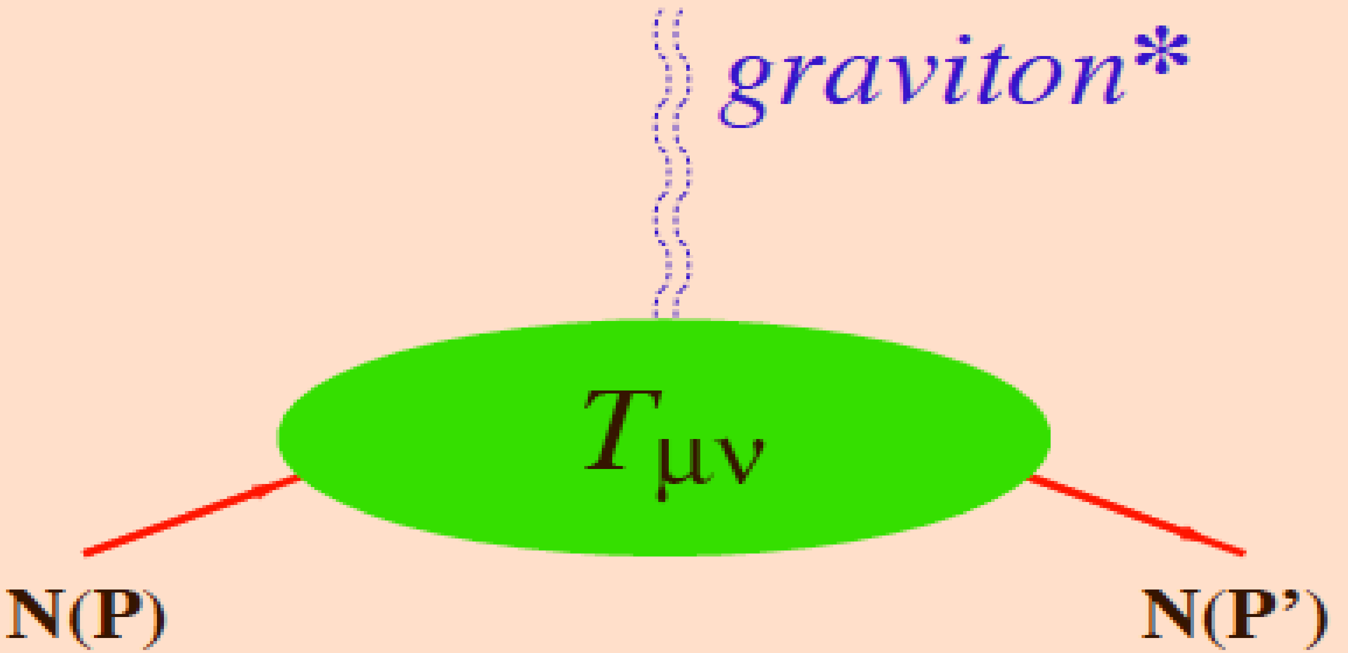
Generalized parton distribution function studies at J-PARC

Kazuhiro Tanaka (Juntendo U/KEK)



$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu}i\overleftrightarrow{D}^{\nu)}q \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\equiv \quad T_q^{\mu\nu} \quad + \quad T_g^{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q + F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4} F^2$$

$$\langle p' | T^{\mu\nu}_{q,g} | p \rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu} P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha}{2m_N} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{m_N} + \bar{C}_{q,g}(t) m_N g^{\mu\nu} \Big] u(p)$$

$$P = \frac{p + p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle p | T^{\mu\nu} | p \rangle = 2 p^\mu p^\nu$$

