Introduction:
Hadron tomography and quark-gluon energy-momentum tensor as a source of gravity

(Introduction to the afternoon talks)

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http://j-parc-th.kek.jp/workshops/2017/11-30/

November 30, 2017

Hadron Tomography

Tomography

- **PET (Positron Emission Tomography)**

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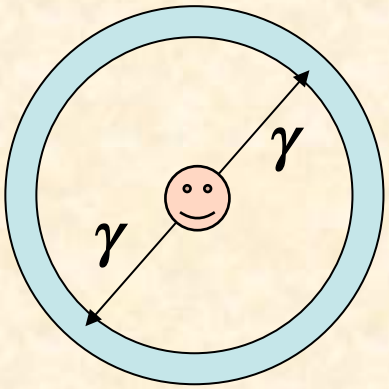
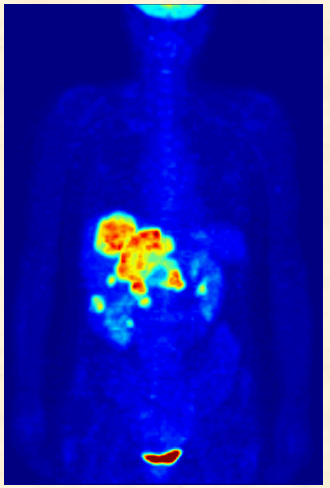
β^+ decay: $^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + e^+ + \nu_e$ $(\tau = 110\ \mathrm{min})$

$e^+ + e^- \rightarrow \gamma + \gamma$ $(E_\gamma = 511\ \mathrm{keV})$



Hadron tomography

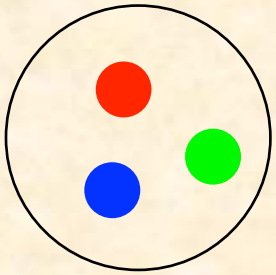
PET (Positron Emission Tomography)



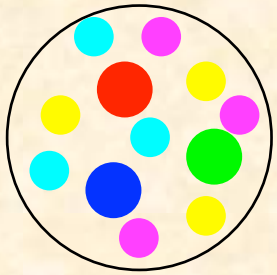
Classical density distribution

3D picture of nucleon
(Density distribution of quantum system: Quantum tomography)

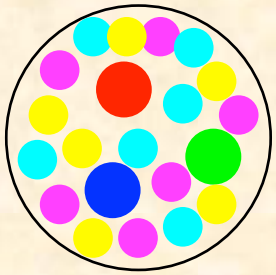
Low energy



Intermediate energy

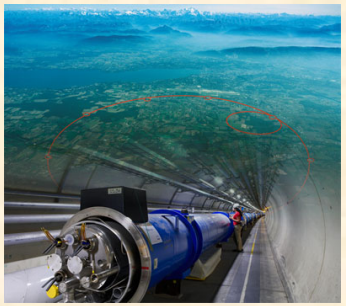


High energy



1D(Bjorken-*x*) picutre@HERA

Test apparatus corresponds to "PET"



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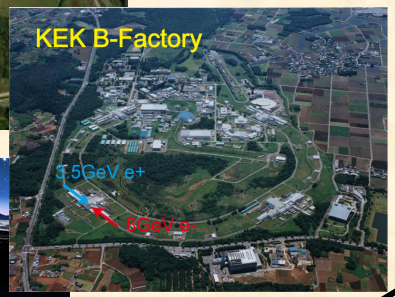

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© GSI

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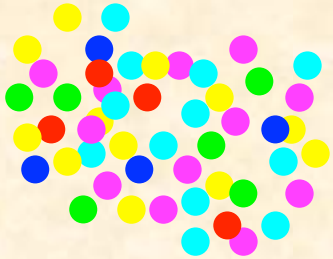
T

μ

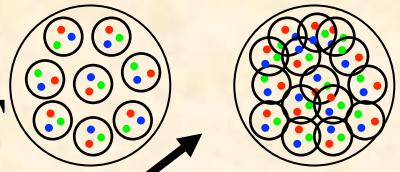
Q^2

LHC
RHIC



GSI-FAIR

RIBF



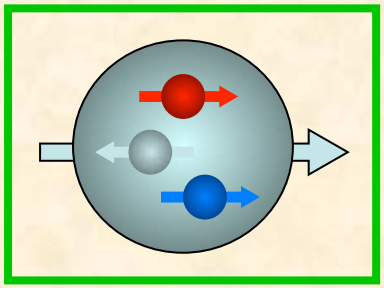
JLab

J-PARC

HERA, EIC
LHC, RHIC

Recent progress on origin of nucleon spin

"old" standard model

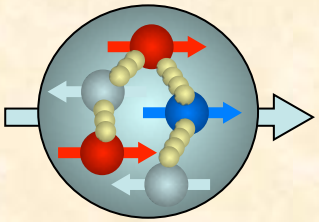


$$p_\uparrow = \frac{1}{3\sqrt{2}}\left(uud\left[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\right] + \text{permutations}\right)$$

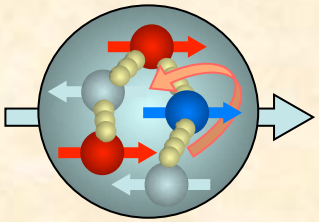
$$\Delta q(x) \equiv q_\uparrow(x) - q_\downarrow(x)$$

$$\Delta\Sigma = \sum_i \int dx\left[\Delta q_i(x) + \Delta\bar{q}_i(x)\right] \rightarrow 1\ (100\%)$$

$$= \text{actually 20-30\%}$$

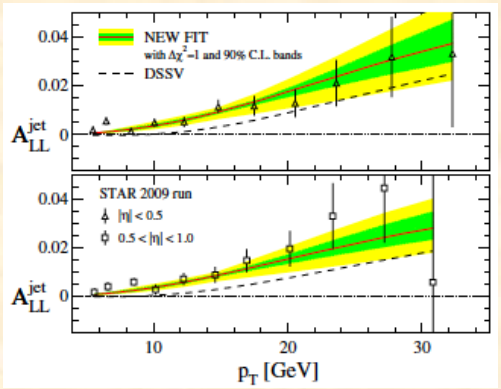


gluon spin



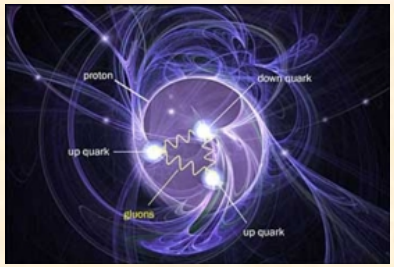
angular momentum

Gluon spin may carry 40% of the proton spin.



