In-medium $\eta \rightarrow 3\pi$ decay width and chiral restoration in nuclear medium

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Quantum Chromodynamics (QCD): basic theory of hadrons

$$\mathcal{L}_{\rm QCD} = \underline{\bar{q}\left(i\not{D} - m_f\right)q} - \frac{1}{4}{\rm tr}F_{\mu\nu}F^{\mu\nu}$$

Approximate chiral symmetry $\left(q_{L,R} \to e^{i\theta^a_{L,R}\lambda^a/2}q_{L,R}\right)$

Existence of degenerate chiral multiplet

- Real world

  ρ(770)⟷a_1(1260), N(939)⟷N^*(1535),…

Spontaneous breaking of chiral symmetry

Characterized by $\langle\bar{q}q\rangle$

→ Relationship with the hadron properties

ρ-a_1 mass(Weinberg,1967), N-N* mass(DeTar and Kunihiro,1989),…

※ Symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

↑

Explicitly broken by non-degenerate quark mass

→ Gell-Mann-Okubo mass formula,…

○ $\eta \to 3\pi$ ($\pi^+\pi^-\pi^0$, $3\pi^0$) decay

✓ Isospin-symmetry breaking in QCD (*u*-*d* quark mass difference)

- G parity violating process (η:even, π:odd)

※ Small QED corrections (Sutherland(1966), Baur et al.(1996), Ditsche et al.(2009))

Small decay width (~70 eV from current algebra ⇔ ~300eV(observation))

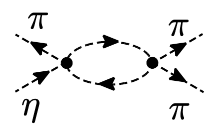
Osborn and Wallace (1970)

✓ Final-State Interaction among π

← Significance of 2π correlation in s-wave (σ channel)

- Perturbative approach (chiral perturbation theory)
  : Gasser and Leutwyler(1985), Bijnens and Ghorbani(2007)

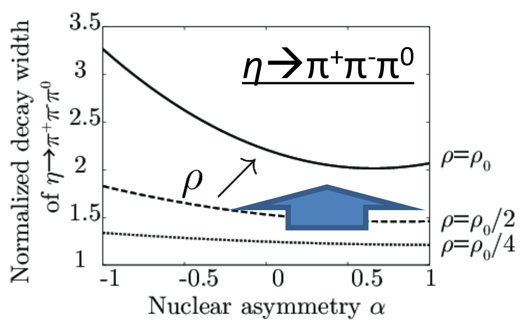
Ex.)



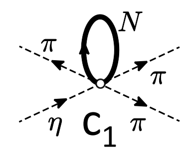
- Non-perturbative approach
  - Chiral Unitary approach (resummation scheme): Borasoy and Nissler(2005)
  - Dispersive approach (Roiesnel and Truong(1981), Kambor et al.(1996), Anisovich and Leutwyler(1996),…)

■ Analysis of $\eta \to 3\pi$ width in asymmetric nuclear medium ($\rho_n \neq \rho_p$)

S.S. and Kunihiro (2015)



$$\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$$



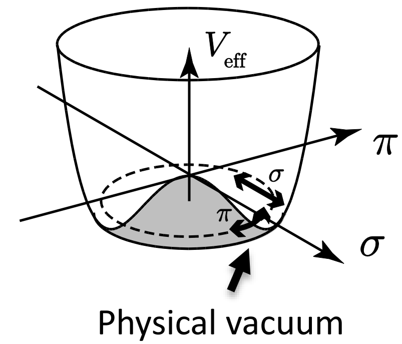
Enhancement by $\rho = \rho_n + \rho_p$

in addition to $\delta\rho = \rho_n - \rho_p$

σ meson and chiral restoration

chiral partner of π

Inevitable existence of the massive σ meson associated with SSB in chiral model



※ σ: coupling with 2π state
➔ relevance to s-wave 2π correlation

Ex.) NJL in chiral limit: $m_\sigma \propto \langle \bar{q}q \rangle$ (Nambu and Jona-Lasinio(1961),Hatsuda and Kunihiro(1994))

➔ Possible effect of <u>chiral restoration</u> (softening of σ) (Hatsuda and Kunihiro(1985))

☐ Reduction of quark condensate in nuclear medium

<u>@ low density</u>

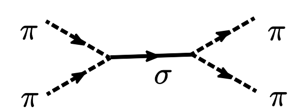
$$\langle \bar{q}q \rangle^* = \left(1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2}\rho\right)\langle \bar{q}q \rangle + \mathcal{O}(\rho^{n>1})$$

Durkarev and Levin (1990), Cohen et al.(1992)