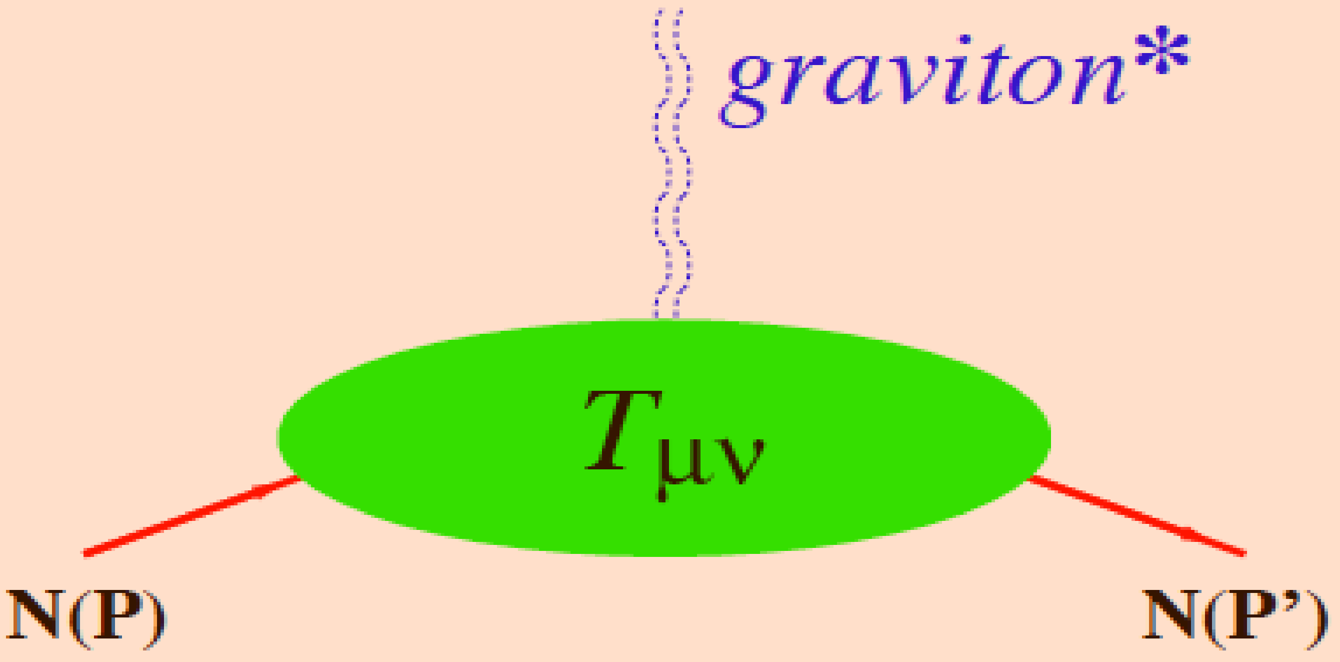
$$\frac{1}{2}\Big(A_q(0) + B_q(0) + A_g(0) + B_g(0) \Big) = \frac{1}{2}$$

$$B_q(0) + B_g(0) = 0$$

$$\frac{\langle pS | J^i | pS \rangle}{\langle pS | pS \rangle} = \frac{1}{2} S^i$$

$$J^i = \frac{1}{2}\epsilon^{ijk} \int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho}$$



$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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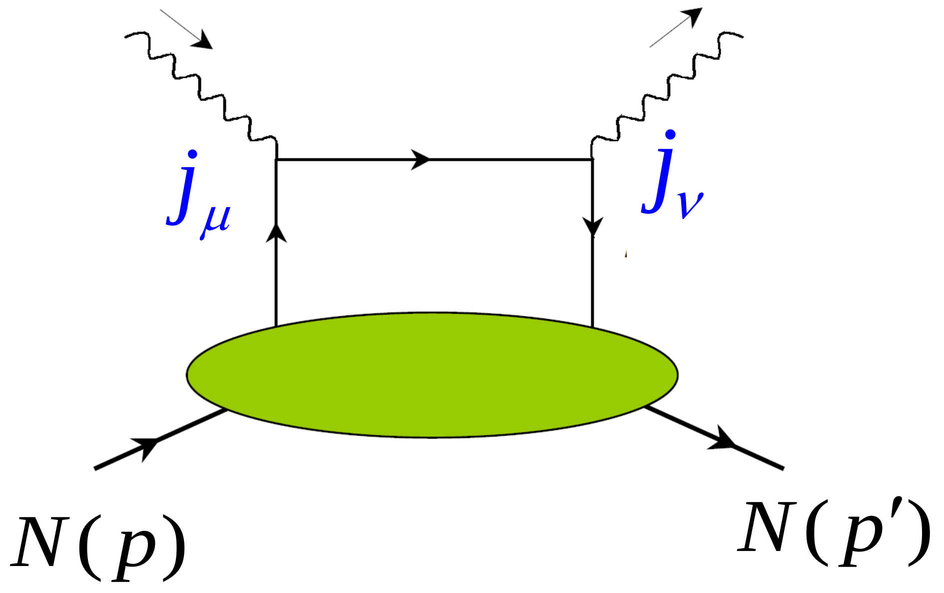


