

Iterative Monte Carlo analysis of spin-dependent parton distributions

(arXiv:1601.07782)

Nobuo Sato

In Collaboration with:

W. Melnitchouk, S. E. Kuhn, J. J. Ethier, A. Accardi



J-PARC hadron physics in 2016

Spin structure of nucleons

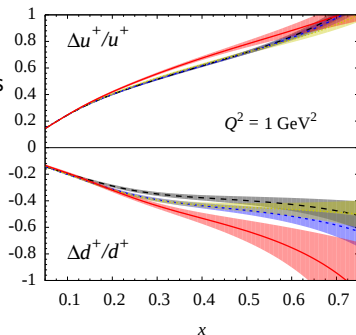
- Spin sum rule $\rightarrow \frac{1}{2} = \frac{1}{2}\Delta\Sigma^{(1)} + \Delta g^{(1)} + \mathcal{L}$
- The spin contribution from quarks: $\rightarrow \Delta\Sigma^{(1)} = \Delta u_+^{(1)} + \Delta d_+^{(1)} + \Delta s_+^{(1)}$
- From existing global analysis $\rightarrow \Delta\Sigma_{[10^{-3},1]}^{(1)} \sim 0.3, \quad \Delta g_{[0.05,0.2]}^{(1)} \sim 0.1$

Higher twists

- d_2 matrix element
 $\rightarrow d_2 = 2g_1^{(3)}(Q^2) + 3g_2^{(3)}(Q^2)$
- Color forces experienced by struck quarks
 $\rightarrow \tilde{F}_E = 2d_2 + f_2, \quad \tilde{F}_B = 4d_2 - f_2$
- Dedicated global QCD analysis is required

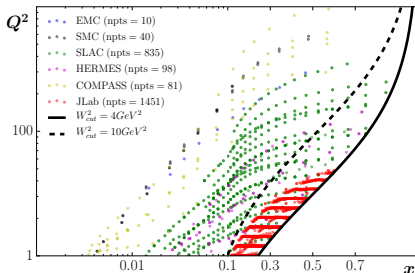
High x

- $SU(6)$ spin-flavor symmetry:
 $\rightarrow \Delta u/u \rightarrow 2/3, \quad \Delta d/d \rightarrow -1/3$
- pQCD $\rightarrow \Delta q/q \rightarrow 1$



Data

- ✓ Polarized DIS $\rightarrow \Delta u, \Delta d$
- Polarized SIDIS: $\rightarrow \Delta \bar{d}, \Delta \bar{u}, \Delta s$
- Inclusive Jets/ π^0 : $\rightarrow \Delta g$
- W production $\rightarrow \Delta \bar{d}, \Delta \bar{u}$
- ? Polarized Drell-Yan data
 $\rightarrow$ Future JPARC data?



Theory

- ✓ Target mass corrections
- ✓ Twist-3 and twist-4 contributions in polarized structure functions
- ✓ Nuclear corrections for ^3He and deuteron targets
 - Threshold resummation $\rightarrow (\alpha_S^m \log(1-x)^n)$

Tools

- ✓ Numerical codes developed within python framework
- ✓ Development of DGLAP evolution equations in Mellin space
- ✓ Fast calculation of observables $\rightarrow$ Mellin space techniques

Asymmetries

$$A_{||} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\downarrow\downarrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\downarrow\downarrow}} = D(A_1 + \eta A_2)$$
$$A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\downarrow\Rightarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\downarrow\Rightarrow}} = d(A_2 - \xi A_1)$$
$$A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \quad A_2 = \gamma \frac{(g_1 + g_2)}{F_1} \quad \gamma^2 = \frac{4M^2 x^2}{Q^2}$$

Theory

$$g_1(x, Q^2) = g_1^{\text{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \dots) + g_1^{\text{T3+TMC}}(D_u, D_d) + g_1^{\text{T4}}(H_{p,n})$$
$$g_2(x, Q^2) = g_2^{\text{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \dots) + g_2^{\text{T3+TMC}}(D_u, D_d)$$