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_{q,g}$$

$$\vec{p} + \vec{p} \rightarrow \pi + X$$



CNN (2014) Scientific American (2014)

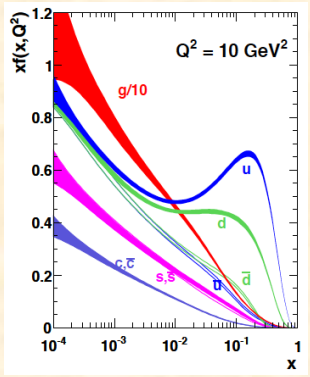
Wigner distribution and various structure functions

Form factor

$\int dx\, d^2k_T$

PDF (Parton Distribution Function)

$\int d^2k_T,\ \Delta \to 0$



GTMD $W(x, \vec{k}_T, \Delta) \xrightarrow[\Delta^+ \to 0]{}$ Wigner $W(x, \vec{k}_T, \vec{r}_T)$

$\Delta = p' - p$

$\int d^2k_T$

3D world

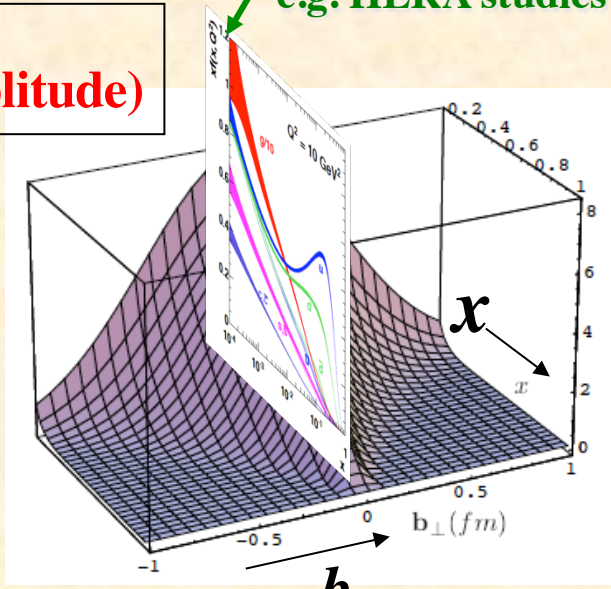
GPD (Generalized Parton Distribution)

$\xrightarrow{s \leftrightarrow t}$ **GDA (Generalized Distribution Amplitude)**

e.g. HERA studies

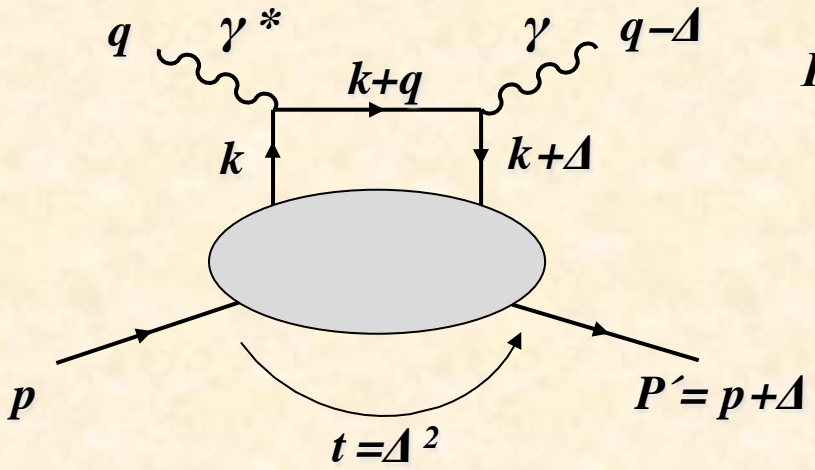
$\Delta \to 0$

TMD (Transverse Momentum Dependent) parton distribution



Our recent work by the two-photon process $\gamma^*\gamma \to h\bar{h}$.

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \dfrac{Q^2}{2p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewdness parameter $\xi = \dfrac{p^+ - p'^+}{p^+ + p'^+} = -\dfrac{\Delta^+}{2P^+}$

GPDs (for the nucleon) are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle p' \middle| \bar{\psi}(-z/2)\gamma^+\psi(z/2) \middle| p \right\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

Forward limit: PDFs

$$H(x,\xi,t)\Big|_{\xi=t=0} = f(x)$$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2

$$\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$$

Second moments: Angular momenta

Sum rule: $J_q = \dfrac{1}{2}\displaystyle\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \dfrac{1}{2}\Delta q + L_q$

GPD $H_q^h(x,\xi,t)$ and GDA $\Phi_q^{hh}(z,\zeta,W^2)$

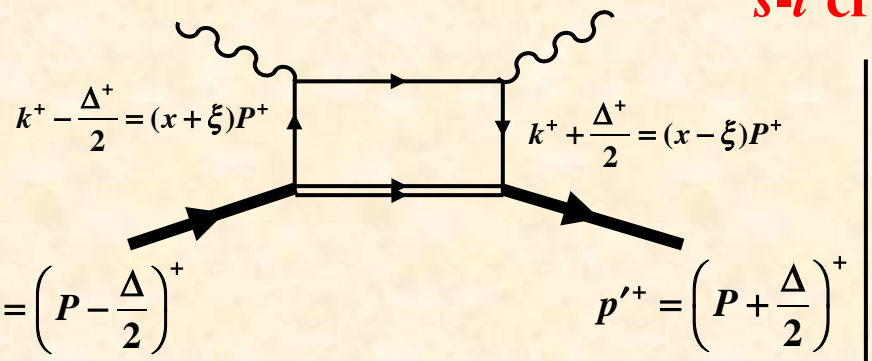
GPD: $$H_q(x,\xi,t)=\int\frac{dy^-}{4\pi}e^{ixP^+y^-}\langle h(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|h(p)\rangle\Big|_{y^+=0,\vec y_\perp=0},\qquad P^+=\frac{(p+p')^+}{2}$$

GDA: $$\Phi_q(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle h(p)\bar h(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|0\rangle\Big|_{y^+=0,\vec y_\perp=0}$$

DA: $$\Phi_q^\pi(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle \pi(p)|\bar\psi(-y/2)\gamma^+\gamma_5\psi(y/2)|0\rangle\Big|_{y^+=0,\vec y_\perp=0}$$

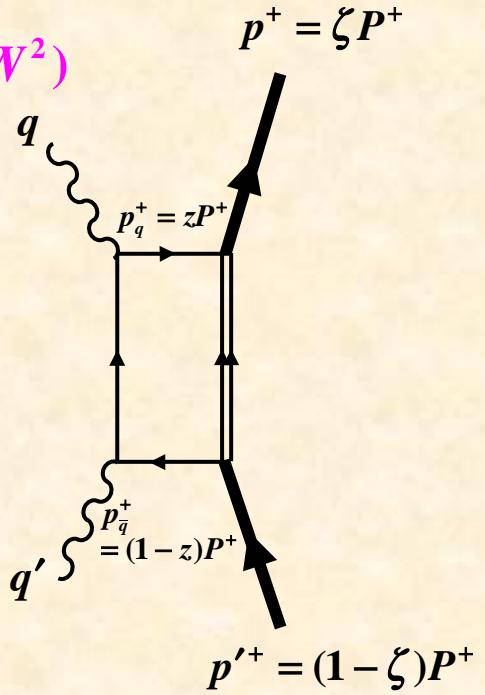
$H_q^h(x,\xi,t)$ $\longleftrightarrow$ $\Phi_q^{hh}(z,\zeta,W^2)$

***s-t* crossing**



$$P=\frac{p+p'}{2},\quad \Delta=p'-p$$

$$z\Leftrightarrow\frac{1-x/\xi}{2}$$
$$\zeta\Leftrightarrow\frac{1-1/\xi}{2}$$
$$W^2\Leftrightarrow t$$