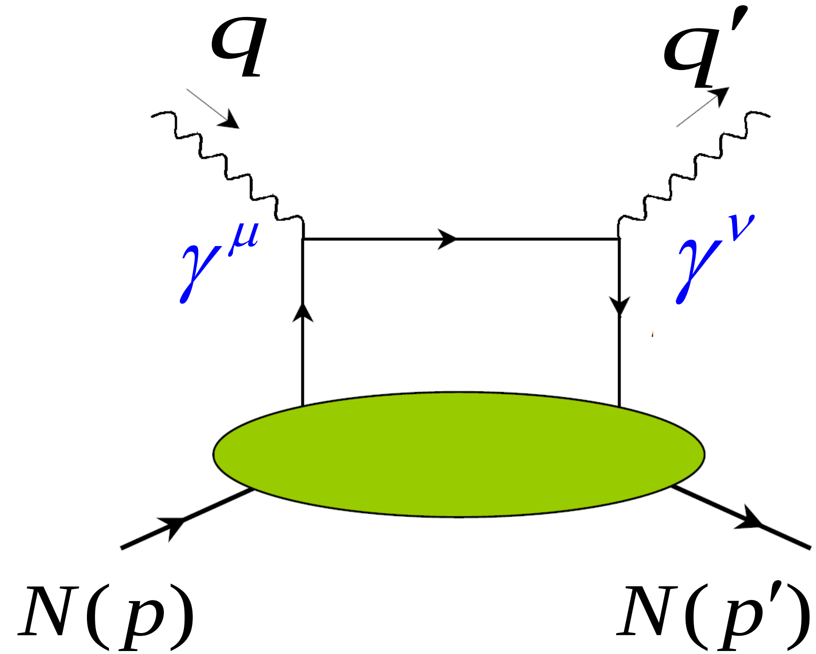
$$j_\mu(x)\, j_\nu(0) \sim \sum_i C_i(x) O_i(0)$$

$$C^q_{\mu\nu;\alpha\beta}(x) T_q^{\alpha\beta}$$

$$C^g_{\mu\nu;\alpha\beta}(x) T_g^{\alpha\beta}$$

DVCS

$$\int d^4x e^{iq'\cdot x}\langle p'|\mathrm{T}\, j_\mu^{\rm em}(0)\, j_\nu^{\rm em}(x)|p\rangle$$



$$\bar{P}=\frac{p+p'}{2},\quad \bar{q}=\frac{q+q'}{2},\quad \Delta=p'-p=q-q',$$

$$t=\Delta^2,\quad \xi=\frac{-\bar{q}^2}{2\bar{P}\cdot\bar{q}},\quad \eta=\frac{-\Delta\cdot n}{2\bar{P}\cdot n}=\frac{-\Delta\cdot\bar{q}}{2\bar{P}\cdot\bar{q}}+O(\text{twist-4}),$$

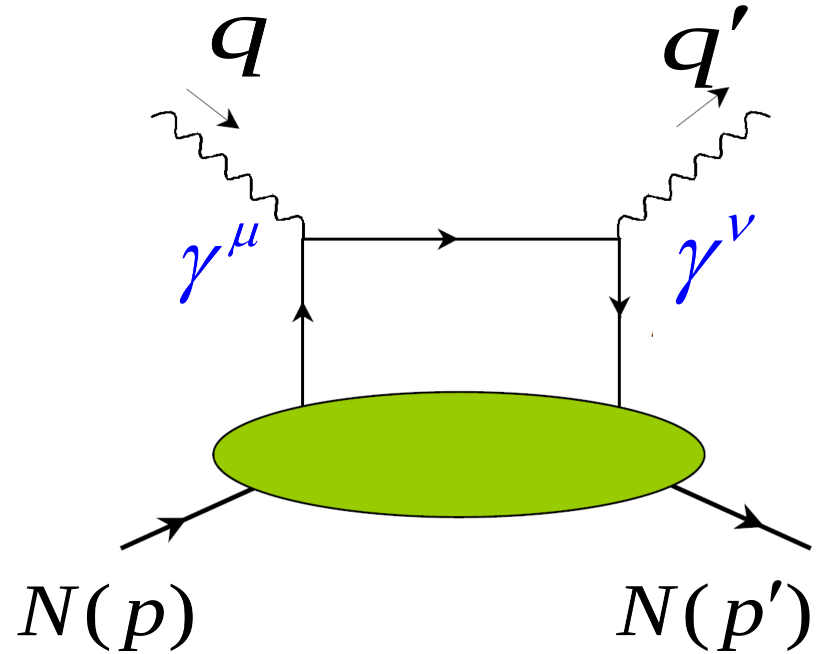
$$n_\mu=-\frac{2\xi}{\bar{q}^2}\left(\bar{q}_\mu+\xi\bar{P}_\mu\right),\quad \tilde{n}_\mu=\bar{P}_\mu,\qquad \left(n^2=\tilde{n}^2=0,\quad n\cdot\tilde{n}=1\right)$$

