Appearance of novel modes of $\bar{D}$ mesons in the dual chiral density wave

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arXiv:1509.08578

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2. Heavy Meson Effective Theory

3. EoMs for $\bar{D}$ mesons in the DCDW

4. Results and a summary

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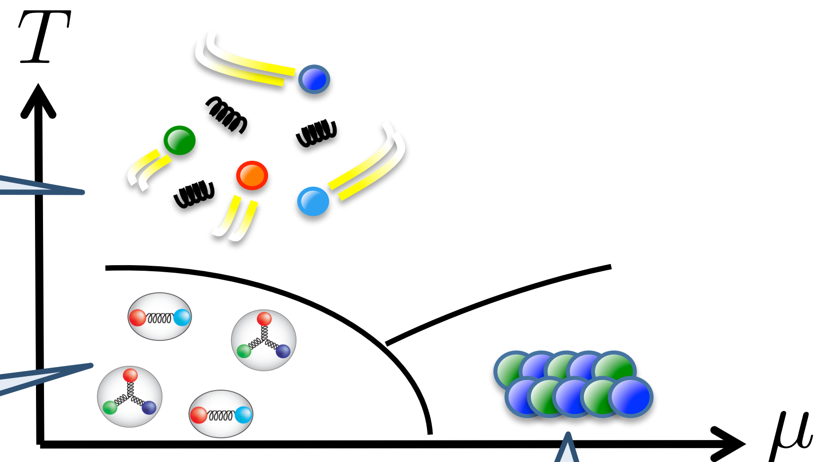
1. Introduction

- **QCD phase diagram**



- Quark Gluon Plasma
- Chiral restored phase
  (RHIC, LHC)

- Quarks are confined
- Chiral broken phase
  (J-PARC, and so on)

- Color superconductivity (?)
- Chiral restored phase (?)
  (Neutron star (?))

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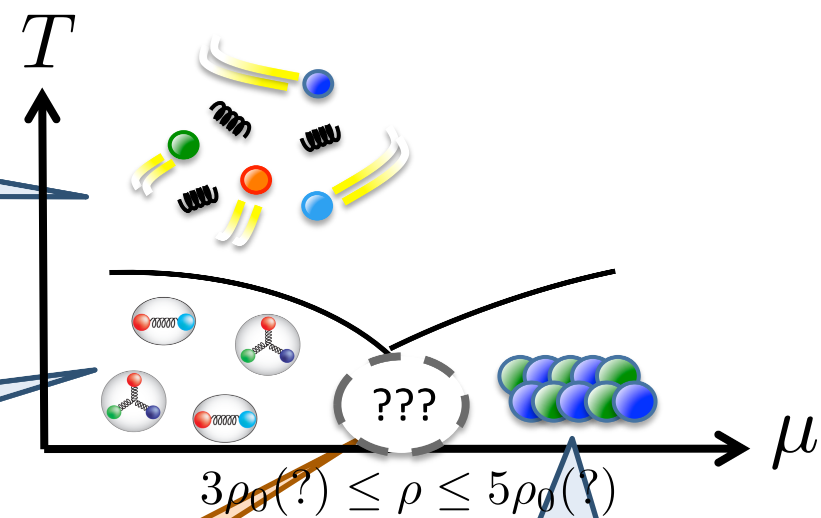
1. Introduction

- **QCD phase diagram**



- Quark Gluon Plasma
- Chiral restored phase
  (RHIC, LHC)

- Quarks are confined
- Chiral broken phase
  (J-PARC, and so on)

- Inhomogeneous chiral broken phase (!?)
  (Dual Chiral Density Wave ?)

- Color superconductivity (?)
- Chiral restored phase (?)
  (Neutron star (?))

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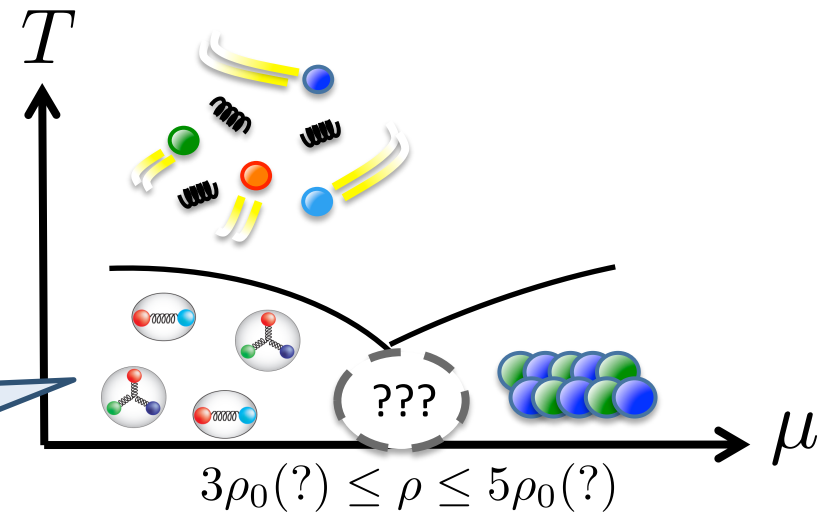
1. Introduction

- **QCD phase diagram**



– The VEV of $\langle \bar{q}q \rangle_0$ does not depend on the space-time

$$\langle \bar{q}q \rangle_0 = \text{const.}$$

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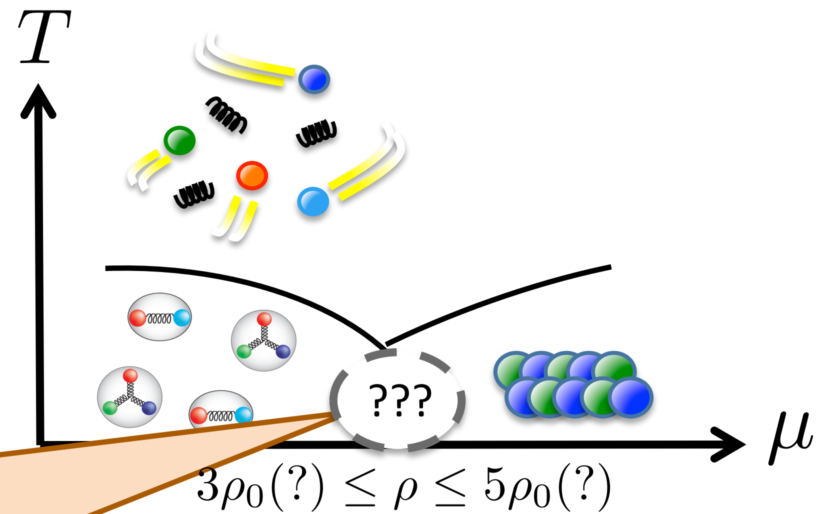
1. Introduction