➔ ***Possible Modification of Hadron Properties***

Ex.) di-lepton spectrum of vector meson, deeply bound π-atom,…

Large modification of s-wave 2π correlation in association with chiral restoration



Ex.) Hatsuda, Kunihiro, and Shimizu (1998): analysis of 2π system

<u>**Possible effect on in-medium *η*➔3π decay from chiral restoration in nuclear medium**</u>

Analysis of $\eta \rightarrow 3\pi$ decay width in nuclear medium using linear sigma model

- low-energy effective model of QCD (possess chiral symmetry)

Lagrangian

$$\mathcal{L}_{\mathrm{L}\sigma\mathrm{M}} = \frac{1}{2}\mathrm{tr}[\partial_\mu M \partial^\mu M^\dagger] - \frac{\mu^2}{2}\mathrm{tr}[MM^\dagger] - \frac{\lambda}{4}\mathrm{tr}[(MM^\dagger)^2] - \frac{\lambda'}{4}[\mathrm{tr}(MM^\dagger)]^2 + \frac{B}{2}(\det M + \det M^\dagger) + \frac{A}{2}\mathrm{tr}[\mathcal{M}M^\dagger + M\mathcal{M}^\dagger] + \bar{N}(i\partial\!\!\!/ - gM_5)N$$

$$M = M_s + iM_{ps} = \sum_{a=0}^{8}\frac{\sigma^a\lambda^a}{\sqrt{2}} + i\sum_{a=0}^{8}\frac{\pi^a\lambda^a}{\sqrt{2}} \quad N = {}^t(p,n) \quad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$$

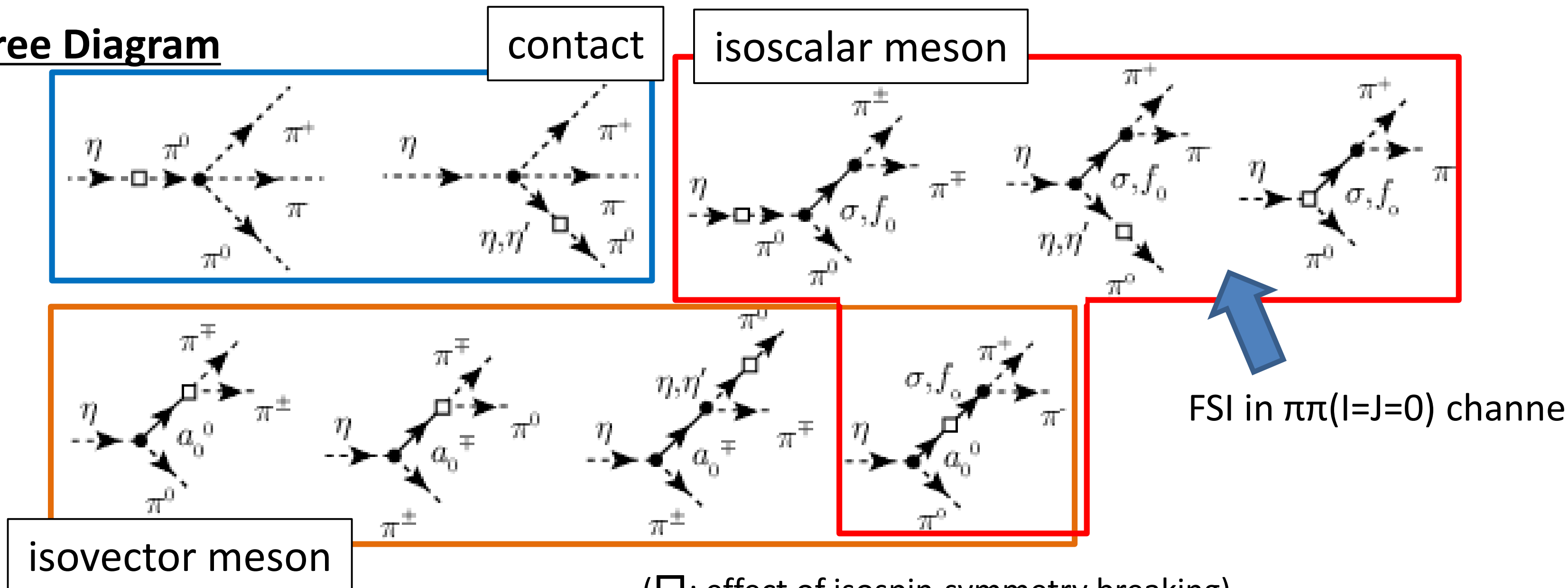
$$M_5 = \sum_{a=0}^{3}\left(\frac{\tau_a\sigma_a}{\sqrt{2}} + i\gamma_5\frac{\tau_a\pi_a}{\sqrt{2}}\right)$$

$$m_d - m_u \propto m_{K^0}^2 - m_{K^\pm}^2 - m_{\pi^0}^2 + m_{\pi^\pm}^2$$

- Chiral restoration: decrease of ⟨σ⟩ (⟨σ⟩: chiral order parameter)
  - ⟨σ⟩: minimum of the effective potential
  - ← 30% reduction at $\rho=\rho_0$ from deeply bound π-atom data (Suzuki et al.,(2004))
- Explicit σ dof → natural inclusion of softening of σ meson