Leading twist structure functions:

$$g_1^{\text{LT+TMC}}(x, Q^2) = \frac{x}{\xi} \frac{g_1^{\text{LT}}(\xi)}{(1+4\mu^2 x^2)^{3/2}} + 4\mu^2 x^2 \frac{x+\xi}{\xi(1+4\mu^2 x^2)^2} \int_{\xi}^1 \frac{dz}{z} g_1^{\text{LT}}(z)$$

$$- 4\mu^2 x^2 \frac{2-4\mu^2 x^2}{2(1+4\mu^2 x^2)^{5/2}} \int_{\xi}^1 \frac{dz}{z} \int_{z'}^1 \frac{dz'}{z'} g_1^{\text{LT}}(z')$$

$$g_2^{\text{LT+TMC}}(x, Q^2) = -\frac{x}{\xi} \frac{g_1^{\text{LT}}(\xi)}{(1+4\mu^2 x^2)^{3/2}} + \frac{x}{\xi} \frac{(1-4\mu^2 x\xi)}{(1+4\mu^2 x^2)^2} \int_{\xi}^1 \frac{dz}{z} g_1^{\text{LT}}(z)$$

$$+ \frac{3}{2} \frac{4\mu^2 x^2}{(1+4\mu^2 x^2)^{5/2}} \int_{\xi}^1 \frac{dz}{z} \int_{z'}^1 \frac{dz'}{z'} g_1^{\text{LT}}(z')$$

In the Bjorken limit ($Q^2 \rightarrow \infty$):

$$g_1^{\text{LT+TMC}}(x, Q^2) \simeq g_1^{\text{LT}}(x), \quad g_2^{\text{LT+TMC}}(x, Q^2) \simeq -g_1^{\text{LT}}(x) + \int_{\xi}^1 \frac{dz}{z} g_1^{\text{LT}}(z)$$

Leading twist quark distributions:

$$g_1^{\text{LT}}(x) = \frac{1}{2} \sum_q e_q^2 [\Delta C_{qq} \otimes \Delta q(x) + \Delta C_{qg} \otimes \Delta g(x)]$$

Twist-3 structure functions:

$$\begin{aligned}
 g_1^{\text{T3+TMC}}(x, Q^2) &= 4\mu^2 x^2 \frac{D(\xi)}{(1 + 4\mu^2 x^2)^{3/2}} - 4\mu^2 x^2 \frac{3}{(1 + 4\mu^2 x^2)^2} \int_{\xi}^1 \frac{dz}{z} D(z) \\
 &\quad + 4\mu^2 x^2 \frac{2 - 4\mu^2 x^2}{(1 + 4\mu^2 x^2)^{5/2}} \int_{\xi}^1 \frac{dz}{z} \int_{z'}^1 \frac{dz'}{z'} D(z') \\
 g_2^{\text{T3+TMC}}(x, Q^2) &= \frac{D(\xi)}{(1 + 4\mu^2 x^2)^{3/2}} - \frac{1 - 8\mu^2 x^2}{(1 + 4\mu^2 x^2)^2} \int_{\xi}^1 \frac{dz}{z} D(z) \\
 &\quad - \frac{12\mu^2 x^2}{(1 + 4\mu^2 x^2)^{5/2}} \int_{\xi}^1 \frac{dz}{z} \int_{z'}^1 \frac{dz'}{z'} D(z')
 \end{aligned}$$

Bjorken limit ($Q^2 \rightarrow \infty$):

$$g_1^{\text{T3+TMC}}(x, Q^2) \simeq 0 \qquad g_2^{\text{T3+TMC}}(x, Q^2) \simeq D(x) - \int_{\xi}^1 \frac{dz}{z} D(z)$$

Twist-3 quark distributions:

$$D(x, Q^2) = \frac{4}{9} D_u(x, Q^2) + \frac{1}{9} D_d(x, Q^2)$$

Twist-4 structure function:

$$g_1^{\text{T4(p,n)}}(x, Q^2) = H^{(p,n)}(x)/Q^2$$

Nuclear corrections: → nuclear smearing functions

$$g_i^A(x, Q^2) = \sum_N \int \frac{dy}{y} f_{ij}^N(y, \gamma) g_j^N(x/y, Q^2)$$

Curse of dimensionality → Mellin trick (Stratmann, Vogelsang)

$$\begin{aligned} I(x) &= \int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} g\left(\frac{x}{yz}\right) \quad \leftarrow \quad g(\xi) = \frac{1}{2\pi i} \int dN \xi^{-N} g_N \\ &= \frac{1}{2\pi i} \int dN g_N \left[\int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} \left(\frac{x}{yz}\right)^{-N} \right] \\ &= \frac{1}{2\pi i} \int dN g_N \mathcal{M}_N \\ &= \sum_{i,k} w_i^k j^k \text{Im} \left(e^{i\phi} g_{N_j^k} \mathcal{M}_{N_j^k} \right) \quad \leftarrow \quad \text{Gaussian quadrature} \end{aligned}$$

→ time consuming part can be precalculated prior to the fit

Fitting strategy

Parametrization

- $xf(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$
- LT quark distributions $\rightarrow \Delta u^+, \Delta d^+, \Delta s^+, \Delta g$
- T3 quark distributions $\rightarrow D_u, D_d$
- T4 structure functions $\rightarrow H_p, H_n$