- **QCD phase diagram**



– The $\langle\bar{q}q\rangle$ and $\langle\bar{q}i\gamma_5\tau^3 q\rangle$ depend on the space-time

e.g. $\langle\bar{q}q\rangle = \Delta\cos(2\vec{f}\cdot\vec{x})$

$\langle\bar{q}i\gamma_5\tau^3 q\rangle = \Delta\sin(2\vec{f}\cdot\vec{x})$

Dual Chiral Density Wave (DCDW phase)

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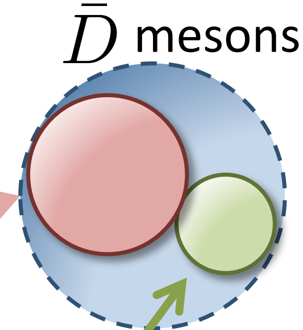
1. Introduction

- **How can we observe such a phase ?**
  - We propose the $\bar{D}$ mesons as probes to observe it

- **Why the $\bar{D}$ mesons ?**



$\bar{c}$ quark

- $\bar{c}$ quark does not interact with nucleons directly
- $1/m_Q$ expansion is applicable

u, d quarks

- we can extract the information of chiral symmetry

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1. Introduction

- **Short summary of what we have done**

  - Assume the existence of the DCDW phase

    ⇩

  - Consider $\bar{D}$ mesons in the DCDW phase as probes

    ⇩

  - Calculate the dispersion relations for $\bar{D}$ mesons

- **$\bar{D}$ mesons can be probes of exploring the Dual Chiral Density Wave phase**

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4. Results and a summary

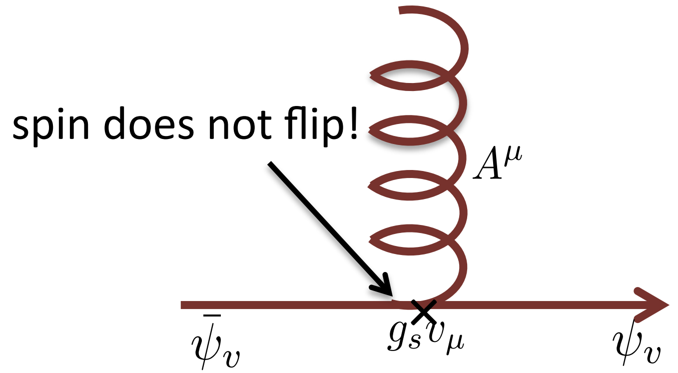
2. Heavy Meson Effective Theory

- **Heavy quark spin symmetry**
  - Magnetic gluon does not change the spin of heavy quarks:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - M_Q)\psi$$
$$= \bar{\psi}_v \underline{(iv \cdot D)} \psi_v + O(1/M_Q)$$

no γ- matrices !

where $\psi_v = \mathrm{e}^{iM_Q v\cdot x} \dfrac{1+\gamma^\mu v_\mu}{2}\psi$



- Spin-up and down states are independent and equivalent
- Heavy quark symmetry is valid in the limit of $M_Q \to \infty$

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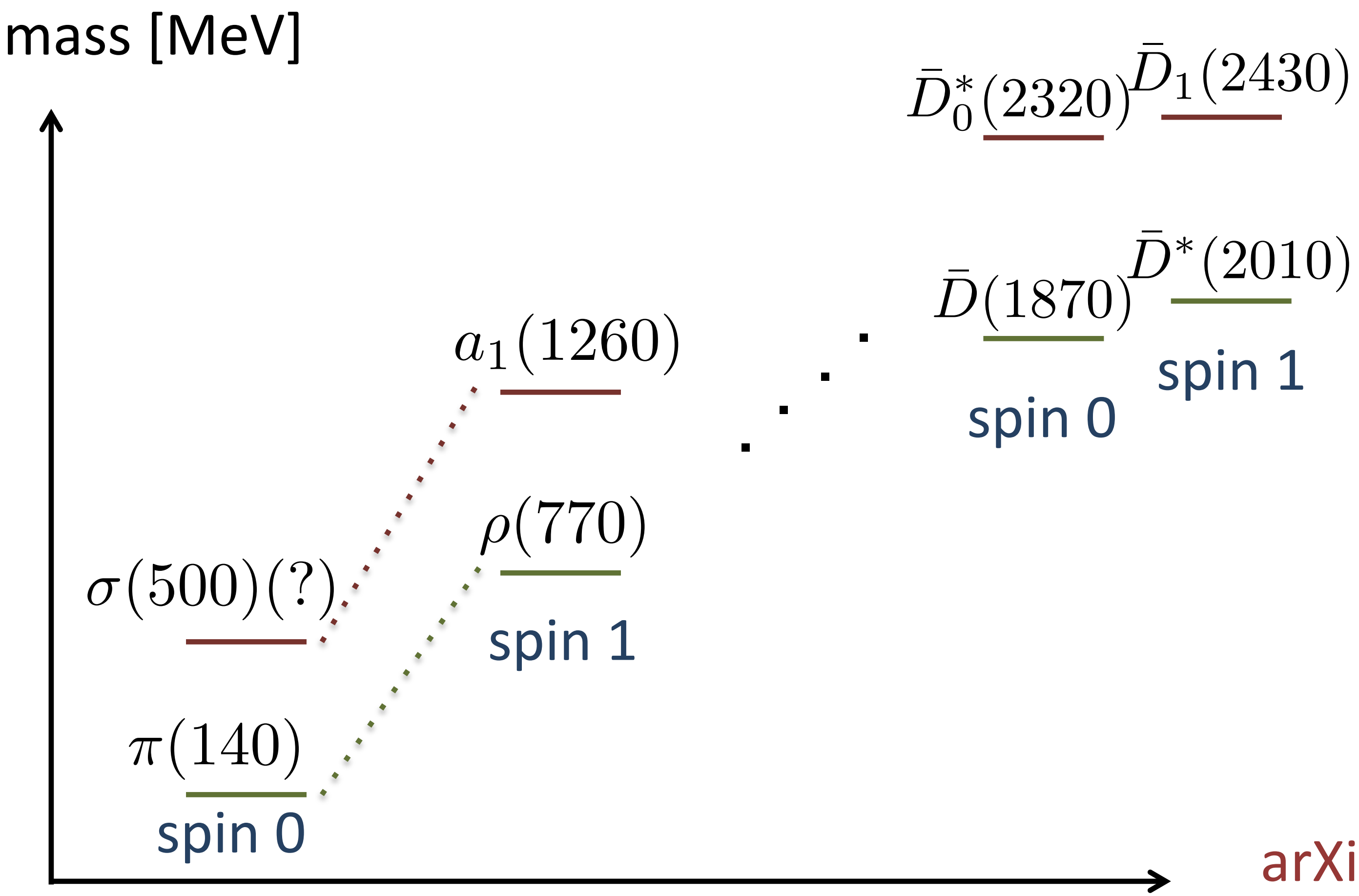
2. Heavy Meson Effective Theory