Bjorken variable: $x=\dfrac{Q^2}{2p\cdot q}$

Momentum transfer squared: $t=\Delta^2$

Skewdness parameter: $\xi=\dfrac{p^+-p'^+}{p^++p'^+}=-\dfrac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z=\dfrac{Q^2}{2q\cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar h$: $\zeta=\dfrac{p^+}{P^+}=\dfrac{1+\beta\cos\theta}{2}$

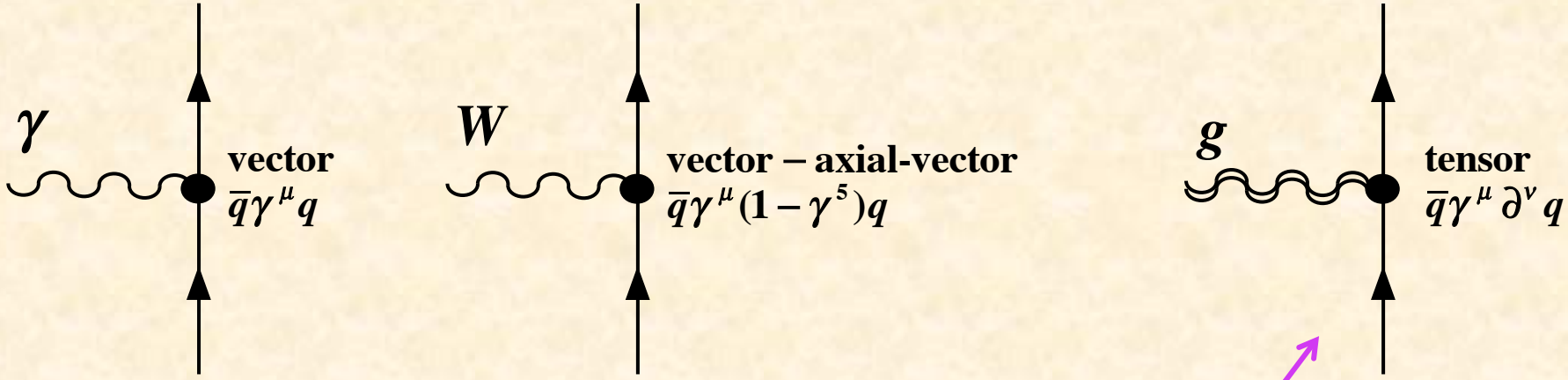
Invariant mass of $h\bar h$: $W^2=(p+p')^2$

Quark-gluon energy-momentum tensor as a source of gravity in hadron tomography

For the details, see

S. Kumano, Q.-T. Song, O. Teryaev, arXiv:1711.08088 (hep-ph).

Why gravitational interactions with hadrons ?



Electron-proton elastic scattering cross section:

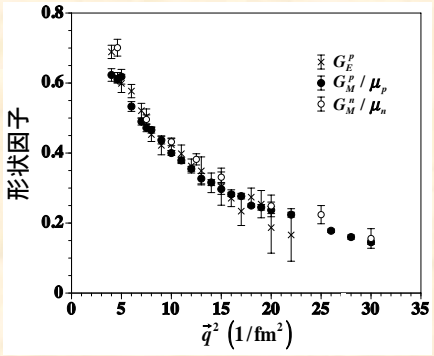
$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2 E_f\cos^2\frac{\theta}{2}}{4E_i^3\sin^4(\theta/2)}\left[\frac{G_E^2+\tau G_M^2}{1+\tau}+2\tau G_M^2\tan^2\frac{\theta}{2}\right],\quad \tau=-\frac{q^2}{4M^2}$$

$$F(\vec{q})=\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\rho(\vec{x})=\int d^3x\left[1-\frac{1}{2}(\vec{q}\cdot\vec{x})^2+\cdots\right]\rho(\vec{x})$$

$$\langle r^2\rangle=\int d^3x\, r^2\rho(\vec{x}),\quad r=|\vec{x}|$$

$\sqrt{\langle r^2\rangle}$ = root-mean-square (rms) radius

$$F(\vec{q})=1-\frac{1}{6}\vec{q}^{\,2}\langle r^2\rangle+\cdots,\quad \langle r^2\rangle=-6\left.\frac{dF(\vec{q})}{d\vec{q}^{\,2}}\right|_{\vec{q}^{\,2}\to 0}$$



$$\rho(r)=\frac{\Lambda^3}{8\pi}e^{-\Lambda r}\ \Leftrightarrow\ \text{Dipole form: } F(q)=\frac{1}{\left(1+|\vec{q}|^2/\Lambda^2\right)^2},\quad \Lambda^2\simeq 0.71\ \text{GeV}^2$$

How about gravitational radius?

Proton-charge-radius puzzle:

$R_{\text{electron scattering}} = 0.8775$ fm

⇕

$R_{\text{muonic atom}} = 0.8418$ fm

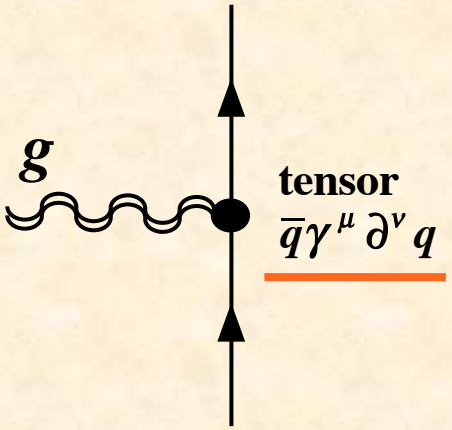
Gravitational interaction and GDAs

GPD: $H_q(x,\xi,t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-/2} \langle h(p') | \bar{\psi}(-y/2)\gamma^+ \psi(y/2) | h(p) \rangle \big|_{y^+=0,\vec{y}_\perp=0}$, $\quad P = p + p'$

GDA: $\Phi_q(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{i(2z-1)P^+y^-/2} \langle h(p)\bar{h}(p') | \bar{\psi}(-y/2)\gamma^+ \psi(y/2) | 0 \rangle \big|_{y^+=0,\vec{y}_\perp=0}$

Lightcone notation:

$a^+ \equiv \frac{a_0 + a_3}{\sqrt{2}}$



Non-local operator of GDAs (GPDs):

$$2\left(P^+/2\right)^n \int dx (2z-1)^{n-1} \int \frac{dy^-}{2\pi} e^{i(2z-1)P^+y^-/2} \left[\bar{q}(-y/2)\gamma^+ q(y/2)\right]_{y^+=0,\vec{y}_\perp=0}$$

$$= \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+\right)^{n-1} q(0)$$

= Energy-momentum tensor for $n=2$ (Vector for $n=1$)

Sum rule for the GDAs:

$$\int_0^1 dz (2z-1) \Phi_q^{\pi^0\pi^0}(x,\zeta,W^2) = \frac{2}{(P^+)^2} \langle \pi^0(p)\pi^0(p') | T_q^{++}(0) | 0 \rangle$$

$\langle \pi^0(p)\pi^0(p') | T_q^{++}(0) | 0 \rangle$ = Matrix element energy-momentum tensor