$\eta \rightarrow 3\pi$ decay in free space

Tree Diagram



(□: effect of isospin-symmetry breaking)

- Tree-level approximation
- Effect of isospin-symmetry breaking: Leading order
- Final-state interaction in ππ(I=J=0) channel: pole of the sigma meson
  - Width of sigma meson: included using the tree-level approximation

$$G_\sigma(p^2) = \frac{1}{p^2 - m_\sigma + i\Theta_{\rm tree}(p^2)} \quad \left(\Theta(p^2) = \frac{g^2_{\sigma\pi\pi}}{16\pi}\sqrt{1-\frac{4m_\pi^2}{p^2}}\,\theta(p^2-4m_\pi^2)\right)$$

➔ Fairly good accordance of $\eta \rightarrow 3\pi$ width with the observed value (~70%)

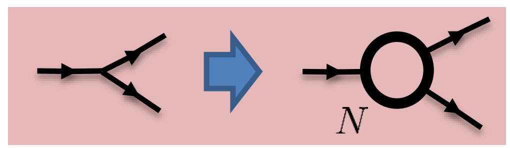
Nuclear-medium effect: perturbative inclusion with respect to Fermi momentum

$$iG(p,k_f) = (\not{p}+m_N)\left\{\frac{i}{p^2-m_N^2+i\epsilon} - 2\pi\delta(p^2-m_N^2)\theta(p_0)\theta(k_f-|\vec{p}|)\right\}$$

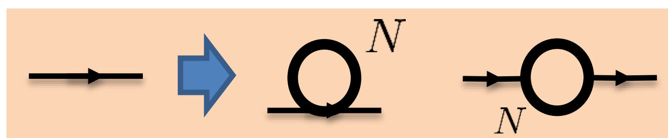
<u>Pauli-Blocking effect</u>

✓ Modification of

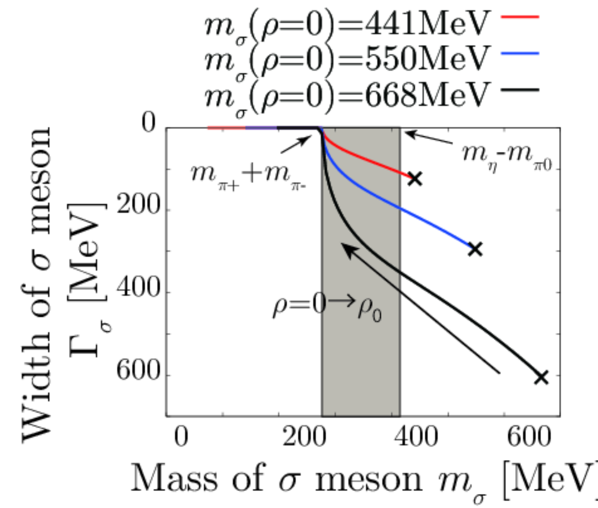
- vacuum $\langle\sigma\rangle_0 \Rightarrow \langle\sigma\rangle_\rho$
- coupling of mesons



- meson mass



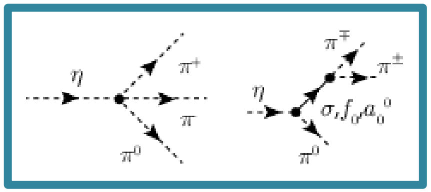
■ m_σ,Γ_σ: reduction along with ρ = softening of σ meson

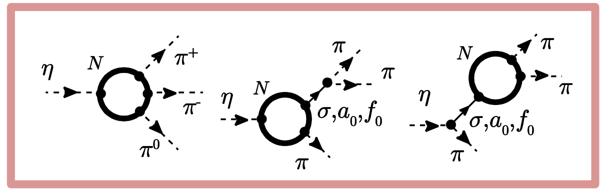


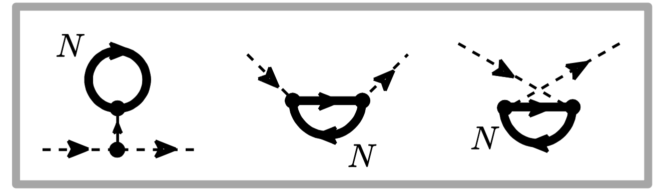
※ Validity of this calculation: small density