- **Heavy-light meson**
  - Two states of $J_{\rm tot} = J_{\rm light} \pm \frac{1}{2}{}_{\rm heavy}$ are degenerated



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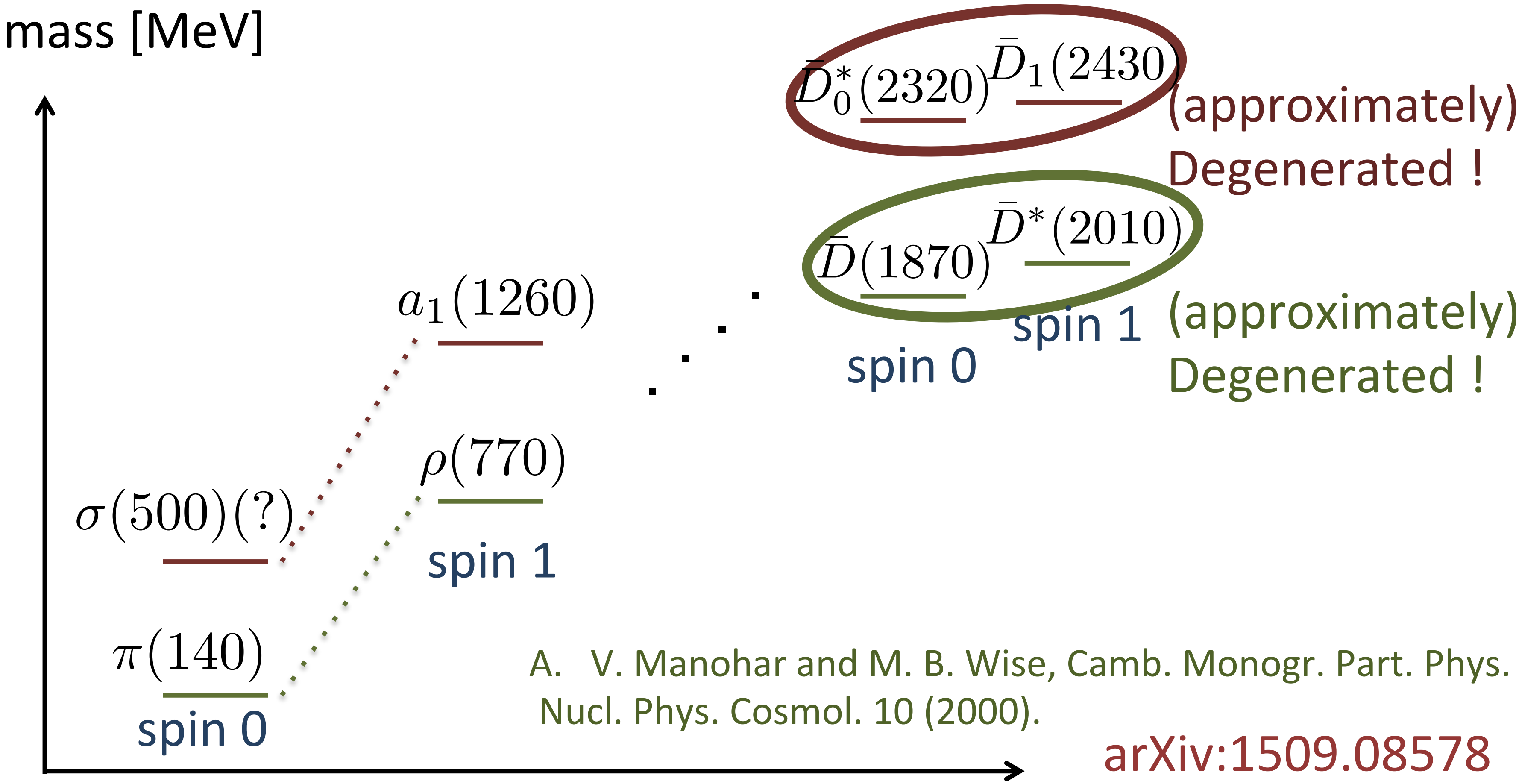
2. Heavy Meson Effective Theory

- **Heavy-light meson**
  - Two states of $J_{\rm tot} = J_{\rm light} \pm \frac{1}{2}{}_{\rm heavy}$ are degenerated



A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 10 (2000).

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2. Heavy Meson Effective Theory

- **Chiral Partner Structure**
  - Mass difference between opposite parity states comes from the breakdown of chiral symmetry

mass

parity-even $\bar{D}_0^*(2320)\ \bar{D}_1(2430) \sim 2.40\ \mathrm{GeV}$

from the chiral symmetry breaking

parity-odd $\bar{D}(1870)\ \bar{D}^*(2010) \sim 1.97\ \mathrm{GeV}$

spin 0 spin 1

M. A. Nowak, M. Rho, and I. Zahed, PRD 48 (1993)
W. A. Bardeen and C. T. Hill, PRD 49 (1994).

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2. Heavy Meson Effective Theory

- **Chiral Partner Structure**
  - Mass difference between opposite parity states comes from the breakdown of chiral symmetry

mass

$(\bar{D}_0^*, \bar{D}_1)$ $(\bar{D}, \bar{D}^*)$ $\sim 2.19\ \mathrm{MeV}$

Chiral restored

M. A. Nowak, M. Rho, and I. Zahed, PRD 48 (1993)
W. A. Bardeen and C. T. Hill, PRD 49 (1994).

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2. Heavy Meson Effective Theory

