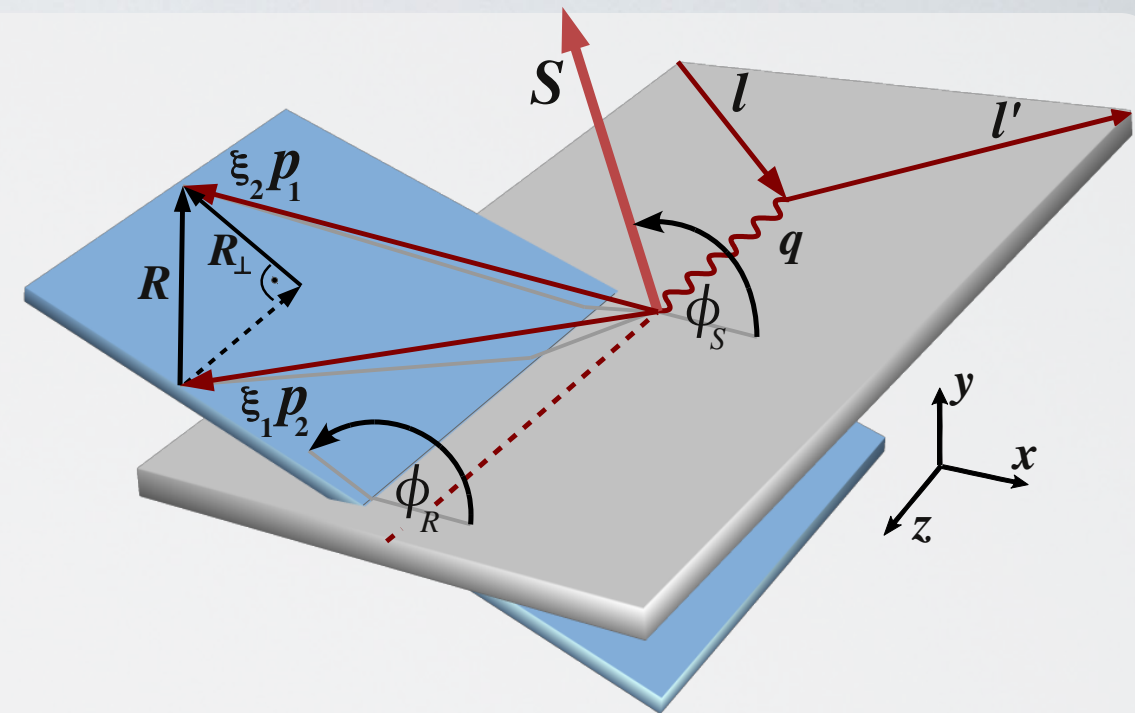
- **NEED Collins Fragmentation Function to access Transversity PDF from SIDIS!** [BELLE (II) , BaBar]



# ACCESS TO TRANSVERSITY PDF From DiFF in *SIDIS*

M. Radici, et al: PRD 65, 074031 (2002).

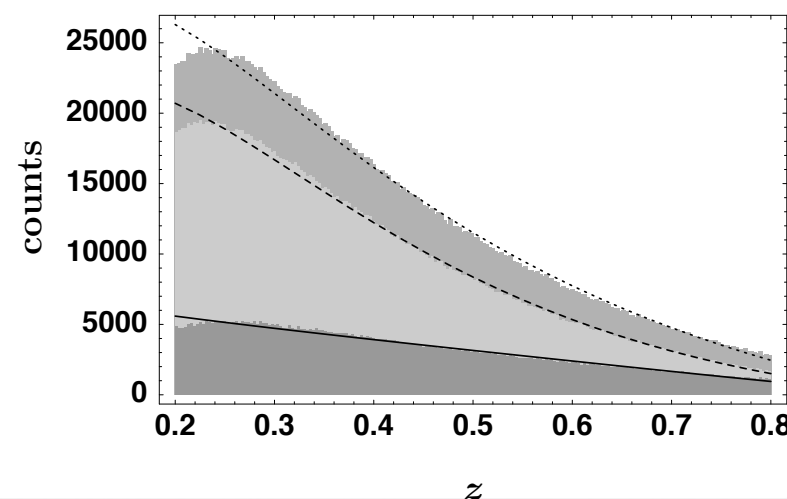
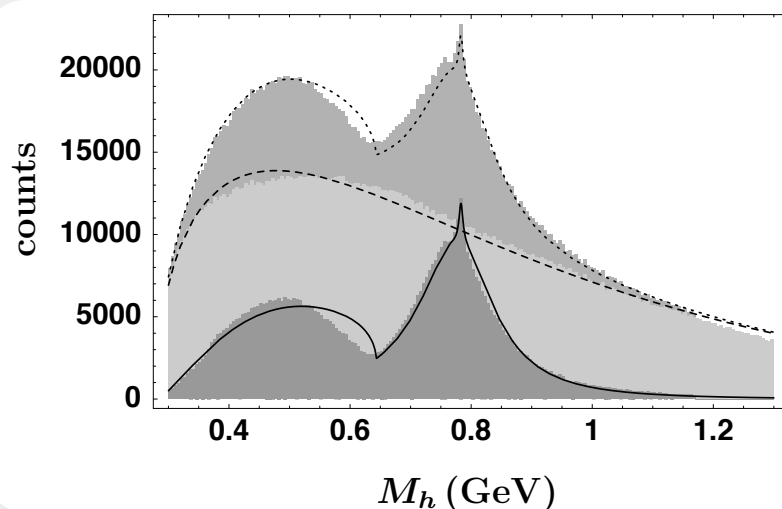
- In two hadron production from polarized target the cross section factorizes **collinearly** - no TMD!
- Allows clean access to **transversity**.
- **Unpolarized** and **Interference** Dihadron FFs are needed!



$$\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 h_1^q(x)/x H_1^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x)/x D_1^q(z, M_h^2)}$$

- Empirical Model for  $D_1^q$  has been fitted to PYTHIA simulations.

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).



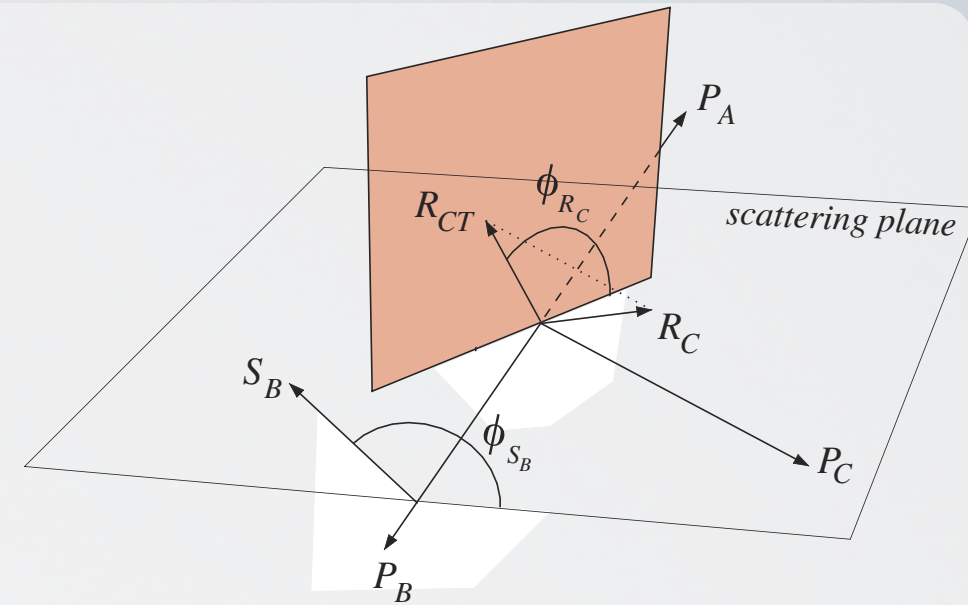
Experiments:  
BELLE,  
HERMES,  
COMPASS.



# ACCESS TO TRANSVERSITY PDF From DiFF in $P^\uparrow P$

Bacchetta, Radici: PRD 70, 094032 (2004).

- In two hadron production from polarized  $P^\uparrow P$  **assume** the cross section factorizes.
- Access to **transversity** in collinear kinematics in hadron pair from the **same quark jet**.



$$\frac{d\sigma^{PP^\uparrow} - d\sigma^{PP^\downarrow}}{d\sigma^{PP^\uparrow} + d\sigma^{PP^\downarrow}} = \frac{d\sigma_{UT}}{d\sigma_{UU}} = \sin(\phi_{RS}) A_{UT}$$

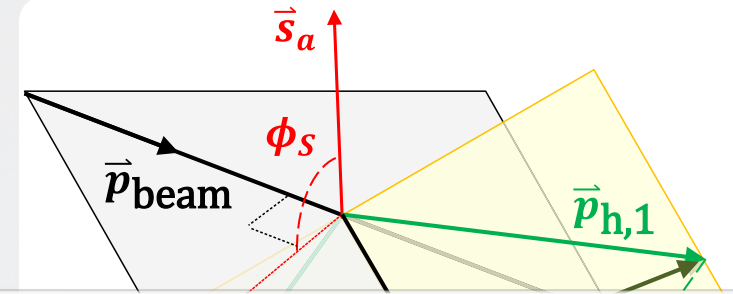
$$A_{UT} \sim \frac{\sum_{a,b,c,d} \int dx_a dx_b f_1^{a/P}(x_a) h_1^{b^\uparrow/P^\uparrow}(x_b) \frac{d\hat{\sigma}^{ab^\uparrow \rightarrow c^\uparrow d}}{d\hat{t}}}{\sum_{a,b,c,d} \int dx_a dx_b f_1^{a/P}(x_a) f_1^{b/P}(x_b) \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}} \frac{H_{1,c}^\triangleleft(z, M)}{D_{1,c}(z, M)}$$

- Again, we **need DiFFs** to extract transversity!
- No  **$u$  dominance for  $P$**  (no **charge weighting** like in SIDIS).

# Measurements in *STAR* for $P^\uparrow P$ at $\sqrt{s} = 200\text{GeV}$

STAR: Phys.Rev.Lett. 115, 242501 (2015).

- First-ever measurements of transversity in *PP*.

