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Analysis of charged lepton flavor violation process $\mu^- e^- \to e^- e^-$ in muonic atoms

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1. INTRODUCTION

Lepton Flavor Violation

- Three lepton flavor numbers (L_e, L_μ, L_τ)

	e^-, ν_e	μ^-, ν_μ	τ^-, ν_τ	others
L_e	+1	0	0	0
L_μ	0	+1	0	0
L_τ	0	0	+1	0

(Anti-leptons have a minus sign.)

- In SM, each lepton flavor # is strictly conserved. (e.g. $\mu^- \to e^- \nu_\mu \bar{\nu}_e$)
  - Neutrino oscillation violates the conservation.

    (e.g. $\nu_\mu \to \nu_\tau$)

- The violation in charged lepton sector has not been observed yet.

  → Charged Lepton Flavor Violation (CLFV)

  e.g. $\mu \to e\gamma,\ \tau \to e\gamma,\ \tau \to e\pi^0 \ldots$

  predicted in many models beyond SM

  ➡ good probes of new physics !

CLFV search using muons

Advantages of using muon for rare process

1. high intensity muon beam (~ 10^8 muons per a second)
2. long lifetime and simple kinematics

Examples of CLFV processes using muons

BR : Branching Ratio

a) $\mu^+ \rightarrow e^+\gamma$ BR $< 5.7 \times 10^{-13}$ by MEG

Phys. Rev. Lett. 110 (2013) 201801.

b) $\mu^+ \rightarrow e^+e^-e^+$ BR $< 1.0 \times 10^{-12}$ by SINDRUM

Nucl. Phys. B 299 (1988) 1.

c) $\mu^- N \rightarrow e^- N$ BR $< 7 \times 10^{-13}$ ($\mu^-\mathrm{Au} \rightarrow e^-\mathrm{Au}$) by SINDRUM II

Eur. Phys. J. C 47 (2006) 337.

$\mu^-\mathrm{Al} \rightarrow e^-\mathrm{Al}$ is planned to be measured by COMET experiment @ J-PARC.

(COherent Muon to Electron Transition)

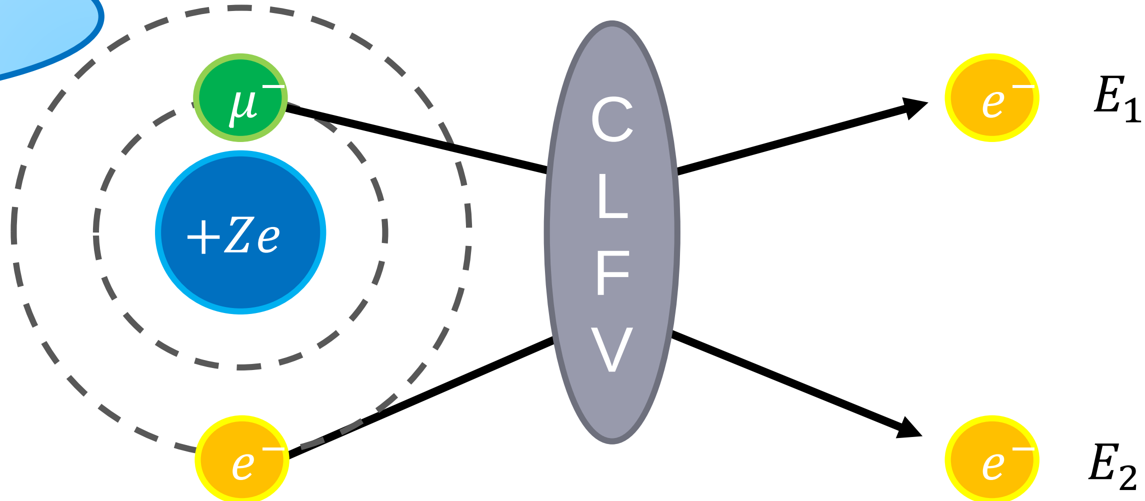
$\mu^- e^- \to e^- e^-$ in a muonic atom

M. Koike, Y. Kuno, J. Sato and M. Yamanaka, Phys. Rev. Lett. 105,121601(2010)

New CLFV search using muonic atoms

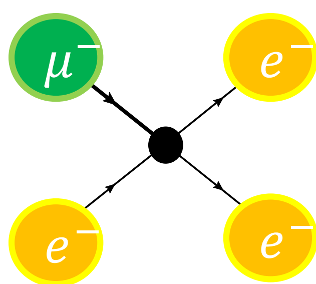
proposed to be measured in **COMET**

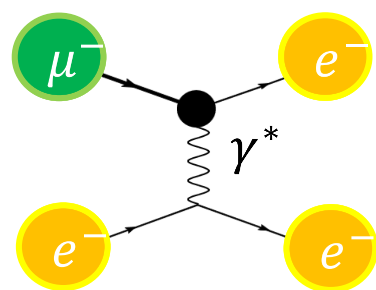
R. Abramishili et al., COMET Phase-I Technical Design Report, KEK Report 2015-1 (2015)



Features

- clear signal : two e^-s $(E_1 + E_2 \simeq m_\mu + m_e - B_\mu - B_e)$
- 2 type interactions
  - ✓ μeee vertex
  - ✓ $\mu e\gamma$ vertex





- atomic # Z : large ⇒ decay rate Γ : large $(\Gamma \propto (Z-1)^3)$

Previous estimation of decay rate

➢ Koike *et al.* Phys. Rev. Lett.105,121601(2010)

Suppose nuclear Coulomb potential is weak

$$\Gamma \sim \sigma v_{\mathrm{rel}} |\psi_{1S}^{e}(0)|^2 \propto (Z-1)^3$$

σv_{rel} : cross section of $\mu^- e^- \to e^- e^-$ (free particles')