- Leading-order calculation with respect to Fermi momentum of nuclear medium

$\eta \rightarrow 3\pi$ decay amplitude in nuclear medium







$$g_{\pi_i\pi_j\pi_k\pi_l}(\rho) = g^0_{\pi_i\pi_j\pi_k\pi_l} + \Gamma_{\pi_i\pi_j\pi_k\pi_l}(\rho)$$

$$g_{\sigma_f\pi_i\pi_j}(\rho) = g^0_{\sigma_f\pi_i\pi_j} + \Gamma_{\sigma_f\pi_i\pi_j}(\rho)$$

$$m^2_{\sigma_i\sigma_j}(\rho) = m^2_{\sigma_i\sigma_j}(\rho=0) + \Sigma_{\sigma_i\sigma_j}(\rho)$$

$$m^2_{\pi_i\pi_j}(\rho) = m^2_{\pi_i\pi_j}(\rho=0) + \Sigma_{\pi_i\pi_j}(\rho)$$

$\eta \rightarrow 3\pi$ decay amplitude in nuclear medium

$\eta \to \pi^+\pi^-\pi^0$ process

$$\mathcal{M}^{\text{tree}}_{\eta\to\pi^+\pi^-\pi^0} = \mathcal{M}^{\text{contact}}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{M}^{\text{isoscalar}}_{\eta\to\pi^+\pi^-\pi^0} + \mathcal{M}^{\text{isovector}}_{\eta\to\pi^+\pi^-\pi^0}$$

$$\mathcal{M}^{\text{contact}}_{\eta\to\pi^+\pi^-\pi^0} = 2(-\sin\theta_{\eta\pi^0})g_{\pi_3\pi_3\pi^+\pi^-} + 2\sin\theta_{\eta\pi^0}g_{\eta\eta\pi^+\pi^-} + \sin\theta_{\eta'\pi^0}g_{\eta\eta'\pi^+\pi^-}$$

$$\mathcal{M}^{\text{isoscalar}}_{\eta\to\pi^+\pi^-\pi^0} \sim -\frac{g_{\sigma\eta\pi}g_{\sigma\pi\pi}}{s - m^2_{\sigma,f_0}}$$

$$\mathcal{M}^{\text{isovector}}_{\eta\to\pi^+\pi^-\pi^0} \sim -\frac{g_{a_0\eta\pi}g_{a_0\pi\pi}}{t - m^2_{a_0^-}} - \frac{g_{a_0\eta\pi}g_{a_0\pi\pi}}{u - m^2_{a_0^+}}$$

※ medium modification appears from the mass and vertex of mesons

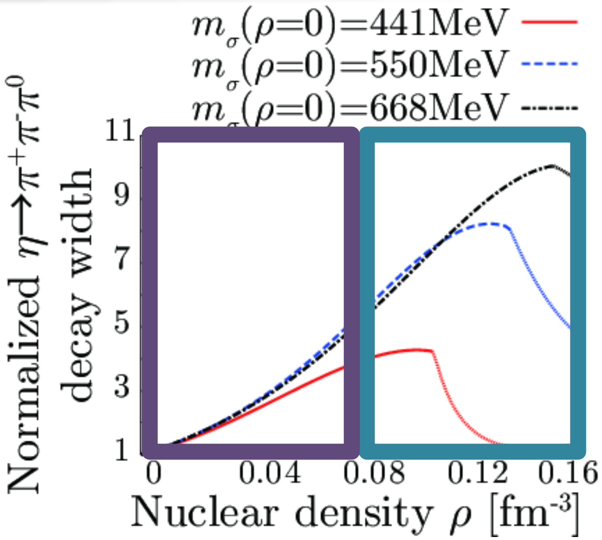
$\eta \to 3\pi^0$ process

Bose symmetry of $3\pi^0$ in final state ← Symmetrization of $\eta \rightarrow \pi^+\pi^-\pi^0$

$$\mathcal{M}_{\eta\to3\pi^0}(s,t,u) = \mathcal{M}_{\eta\to\pi^+\pi^-\pi^0}(s,t,u) + \mathcal{M}_{\eta\to\pi^+\pi^-\pi^0}(t,u,s) + \mathcal{M}_{\eta\to\pi^+\pi^-\pi^0}(u,s,t)$$