- **Lagrangian in relativistic form**
  - Based on chiral partner structure

$$
\begin{aligned}
\mathcal{L} = {} & \partial_\mu \bar{D}_0^* \partial^\mu \bar{D}_0^{*\dagger} - m^2 \bar{D}_0^* \bar{D}_0^{*\dagger} - \partial_\mu \bar{D}_{1\nu} \partial^\mu \bar{D}_1^{\dagger\nu} + \partial_\mu \bar{D}_{1\nu} \partial^\nu \bar{D}_1^{\dagger\mu} + m^2 \bar{D}_{1\mu} \bar{D}_1^{\dagger\mu} \\
& + \partial_\mu \bar{D} \partial^\mu \bar{D}^\dagger - m^2 \bar{D} \bar{D}^\dagger - \partial_\mu \bar{D}_\nu^* \partial^\mu \bar{D}^{*\dagger\nu} + \partial_\mu \bar{D}_\nu^* \partial^\nu \bar{D}^{*\dagger\mu} + m^2 \bar{D}_\mu^* \bar{D}^{*\dagger\mu} \\
& - \frac{1}{2} \frac{m\Delta_m}{f_\pi} \left[\bar{D}_0^* (M + M^\dagger) \bar{D}_0^{*\dagger} - \bar{D}_{1\mu} (M + M^\dagger) \bar{D}_1^{\dagger\mu} - \bar{D} (M + M^\dagger) \bar{D}^\dagger + \bar{D}_\mu^* (M + M^\dagger) \bar{D}^{*\dagger\mu} \right] \\
& - \frac{1}{2} \frac{m\Delta_m}{f_\pi} \left[\bar{D}_0^* (M - M^\dagger) \bar{D}^\dagger - \bar{D}_{1\mu} (M - M^\dagger) \bar{D}^{*\dagger\mu} - \bar{D} (M - M^\dagger) \bar{D}_0^{*\dagger} + \bar{D}_\mu^* (M - M^\dagger) \bar{D}_1^{\dagger\mu} \right] \\
& + \cdots
\end{aligned}
$$

where $M = \sigma + i\tau^a \pi^a$

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2. Heavy Meson Effective Theory

- **Lagrangian in relativistic form**
  - Based on chiral partner structure

$$\mathcal{L} = \partial_\mu \bar{D}_0^* \partial^\mu \bar{D}_0^{*\dagger} - m^2 \bar{D}_0^* \bar{D}_0^{*\dagger} - \partial_\mu \bar{D}_{1\nu} \partial^\mu \bar{D}_1^{\dagger\nu} + \partial_\mu \bar{D}_{1\nu} \partial^\nu \bar{D}_1^{\dagger\mu} + m^2 \bar{D}_{1\mu} \bar{D}_1^{\dagger\mu}$$
$$+ \partial_\mu \bar{D} \partial^\mu \bar{D}^\dagger - m^2 \bar{D}\bar{D}^\dagger - \partial_\mu \bar{D}_\nu^* \partial^\mu \bar{D}^{*\dagger\nu} + \partial_\mu \bar{D}_\nu^* \partial^\nu \bar{D}^{*\dagger\mu} + m^2 \bar{D}_\mu^* \bar{D}^{*\dagger\mu}$$
$$- \frac{1}{2}\frac{m\Delta_m}{f_\pi}\left[\bar{D}_0^*(M+M^\dagger)\bar{D}_0^{*\dagger} - \bar{D}_{1\mu}(M+M^\dagger)\bar{D}_1^{\dagger\mu} - \bar{D}(M+M^\dagger)\bar{D}^\dagger + \bar{D}_\mu^*(M+M^\dagger)\bar{D}^{*\dagger\mu}\right]$$
$$- \frac{1}{2}\frac{m\Delta_m}{f_\pi}\left[\underline{\bar{D}_0^*(M-M^\dagger)\bar{D}^\dagger} - \underline{\bar{D}_{1\mu}(M-M^\dagger)\bar{D}^{*\dagger\mu}} - \underline{\bar{D}(M-M^\dagger)\bar{D}_0^{*\dagger}} + \underline{\bar{D}_\mu^*(M-M^\dagger)\bar{D}_1^{\dagger\mu}}\right]$$

- $\underline{\bar{D}}$ and $\underline{\bar{D}_0^*}$ mesons can mix
- $\underline{\bar{D}^*}$ and $\underline{\bar{D}_1}$ mesons can mix

where $M = \sigma + i\tau^a\pi^a$

- Let us consider only $(\bar{D}_u, \bar{D}^*_{0\,u})$ sector hereafter

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3. EoMs for $\bar{D}$ Mesons in DCDW

- **Derive the EoMs**
  - Use the variational method for $\bar{D}$, $\bar{D}_0^*$ fields and insert the DCDW configuration $\langle M \rangle = \phi \cos(2fx) + i\phi\, \tau^3 \sin(2fx)$

$$(\partial_\mu \partial^\mu + m^2)\bar{D}_u - \tilde{\phi} m \Delta_m \cos(2fx)\bar{D}_u + i\tilde{\phi} m \Delta_m \sin(2fx)\bar{D}^*_{0\,u} = 0$$

$$(\partial_\mu \partial^\mu + m^2)\bar{D}^*_{0\,u} + \tilde{\phi} m \Delta_m \cos(2fx)\bar{D}^*_{0\,u} - i\tilde{\phi} m \Delta_m \sin(2fx)\bar{D}_u = 0$$

$$(\tilde{\phi} = \phi / f_\pi)$$

  - Dispersions are obtained from these EoMs
  - These EoMs contain periodic potentials …

⇨ **the Bloch's theorem**

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3. EoMs for $\bar{D}$ Mesons in DCDW

- **Bloch's theorem**
  - Wave function in periodic potentials (period a) is of the form:

$$\psi = \sum_k \sum_{K'} C_{k-K'} \mathrm{e}^{i(k-K')x}$$

  - $K' = 0, \pm K, \pm 2K, \cdots \ (K = 2\pi/a)$ is reciprocal lattice vector
  - k is so-called 'crystal momentum' restricted in the first Brillouin zone ($-K/2 \leq k \leq +K/2$)

- Momentum q $\left(\psi = \sum_q C_q \mathrm{e}^{iqx}\right)$ is no longer a good quantum number

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3. EoMs for $\bar{D}$ Mesons in DCDW

- **EoMs for $\bar{D}$ in the DCDW**
  - EoMs are

$$(\partial_\mu \partial^\mu + m^2)\bar{D}_u - \tilde{\phi} m \Delta_m \cos(Kx)\bar{D}_u + i\tilde{\phi} m \Delta_m \sin(Kx)\bar{D}^*_{0\,u} = 0$$

$$(\partial_\mu \partial^\mu + m^2)\bar{D}^*_{0\,u} + \tilde{\phi} m \Delta_m \cos(Kx)\bar{D}^*_{0\,u} - i\tilde{\phi} m \Delta_m \sin(Kx)\bar{D}_u = 0$$

where $\langle M \rangle = \phi \cos(2fx) + i\phi\, \tau^3 \sin(2fx)\,,\ K = 2f,\ \tilde{\phi} = \phi / f_\pi$