$\psi_{1S}^{e}(x)$: Schrödinger wave function of a bound electron

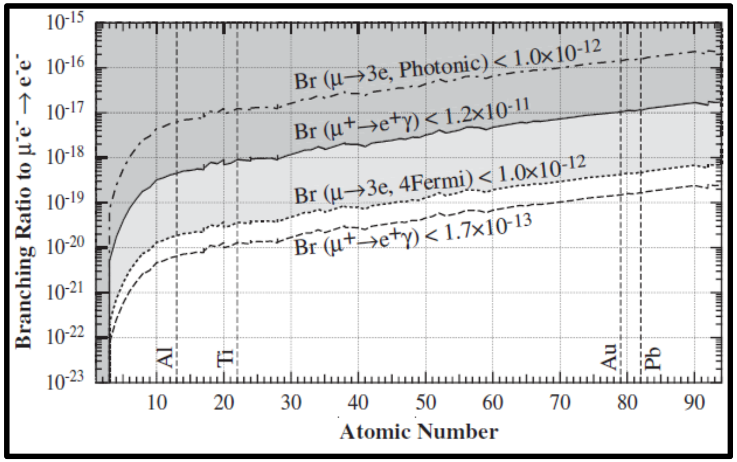
Branching ratio

$$\mathrm{Br}(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-)$$

$\tilde{\tau}_\mu$: lifetime of a muonic atom

➢ increasing as atomic # Z is larger

Using muonic atom with large Z is favored.



Improved estimation of decay rate

Approximations used in the previous work

- The spreads of bound μ^-, e^- are sufficiently large.
- emitted e^- : **plane wave**
- bound electron : **non-rela**
- nucleus : point charge

Those approximations are expected to be worse for large Z.

⇩ for more quantitative estimation

☐ treatment of leptons as **relativistic Coulomb wave**

- **distortion** of emitted e^-s by nuclear Coulomb potential
- **relativistic** treatment of bound leptons
- nuclear charge distribution with a finite size

➡ How will the decay rates be changed by this improvement?

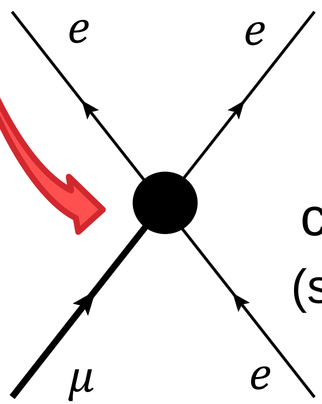
2. FORMULATION

Effective Lagrangian

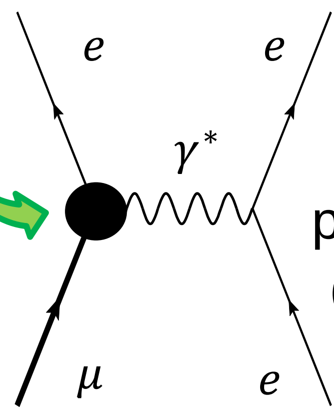
$$\mathcal{L}_I = \underline{\mathcal{L}_{contact}} + \underline{\mathcal{L}_{photo}}$$

$$\mathcal{L}_{contact} = g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L)$$
$$+g_3(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_R}\gamma^\mu e_R) + g_4(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_L}\gamma^\mu e_L)$$
$$+g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + g_6(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_R}\gamma^\mu e_R) + [h.c.]$$

$$\mathcal{L}_{photo} = g_R\overline{e_L}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + g_L\overline{e_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu} + [h.c.]$$



contact interaction
(short range process)



photonic interaction
(long range process)

Calculating method

Decay rate Γ

$$\Gamma = 2\pi \sum_f \sum_{\bar{i}} \delta(E_f - E_i) \left| \langle \psi_e^{s_1}(\boldsymbol{p}_1) \psi_e^{s_2}(\boldsymbol{p}_2) | H | \psi_\mu^{s_\mu}(1s) \psi_e^{s_e}(1s) \rangle \right|^2$$

use partial wave expansion to express the distortion

$$\psi_e^{p,s} = \sum_{\kappa,\mu,m} 4\pi\, i^{l_\kappa} (l_\kappa, m, 1/2, s | j_\kappa, \mu) Y_{l_\kappa,m}^*(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa,\mu}$$

get radial functions by solving Dirac eq. numerically

$$\frac{dg_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - \big(E + m + e\phi(r)\big) f_\kappa(r) = 0$$

$$\frac{df_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + \big(E - m + e\phi(r)\big) g_\kappa(r) = 0$$

$$\psi(\boldsymbol{r}) = \begin{pmatrix} g_\kappa(r) \chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r) \chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