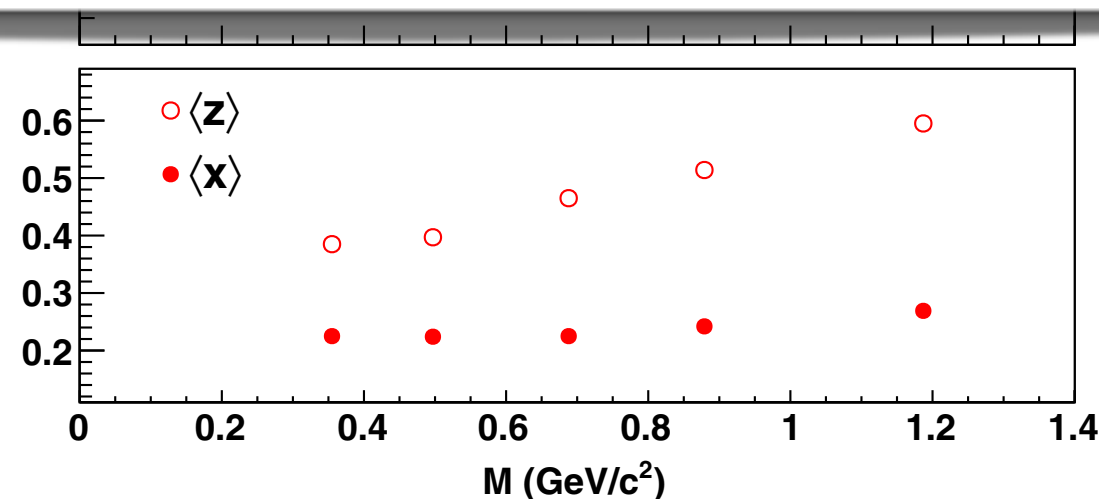


$$\frac{N^\uparrow(\phi_{RS}) - r \cdot N^\downarrow(\phi_{RS})}{N^\uparrow(\phi_{RS}) + r \cdot N^\downarrow(\phi_{RS})} = P_{\text{beam}} A_{UT} \sin(\phi_{RS})$$

- ▶ Hadron - Proton scattering is viable for accessing transversity!
- ▶ Signal in excess of  $5\sigma$  at high  $p_T$ ,  $\eta > 0.5$ .
- ▶ Possible future experimental measurement in medium-x region at J-PARC High-Momentum beam line.

- ▶ Complimentary to *D-Y* measurements proposed in

S. Kumano, arXiv:1504.05264 [hep-ph](2015).

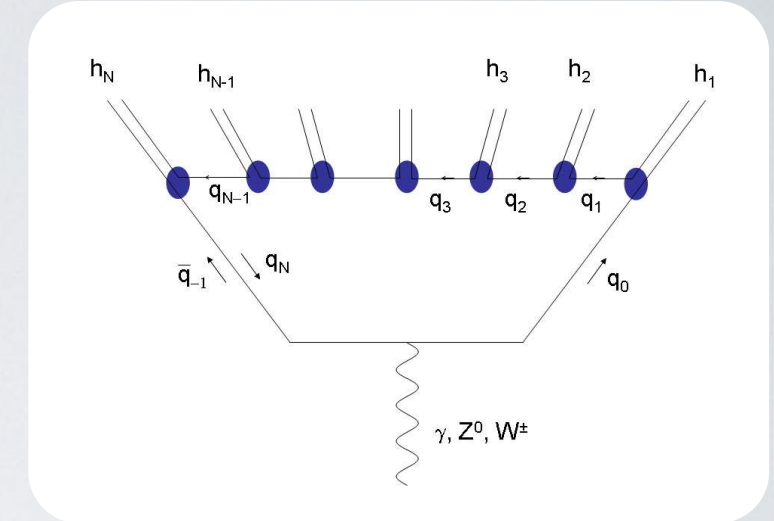




# (SOME of the) MODELS FOR FRAGMENTATION

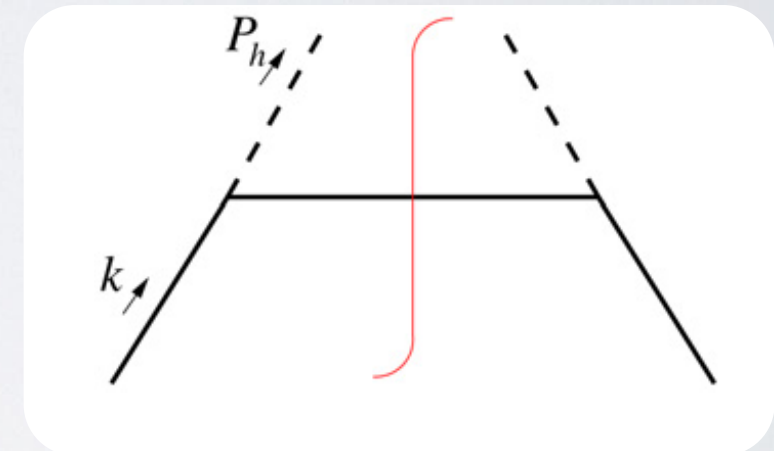
- *Lund String Model*

- Very Successful implementation in *JETSET*, *PYTHIA*.
- Highly Tunable - Limited Predictive Power.
- No Spin Effects - Formal developments by X.Artru et al but no quantitative results!



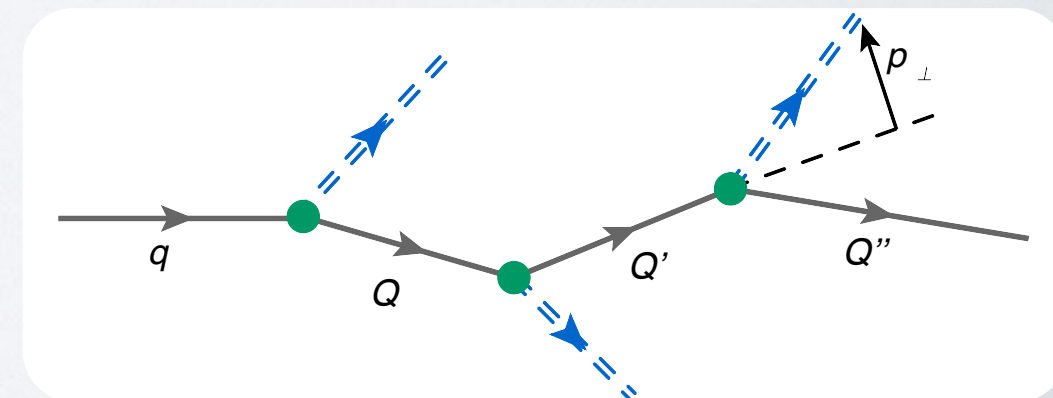
- *Spectator Model*

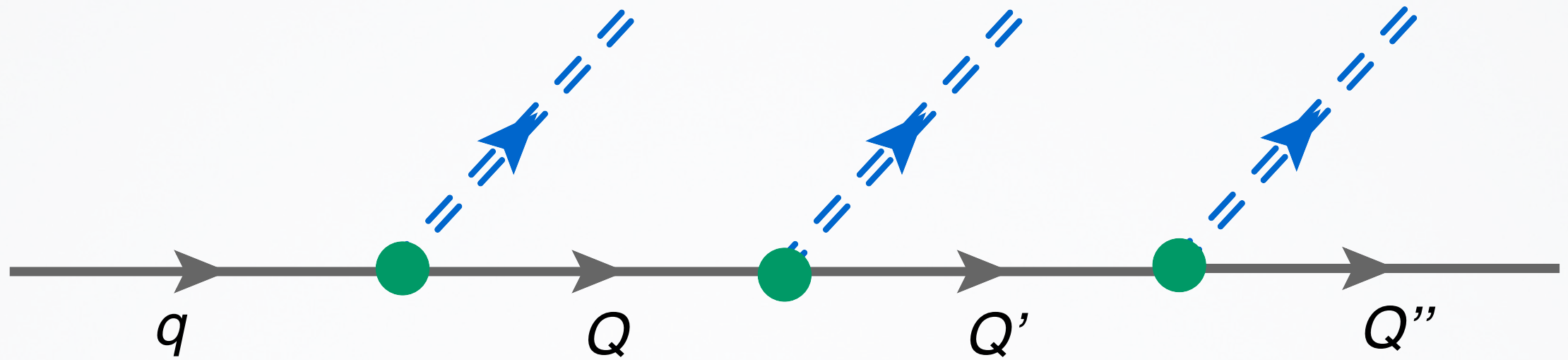
- Quark model calculations with empirical form factors.
- **No unfavored fragmentations.**
- Need to tune parameters for small  $z$  dependence.



- *NJL-jet Model*

- Multi-hadron emission framework with effective quark model input.
- Monte-Carlo framework allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.