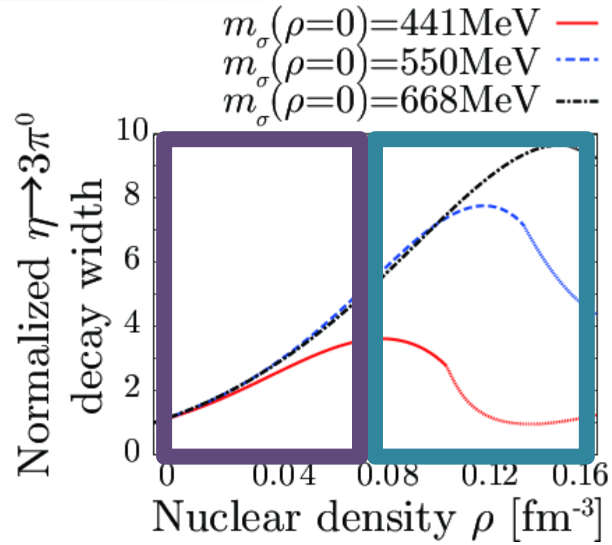
$\eta \rightarrow 3\pi$ decay width in nuclear medium

$\eta \rightarrow \pi^+\pi^-\pi^0$ process

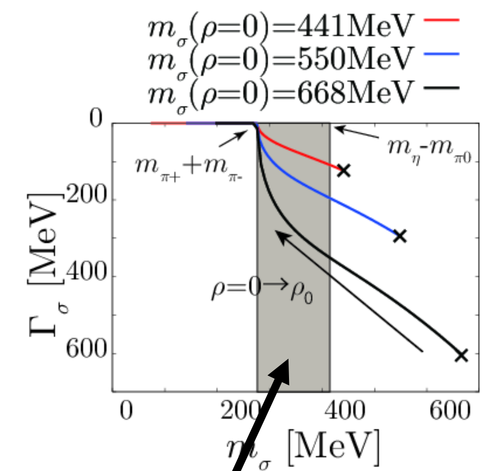


$\Gamma_{\eta\rightarrow\pi^+\pi^-\pi^0}(\rho=0)$~200 eV

$\eta \rightarrow 3\pi^0$ process



$\Gamma_{\eta\rightarrow 3\pi^0}(\rho=0)$~300 eV



kinematically allowed region
in $\eta \rightarrow 3\pi$ Dalitz plot

○ Enhancement of decay width by ρ

- Enhancement by a factor 4-10 at most
- Relatively large $m_\sigma(\rho=0)$ dependence

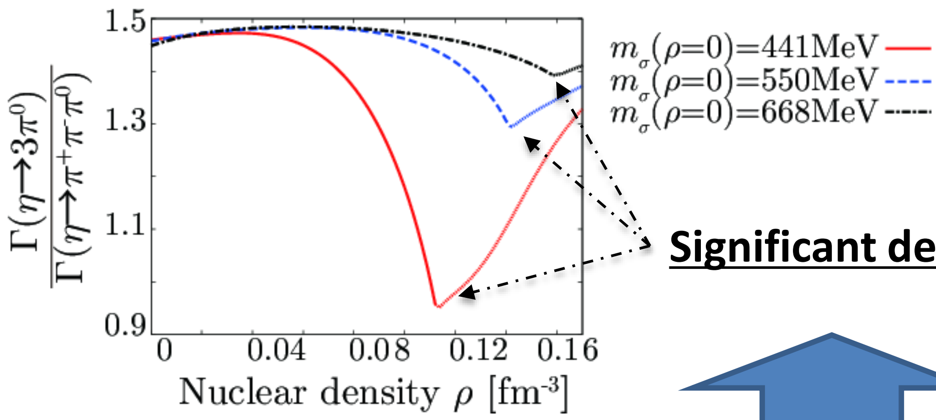
○ Enhancement in the small density (~$\rho_0/2$)

than that in ρ=0

(small dependence on $m_\sigma(\rho=0)$)

※ Similar tendency of in-medium $\eta \rightarrow 3\pi^0$ to $\pi^+\pi^-\pi^0$

Ratio of $\eta \rightarrow 3\pi^0$ to $\pi^+\pi^-\pi^0$ width ($\Gamma_{\eta\rightarrow 3\pi 0}/ \Gamma_{\eta\rightarrow \pi+\pi-\pi 0}$)



Significant decrease of $\Gamma_{\eta\rightarrow 3\pi 0}/ \Gamma_{\eta\rightarrow \pi+\pi-\pi 0}$