generalized Bjorken kinematics:

$$|\bar{q}^2|\to\infty,\quad |\bar{P}\cdot\bar{q}|\to\infty,\quad |\Delta\cdot\bar{q}|\to\infty,\quad \Delta^2=\text{finite}\quad (\xi\ \text{and}\ \eta\ \text{fixed})$$

DVCS

$$\int d^4x e^{iq'\cdot x}\langle p'|\mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x)|p\rangle$$



$$\bar{P}=\frac{p+p'}{2},\quad \bar{q}=\frac{q+q'}{2},\quad \Delta=p'-p=q-q',$$

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generalized Bjorken kinematics: light-cone expansion

$$|\bar{q}^2|\to\infty,\quad |\bar{P}\cdot\bar{q}|\to\infty,\quad |\Delta\cdot\bar{q}|\to\infty,\quad \Delta^2=\text{finite}\quad (\xi \text{ and } \eta \text{ fixed})$$

$$\int d^4x e^{iq'\cdot x}\langle p'|\mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x)|p\rangle$$

$$= \mathcal{T}^{(1)}_{\mu\nu}\frac{ie_q^2}{\bar{q}^2}\int dx\left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2}-\frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2}\right)\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-} < N(p')\left|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})\right|N(p) >\Big|_{z^+=\vec{z}_\perp=0}$$

$$+\mathcal{T}^{(2)}_{\mu\nu}\frac{ie_q^2}{\bar{q}^2}\int dx\left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2}+\frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2}\right)\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-} < N(p')\left|\bar{q}(-\frac{z}{2})\gamma^+\gamma_5 q(\frac{z}{2})\right|N(p) >\Big|_{z^+=\vec{z}_\perp=0}$$

$$+O(\text{twist-3})$$

$$\mathcal{T}^{(1)}_{\mu\nu} = \tilde{n}_\mu\left(\bar{q}_\nu+\xi\tilde{n}_\nu\right)+\tilde{n}_\nu\left(\bar{q}_\mu+\xi\tilde{n}_\mu\right)-g_{\mu\nu}\bar{q}\cdot\tilde{n} = \frac{\bar{q}^2}{2\xi}\left(g_{\mu\nu}-\frac{q'_\mu q_\nu}{q\cdot q'}\right)+2\xi\left(\bar{P}_\mu-\frac{\bar{P}\cdot q}{q\cdot q'}q'_\mu\right)\left(\bar{P}_\nu-\frac{\bar{P}\cdot q'}{q\cdot q'}q_\nu\right)$$

$$\mathcal{T}^{(2)}_{\mu\nu} = i\varepsilon_{\mu\alpha\nu\rho}\bar{q}^\alpha\tilde{n}^\rho = i\varepsilon^{\lambda\beta\rho\sigma}\bar{P}_\rho\bar{q}_\sigma\left(g_{\mu\lambda}-\frac{\bar{P}_\mu q_\lambda}{\bar{P}\cdot q}\right)\left(g_{\nu\beta}-\frac{\bar{P}_\nu q'_\beta}{\bar{P}\cdot q'}\right)$$