Chi-squared minimization $\rightarrow$ with correlated systematic uncertainties

$$\chi^2 = \sum_i \left(\frac{D_i - T_i(1 - \sum_k r^k \beta_i^k / D_i)^{-1}}{\alpha_i} \right)^2 + \sum_k (r^k)^2$$

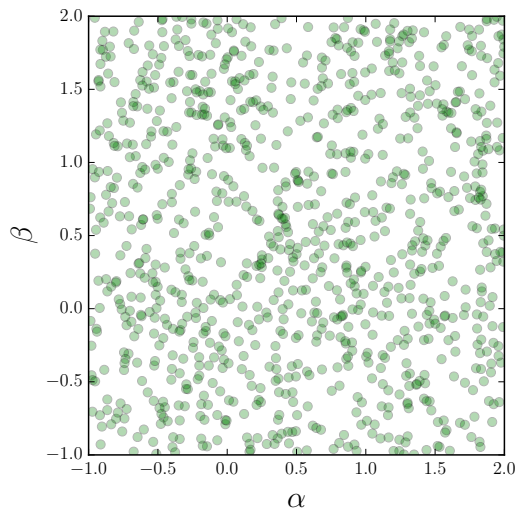
Issues

- Stability in the moments (e.g. $\Delta\Sigma^{(1)}$)
- Is the solution given by a single fit unique? $\rightarrow$ False minima
- Is over-fitting present in our fits?
- Which parameters should be fixed and at which value?
- Determination of uncertainty bands.

Solution $\rightarrow$ MC approach

Iterative Monte Carlo Analysis (IMC)

Toy example $\rightarrow$ fitting 2 model parameters α, β

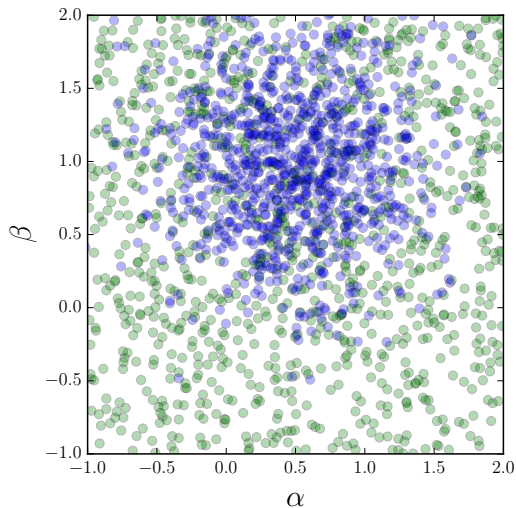


I. Flat sampling

Initial priors $\{(\alpha, \beta)\}$

Iterative Monte Carlo Analysis (IMC)

Toy example $\rightarrow$ fitting 2 model parameters α, β



I. Flat sampling

Initial priors $\{(\alpha, \beta)\}$

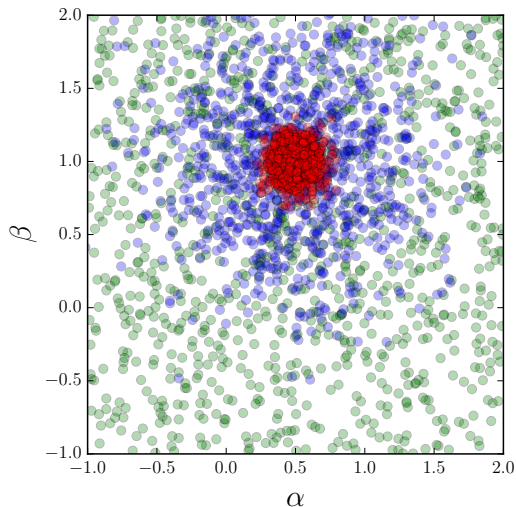
II. First iteration

priors $\{(\alpha, \beta)\}$

posteriors $\{(\alpha, \beta)\}$

Iterative Monte Carlo Analysis (IMC)