= Expressed by form factors energy-momentum tensor

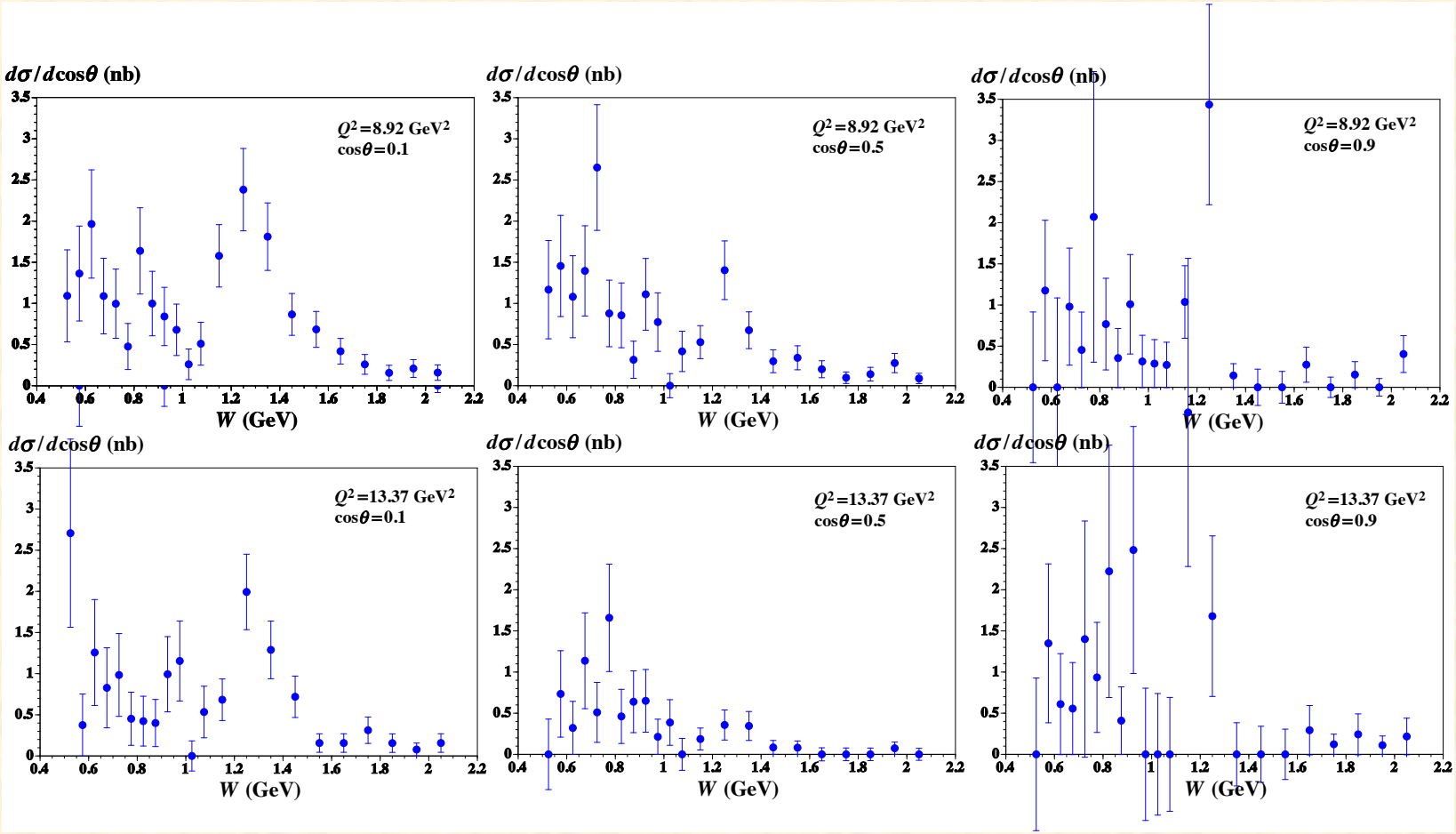
→ gravitational-interaction size ⇔ electric-charge radius

$T_q^{\mu\nu}$ = Energy-momentum tensor of quark

→ Suzuki's talk

Analysis of Belle data on $\gamma^*\gamma \to \pi^0\pi^0$

Belle (M. Masuda *et al.*),
PRD93 (2016) 032003.



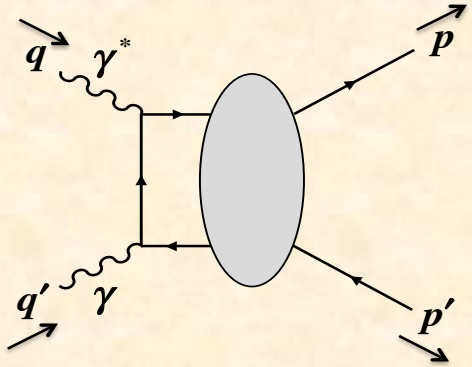
→ Masuda's talk

Gravitational form factors and radii for pion

$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\left\langle \pi^0(p)\pi^0(p')\left|T_q^{++}(0)\right|0\right\rangle$$

$$\left\langle \pi^0(p)\pi^0(p')\left|T_q^{\mu\nu}(0)\right|0\right\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\right]$$

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$



$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$: gravitational form factos for pion

Analyiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section

$\Rightarrow$ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$

$\Rightarrow$ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$

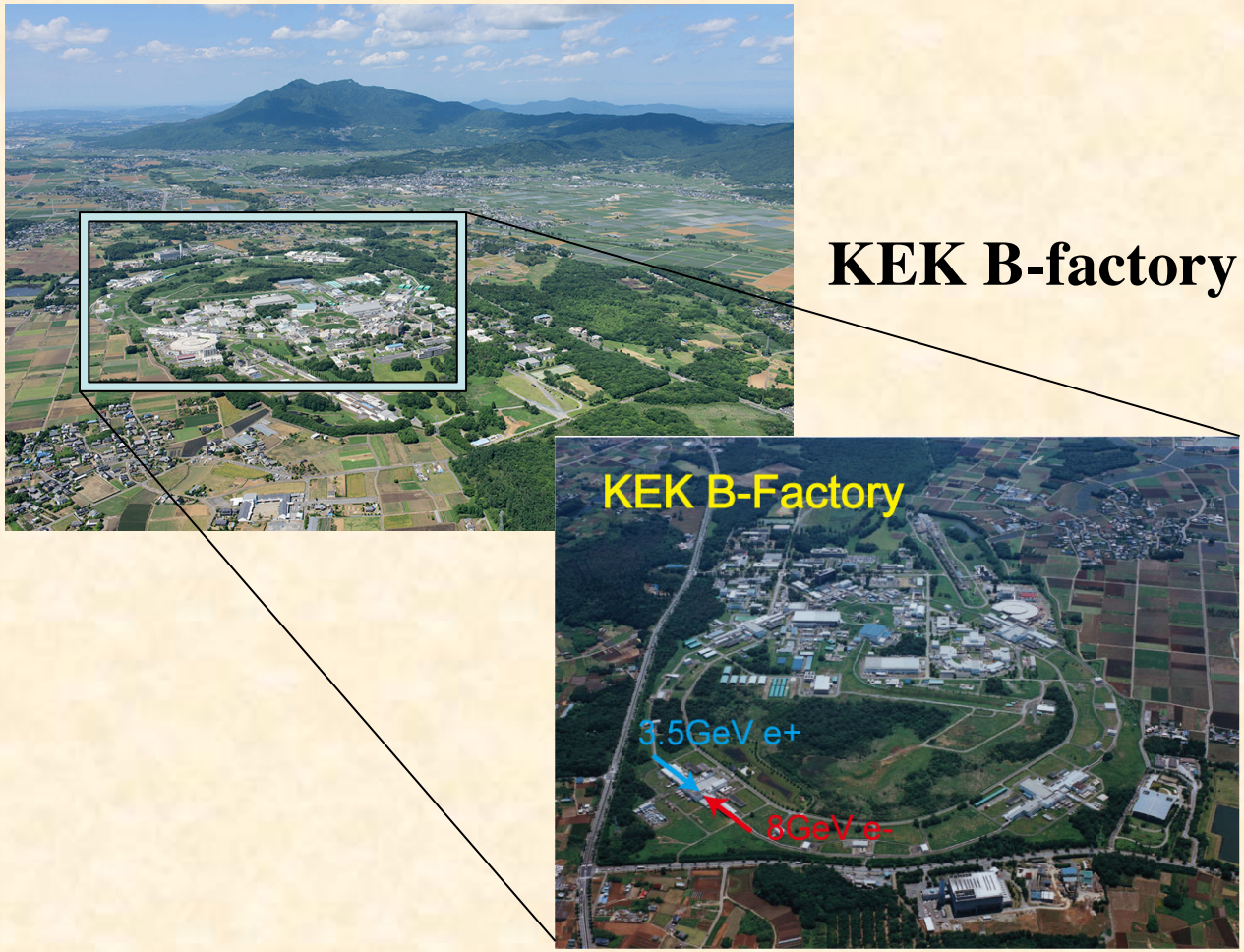
$\Rightarrow$ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$