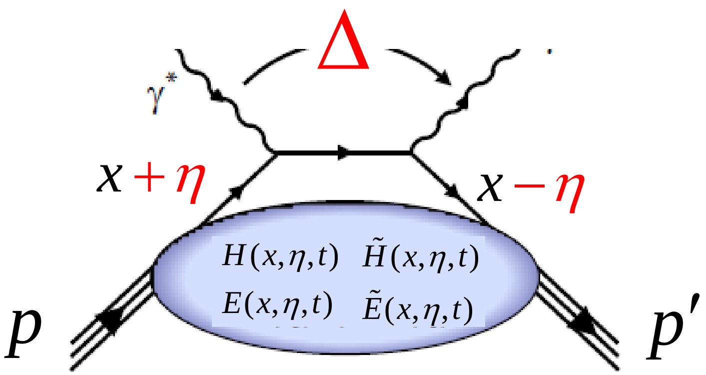
$$q^\mu\mathcal{T}^{(1)}_{\mu\nu} = q'^\nu\mathcal{T}^{(1)}_{\mu\nu} = q^\mu\mathcal{T}^{(2)}_{\mu\nu} = q'^\nu\mathcal{T}^{(2)}_{\mu\nu} = 0\,,$$

$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

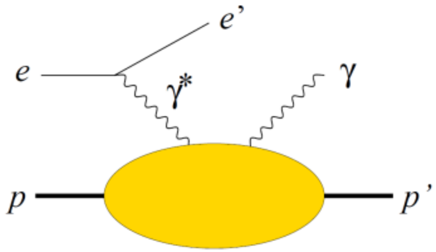
$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



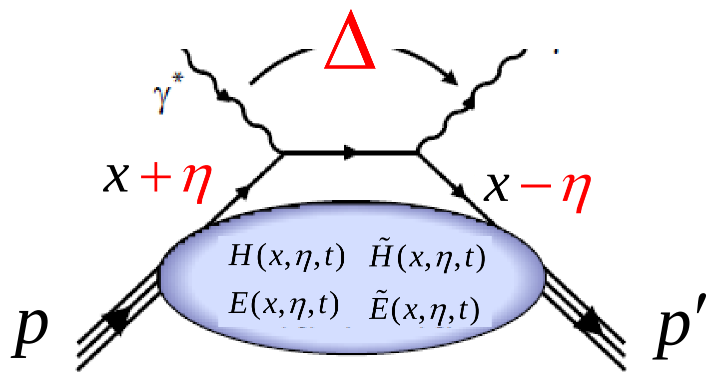
$$\bar{P}=\frac{p+p'}{2}$$

JLab, HERMES, COMPASS,…

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}z}\langle p'|\bar{q}(-\frac{z^-}{2})\gamma^+q(\frac{z^-}{2})|p\rangle=\frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+u(p)+E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar{P}z}\langle p'|\bar{q}(-\frac{z^-}{2})\gamma^+\gamma_5q(\frac{z^-}{2})|p\rangle=\frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5u(p)+\tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

GPD



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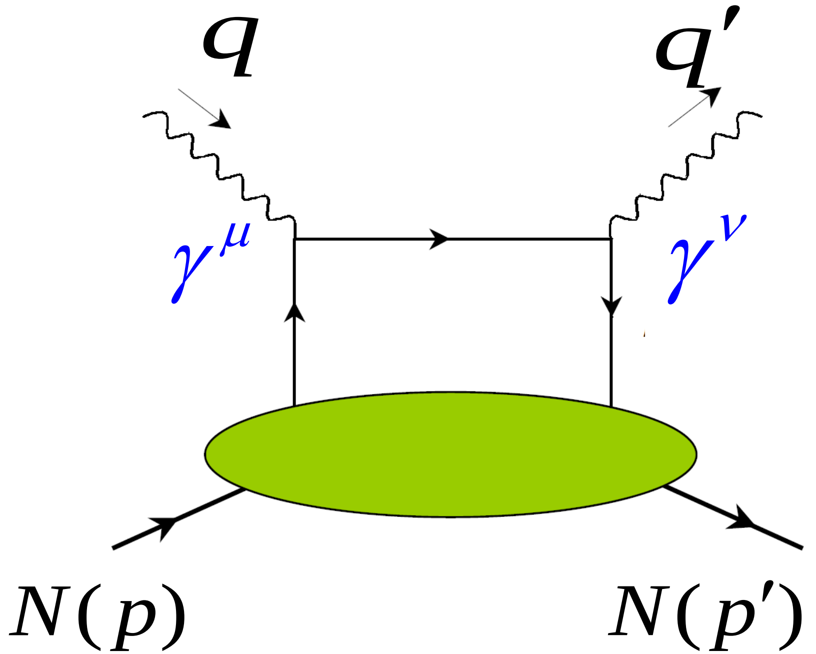
$$\int d^4x e^{iq'\cdot x}\langle p' | \mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x) | p\rangle$$