Toy example $\rightarrow$ fitting 2 model parameters α, β



I. Flat sampling

Initial priors $\{(\alpha, \beta)\}$

II. First iteration

priors $\{(\alpha, \beta)\}$

posteriors $\{(\alpha, \beta)\}$

III. Second iteration

priors $\{(\alpha, \beta)\}$

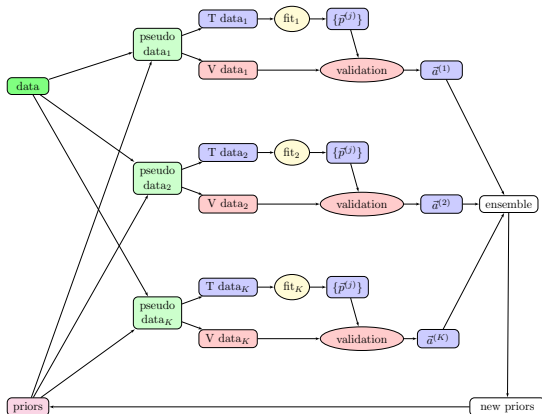
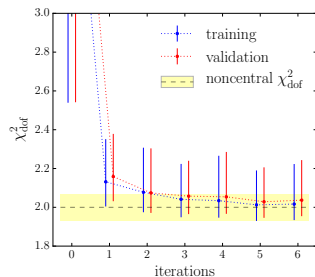
posteriors $\{(\alpha, \beta)\}$

... until convergence

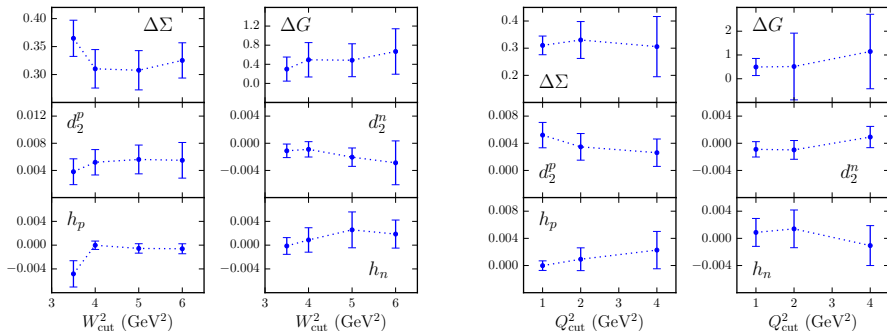
Iterative Monte Carlo Analysis (IMC)

Each iteration

- Generate pseudo data sets via data resampling
- Random data partition $\rightarrow$ Training & Validation
- Fit the training set
- Validation



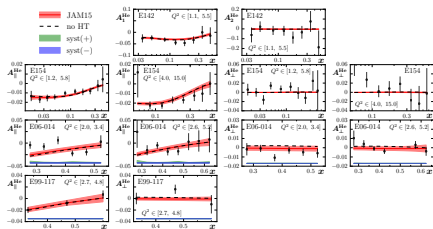
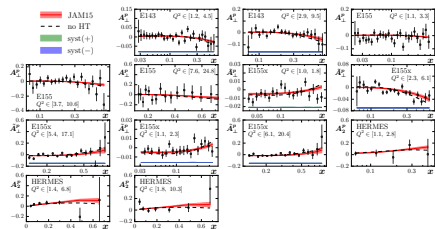
W_{cut}^2 and Q_{cut}^2



W_{cut}^2 (GeV ²)	3.5	4	5	6	8	10
# points	2868	2515	1880	1427	943	854
χ_{dof}^2	1.20	1.07	1.03	1.02	0.99	0.97

Q_{cut}^2 (GeV ²)	1.0	2.0	4.0
# points	2515	1421	611
χ_{dof}^2	1.07	1.08	0.95

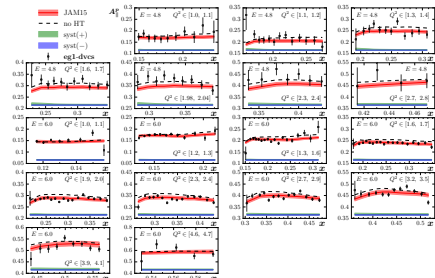
Data vs. Theory



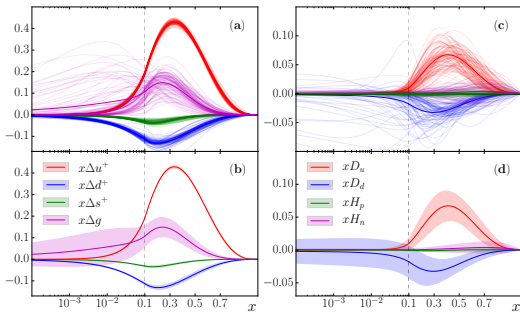
Signal of HT?