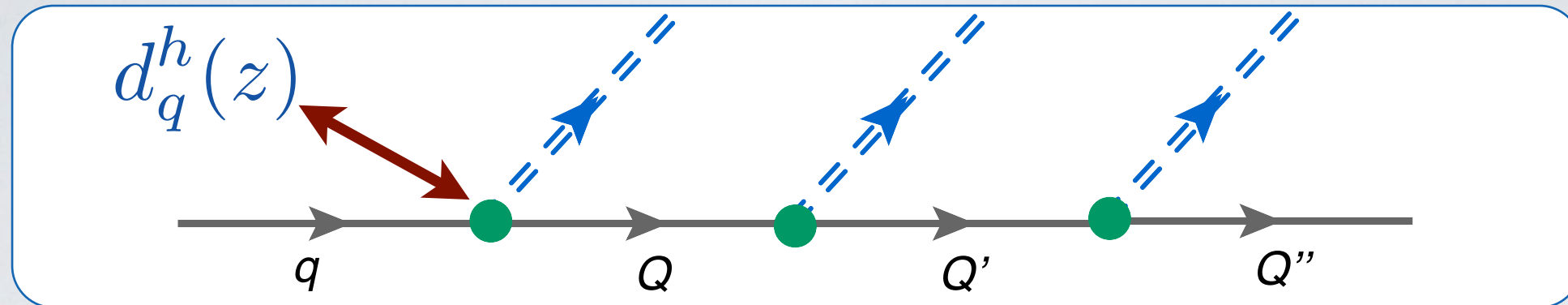
***THE NJL-jet MODEL***



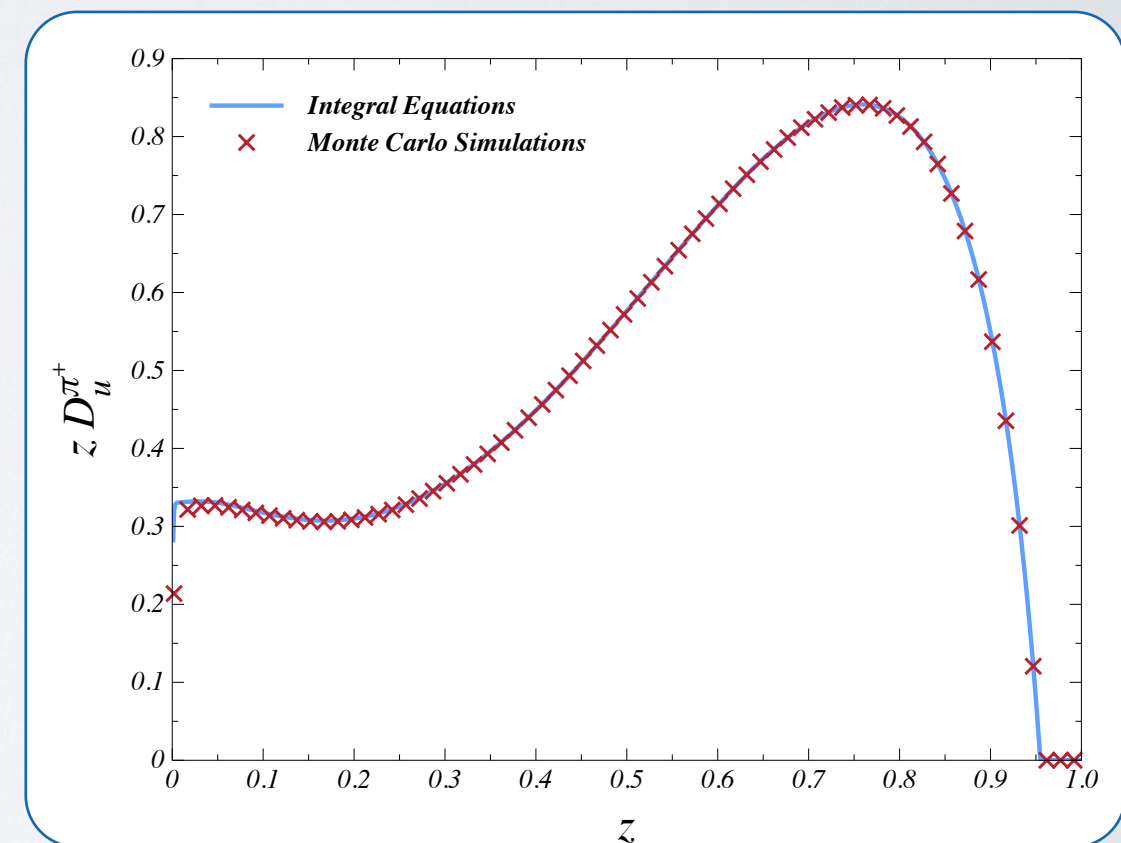
# COLLINEAR FRAGMENTATIONS FROM MC

H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

- Input: One hadron emission probability



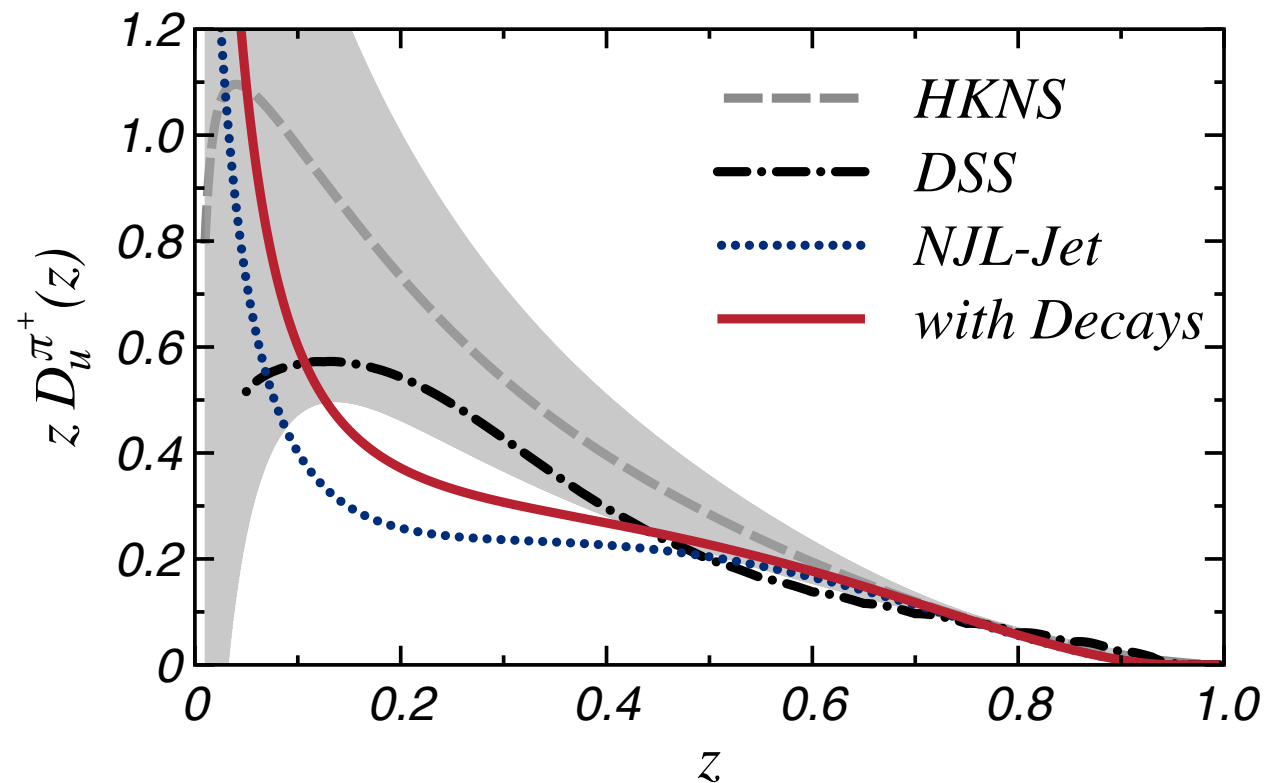
- Sample the emitted hadron type and  $z$  according to input splitting.
- CONSERVE:** Momentum and Quark Flavor in each step.
- Repeat for decay chains with the same initial quark.



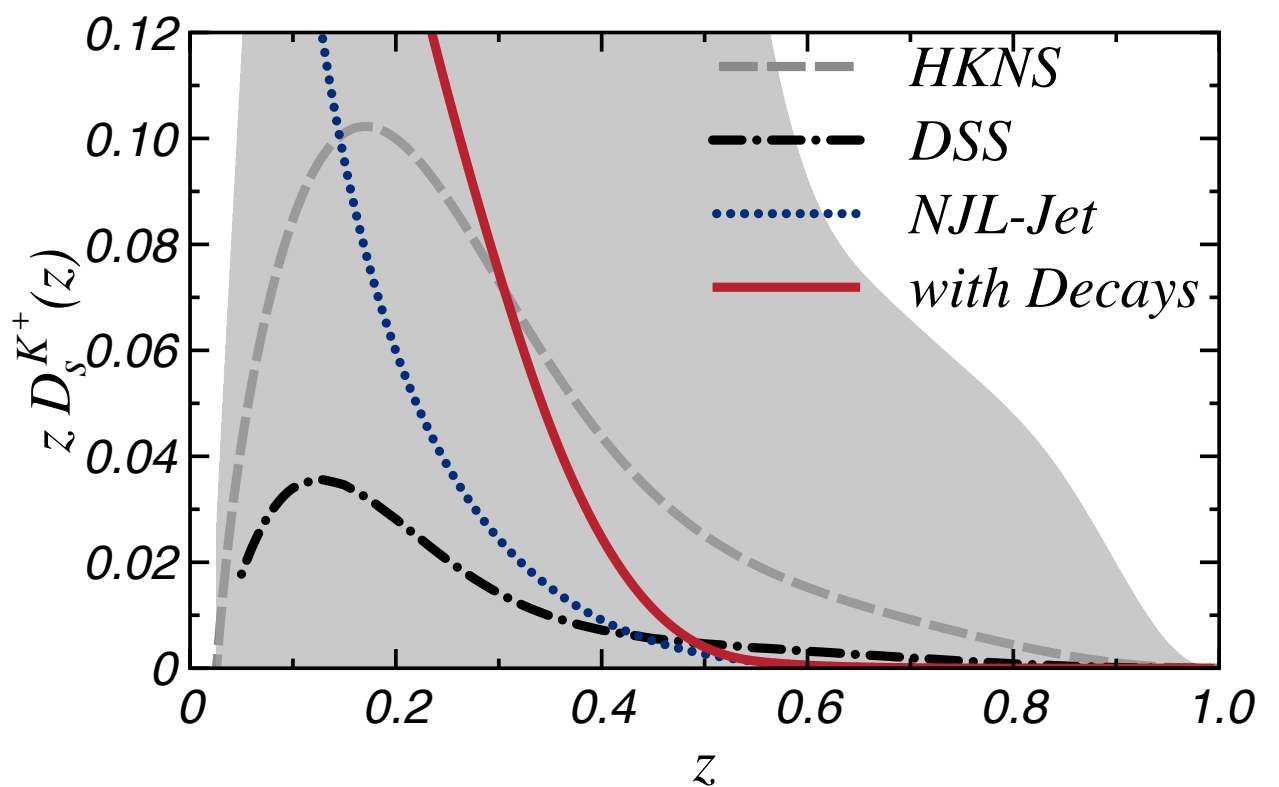
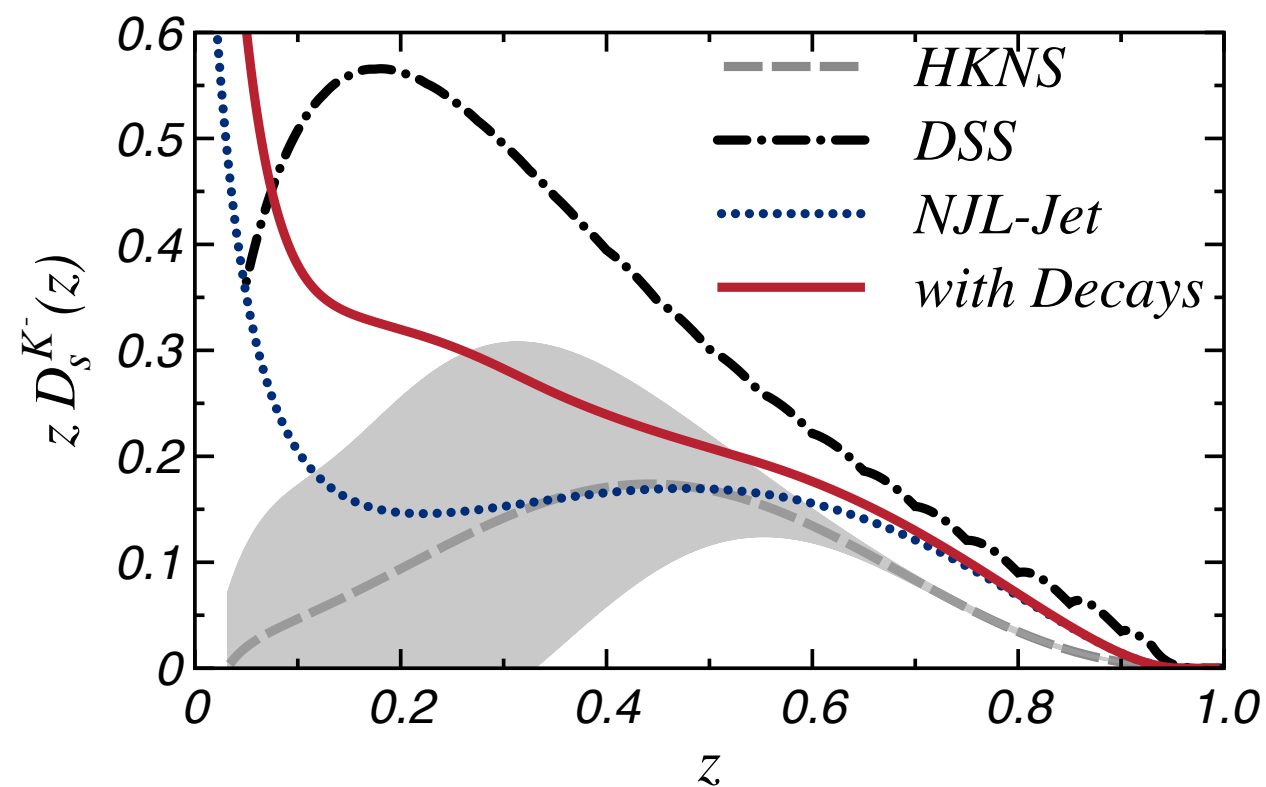
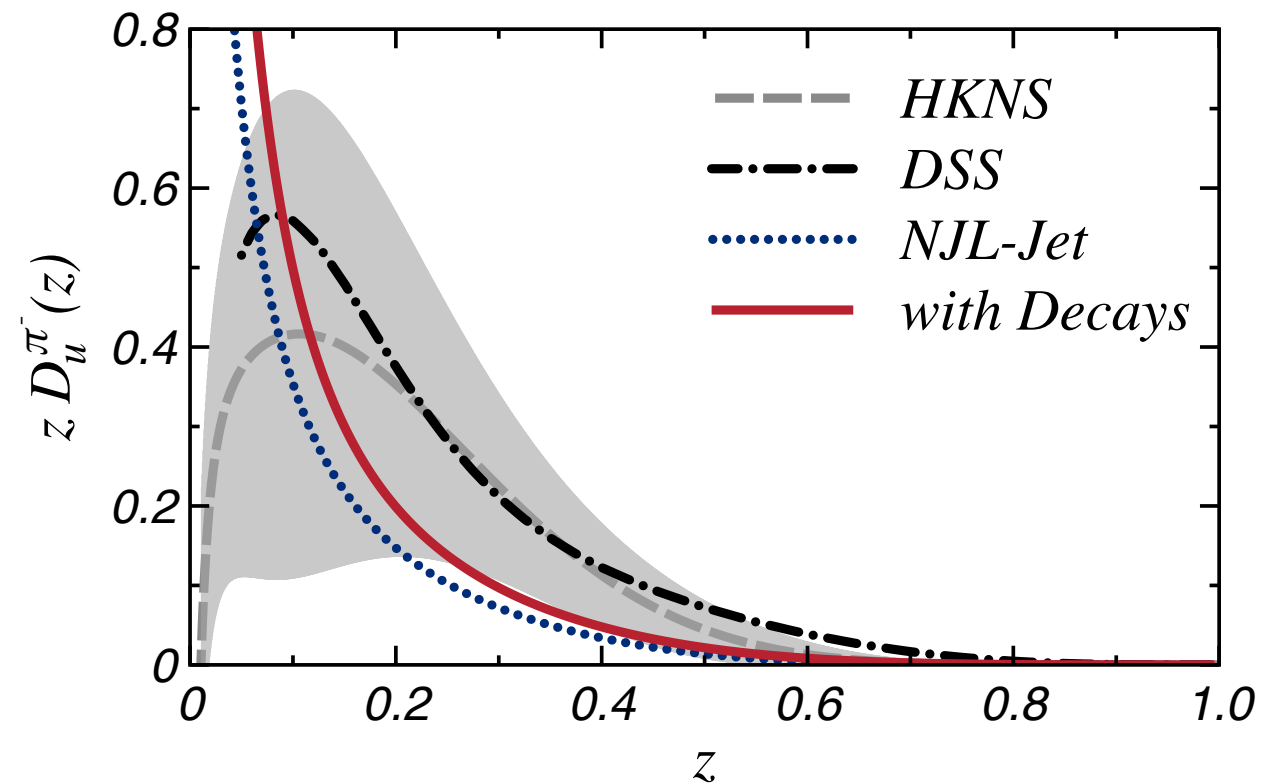
$$D_q^h(z) \Delta z = \langle N_q^h(z, z + \Delta z) \rangle \equiv \frac{\sum_{N_{Sim}s} N_q^h(z, z + \Delta z)}{N_{Sim}s}$$