Cancellation from Bose symmetry of $3\pi^0$ and softening of σ meson

σ meson contribution to $\eta \rightarrow 3\pi^0$

$$\mathcal{M}^{\rm sigma}_{\eta\rightarrow 3\pi^0} = -\frac{g_{\sigma\eta\pi}g_{\sigma\pi\pi}}{s-m_\sigma^2(\rho)} - \frac{g_{\sigma\eta\pi}g_{\sigma\pi\pi}}{t-m_\sigma^2(\rho)} - \frac{g_{\sigma\eta\pi}g_{\sigma\pi\pi}}{u-m_\sigma^2(\rho)}$$

$$t, u \sim \frac{m_\eta^2 - m_\pi^2}{2} \quad \text{when} \quad s \sim 4m_\pi^2$$

$4m_\pi^2 < m_\sigma^2(\rho) < \frac{m_\eta^2 - m_\pi^2}{2}$ ⇒ Cancellation between the s channel contribution and the t,u channel ones

Summary

- Analysis of $\eta \rightarrow 3\pi$ decay in nuclear medium using linear σ model
  - Enhancement of $\eta \rightarrow 3\pi(\pi^+\pi^-\pi^0, 3\pi^0)$ width

⇨ $\eta \rightarrow 3\pi$ decay is one of the possible probe for chiral restoration through the softening of σ meson

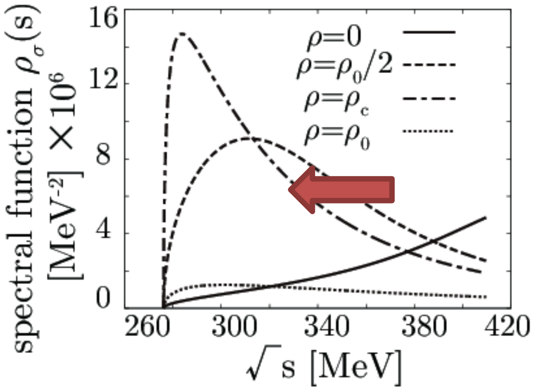
- Decrease of $\Gamma_{\eta \rightarrow 3\pi 0}/\Gamma_{\eta \rightarrow \pi+\pi-\pi 0}$

← ✓ Softening of σ
✓ Bose symmetry of $3\pi^0$ in $\eta \rightarrow 3\pi^0$ decay

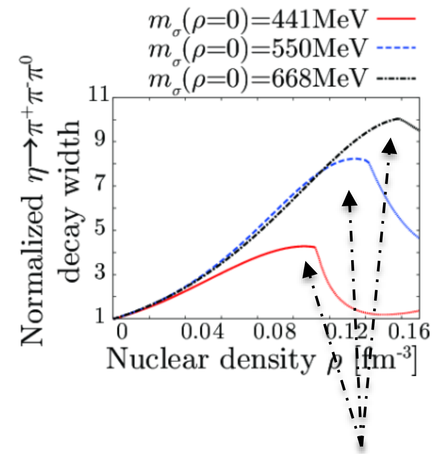
Thank you for your attention.

■Spectral function of σ meson

$$m_\sigma = 441(\rho = 0)\mathrm{MeV}$$



softening of σ
→Enhancement of spectral function



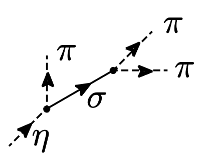
$\rho=\rho_c$

ρ_c=0.1 fm^{-3} with m_σ=441MeV
0.13 fm^{-3} with m_σ=550MeV
0.15 fm^{-3} with m_σ=668MeV

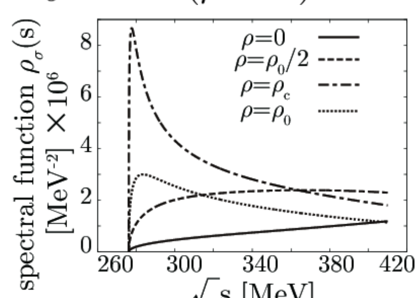
$$\rho_\sigma = -\frac{1}{\pi}\mathrm{Im}G_\sigma \quad G_\sigma = \frac{1}{s - m_\sigma^2(\rho) + i\Theta(s)}$$

$$\left(\Theta(s) = \frac{g^2_{\sigma\pi\pi}}{16\pi}\sqrt{1 - \frac{m_\sigma^2}{s}}\theta(s - 4m_\pi^2)\right)$$