ϕ : nuclear Coulomb potential

3. RESULTS

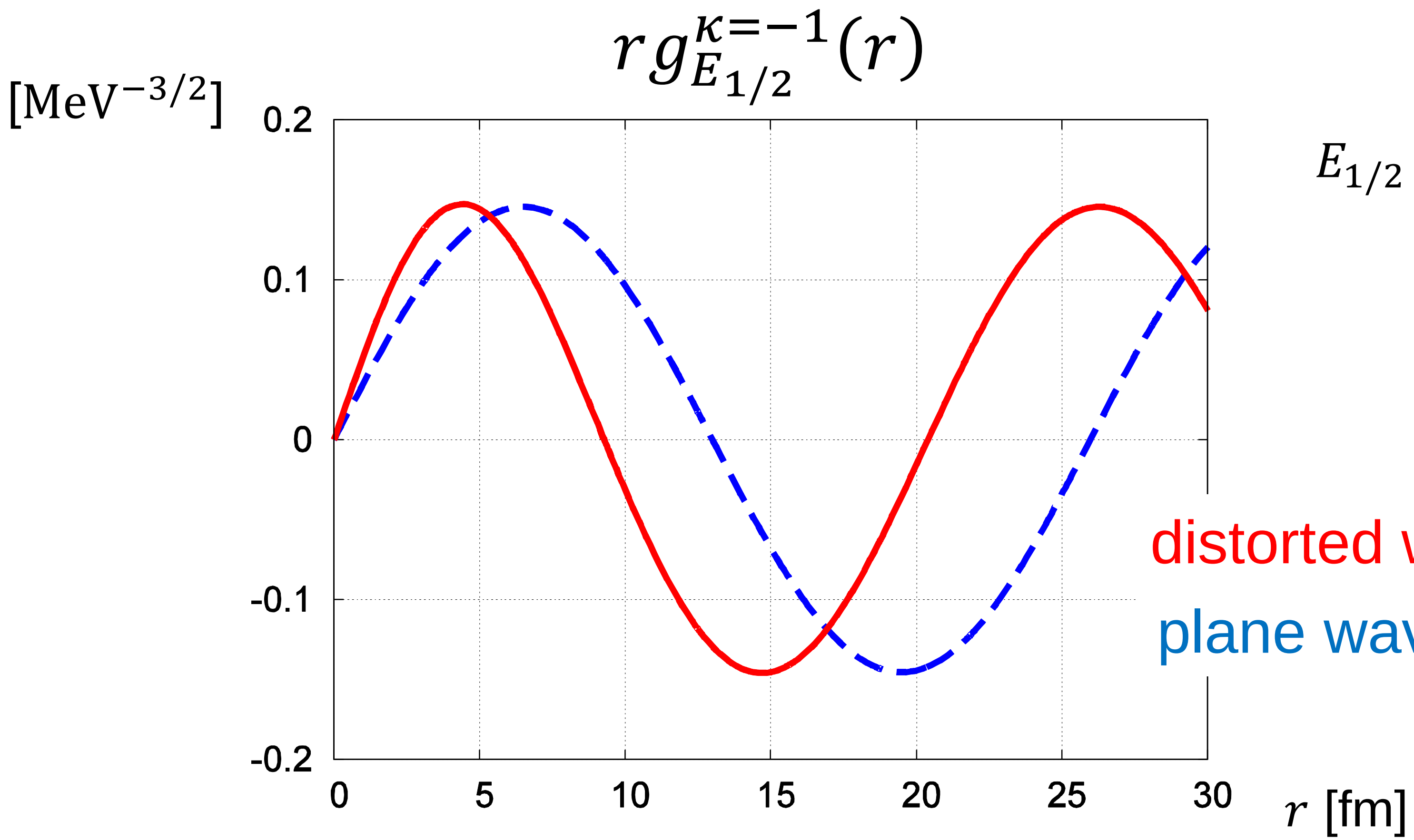
Radial functions (scattering e^-)