- $A_{\perp}^P \rightarrow$ marginally visible at large x
- Not visible in ^3He data
- Significant contribution in eg1-dvcs

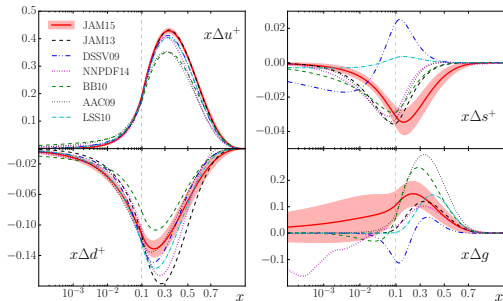
$A_{\parallel}^P$



Results

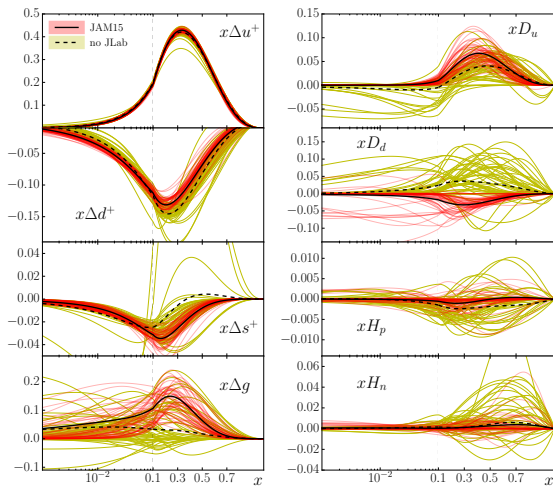


- Significant constraints to large $x \Delta s^+$ and Δg
- Non zero T3 quark distributions
- T4 contribution to g_1 consistent with zero



- Negative Δs^+
- JAM15 Δg compatible with rescaled DSSV fits.

Impact of JLab data



- JLab data $\rightarrow 0.1 < x < 0.7$
- Constraints on small x from large $x \rightarrow$ weak baryon decay constraints
- Large uncertainties in Δs^+ , Δg removed by JLab data
- Non vanishing T3 quark distributions
- T4 distributions consistent with zero

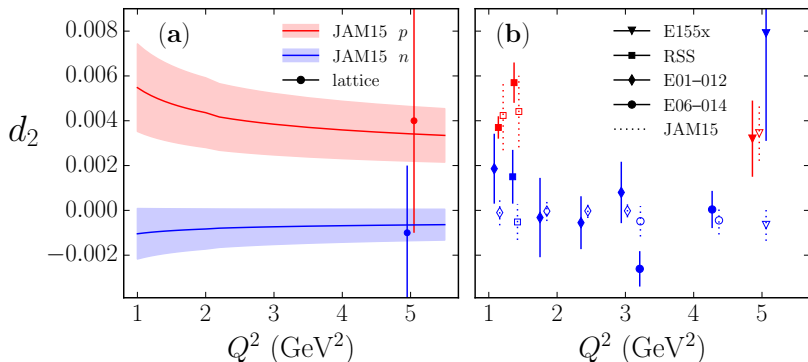
Impact of JLab data

Table: Lowest moments of the twist-2 PDFs Δu^+ , Δd^+ , Δs^+ , $\Delta \Sigma$ and ΔG , the twist-3 d_2^p and d_2^n moments, and the x^2 -weighted moments h_p and h_n of the twist-4 distributions. The truncated moments in the measured region $x \in [0.001, 0.8]$ and the extrapolated full moments are shown at $Q^2 = 1 \text{ GeV}^2$.

moment	truncated	full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta \Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.005 ± 0.002	0.005 ± 0.002
d_2^n	-0.001 ± 0.001	-0.001 ± 0.001
h_p	-0.000 ± 0.001	0.000 ± 0.001
h_n	0.001 ± 0.002	0.001 ± 0.003

d_2 matrix element

- $d_2(Q^2) \equiv \int_0^1 dx x^2 [2g_1^{T3}(x, Q^2) + 3g_2^{T3}(x, Q^2)]$
- d_2 is related to “color polarizability” or the “transverse color force” acting on quarks.
- Existing measurements of d_2 are in the resonance region (contains TMC T4 and beyond.)
- Agreement with data indicates quark-hadron duality

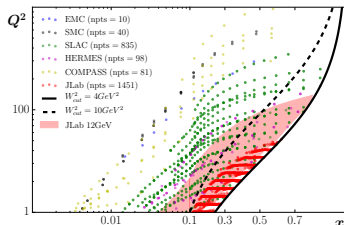


JAM

- ✓ New JAM15 analysis to study impact of all JLab 6 GeV inclusive DIS data at low W and high x
- ✓ New extraction of LT & HT distributions
 - **Upcoming JAM16** analysis to study polarization of sea quarks & gluons.
 - SIDIS for flavor separation.
 - polarized pp cross sections (inclusive jet & π production) for Δg
 - W boson asymmetries
 - Threshold resummation impacts on large x
 - Combined analysis of all inclusive (un)polarized DIS data
 - Fits to helicity distributions