  - Expand $\bar{D}$ mesons fields as Bloch's states

$$\bar{D}_u(x) = \sum_{k_x, y, k_z} \sum_{K'} C^{\bar{D}_u}_{k_x - K'} \mathrm{e}^{i(k_x - K')x} \mathrm{e}^{-iEt + ik_y y + ik_z z}$$

$$\bar{D}^*_{0\,u}(x) = \sum_{k_x, y, k_z} \sum_{K'} C^{\bar{D}^*_{0\,u}}_{k_x - K'} \mathrm{e}^{i(k_x - K')x} \mathrm{e}^{-iEt + ik_y y + ik_z z}$$

$$\cos(Kx) = \frac{\mathrm{e}^{iKx} + \mathrm{e}^{-iKx}}{2} \qquad \sin(Kx) = \frac{\mathrm{e}^{iKx} - \mathrm{e}^{-iKx}}{2i}$$

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3. EoMs for $\bar{D}$ Mesons in DCDW

- **Matrix form of EoMs**
  - Momentum $q = \cdots, k_x - K, k_x, k_x + K, \cdots$ states can mix

$$\begin{pmatrix} \ddots & \ddots & 0 & 0 & 0 \\ \ddots & \Delta_{k_x-K} & V & 0 & 0 \\ 0 & \tilde{V} & \Delta_{k_x} & V & 0 \\ 0 & 0 & \tilde{V} & \Delta_{k_x+K} & \ddots \\ 0 & 0 & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ C_{k_x-K} \\ C_{k_x} \\ C_{k_x+K} \\ \vdots \end{pmatrix} = 0$$

where $C_q = \begin{pmatrix} C_q^{\bar{D}_u} \\ C_q^{\bar{D}^*_0{}_u} \end{pmatrix}$ $\quad \Delta_q = \begin{pmatrix} -E^2 + m^2 + q^2 & 0 \\ 0 & -E^2 + m^2 + q^2 \end{pmatrix}$

$$V = \begin{pmatrix} -\frac{m\Delta_m}{2}\tilde{\phi} & -\frac{m\Delta_m}{2}\tilde{\phi} \\ \frac{m\Delta_m}{2}\tilde{\phi} & \frac{m\Delta_m}{2}\tilde{\phi} \end{pmatrix} \quad \tilde{V} = \begin{pmatrix} -\frac{m\Delta_m}{2}\tilde{\phi} & \frac{m\Delta_m}{2}\tilde{\phi} \\ -\frac{m\Delta_m}{2}\tilde{\phi} & \frac{m\Delta_m}{2}\tilde{\phi} \end{pmatrix}$$

- We can diagonalize this matrix analytically thanks to the internal structures in a given Brillouin zone !!

3. EoMs for $\bar{D}$ Mesons in DCDW

- **Energy eigenvalues**
  - Infinite number of energy eigenvalues

$$E^2 = m^2 + \frac{1}{2}\left[(k_x + (n+1)K)^2 + (k_x + nK)^2\right] + k_\perp^2$$
$$\pm \frac{1}{2}\sqrt{\left[(k_x + (n+1)K)^2 - (k_x^2 + nK)^2\right]^2 + 4\tilde{\phi}^2 m^2 \Delta_m^2}$$

  - Eigenvectors have nonzero values only for C_{k_x+nK} , $C_{k_x+(n+1)K}$

  - Change the variable k_x to q_x by setting $q_x = k_x + nK$ and $q_x = k_x + (n+1)K$

$$E^2 = m^2 + \frac{1}{2}\left[(q_x + K)^2 + q_x^2\right] + k_\perp^2 \pm \frac{1}{2}\sqrt{\left[(q_x + K)^2 - q_x^2\right]^2 + 4\tilde{\phi}^2 m^2 \Delta_m^2}$$

$$E^2 = m^2 + \frac{1}{2}\left[q_x^2 + (q_x - K)^2\right] + k_\perp^2 \pm \frac{1}{2}\sqrt{\left[q_x^2 - (q_x - K)^2\right]^2 + 4\tilde{\phi}^2 m^2 \Delta_m^2}$$

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3. Results and a summary

- **Eigenvalues and plots**
  - We can get four energy eigenvalues

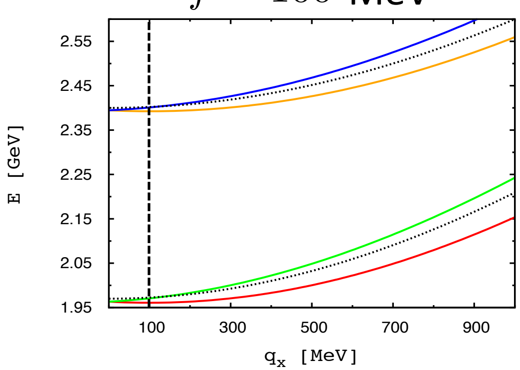
$$E^2 = m^2 + \frac{1}{2}\left[(q_x+K)^2 + q_x^2\right] + k_\perp^2 \pm \frac{1}{2}\sqrt{\left[(q_x+K)^2 - q_x^2\right]^2 + 4\tilde{\phi}^2 m^2 \Delta_m^2}$$

$$E^2 = m^2 + \frac{1}{2}\left[q_x^2 + (q_x-K)^2\right] + k_\perp^2 \pm \frac{1}{2}\sqrt{\left[q_x^2 - (q_x-K)^2\right]^2 + 4\tilde{\phi}^2 m^2 \Delta_m^2}$$

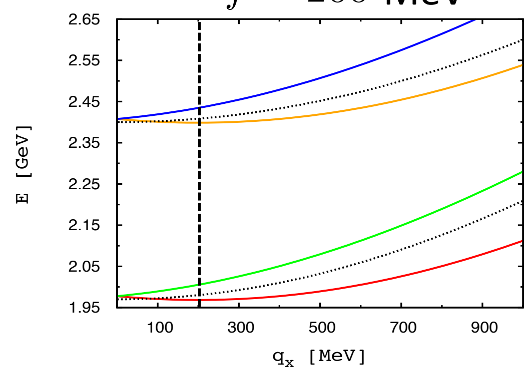
$$\langle M\rangle = \phi\cos(2fx) + i\phi\,\tau^3 \sin(2fx) \quad (K = 2f)$$