$\Rightarrow$ Gravitational radii of pion

→ Song's talk

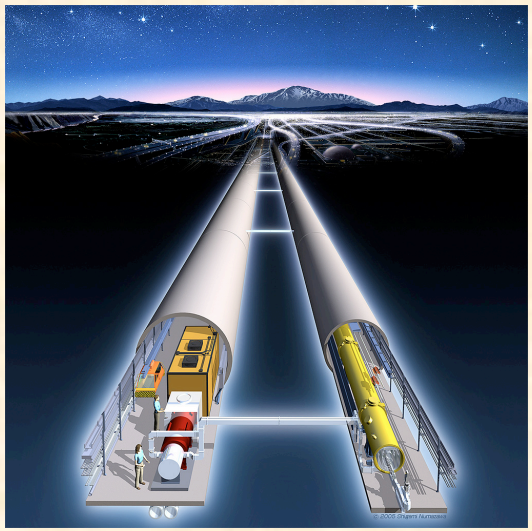
Experimental studies of GDAs in future

$\gamma\gamma \to h\bar{h}$ for internal structure of exotic hadron candidate h

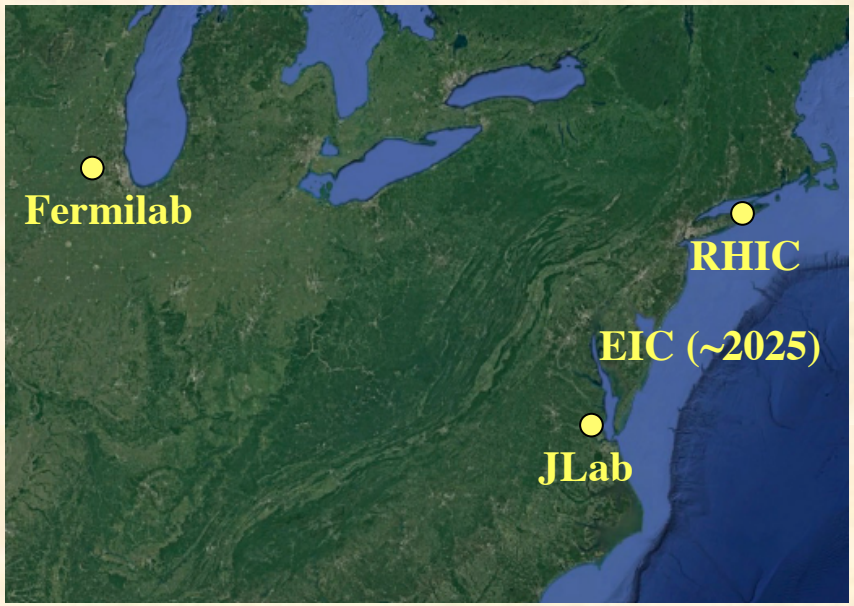


KEK B-factory

Linear Collider ?



Hadron-tomography studies in US and Europe



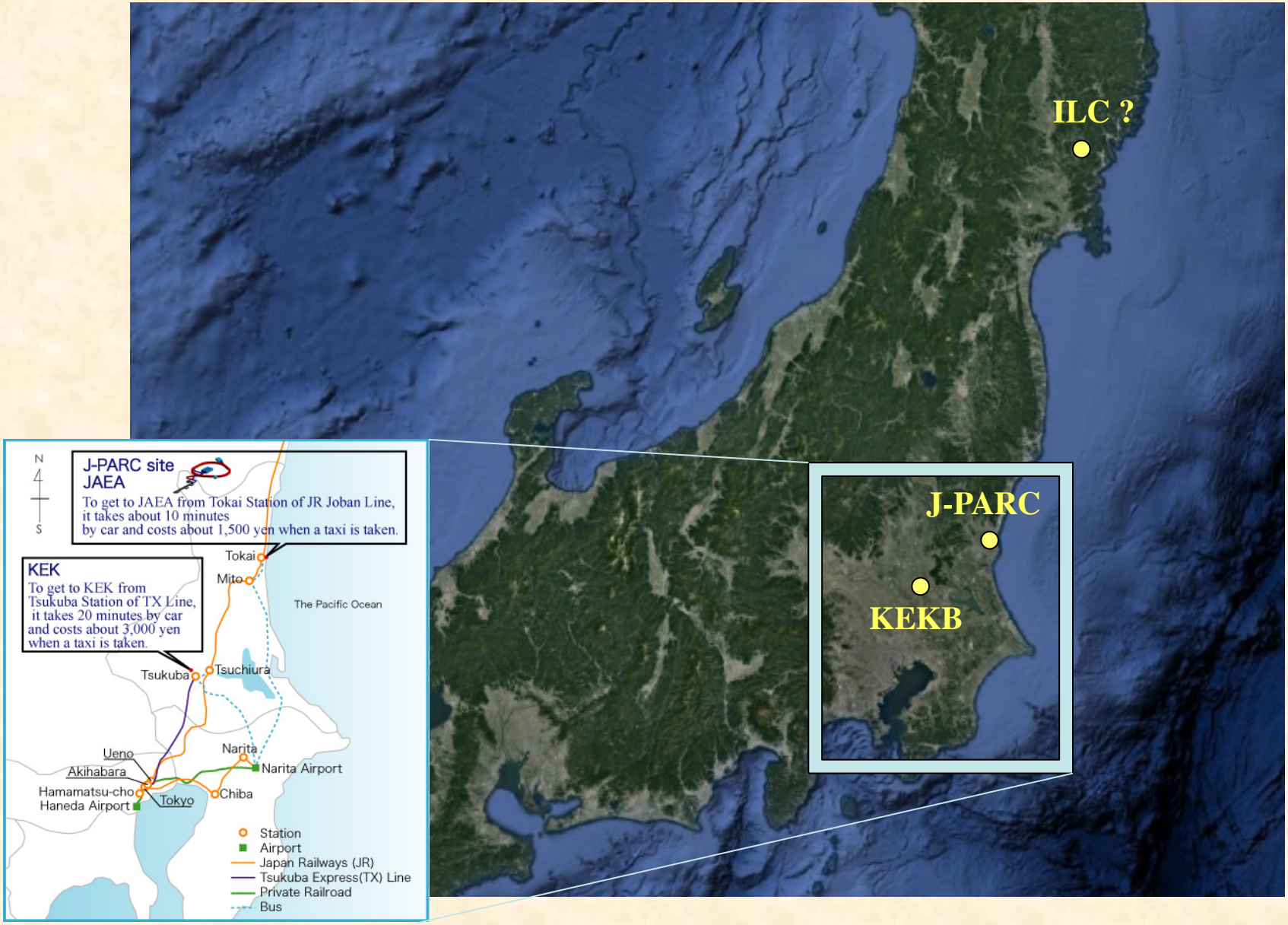
Fermilab: Main Injector (120 GeV proton),
Neutrino (Minerva, several GeV)