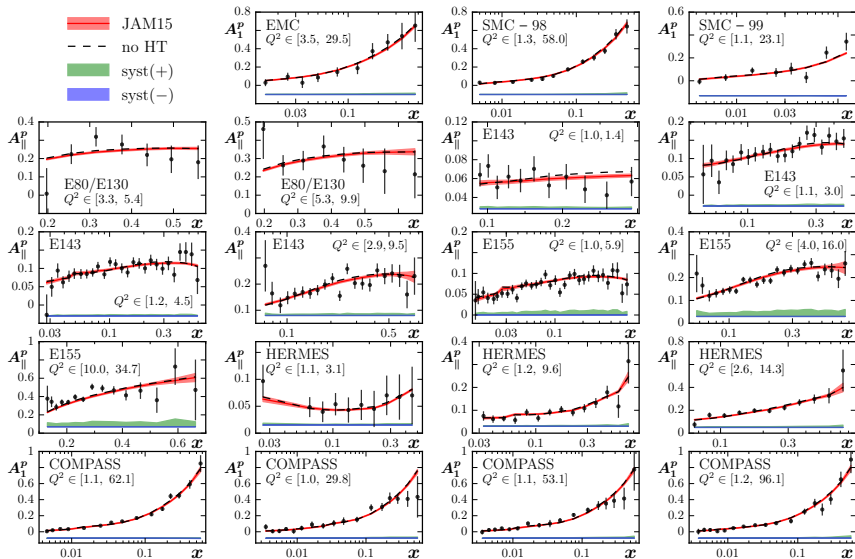
JLab 12

- Measurements at high- $x \rightarrow \Delta q/q$
- Wider coverage in $Q^2 \rightarrow \Delta g$
- Determination of pure twist-3 d_2 in DIS