  - Plot with $\tilde{\phi}(=\phi/f_\pi) = 1$

$f = 100$ MeV



$f = 200$ MeV



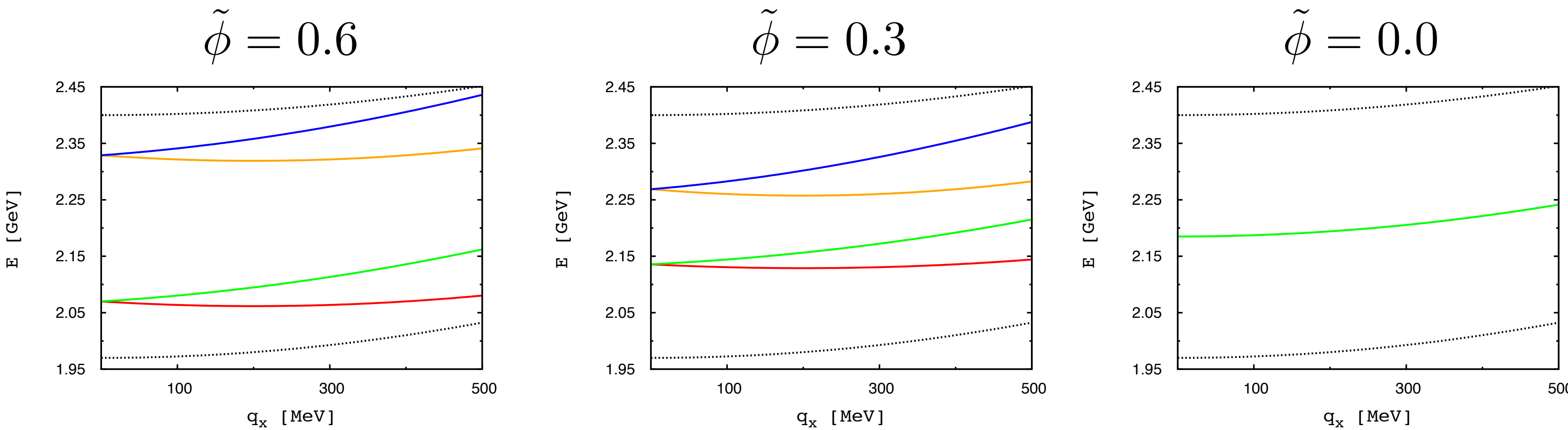
  - Red and orange curves have minimum at $q_x = f$

Results

- **Plots**

$$\langle M \rangle = \phi\ \cos(2fx) + i\phi\ \tau^3 \sin(2fx)$$

  - Vary the chiral condensate $\tilde{\phi} = \phi / f_\pi$ with $f = 200$ MeV



(chiral restoration)

  - Mass difference between two modes have the information of the magnitude of chiral condensate

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A summary

- **We calculated dispersions for $\bar{D}$ in DCDW**
  - DCDW phase has a periodic chiral condensate so that we have to employ the Bloch's theorem to get dispersions
  - the number of modes of $\bar{D}$ mesons increases
  - Some modes have negative velocities and energy minimum is realized at $q_x = f$

- **These modes are the consequences of existence of the DCDW phase at finite density region**

Back up

1. Introduction

- **Representation in effective theory**
  - **in vacuum**
    - Mean field of Chiral field $M = \sigma + i\tau^a\pi^a$ is expressed as

$$\langle M \rangle = \sigma_0$$

  - **in DCDW phase**
    - Mean field of Chiral field $M = \sigma + i\tau^a\pi^a$ is expressed as

$$\langle M \rangle = \phi\cos(2fx) + i\phi\,\tau^3\sin(2fx)$$

    - ϕ measures the magnitude of chiral condensate
    - f is the wave number of DCDW modulation

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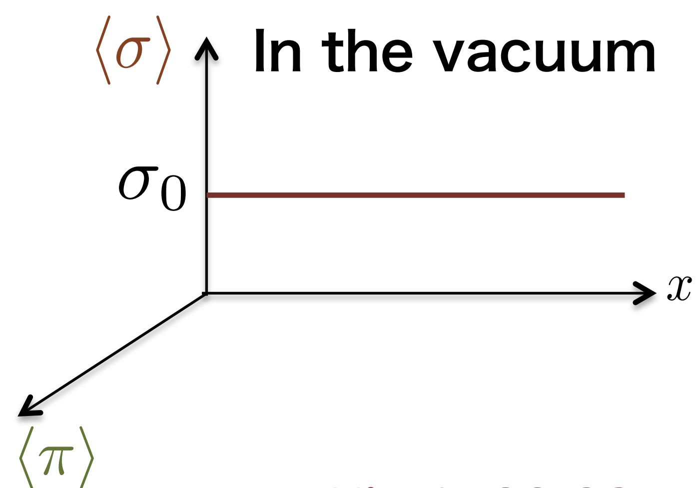
1. Introduction

- **Chiral symmetry in the vacuum**
  - Chiral symmetry is homogeneously broken
  - Mean filed of Chiral field $M = \sigma + i\tau^a\pi^a$ is expressed as

$$\langle M \rangle = \sigma_0$$

  - Pion fields do not condense
  - Mean field of sigma does not depend on the space time



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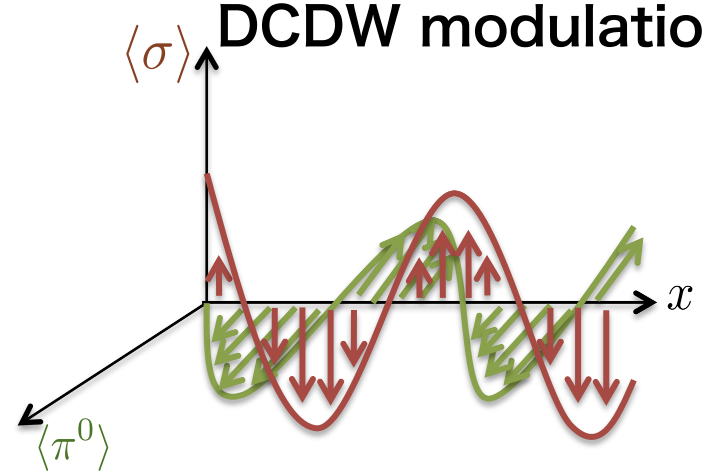
1. Introduction

- **Dual Chiral Density Wave (DCDW)**
  - DCDW is one of the inhomogeneous chiral broken phase
  - Mean filed of Chiral field $M = \sigma + i\tau^a\pi^a$ is expressed as

$$\langle M \rangle = \phi \cos(2fx) + i\phi\, \tau^3 \sin(2fx)$$

  - ϕ measures the magnitude of chiral condensate
  - f is the wave number of DCDW modulation

DCDW modulation



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Back up

- Interaction among all four $\bar{D}$ mesons

$$\mathcal{L} = \partial_\mu \bar{D}_0^* \partial^\mu \bar{D}_0^{*\dagger} - m^2 \bar{D}_0^* \bar{D}_0^{*\dagger} - \partial_\mu \bar{D}_{1\nu} \partial^\mu \bar{D}_1^{\dagger\nu} + \partial_\mu \bar{D}_{1\nu} \partial^\nu \bar{D}_1^{\dagger\mu} + m^2 \bar{D}_{1\mu} \bar{D}_1^{\dagger\mu}$$
$$+\partial_\mu \bar{D} \partial^\mu \bar{D}^\dagger - m^2 \bar{D}\bar{D}^\dagger - \partial_\mu \bar{D}_\nu^* \partial^\mu \bar{D}^{*\dagger\nu} + \partial_\mu \bar{D}_\nu^* \partial^\nu \bar{D}^{*\dagger\mu} + m^2 \bar{D}_\mu^* \bar{D}^{*\dagger\mu}$$