# Results with VM decays: $Q^2 = 4 \text{ GeV}^2$

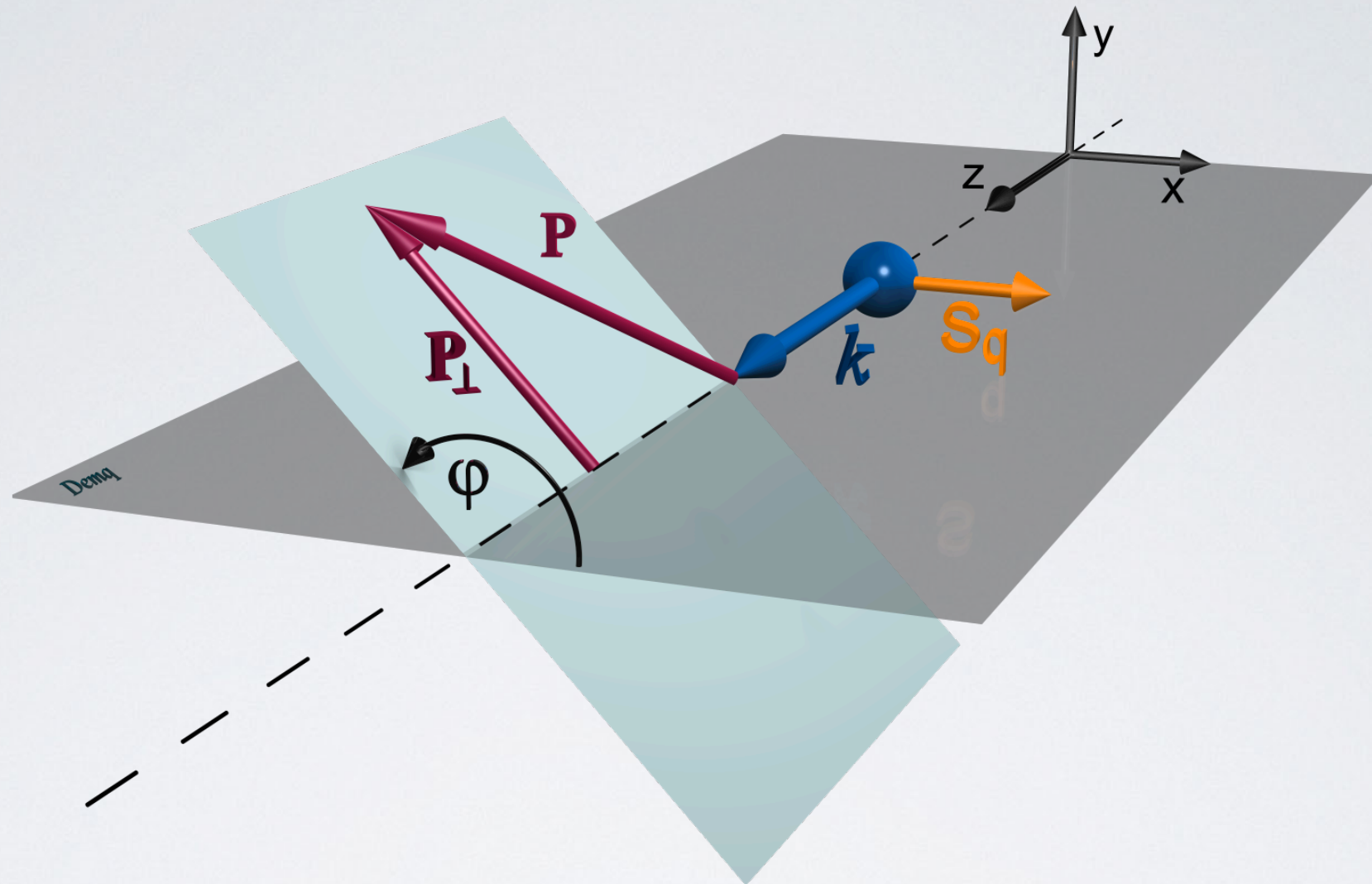
Favored



Unfavored





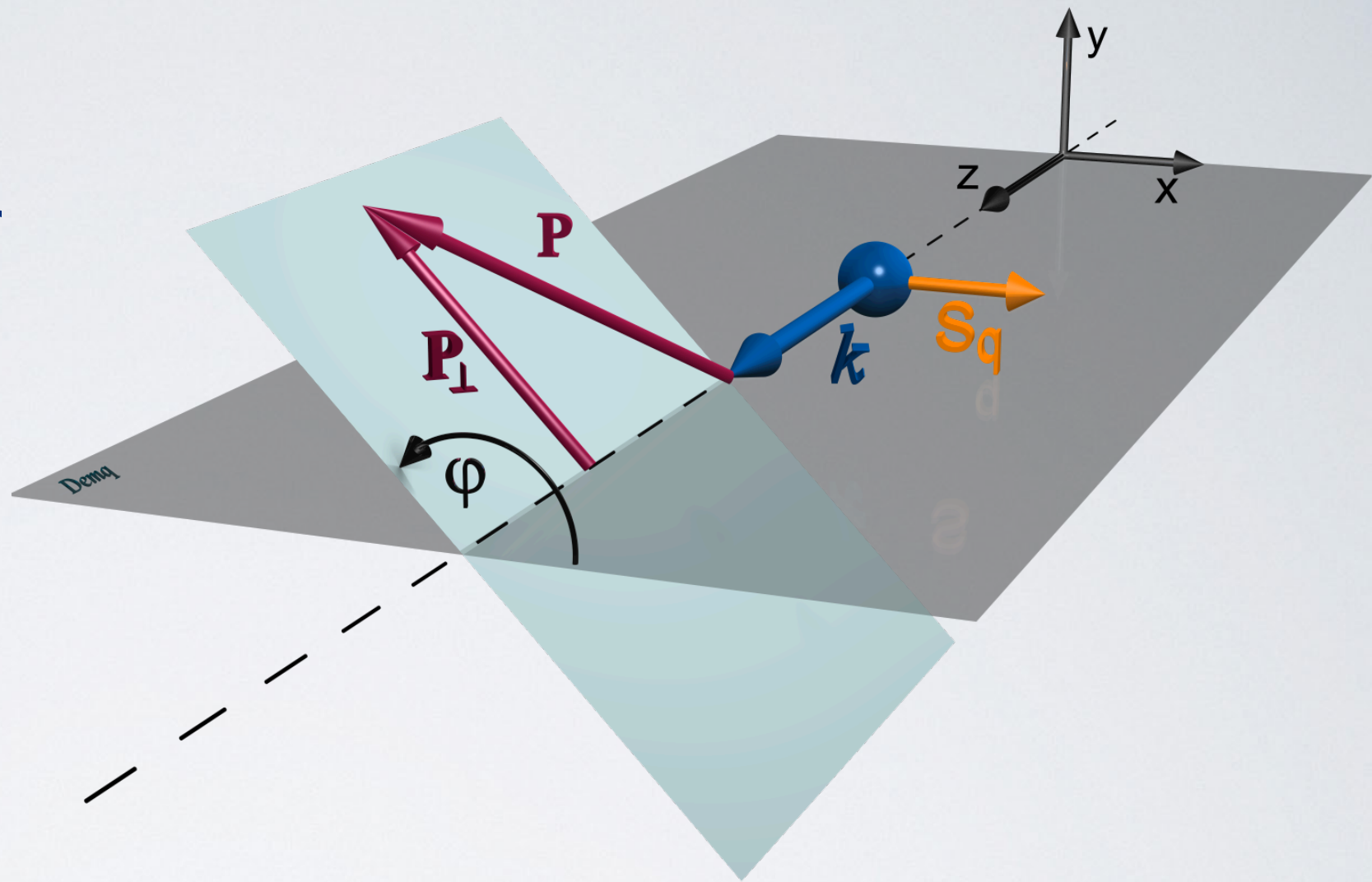


**TRANSVERSELY POLARIZED QUARK FRAGMENTATION:**  
**COLLINS EFFECT AND TWO-HADRON CORRELATIONS**

# COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarized

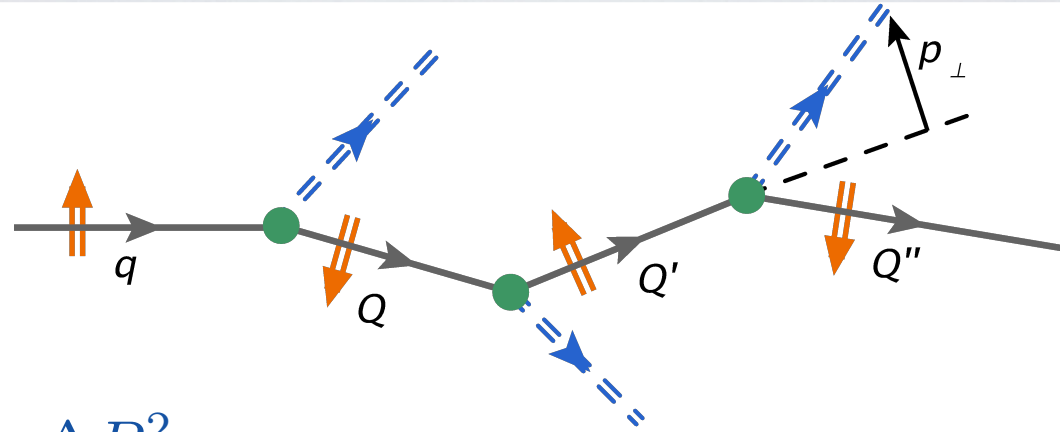
$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2) \frac{P_\perp S_q}{zm_h} \sin(\varphi)$$

Collins

- **Chiral-ODD:** Needs to be coupled with another chiral-odd quantity to be observed.