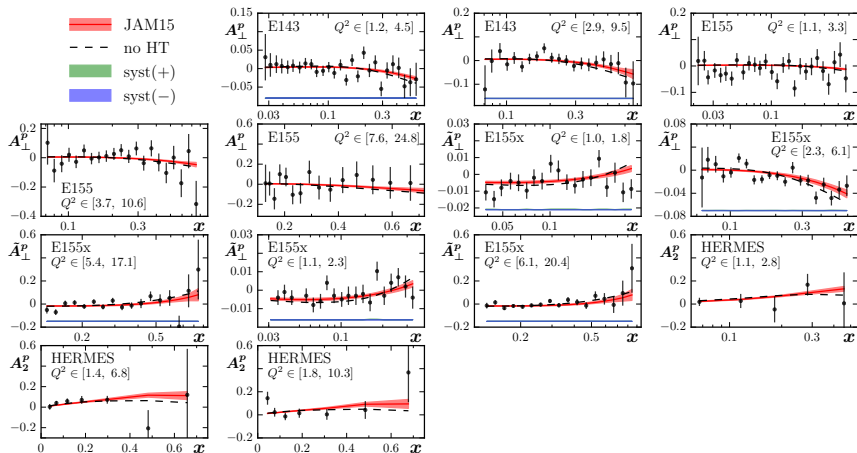


Backup

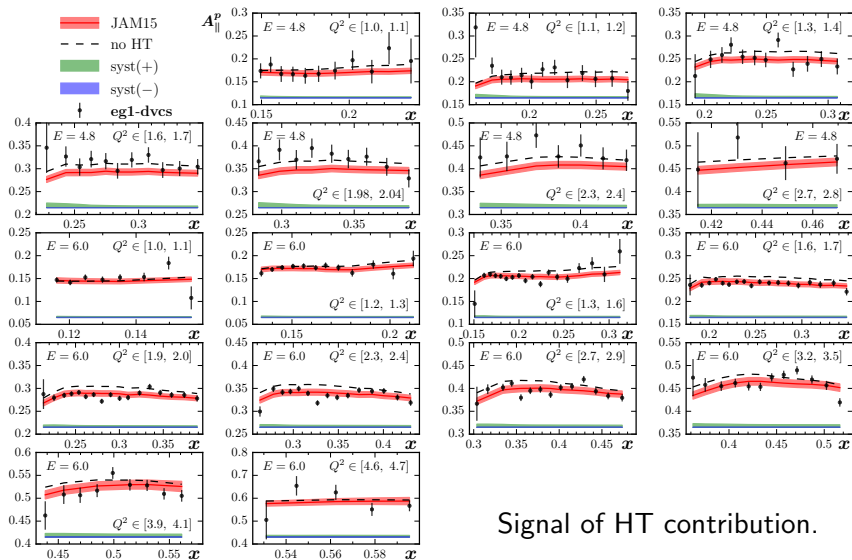
Data vs. Theory: proton



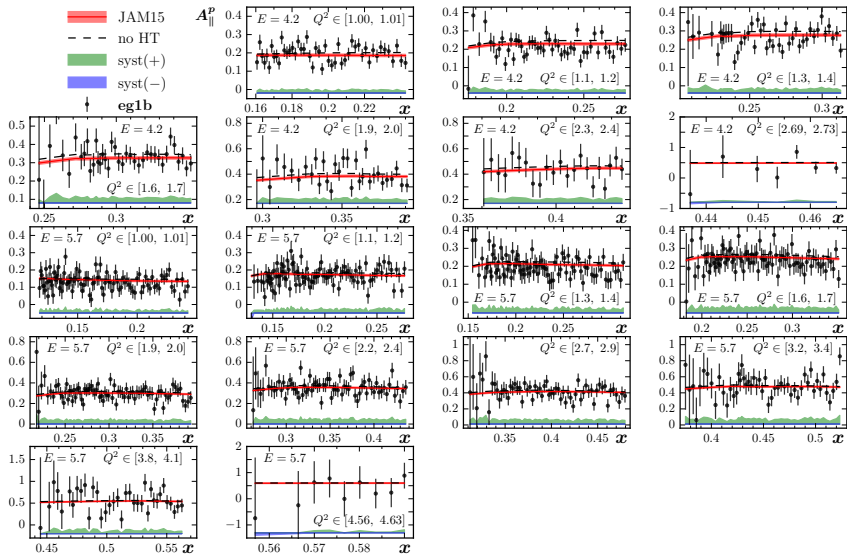
Data vs. Theory: proton



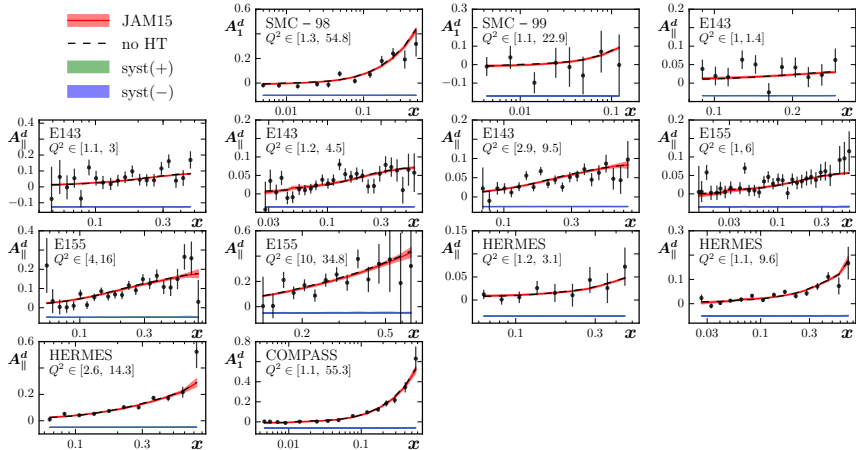
Data vs. Theory: proton DVCS



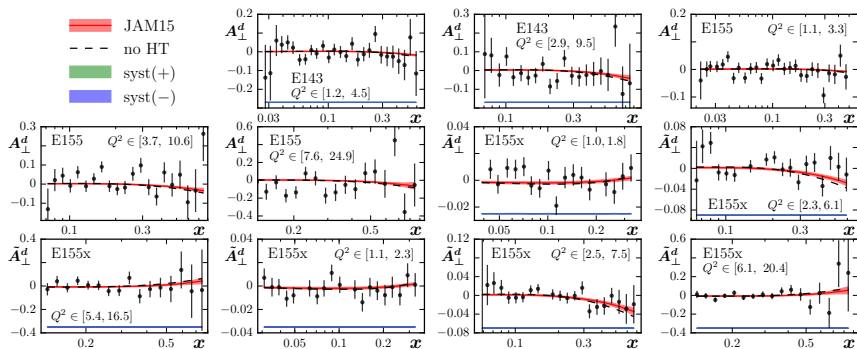
Data vs. Theory: proton EG1b



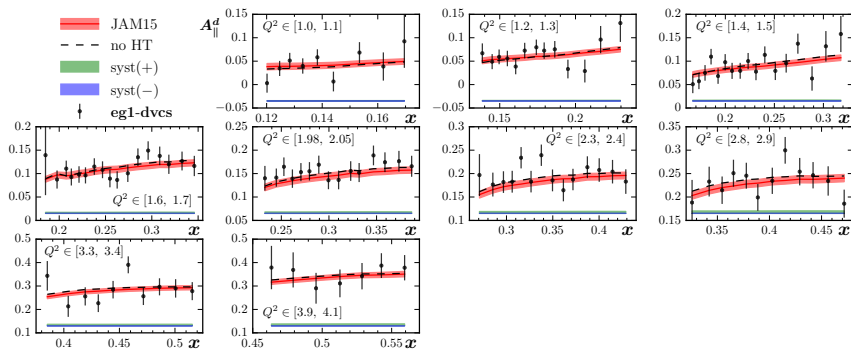
Data vs. Theory: deuteron



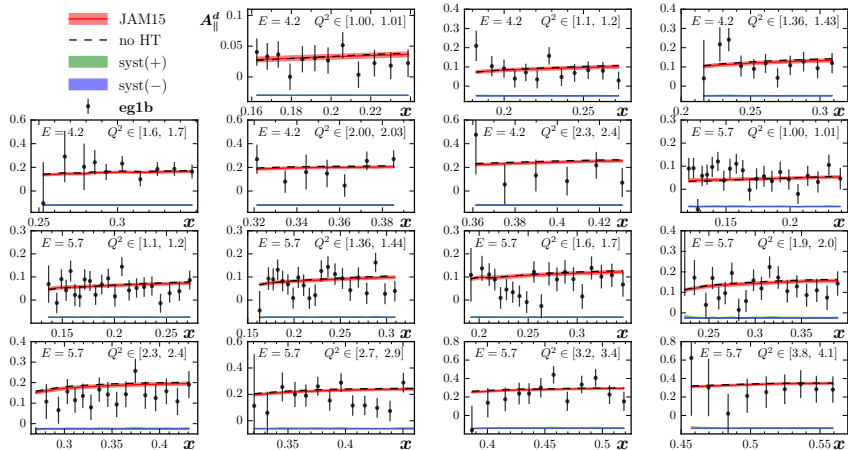