All mesons can mix

$$\boxed{\begin{aligned}
&-\frac{1}{2}\frac{m\Delta_m}{f_\pi}\left[\bar{D}_0^*(M+M^\dagger)\bar{D}_0^{*\dagger} - \bar{D}_{1\mu}(M+M^\dagger)\bar{D}_1^{\dagger\mu} - \bar{D}(M+M^\dagger)\bar{D}^\dagger + \bar{D}_\mu^*(M+M^\dagger)\bar{D}^{*\dagger\mu}\right]\\
&-\frac{1}{2}\frac{m\Delta_m}{f_\pi}\left[\bar{D}_0^*(M-M^\dagger)\bar{D}^\dagger - \bar{D}_{1\mu}(M-M^\dagger)\bar{D}^{*\dagger\mu} - \bar{D}(M-M^\dagger)\bar{D}_0^{*\dagger} + \bar{D}_\mu^*(M-M^\dagger)\bar{D}_1^{\dagger\mu}\right]\\
&-\frac{g}{2}\frac{m}{f_\pi}[\bar{D}_1^\mu(\partial_\mu M^\dagger - \partial_\mu M)\bar{D}_0^{*\dagger} - \bar{D}_0^*(\partial_\mu M^\dagger - \partial_\mu M)\bar{D}_1^{\dagger\mu} - \frac{1}{m}\epsilon^{\mu\nu\rho}\bar{D}'_{1\mu}(\partial_\nu M^\dagger - \partial_\nu M)i\partial\bar{D}_{1\rho}^\dagger]\\
&+\frac{g}{2}\frac{m}{f_\pi}[\bar{D}^{*\mu}(\partial_\mu M^\dagger - \partial_\mu M)\bar{D}^\dagger - \bar{D}(\partial_\mu M^\dagger - \partial_\mu M)\bar{D}^{*\dagger\mu} - \frac{1}{m}\epsilon^{\mu\nu\rho}\bar{D}_\mu^*(\partial_\nu M^\dagger - \partial_\nu M)i\partial\bar{D}_\rho^{*\dagger}]\\
&+\frac{g}{2}\frac{m}{f_\pi}[\bar{D}_1^\mu(\partial_\mu M^\dagger + \partial_\mu M)\bar{D} + \bar{D}(\partial_\mu M^\dagger + \partial_\mu M)\bar{D}_1^{\dagger\mu}]\\
&-\frac{g}{2}\frac{m}{f_\pi}[\bar{D}_0^*(\partial_\mu M^\dagger + \partial_\mu M)\bar{D}^{*\dagger\mu} + \bar{D}^{*\mu}(\partial_\mu M^\dagger + \partial_\mu M)\bar{D}_0^{*\dagger}]\\
&-\frac{g}{2}\frac{1}{f_\pi}[\epsilon^{\mu\nu\rho}\bar{D}_{1\nu}(\partial_\rho M^\dagger + \partial_\rho M)i\partial\bar{D}_\mu^{*\dagger} + \epsilon^{\mu\nu\rho}\bar{D}_\mu^*(\partial_\rho M^\dagger + \partial_\rho M)i\partial_\sigma\bar{D}_{1\nu}^\dagger]
\end{aligned}}$$

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3. EoMs for $\bar{D}$ Mesons in DCDW

- **Bloch's theorem**
  - Wave function in periodic potentials (period a) is of the form:

$$\psi = \sum_{k} \sum_{K'} C_{k-K'} \mathrm{e}^{i(k-K')x}$$

  - $K' = 0, \pm K, \pm 2K, \cdots \ (K = 2\pi/a)$ is reciprocal lattice vector
  - k is so-called 'crystal momentum' restricted in the first Brillouin zone ($-K/2 \leq k \leq +K/2$)

- 'Crystal momentum k' is different from 'momentum q'

arXiv:1509.08578

3. EoMs for $\bar{D}$ Mesons in DCDW

- **Bloch's theorem**
  - Wave function in periodic potentials (period a) is of the form:

$$\psi = \sum_{k} \sum_{K'} C_{k-K'} \mathrm{e}^{i(k-K')x}$$

  - Infinite states with momentum $q = \cdots, k-K, k, k+K, \cdots$ can mix each others
  - Momentum q is no longer a good quantum number

- 'Crystal momentum k' is different from 'momentum q'

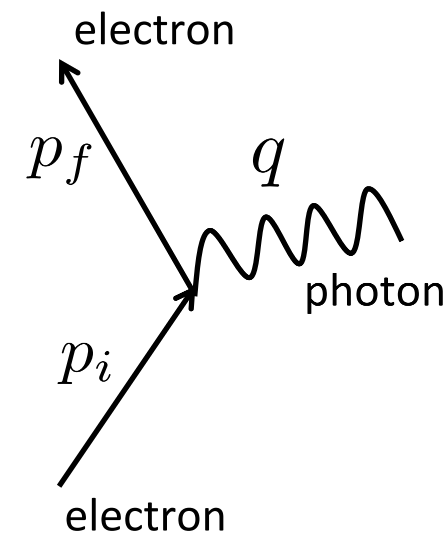
arXiv:1509.08578

3. EoMs for $\bar{D}$ Mesons in DCDW

- **What is the crystal momentum k ?**
  - Let us consider an electron interacting with a photon in the vacuum
  - Continuous translational symmetry exists

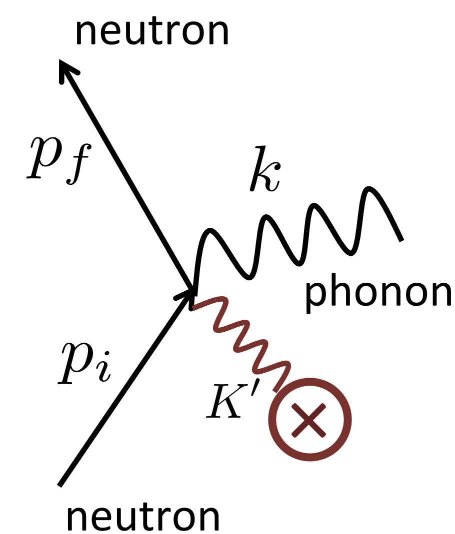
Momentum conservation is

$$p_f - p_i = q$$



arXiv:1509.08578

- **What is the crystal momentum k ?**
  - Let us consider a neutron interacting with a phonon in the crystal
  - Discrete translational symmetry exists