➤ $\kappa = -1$ partial wave

^{208}Pb case

$Z = 82$

$E_{1/2} \approx 48$MeV

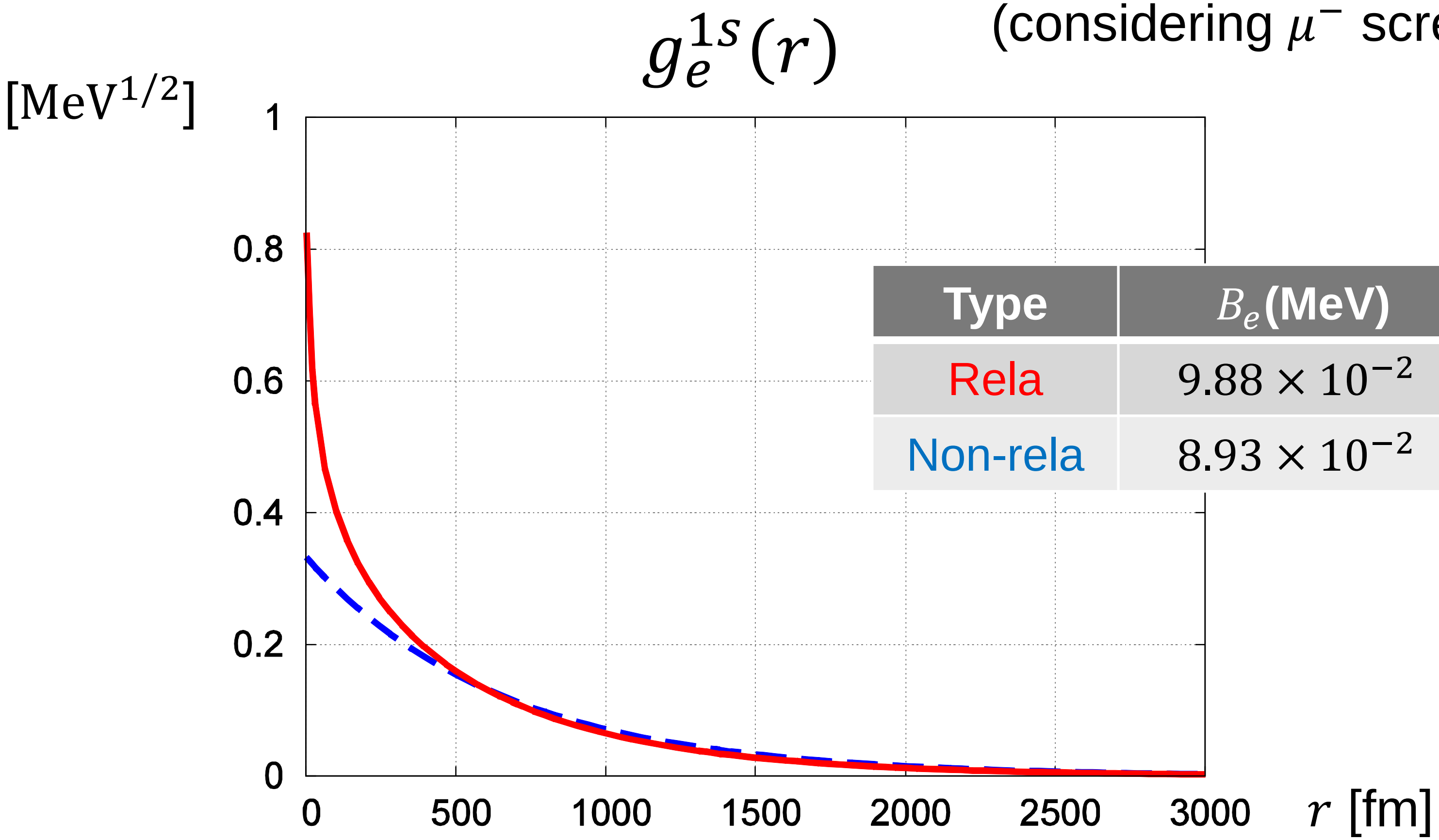


Attracted by nuclear Coulomb potential

Radial functions (bound e^-)

^{208}Pb case $Z = 81$

(considering μ^- screening)



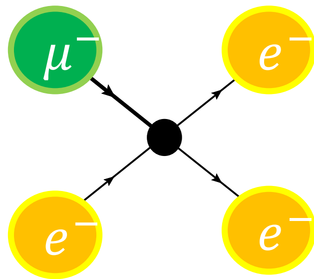
Type	B_e(MeV)
Rela	9.88×10^{-2}
Non-rela	8.93×10^{-2}

Relativity enhances the value near the origin.

Contact process

✓ bound μ^-

✓ bound e^-

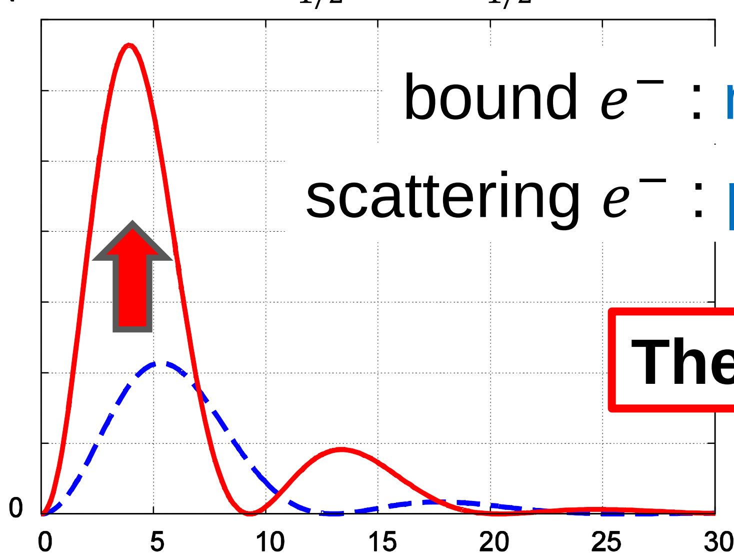


✓ scattering e^-

✓ scattering e^-

- overlap of bound μ^-, bound e^-, and two scattering e^-s

$r^2 g_\mu^{1s}(r) g_e^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) g_{E_{1/2}}^{\kappa=-1}(r)$



r [fm]

bound e^- : non-rela → rela

scattering e^- : plane → distorted

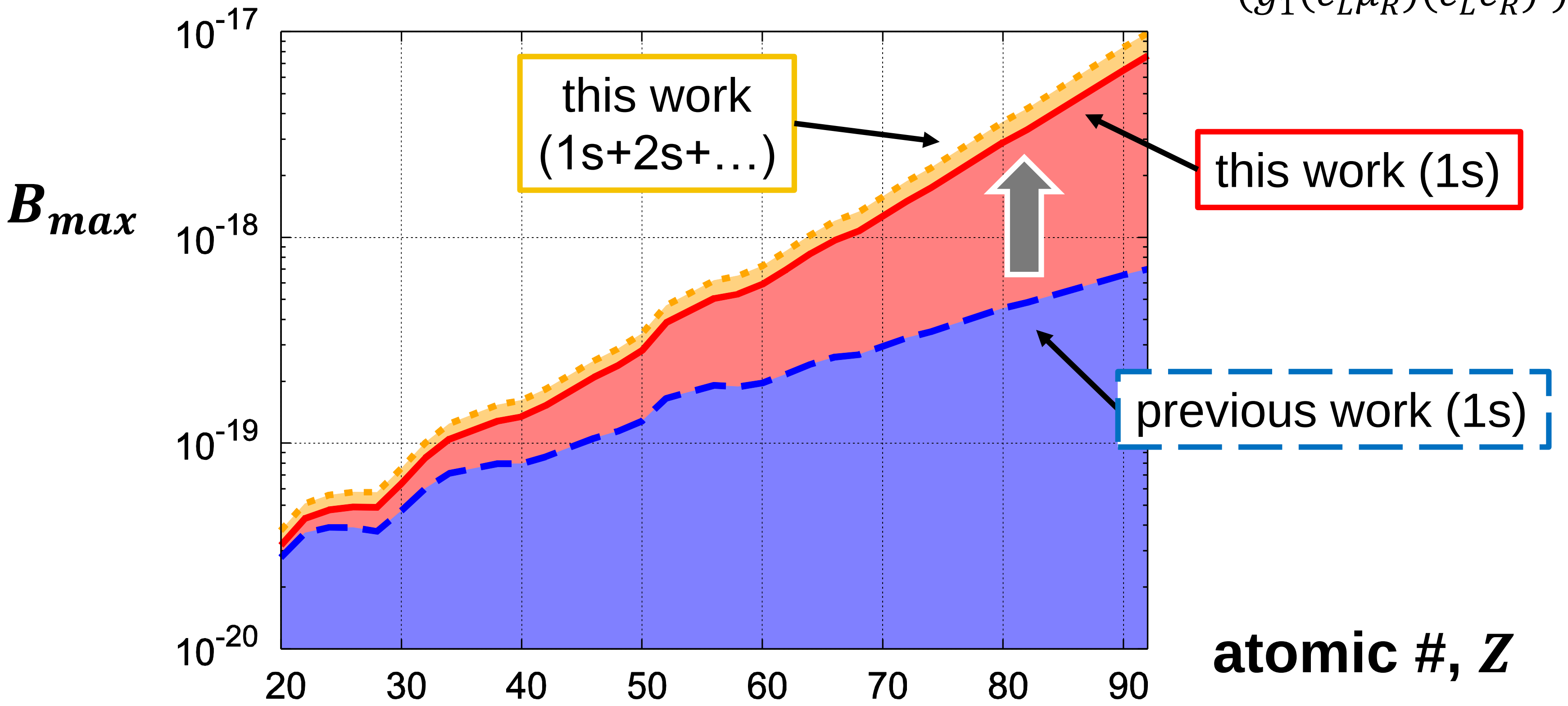
The transition rate is enhanced!