RHIC: Spin (polarized p + polarized p)
Heavy ion (*e.g.* UPC: Ultra-Peripheral Collision) ??

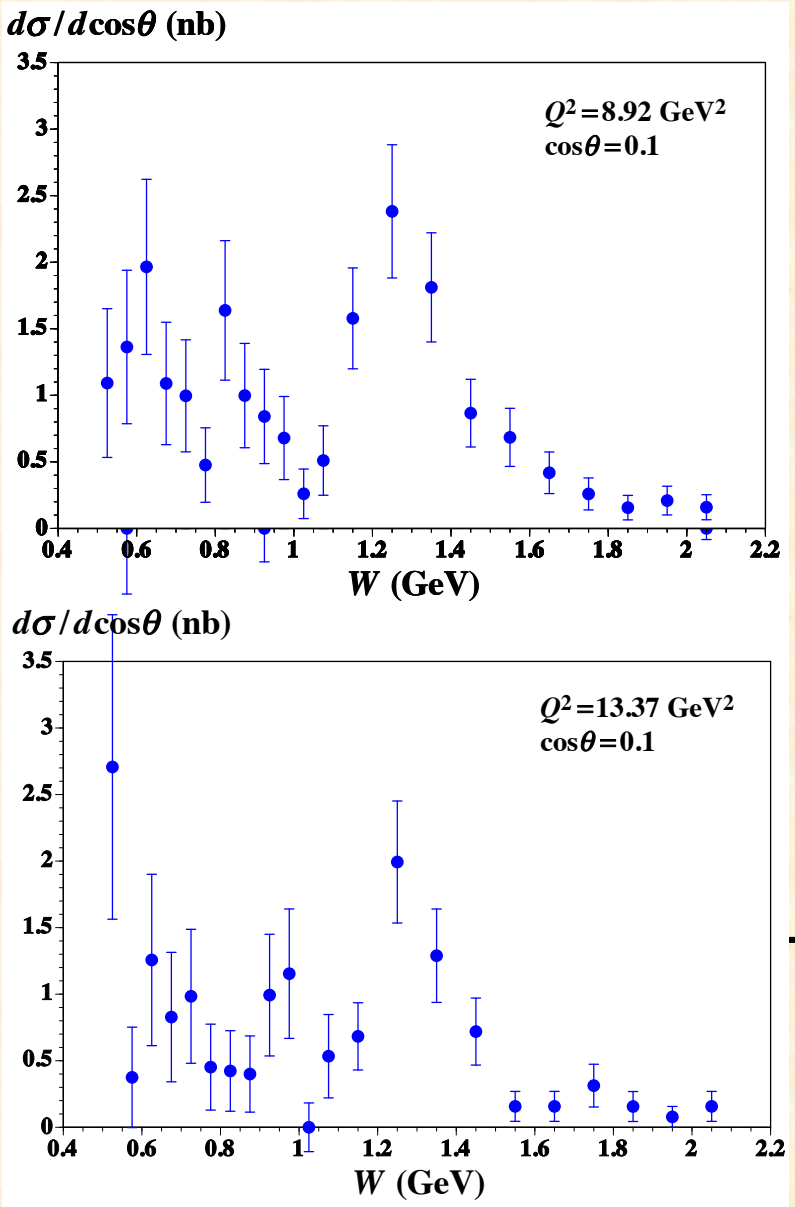
CERN: COMPASS (μ, π beams)
LHC Heavy ion (*e.g.* UPC ??)

EIC (Electron Ion Collider, ~2025)

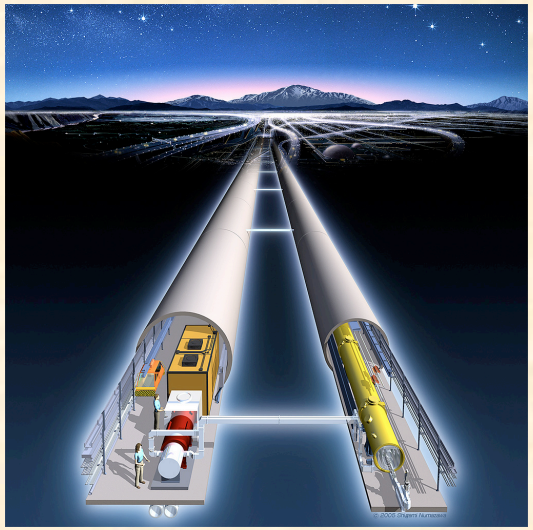
Possible hadron-tomography studies at J-PARC, KEKB, ILC?

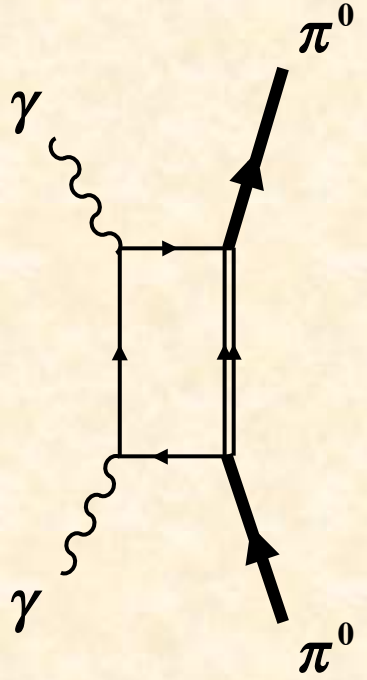


From KEKB to ILC



Linear Collider ?