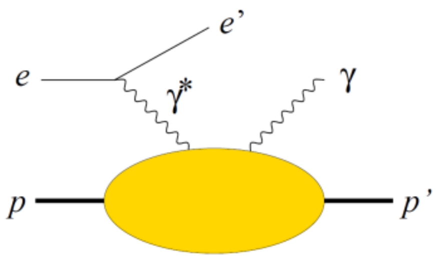
$$\bar{P} = \frac{p+p'}{2}, \quad \bar{q} = \frac{q+q'}{2}, \quad \Delta = p' - p = q - q',$$

$$t = \Delta^2, \quad \xi = \frac{-\bar{q}^2}{2\bar{P}\cdot\bar{q}}, \quad \eta = \frac{-\Delta\cdot n}{2\bar{P}\cdot n} = \frac{-\Delta\cdot\bar{q}}{2\bar{P}\cdot\bar{q}} + O(\text{twist-4}),$$

$$n_\mu = -\frac{2\xi}{\bar{q}^2}\left(\bar{q}_\mu + \xi\bar{P}_\mu\right), \quad \tilde{n}_\mu = \bar{P}_\mu, \qquad \left(n^2 = \tilde{n}^2 = 0, \quad n\cdot\tilde{n} = 1\right)$$

generalized Bjorken kinematics: light-cone expansion

$$|\bar{q}^2| \to \infty, \quad |\bar{P}\cdot\bar{q}| \to \infty, \quad |\Delta\cdot\bar{q}| \to \infty, \quad \Delta^2 = \text{finite} \quad (\xi \text{ and } \eta \text{ fixed})$$



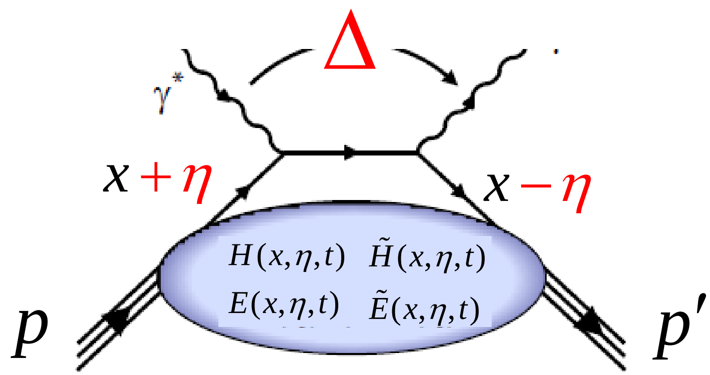
$$\bar{P} = \frac{p+p'}{2}$$

JLab, HERMES, COMPASS,...

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

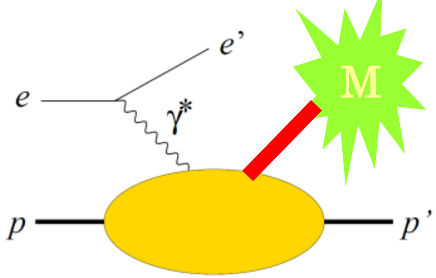
$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



$$\bar{P} = \frac{p+p'}{2}$$

JLab, HERMES, COMPASS,...

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + Eq(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

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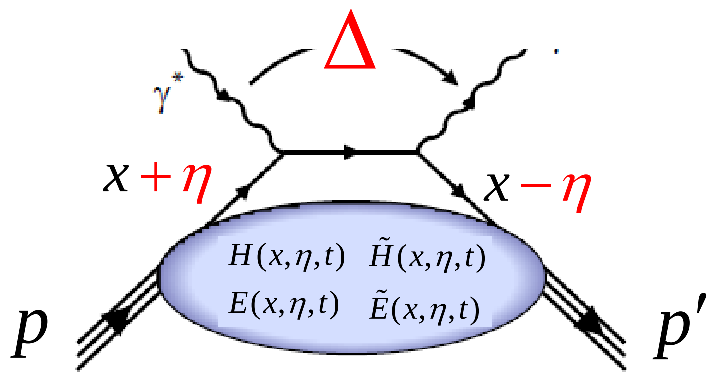