Momentum conservation is

$$p_f - p_i = k + \underline{K'}$$

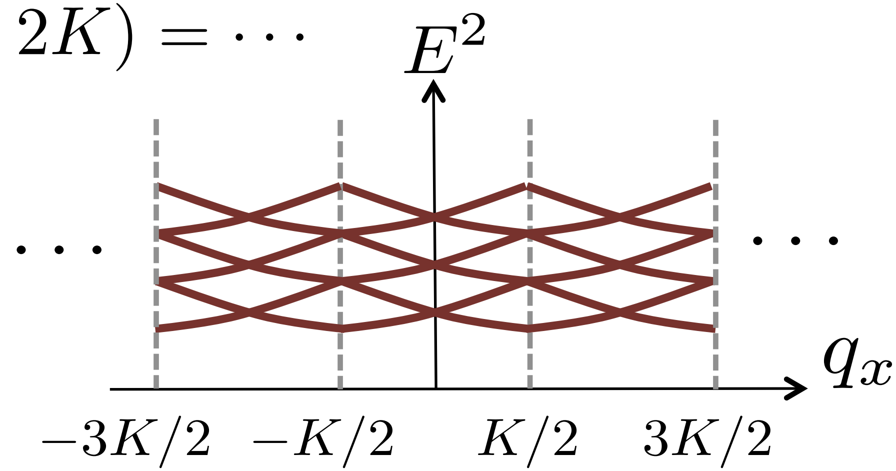
  - We cannot control the reciprocal lattice vector $K' = 0, \pm K, \pm 2K, \cdots \ (K = 2\pi/a)$

arXiv:1509.08578

3. EoMs for $\bar{D}$ Mesons in DCDW

- **What is the crystal momentum k ?**
  - Energy eigenvalue is also periodic in the momentum space:

$$\epsilon(k) = \epsilon(k \pm K) = \epsilon(k \pm 2K) = \cdots$$

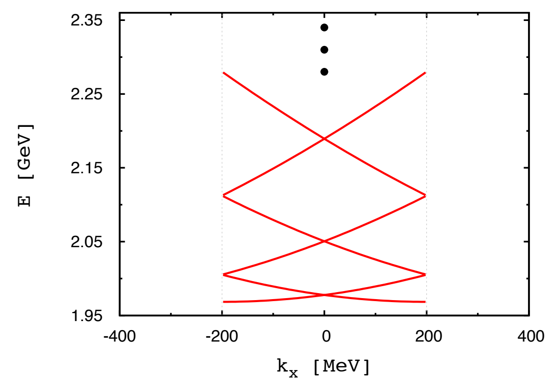


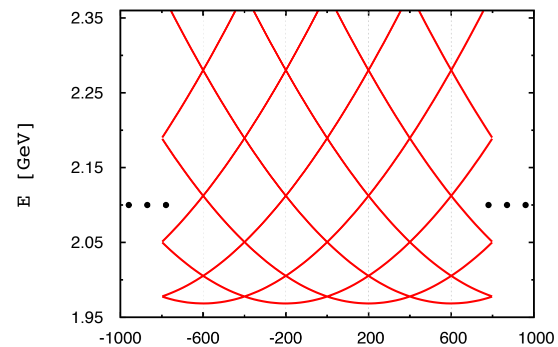
It is enough to consider only the first Brillouin zone
$-K/2 \leq k \leq +K/2$

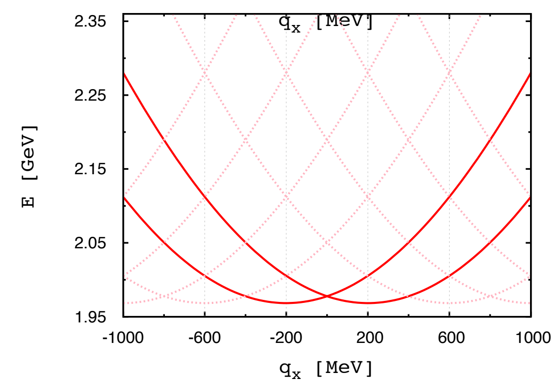
arXiv:1509.08578

3. EoMs for $\bar{D}$ Mesons in DCDW

- **What have I done ?**
  - Dispersion is defined in the first zone in terms of k_x
  - Extend it outside of the first zone in terms of q_x via $\epsilon(k_x) = \epsilon(k_x \pm K) = \epsilon(k_x \pm 2K) = \cdots$
  - Take into account the eigenvector and some curves vanish

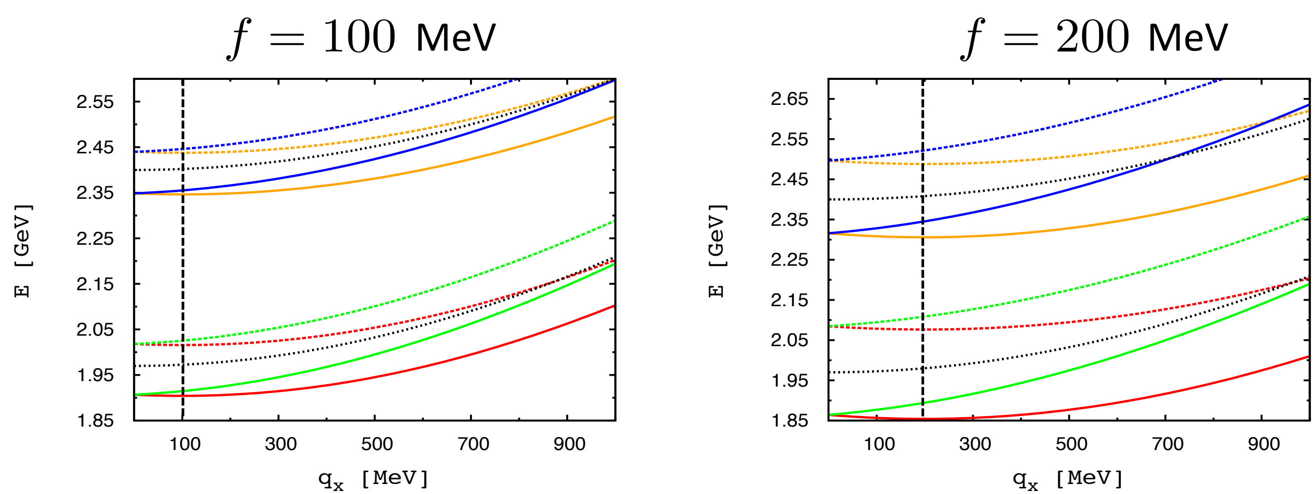






Back up

- The results with four $\bar{D}$ mesons



  - Splitting between solid and dotted curves are caused by spin−0 and spin−1 mixing

arXiv:1509.08578