Upper limits of BR (contact process)

$BR(\mu^+ \to e^+e^-e^+) < 1.0 \times 10^{-12}$ → $BR(\mu^- e^- \to e^- e^-) < B_{max}$

(SINDRUM, 1988)

$(g_1(\bar{e_L}\mu_R)(\bar{e_L}e_R))$

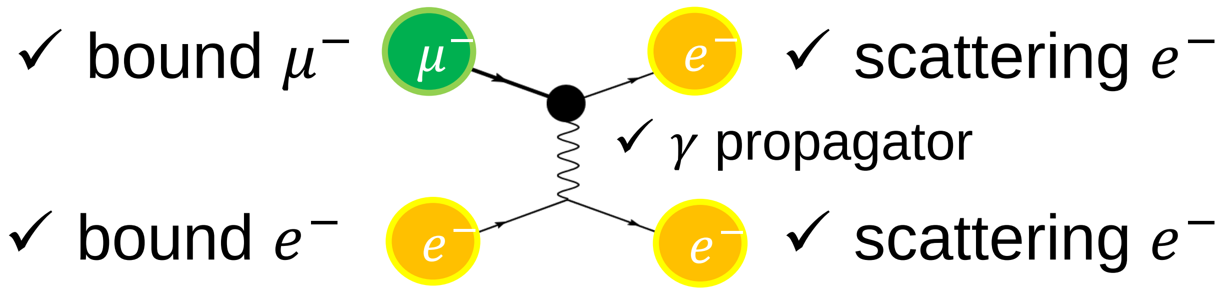


needed # of muonic atoms ($Z = 82$)

2.1×10^{18} → 3.0×10^{17}

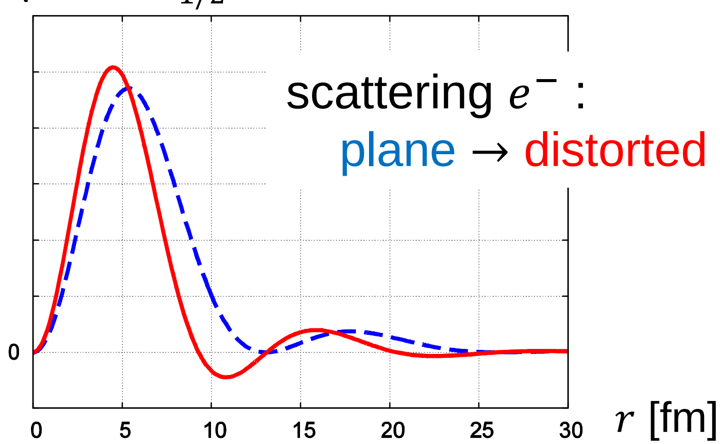
cf. COMET ($\mu^-\mathrm{Al} \to e^-\mathrm{Al}$)
$O(10^{18})$ muonic atoms

Photonic process



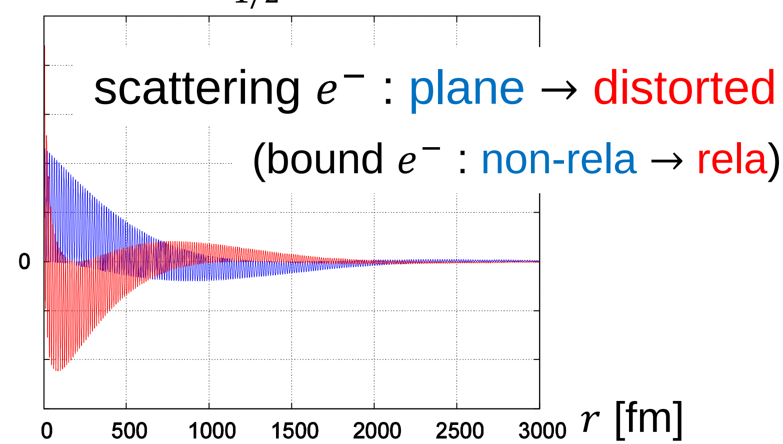
◆ overlap of bound μ^-, scattering e^-, and γ