# COLLINS EFFECT - NJL-jet MKII

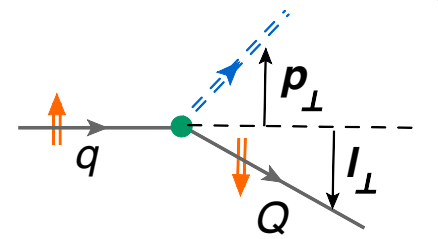


$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) \Delta z \frac{\Delta P_\perp^2}{2} \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z + \Delta z; P_\perp^2, P_\perp^2 + \Delta P_\perp^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

## MKII Model Assumptions:

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

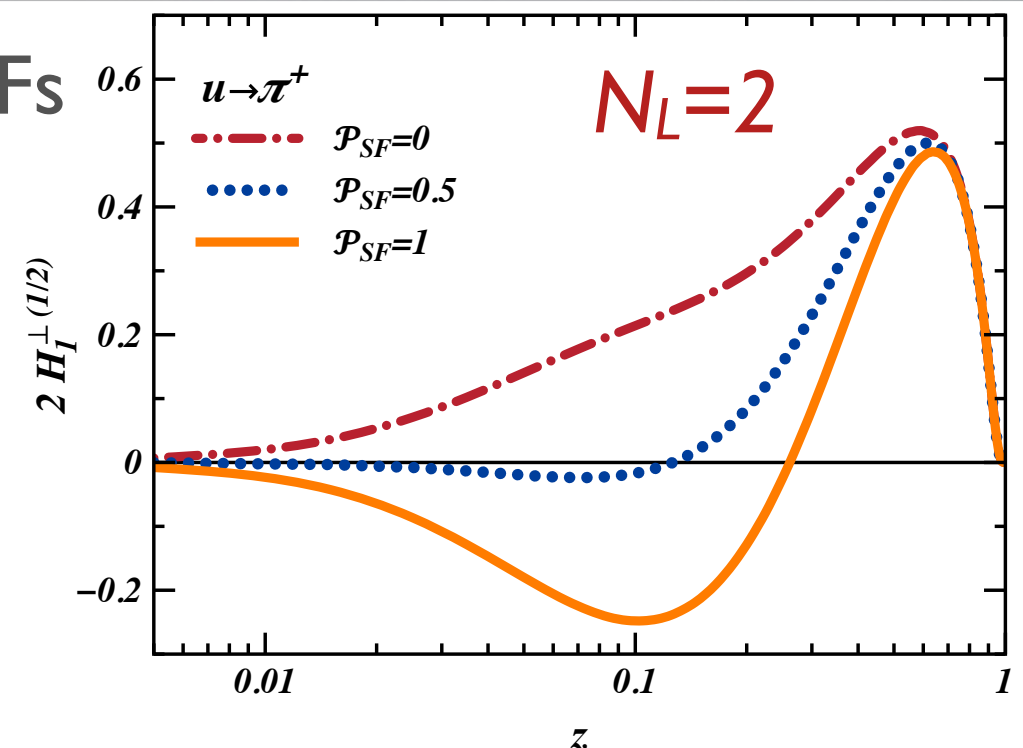
1. Allow for Collins Effect only in a SINGLE emission vertex ( $N_L^{-1}$  scaling of the resulting Collins function).
2. Use constant values for spin flip probability:  $\mathcal{P}_{SF}$ .



- ◆ Use fit form to extract unpol. and Collins FFs from  $D_{h/q^\uparrow}$ .

$$F(c_0, c_1) = c_0 - c_1 \sin(\varphi)$$

- ◆ The results for  $N_L=2$ ,  $\mathcal{P}_{SF} = 1$ .



# TWO-HADRON FRAGMENTATION

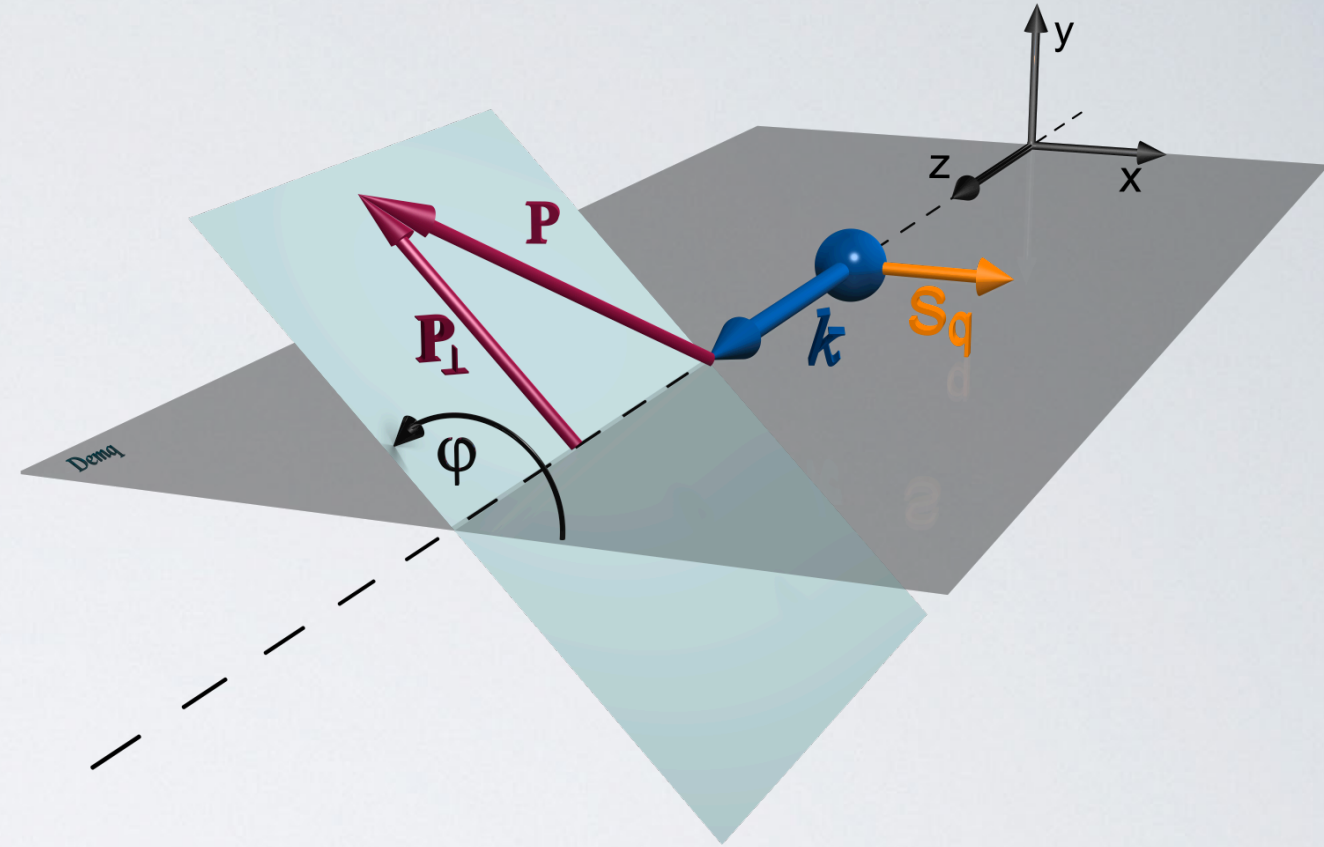
## ♦ Transformation to frame $\mathbf{k}_T = 0$

$$k = (k^-, k^+, \mathbf{0})$$

$$\mathbf{k}_T = -\mathbf{P}_T / z_h$$

$$\mathbf{P}_T = \mathbf{P}_{h_1}^\perp + \mathbf{P}_{h_2}^\perp$$

$$\mathbf{R} = (\mathbf{P}_{h_1}^\perp - \mathbf{P}_{h_2}^\perp) / 2$$



## ♦ Integrate over one or other momentum:

$$D_{q^\uparrow}^{h_1 h_2}(\varphi_R) = D_{1,q}^{h_1 h_2} + \sin(\varphi_R - \varphi_S) \mathcal{F}[H_1^\triangleleft, H_1^\perp]$$

$$D_{q^\uparrow}^{h_1 h_2}(\varphi_T) = D_{1,q}^{h_1 h_2} + \sin(\varphi_T - \varphi_S) \mathcal{F}'[H_1^\triangleleft, H_1^\perp]$$

## ♦ The IFF surviving after $\mathbf{k}_T$ integration is redefined as

**A. Bacchetta, M. Radici: PRD 69, 074026 (2004).**

$$H_1^\triangleleft(z_h, \xi, M_h^2) \equiv \int d^2 \mathbf{k}_T \left[ H_1^{\triangleleft'e}(z_h, \xi, M_h^2, k_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{k_T^2}{2M_h^2} H_1^{\perp o}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \right]$$



# RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

◆ SIDIS with transversely polarized target.