Data vs. Theory: deuteron



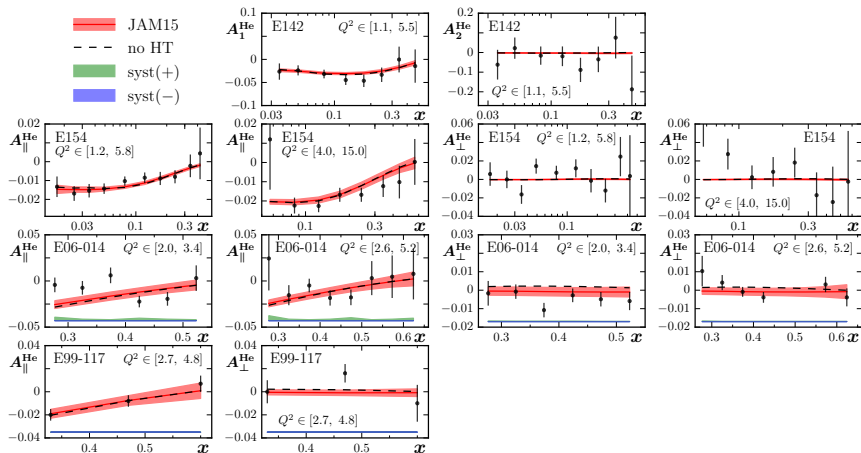
Data vs. Theory: deuteron DVCS



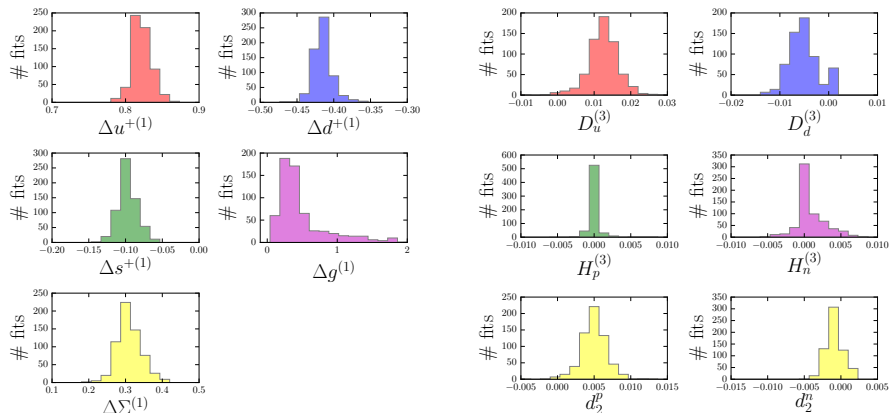
Data vs. Theory: deuteron EG1b



Data vs. Theory: ^3He



JAM: moments



- **New** extraction of $\Delta \Sigma^{(1)} = 0.31 \pm 0.03$ (Only truncated moments are shown $x \in [10^{-3}, 0.8]$)
- First extraction of d_2 matrix element in global QCD analysis
- **JAM16**: reduction of gluon uncertainties (jet data)