$$r^2 g_\mu^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)$$



◆ overlap of bound e^-, scattering e^-, and γ

$$r^2 g_e^{1s}(r) g_{E_{1/2}}^{\kappa=-1}(r) j_0(q_0 r)$$



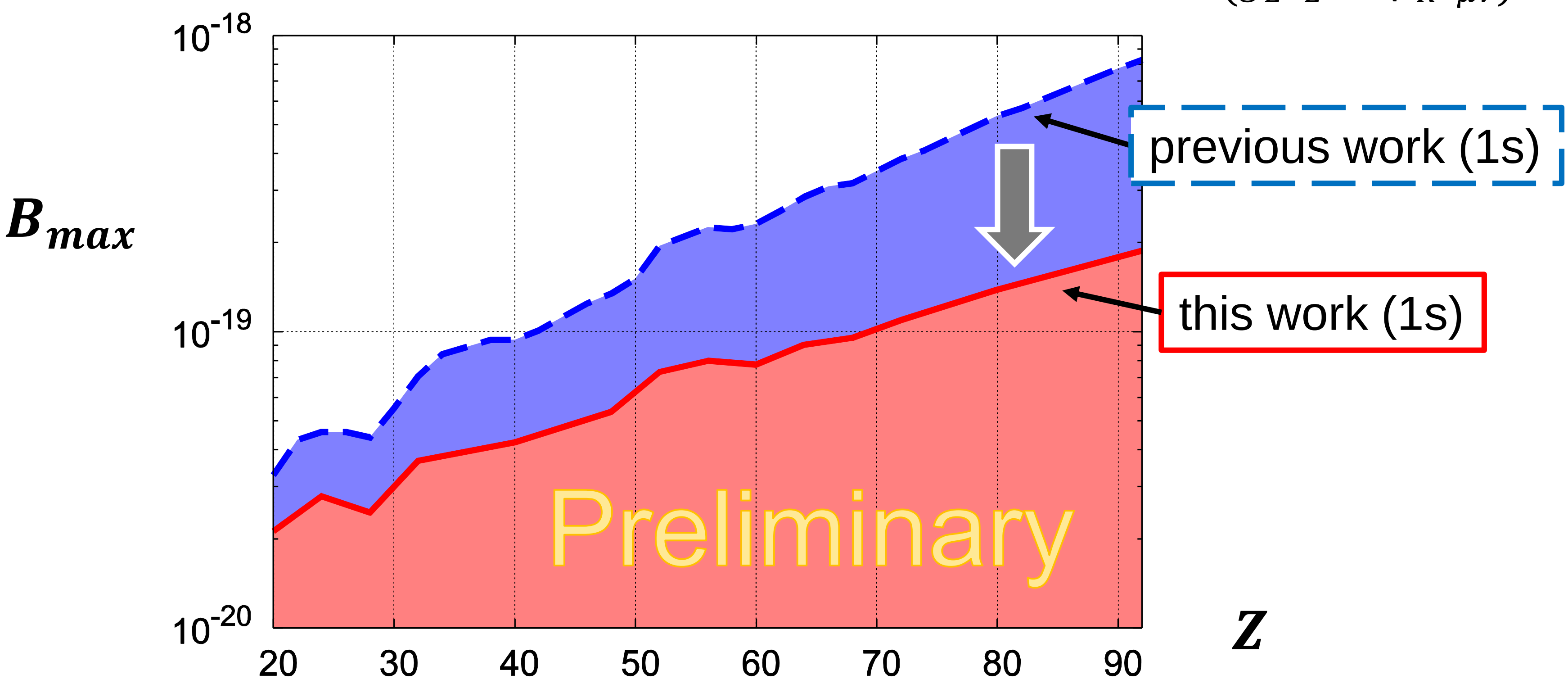
⇒ The distortion makes these overlaps smaller.

Upper limits of BR (photonic process)

$BR(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13}$ → $BR(\mu^- e^- \to e^- e^-) < B_{max}$

(MEG, 2013)

$\left(g_L \overline{e_L} \sigma^{\mu\nu} \mu_R F_{\mu\nu}\right)$



needed # of muonic atoms ($Z = 82$)

1.8×10^{18} → 7.1×10^{18}

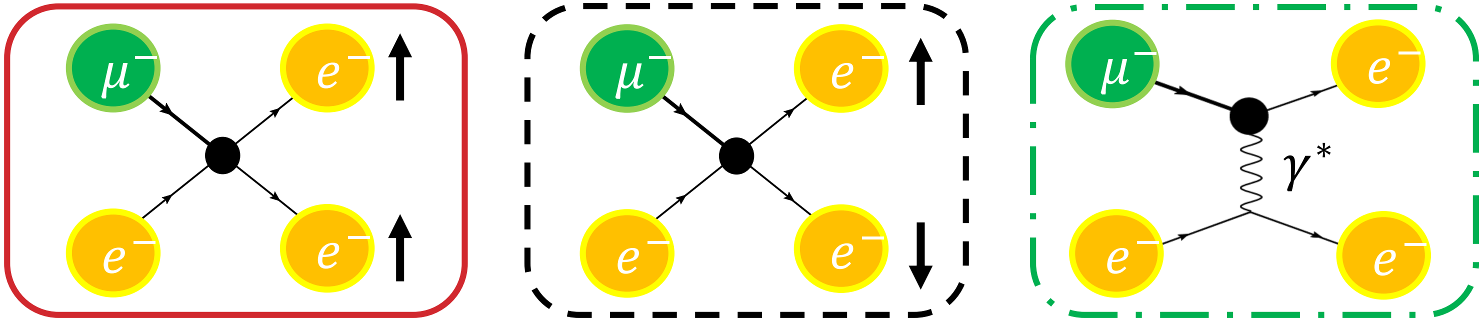
Model-discriminating power

After finding CLFV transition,
which CLFV interaction exists would be an important issue.