◆ Collins single spin asymmetry

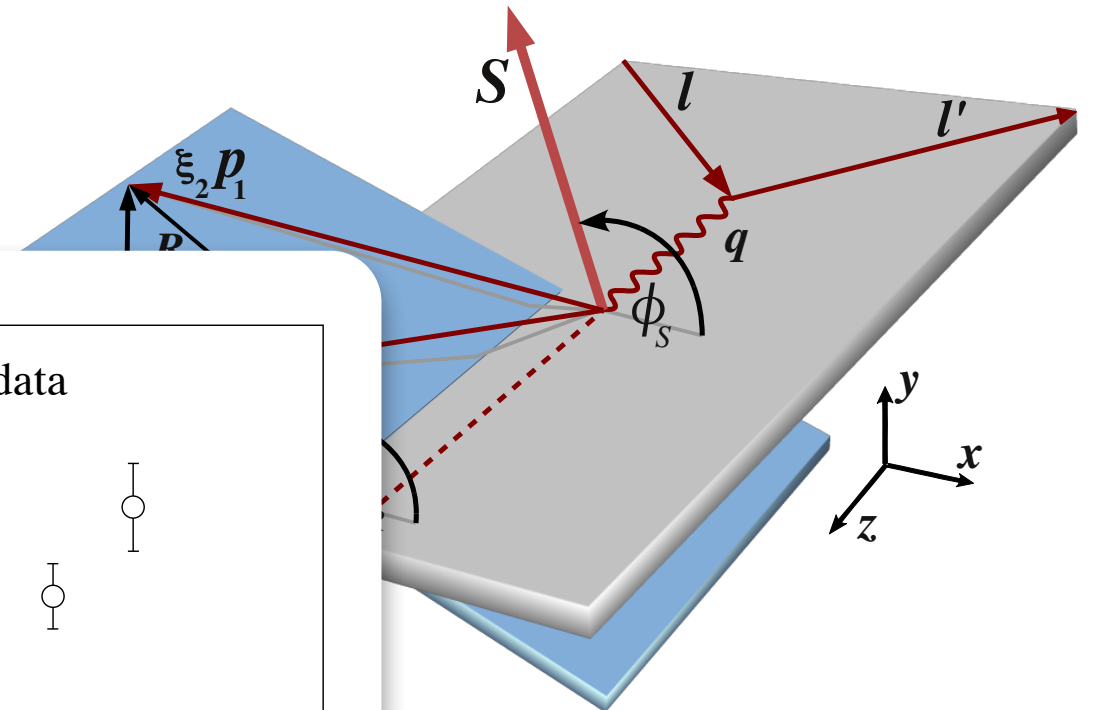
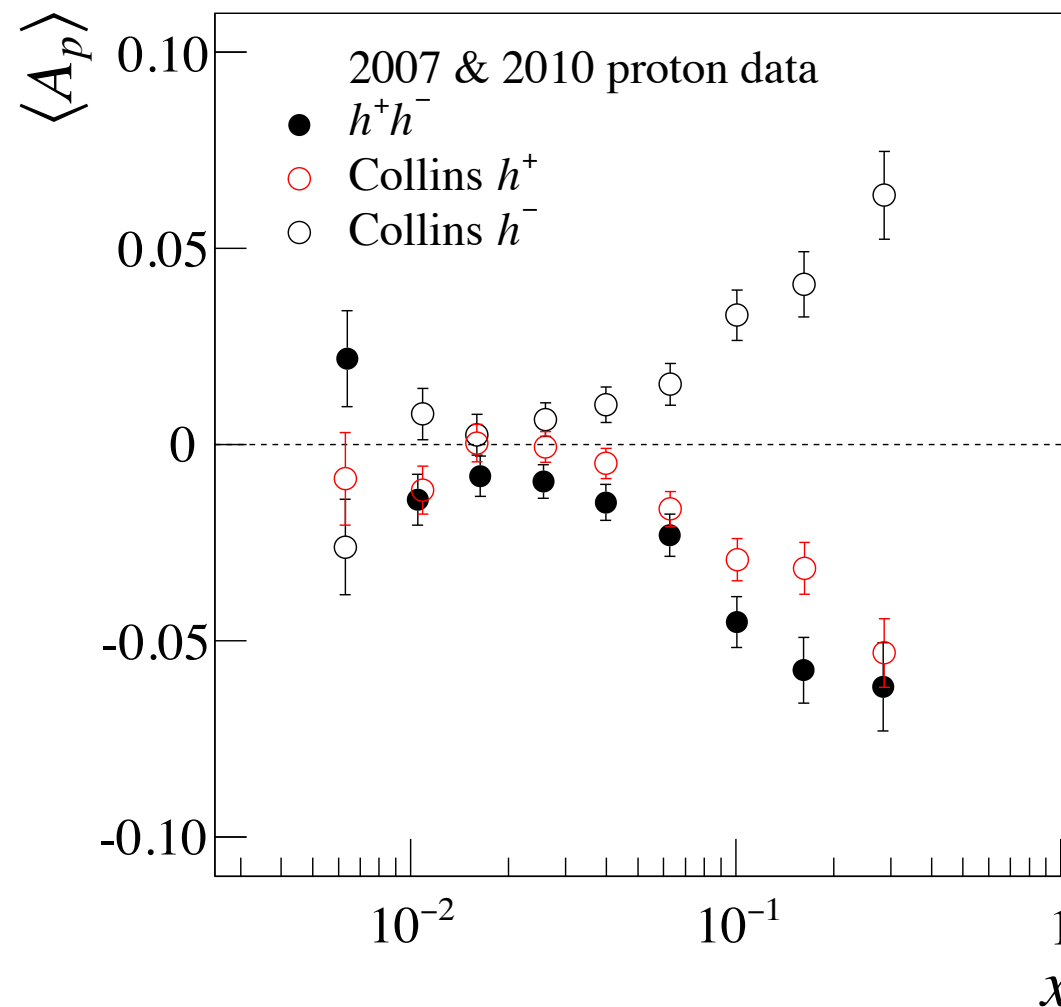
$$A_{Coll} = \frac{\sum_q e_q^2 h_1^q}{\sum_q e_q^2 f_1^q}$$

◆ Two hadron single spin asymmetry

$$A_{UT}^{\sin \phi_{RS}} = \frac{|\mathbf{p}_{\perp}|}{2M^2_{h^+h^-} \cos \theta}$$

◆ Note the choice of the vector

$$\mathbf{R}_{Artru} = \frac{z_2 \mathbf{P}_1 - z_1 \mathbf{P}_2}{z_1 + z_2}$$

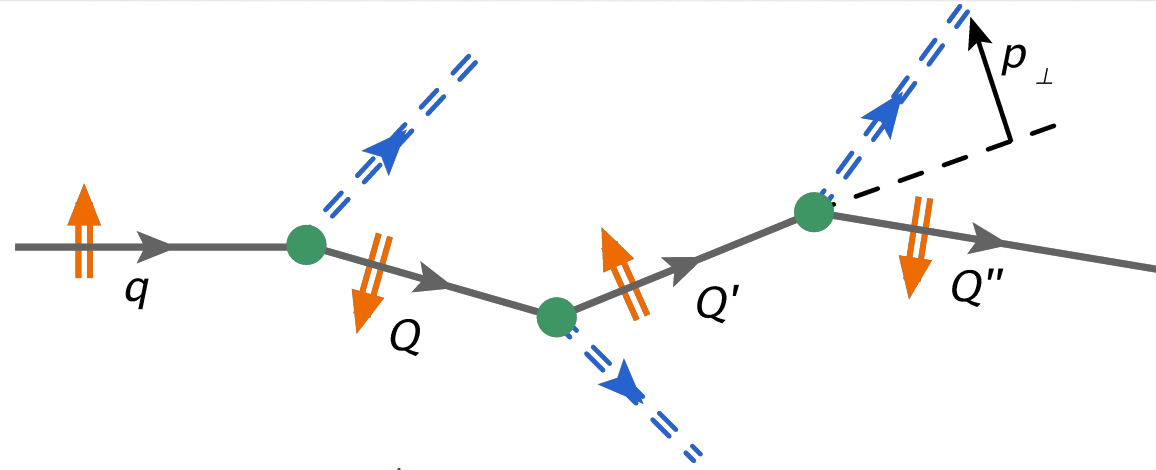


$$\frac{M^2_{h^+h^-}(\cos \theta)}{M^2_{h^+h^-}(\cos \theta)}$$

# POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- Use the NJL-jet Model including Collins effect (MKII) to study DiFFs.



$$D_{q\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R) \Delta z \Delta M_h^2 \Delta \varphi_R = \left\langle N_{q\uparrow}^{h_1 h_2}(z, z + \Delta z; M_h^2, M_h^2 + \Delta M_h^2; \varphi_R, \varphi_R + \Delta \varphi_R) \right\rangle.$$

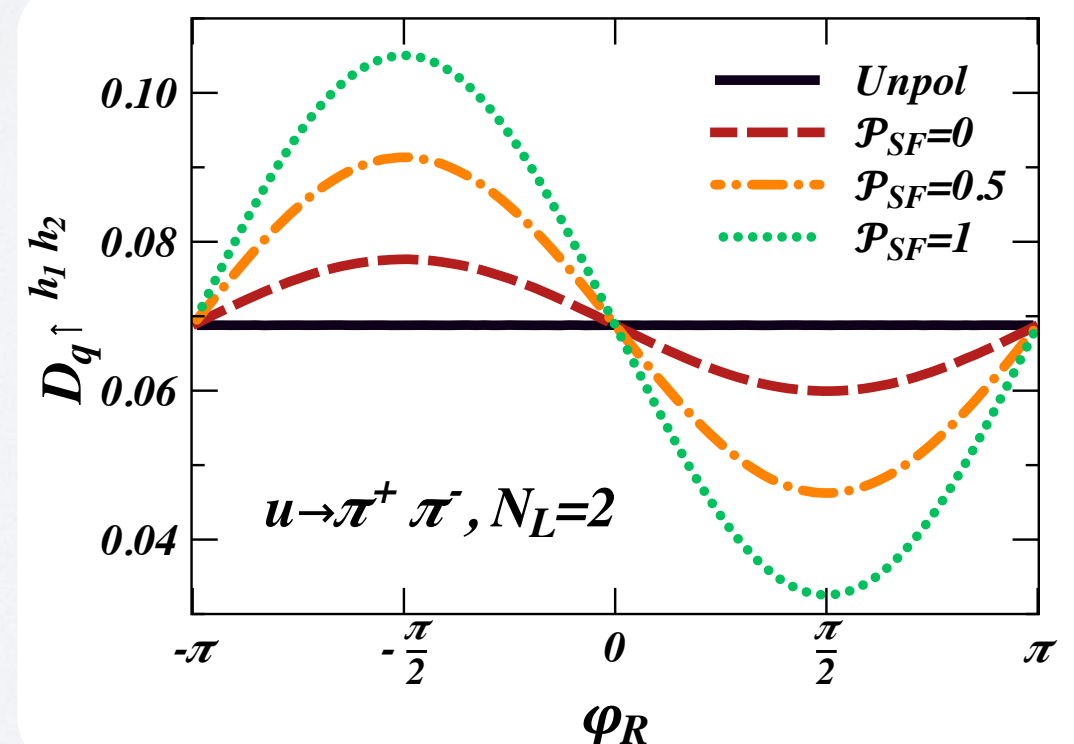
- Choose a constant Spin flip probability:

- Simple model to start with:

Only pions and extreme ansatz for the Collins term in elementary function.