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$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

$$\left(t = \Delta^2 \to 0 , \quad \eta = \frac{-\Delta \cdot n}{2\bar{P} \cdot n} \to 0 \right)$$

$$H^q(x,0,0) = q(x) , \qquad \tilde{H}^q(x,0,0) = \Delta q(x).$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

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$$H^q(x,0,0) = q(x) , \qquad \tilde{H}^q(x,0,0) = \Delta q(x).$$

$$\int_{-1}^{1} dx H^q(x,\eta,t) = F_1^q(t), \qquad \int_{-1}^{1} dx E^q(x,\eta,t) = F_2^q(t),$$

$$\int_{-1}^{1} dx \tilde{H}^q(x,\eta,t) = g_A^q(t), \qquad \int_{-1}^{1} dx \tilde{E}^q(x,\eta,t) = g_P^q(t)$$

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$$H^q(x,0,0) = q(x), \qquad \tilde{H}^q(x,0,0) = \Delta q(x).$$

$$\int_{-1}^{1} dx H^q(x,\eta,t) = F_1^q(t), \qquad \int_{-1}^{1} dx E^q(x,\eta,t) = F_2^q(t),$$

$$\int_{-1}^{1} dx \tilde{H}^q(x,\eta,t) = g_A^q(t), \qquad \int_{-1}^{1} dx \tilde{E}^q(x,\eta,t) = g_P^q(t)$$

$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = \cdots \quad , \quad \int_{-1}^{1} dx\, x E^q(x,\eta,t) = \cdots$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\frac{1}{2P^+} \left\langle p' \middle| \bar{q}(0) \gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| p \right\rangle = \bar{u}(p') \gamma^+ u(p) \int_{-1}^{1} dx x H^q(x,\xi,t) + \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_N} u(p) \int_{-1}^{1} dx x E^q(x,\xi,t)$$

$$= \frac{1}{P^+} \left\langle p' \middle| T_q^{++}(0) \middle| p \right\rangle$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q + F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\langle p' | T^{\mu\nu}_{q,g} | p\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu} i\sigma^{\nu)\alpha}\Delta_{\alpha}}{2m_N}$$
$$+ D_{q,g}(t)\frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2}{m_N} + \bar{C}_{q,g}(t) m_N g^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_{\mu} T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle p | T^{\mu\nu} | p\rangle = 2 p^{\mu} p^{\nu}$$

$$\frac{1}{2}\left(A_q(0) + B_q(0) + A_g(0) + B_g(0)\right) = \frac{1}{2}$$

$$B_q(0) + B_g(0) = 0$$

$$\frac{\langle pS | J^i | pS\rangle}{\langle pS | pS\rangle} = \frac{1}{2}S^i$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^{\rho}T^{\mu\sigma} - x^{\sigma}T^{\mu\rho}$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\frac{1}{2P^+} \left\langle p' \middle| \bar{q}(0) \gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| p \right\rangle = \bar{u}(p') \gamma^+ u(p) \int_{-1}^{1} dx x H^q(x,\xi,t) + \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_N} u(p) \int_{-1}^{1} dx x E^q(x,\xi,t)$$

$$= \frac{1}{P^+} \left\langle p' \middle| T_q^{++}(0) \middle| p \right\rangle$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\frac{1}{2P^+} \left\langle p' \middle| \bar{q}(0) \gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| p \right\rangle = \bar{u}(p') \gamma^+ u(p) \int_{-1}^{1} dx x H^q(x,\xi,t) + \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_N} u(p) \int_{-1}^{1} dx x E^q(x,\xi,t)$$

$$= \frac{1}{P^+} \left\langle p' \middle| T_q^{++}(0) \middle| p \right\rangle == \bar{u}(p') \gamma^+ u(p) \left(A_q(t) + 4\xi^2 D_q(t) \right) + \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_N} u(p) \left(B_q(t) - 4\xi^2 D_q(t) \right)$$

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$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

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$$H^q(x,0,0) = q(x)\,, \qquad \tilde{H}^q(x,0,0) = \Delta q(x).$$

$$\int_{-1}^{1} dx H^q(x,\eta,t) = F_1^q(t), \qquad \int_{-1}^{1} dx E^q(x,\eta,t) = F_2^q(t),$$

$$\int_{-1}^{1} dx \tilde{H}^q(x,\eta,t) = g_A^q(t), \qquad \int_{-1}^{1} dx \tilde{E}^q(x,\eta,t) = g_P^q(t)$$