$$\mathcal{L}_{contact}^{\uparrow\uparrow} = g_1(\overline{e_L}\mu_R)(\overline{e_L}e_R) + g_2(\overline{e_R}\mu_L)(\overline{e_R}e_L) + g_3(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_R}\gamma^\mu e_R) + g_4(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_L}\gamma^\mu e_L) + [h.c.]$$

$$\mathcal{L}_{contact}^{\uparrow\downarrow} = g_5(\overline{e_R}\gamma_\mu\mu_R)(\overline{e_L}\gamma^\mu e_L) + g_6(\overline{e_L}\gamma_\mu\mu_L)(\overline{e_R}\gamma^\mu e_R) + [h.c.]$$

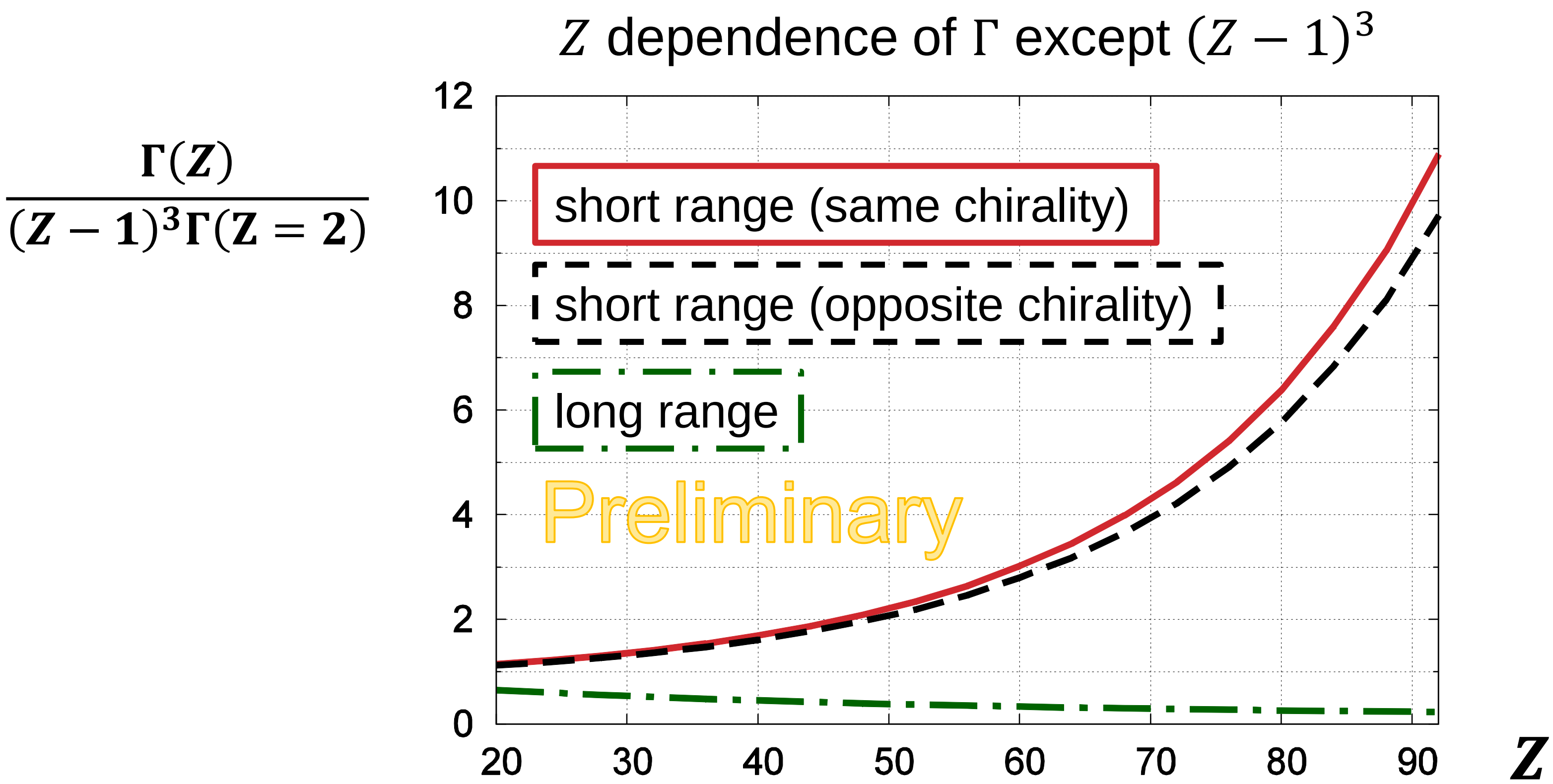
$$\mathcal{L}_{photo} = g_R\overline{e_L}\sigma^{\mu\nu}\mu_R F_{\mu\nu} + g_L\overline{e_R}\sigma^{\mu\nu}\mu_L F_{\mu\nu} + [h.c.]$$



Can experiments discriminate those?

Discriminating method 1

~ atomic # dependence of decay rates ~



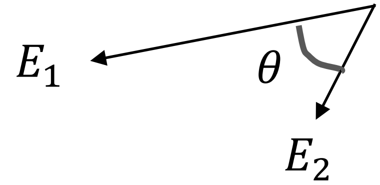
- The Z dependences are different among each interactions.
- Compared to $(Z-1)^3$, that of short range process is larger while that of long range process is smaller.