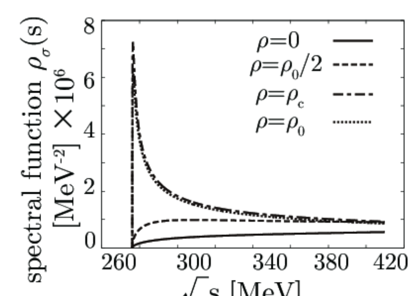
➡ Enhancement from σ-contribution

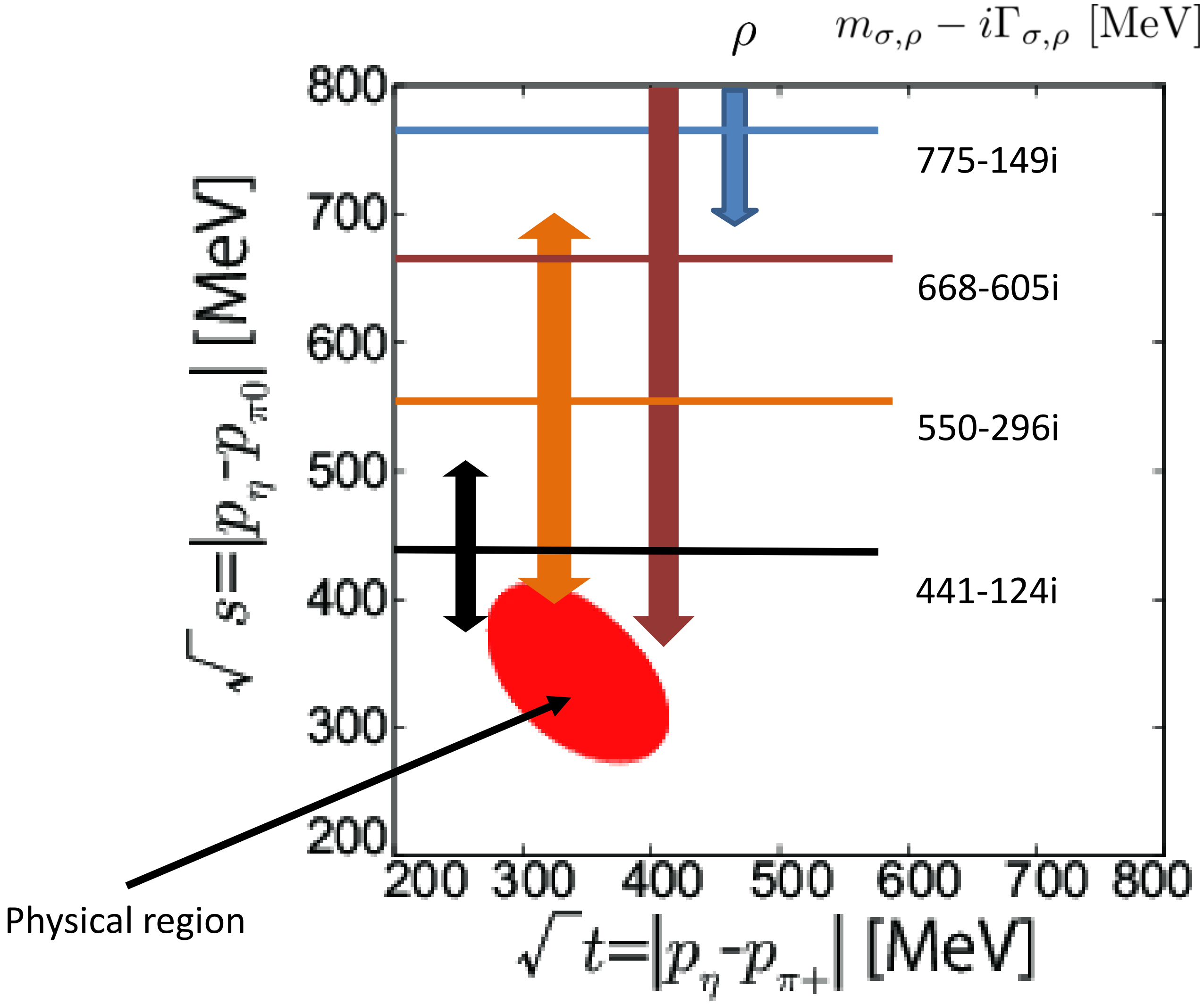


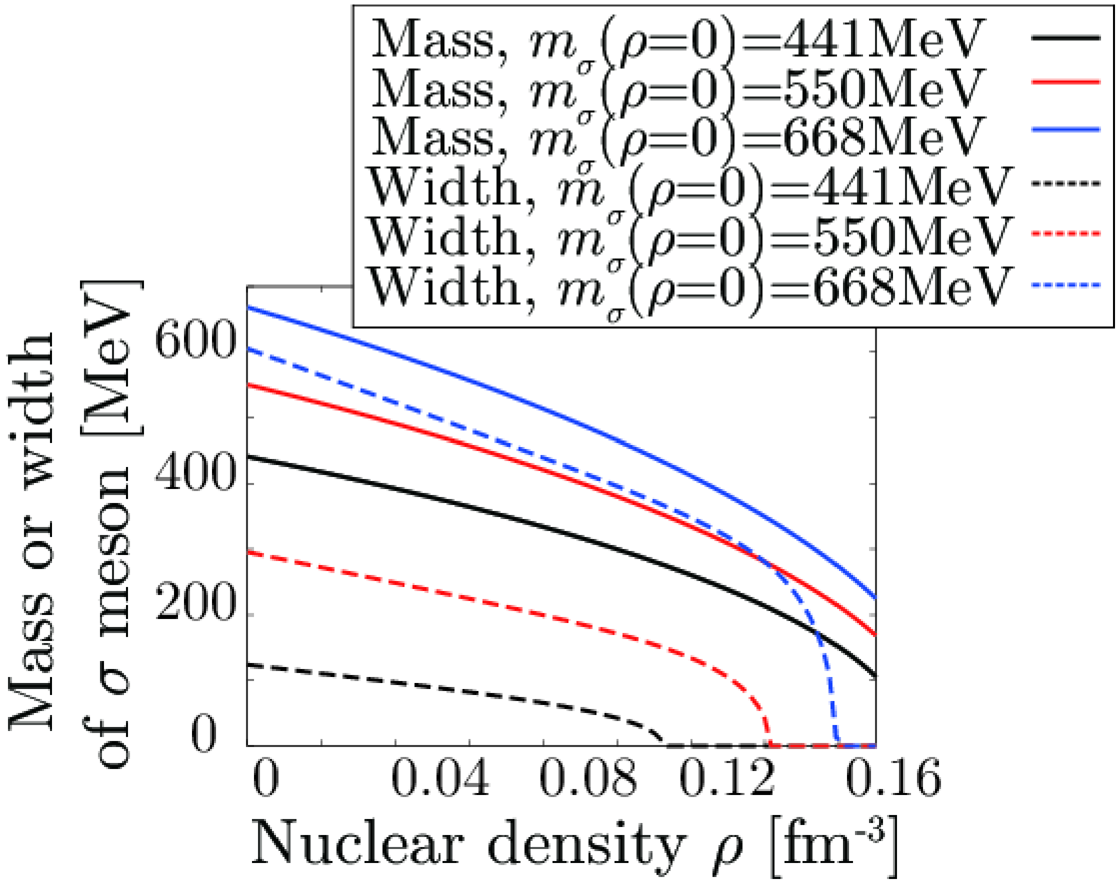
$$m_\sigma = 550(\rho = 0)\mathrm{MeV}$$



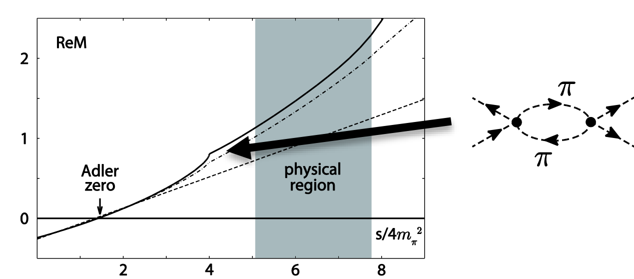
$$m_\sigma = 668(\rho = 0)\mathrm{MeV}$$




ρ
$m_{\sigma,\rho} - i\Gamma_{\sigma,\rho}$ [MeV]
800
700
600
500
400
300
200
775-149i
668-605i
550-296i
441-124i
200 300 400 500 600 700 800
$\sqrt{s}=|p_\eta - p_{\pi^0}|$ [MeV]
$\sqrt{t}=|p_\eta - p_{\pi^+}|$ [MeV]
Physical region


Mass, $m_\sigma(\rho=0)$=441MeV
Mass, $m_\sigma(\rho=0)$=550MeV
Mass, $m_\sigma(\rho=0)$=668MeV
Width, $m_\sigma(\rho=0)$=441MeV
Width, $m_\sigma(\rho=0)$=550MeV
Width, $m_\sigma(\rho=0)$=668MeV
Mass or width of σ meson [MeV]
600
400
200
0
0
0.04
0.08
0.12
0.16
Nuclear density ρ [fm^{-3}]

Plot of Real part of matrix element of $\eta \rightarrow 3\pi$



$$\sim \sqrt{1 - \frac{4m_\pi^2}{s}}$$

Anisovich and Leutwyler(1996)