$$d_{h/q\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9 \sin \varphi)$$

$\mathcal{P}_{SF}$

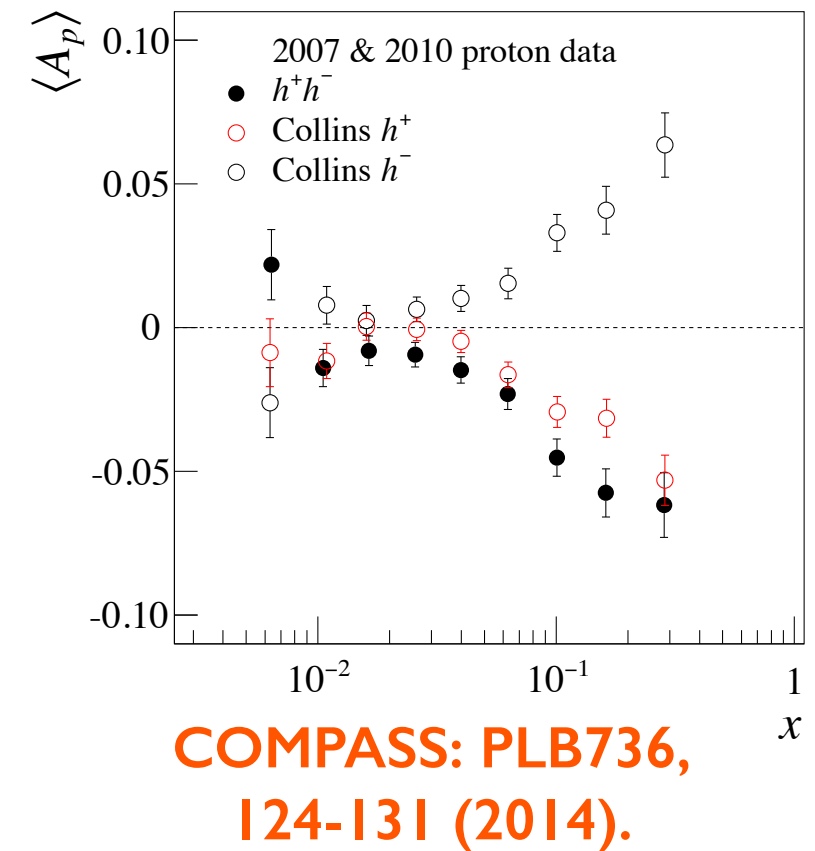
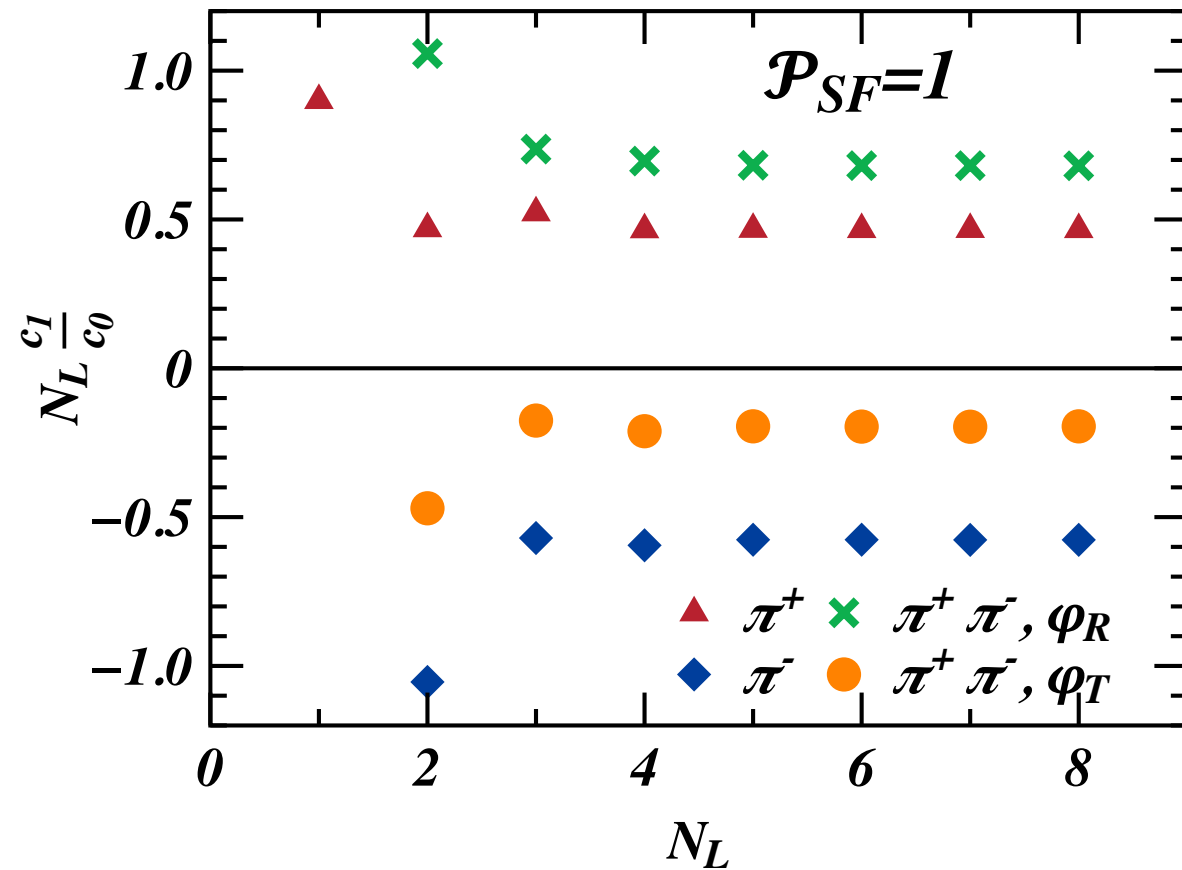




# INTEGRATED ANALYZING POWERS

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

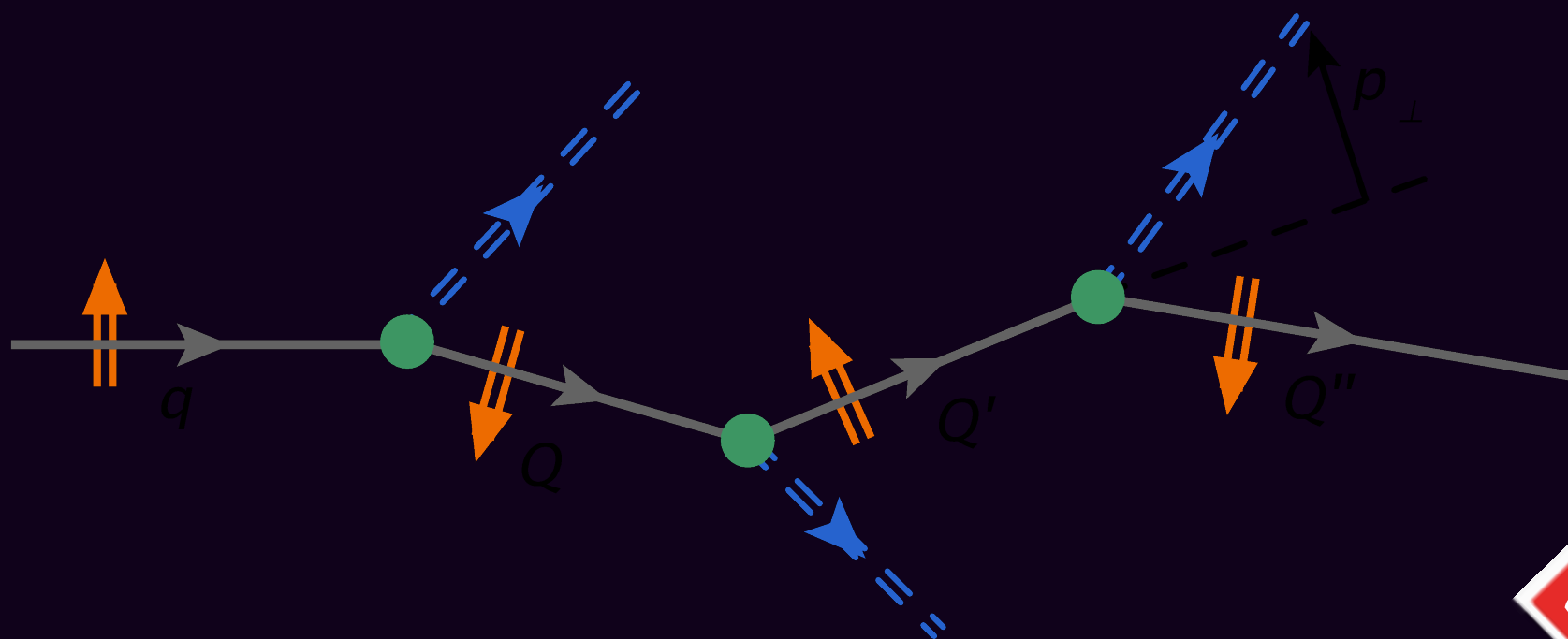
$$z_{1,2} > 0.2, z > 0.2$$



✓ *NJL-jet* model results are **consistent** with **COMPASS** measurements on interplay between one- and two- hadron SSAs.

# NJL-jet MKIII

## New Developments



27.3% Extra Free!



# What's Inside the Green BoX?

## ◆ TMD Polarized Fragmentation Functions at LO.

► Only *two* for unpolarised final state hadrons.

PDFs

N/q	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1 h_{1T}^\perp$

Fragment. Functions

h/q	U	L	T
U	$D_1$		$H_1^\perp$

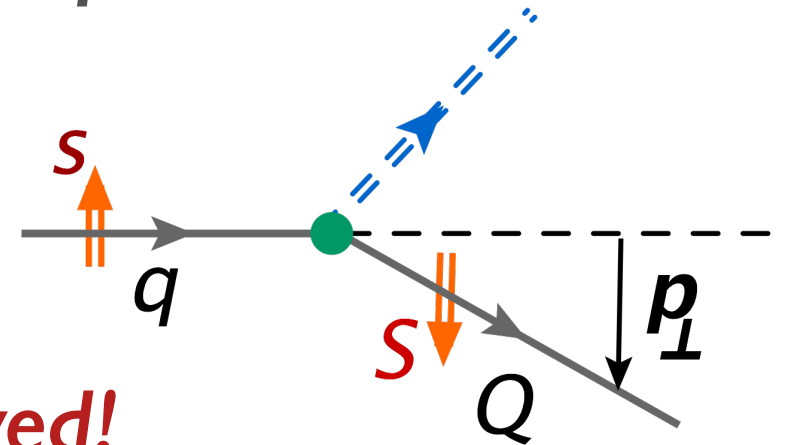
► 8 for spin 1/2 final state (including quark). Similar to TMD PDFs.

# Spin Transfer in quark-jet Framework.

## ◆NJL-jet MKIII:

- ▶ The probability for the process  $q \rightarrow Q$ , initial spin  $s$  to  $S$

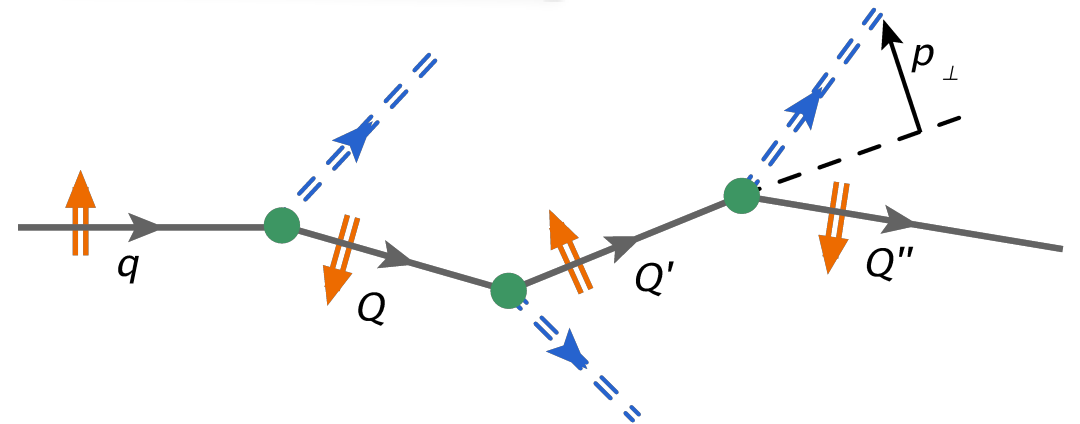
$$F^{q \rightarrow Q}(z, \mathbf{p}_\perp; s, \mathbf{S}) = \alpha_s + \beta_s \cdot \mathbf{S}$$



- ▶ **Intermediate** quarks in quark-jet are unobserved!

Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii: **QUANTUM ELECTRODYNAMICS (1982).**

$$\mathbf{S}_{unob} = \frac{\beta_s}{\alpha_s}$$



- ▶ Remnant quark's  $\mathbf{S}$  uniquely determined by  $z, \mathbf{p}_\perp$  and  $s$  !
- ▶ Process probability is **the same** as transition to **unpolarized state**.

$$F^{q \rightarrow Q}(z, \mathbf{p}_\perp; s, \mathbf{0}) = \alpha_s$$

# Example: Pion production.

*Fragment. Functions*

$$F^{q \rightarrow Q}(z, \mathbf{p}_\perp; s, S)$$

Q/q	U	L	T
U	$D_1$		$H_1^\perp$
L		$G_{1L}$	$H_{1L}^\perp$
T	$D_{1T}^\perp$	$G_{1T}$	$H_{1T}H_{1T}^\perp$

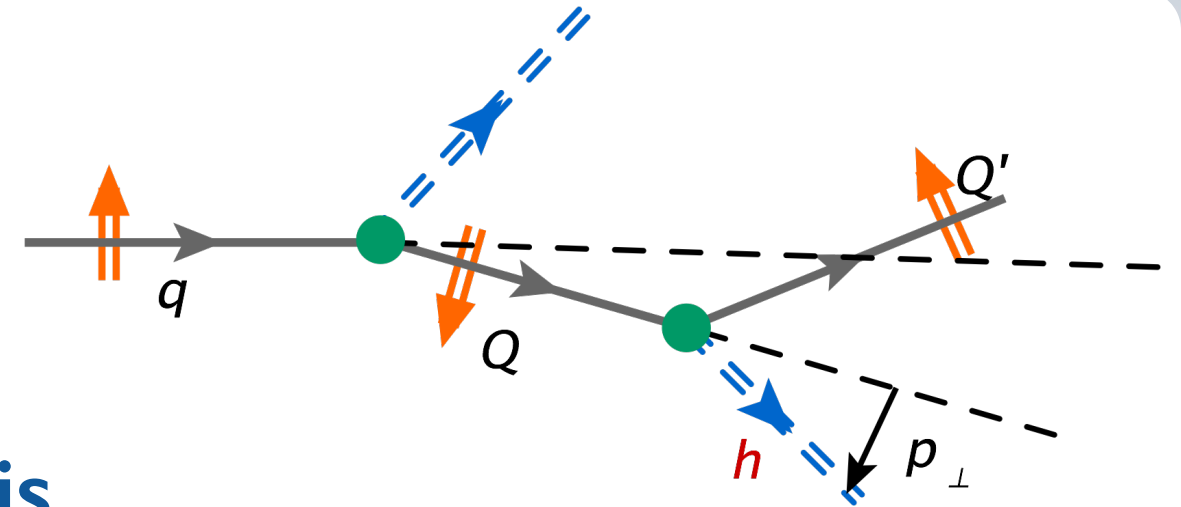
$$F^{q \rightarrow \pi}(z, \mathbf{p}_\perp; s)$$

$\pi/q$	U	L	T
U	$D_1$		$H_1^\perp$



# Example: Pion prod. up to Rank 2

◆ Only consider pion produced in the first two emission steps!



◆ Then the polarised number density is

$$F^{(2)q \rightarrow \pi} = \overset{\text{1st rank}}{\boxed{f^{q \rightarrow \pi}}} + \overset{\text{2nd rank}}{\boxed{f^{q \rightarrow Q} \otimes f^{Q \rightarrow \pi}}}$$

◆ “Elementary” number densities: *only favoured types are non-zero.*

$$\boxed{f^{q \rightarrow \pi} = d^{q \rightarrow \pi} - \frac{p_{\perp}}{z M_h} s_T h_1^{\perp q \rightarrow \pi}}$$

$$\boxed{f^{u \rightarrow \pi^-} = 0}$$

◆ It is shown analytically that only Collins modulations appear!

$$\boxed{F^{(2)q \rightarrow \pi}(z, p_{\perp}^2, \varphi_C) = F_0^{(2)}(z, p_{\perp}^2) - \sin(\varphi_C) F_1^{(2)}(z, p_{\perp}^2)}$$

# Example: Pion prod. up to Rank 2

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◆ Up to unspecified coefficients, using.

## Unpolarised term:

## From TM-induced Spin of intermediate quark

$$F_0^{(2)q \rightarrow \pi} = d^{q \rightarrow \pi} + (d^{q \rightarrow Q} \otimes d^{Q \rightarrow \pi} + d_T^{\perp q \rightarrow Q} \otimes h^{\perp Q \rightarrow \pi})$$

## Collins term:

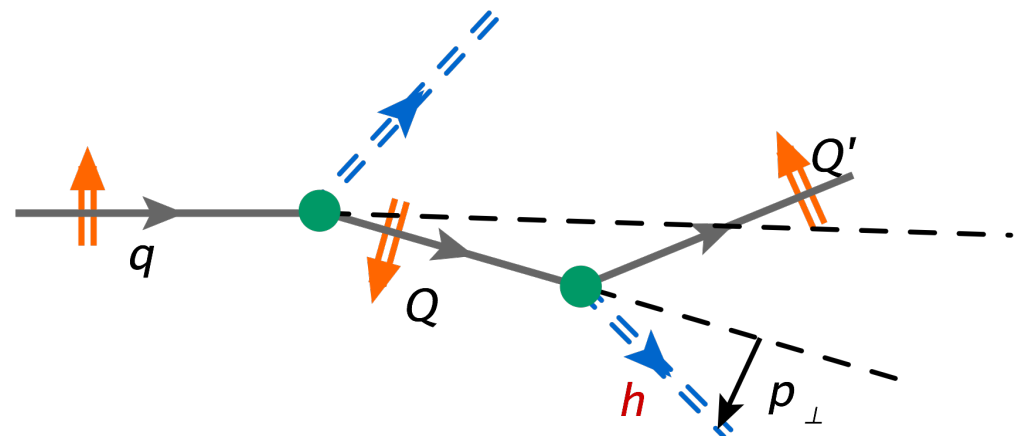
## “Recoil” TM contribution

$$F_1^{(2)q \rightarrow \pi} \sim h^{\perp q \rightarrow \pi} + [h^{\perp q \rightarrow Q} \otimes d^{Q \rightarrow \pi} + (h_T^{q \rightarrow Q} + h_T^{\perp q \rightarrow Q}) \otimes h^{\perp Q \rightarrow \pi}]$$

## Transferred Spin of intermediate quark

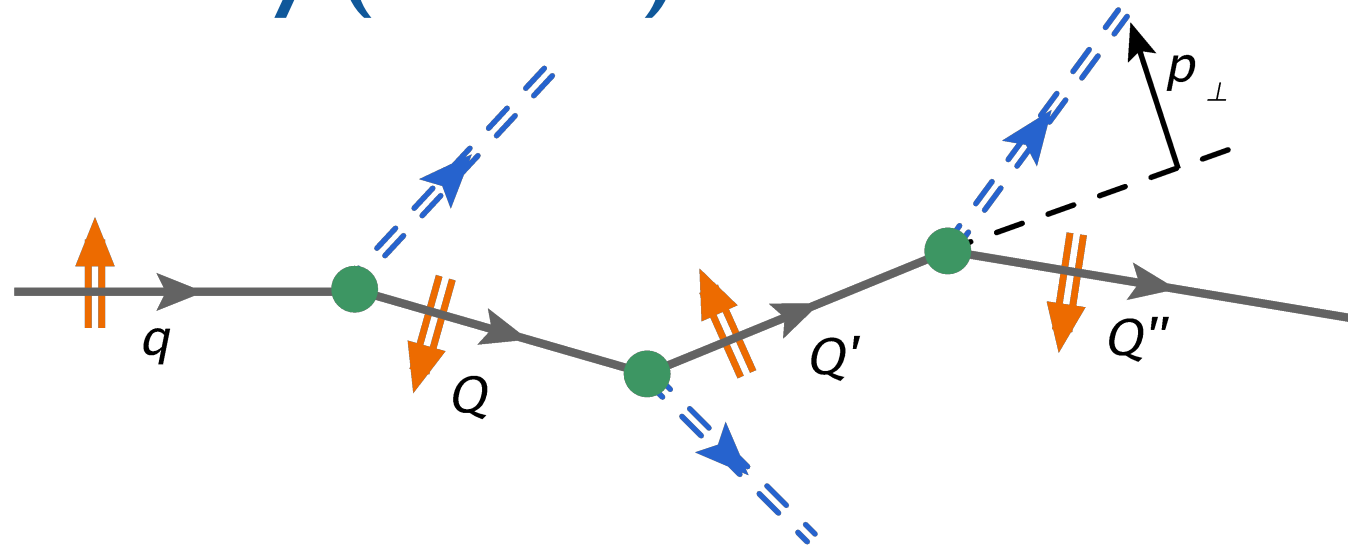
## ◆ *Reminder*

Q/q	U	L	T
U	$D_1$		$H_1^\perp$
L		$G_{1L}$	$H_{1L}^\perp$
T	$D_{1T}^\perp$	$G_{1T}$	$H_{1T}H_{1T}^\perp$



# Full Hadronization

- ◆ We can consider many (infinite) number of emissions.



- ◆ Then the polarised number density is

$$F^{q \rightarrow h} = f^{q \rightarrow h} + f^{q \rightarrow Q} \otimes F^{Q \rightarrow h}$$

- ◆ Again, only Collins modulations appear for unpolarised  $h$ !

$$F^{q \rightarrow h}(z, p_{\perp}^2, \varphi_C) = F_0(z, p_{\perp}^2) - \sin(\varphi_C) F_1(z, p_{\perp}^2)$$

- ◆ Also applicable for polarised hadron production.

- ◆ We can also employ MC approach.

- ◆ We only need the “elementary” splittings.

$$f^{q \rightarrow h}$$

$$f^{q \rightarrow Q}$$



# Conclusions

- ❖ Polarised TMD FFs provide a wealth of information about the *spin-spin* and *spin-momentum* correlations in hadronisation.
- ❖ Modelling Polarised Quark Hadronization is needed for both calculations of polarised FFs and phenomenological studies of various correlations (Collins and IFF, etc).
- ❖ Incorporating polarised parton hadronisation into *MC generators* is needed for *supporting future experiments* in mapping out the 3D structure of nucleon (*JLab I 2, BELLE II, EIC*).
- ❖ The *NJL-jet* model provides a robust and extendable framework for *microscopic* description of various fragmentation phenomena using MC simulations: *TMD, Collins, DiHadron*.
- ❖ The extension of the underlying *quark-jet* mechanism to include polarisation can be readily incorporated in other MC frameworks.

*Thanks!*

*Photo by Jun Zhang*