- Very Large Q^2
- Large W^2

for extracting GDAs

→ **ILC**

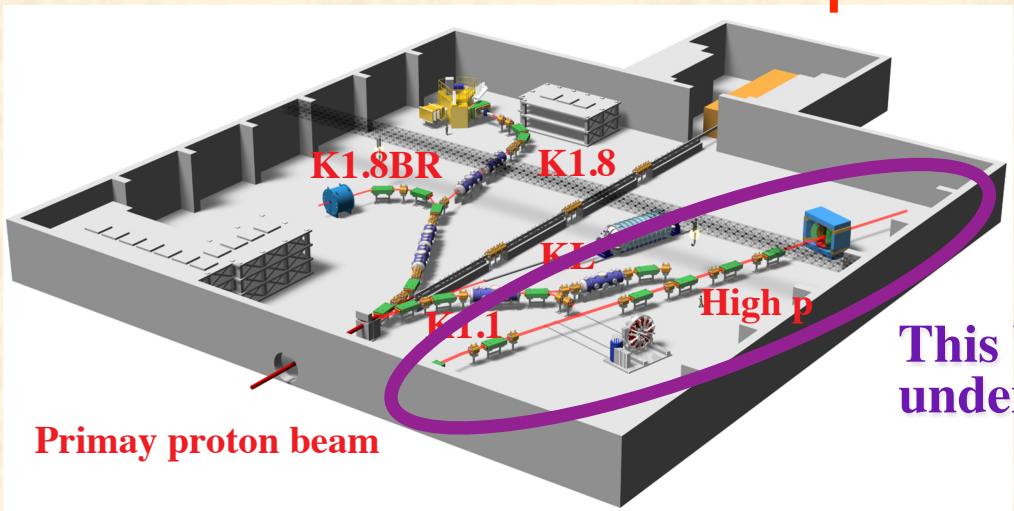
→ Sanuki's talk

Possible GPD studies at J-PARC

KEK Tokai campus

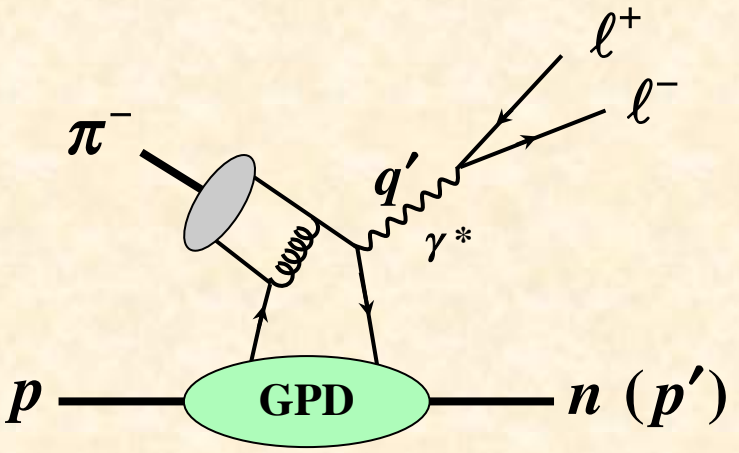
ν facility

Hadron facility



This beam line is now under construction.

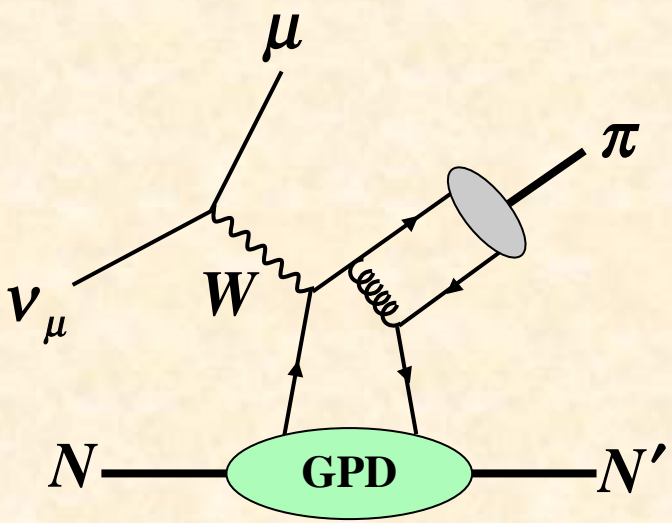
Exclusive Drell-Yan $\pi^- + p \to \mu^+\mu^- + n$ and GPDs

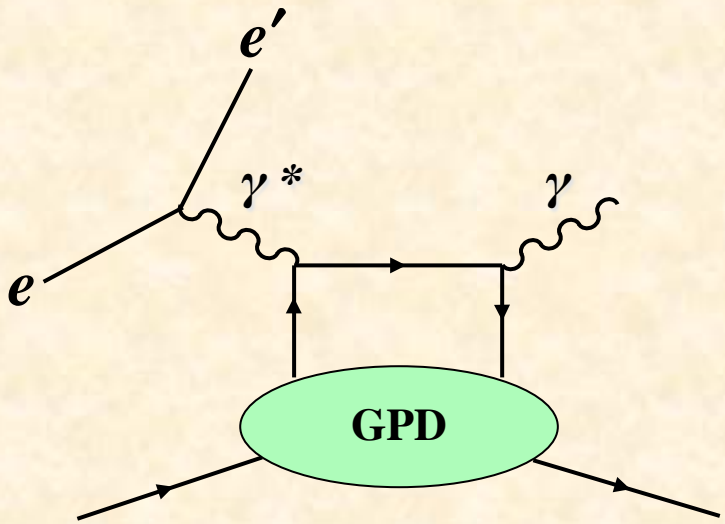


$\pi^-(\bar{u}d) + p(uud) \to n(udd) + \gamma^*(\to \ell^+\ell^-)$

→ Tanaka's talk

Neutrino-induced pion production for hadron tomography and gravitational physics (high-energy neutrino)





$$\nu_\mu + N \rightarrow \mu + N' + \pi$$

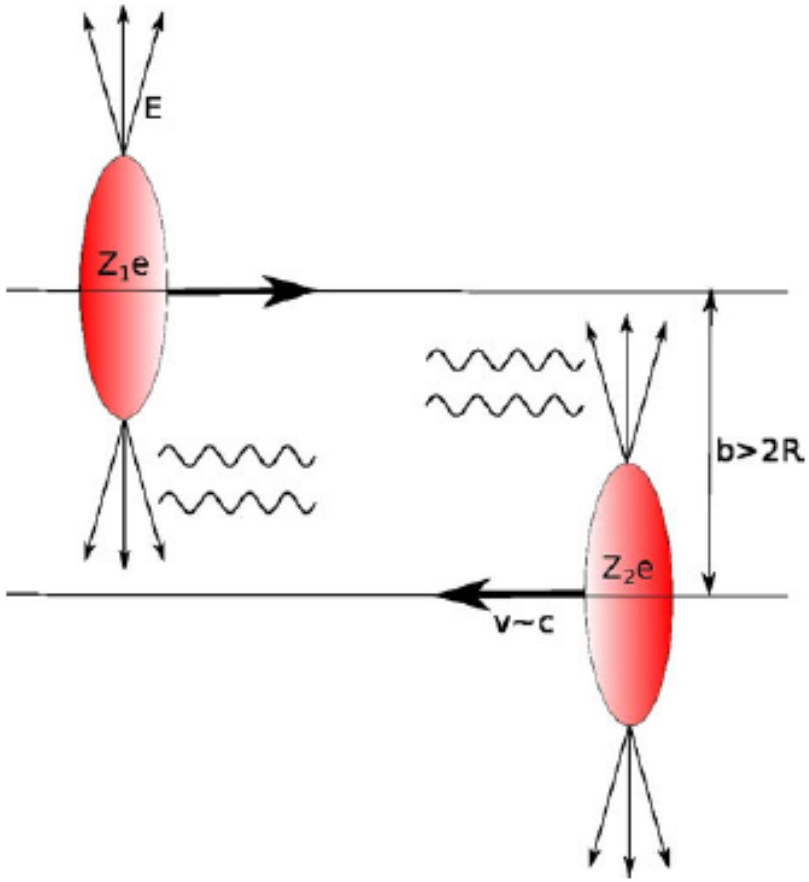
DVCS (Deeply virtual Compton scattering)

Ultra-Peripheral Collision (UPC) @ LHC/RHIC ?