Discriminating method 2

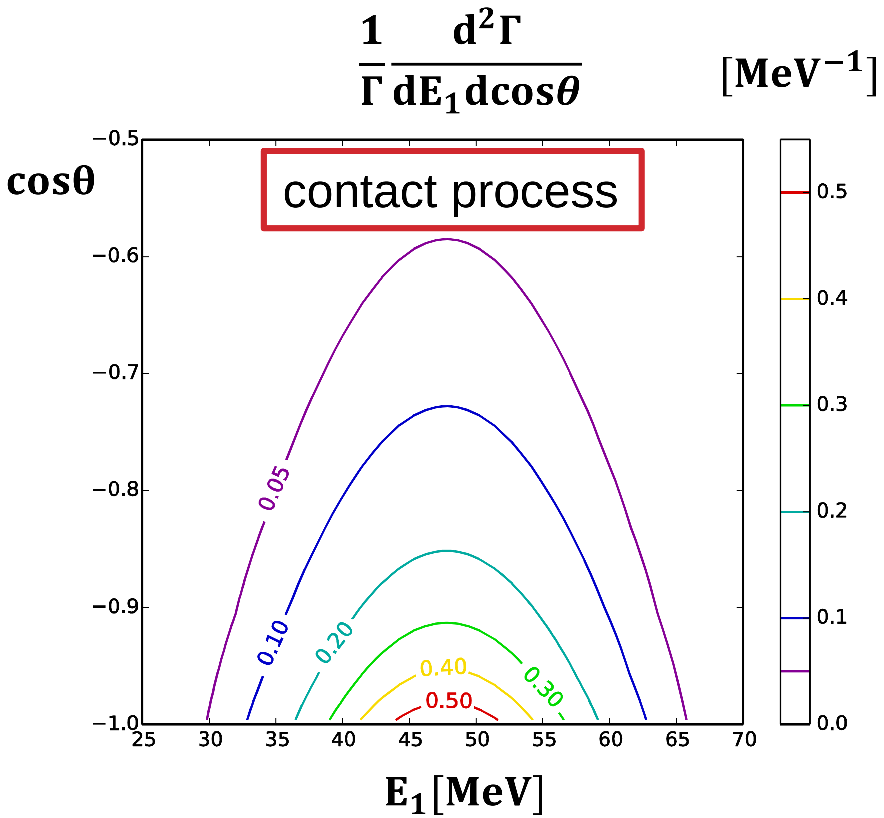
~ energy and angular distributions ~

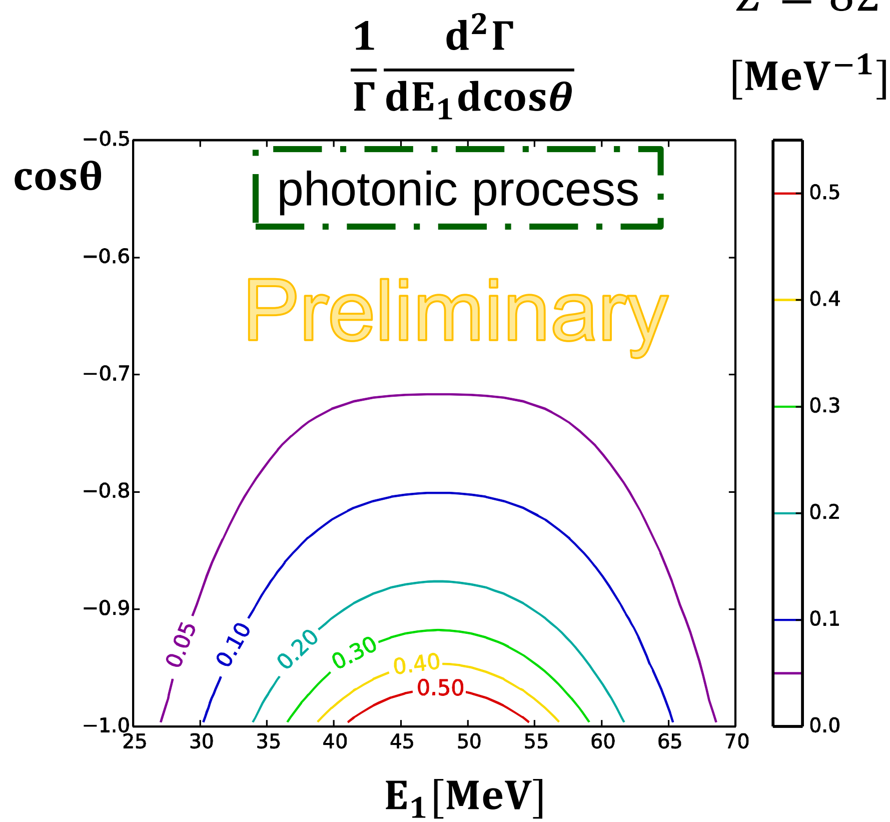
E_1 : energy of an emitted electron

θ : angle between two emitted electrons



$Z = 82$





4. SUMMARY

Summary

- $\mu^- e^- \to e^- e^-$ process in a muonic atom
  - ✓ interesting candidate for CLFV search
  - ✓ Our finding
    - Distortion of emitted electrons
    - Relativistic treatment of a bound electron

are important in calculating decay rates.

⇩

- ■ contact process：decay rate Enhanced (7 times in $Z = 82$)
- ■ photonic process：decay rate suppressed (1/4 times in $Z = 82$)

◆ How to identify interaction types, found by this analyses

- ✓ atomic # dependence of the decay rate
- ✓ energy and angular distributions of emitted electrons