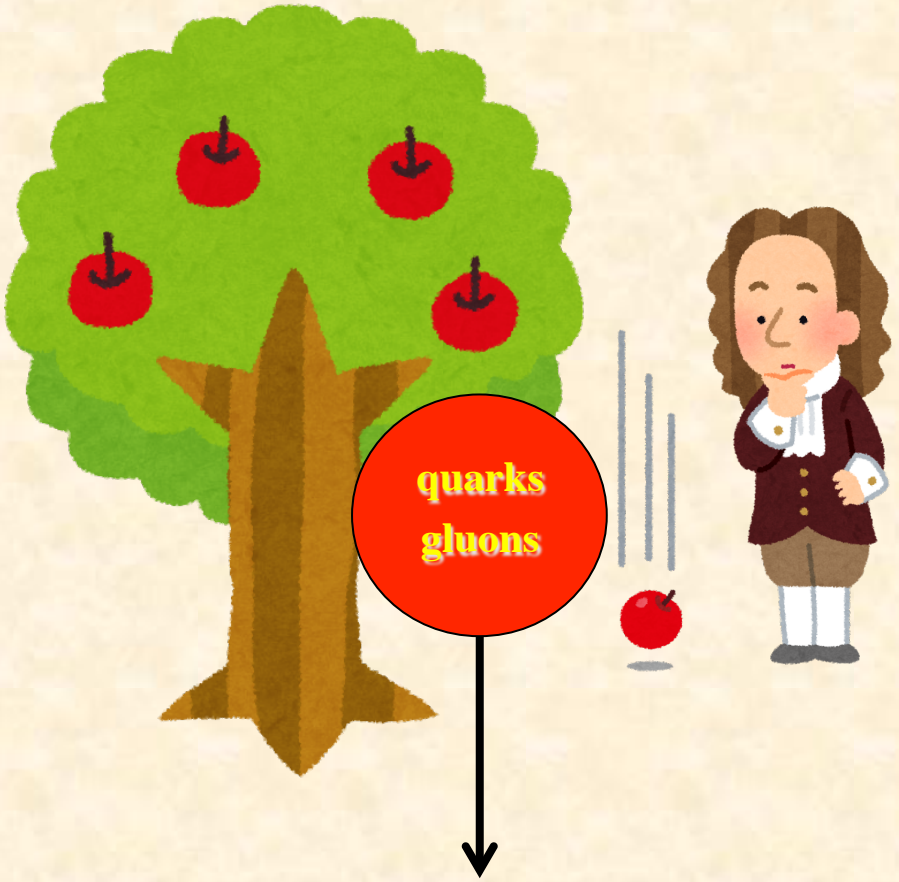
INT Workshop INT-17-65W

Probing QCD in Photon-Nucleus Interactions at RHIC and LHC: the Path to EIC

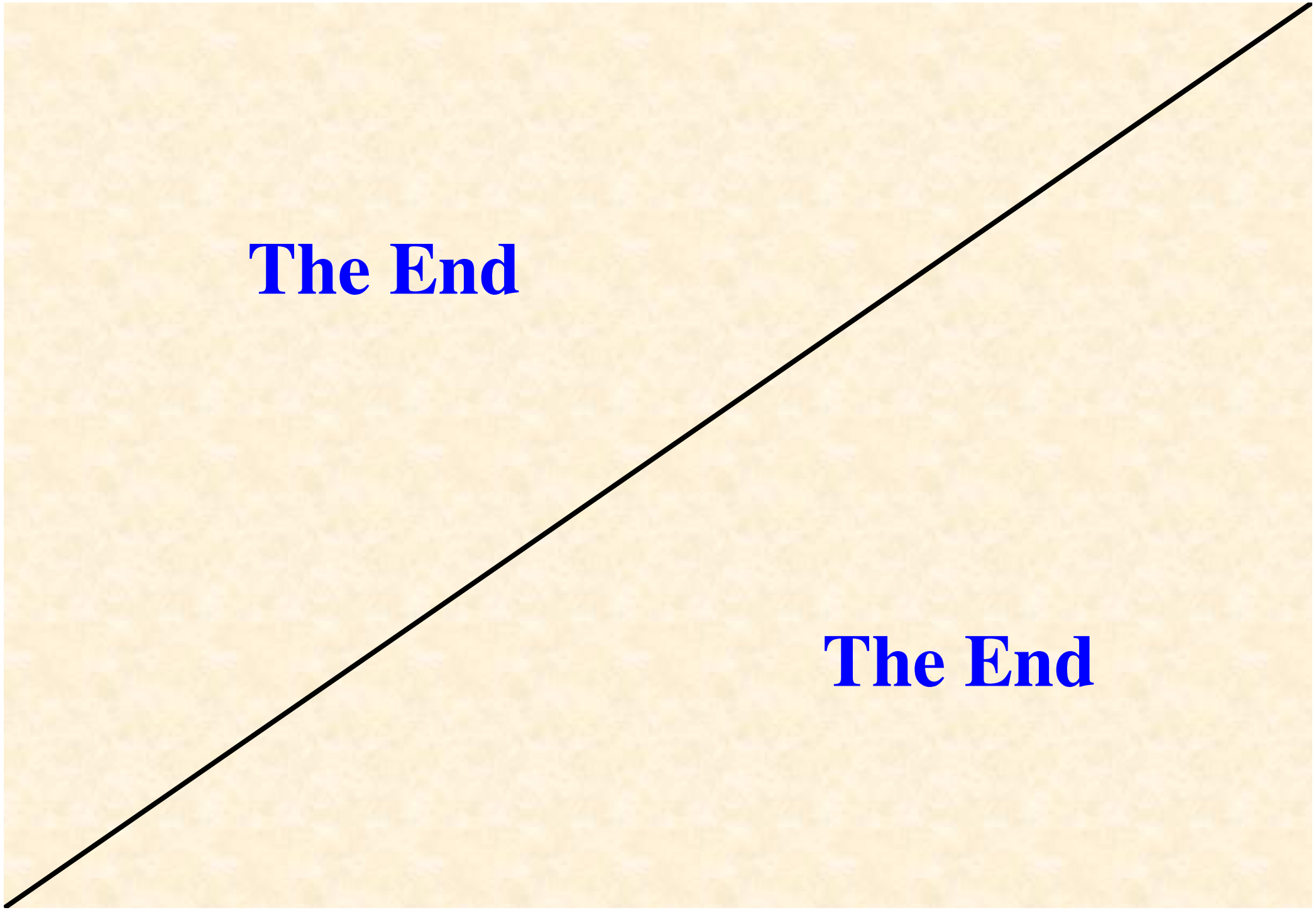
February 13 - 17, 2017



Origin of gravity in terms of quarks and gluons…



By the tomography, we determine gravitational sources (interactions) with quarks and gluons.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