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$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

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$$\langle p' | T_{q,g}^{\mu\nu} | p\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha}{2m_N} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{m_N} + \bar{C}_{q,g}(t)m_N g^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

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$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle p | T^{\mu\nu} | p\rangle = 2p^\mu p^\nu$$

$$\frac{1}{2}\Big(A_q(0) + B_q(0) + A_g(0) + B_g(0)\Big) = \frac{1}{2}$$

$$B_q(0) + B_g(0) = 0$$

$$\frac{\langle pS | J^i | pS\rangle}{\langle pS | pS\rangle} = \frac{1}{2}S^i$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho}$$

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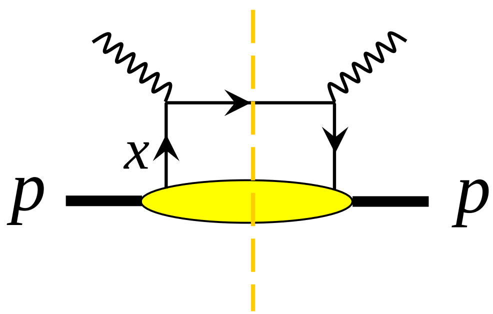

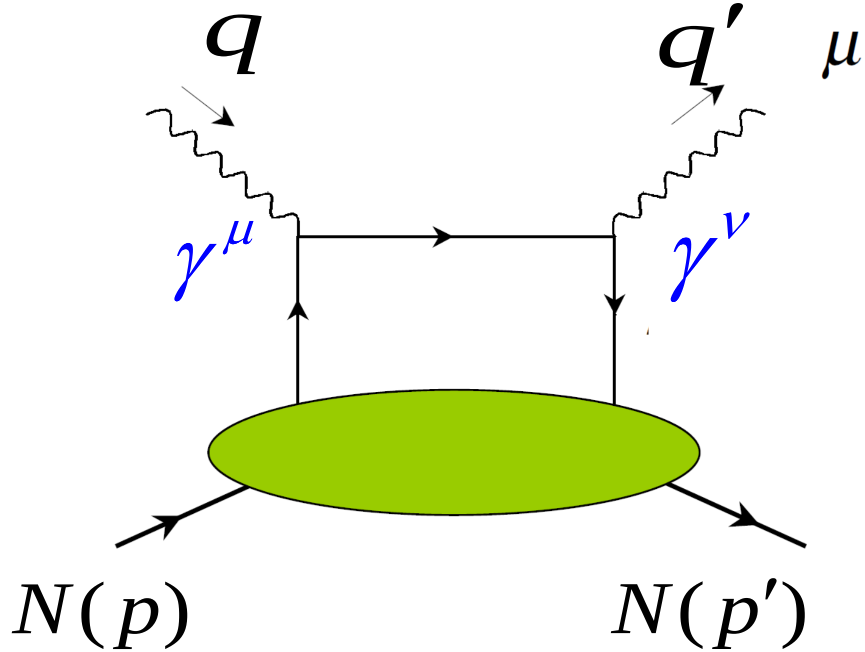

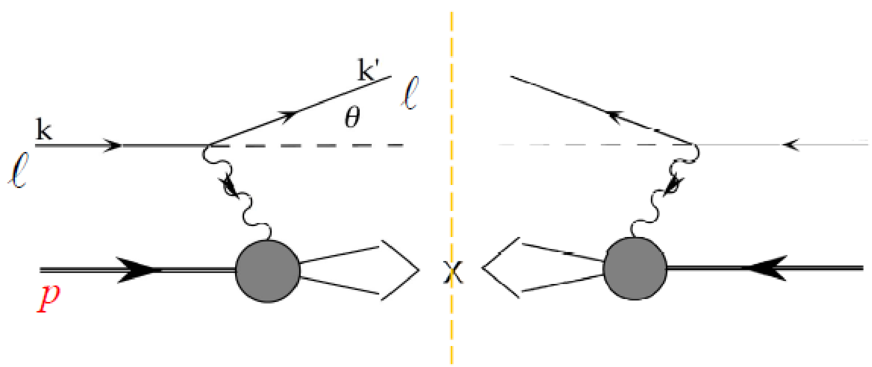

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$$\simeq 1/2$$

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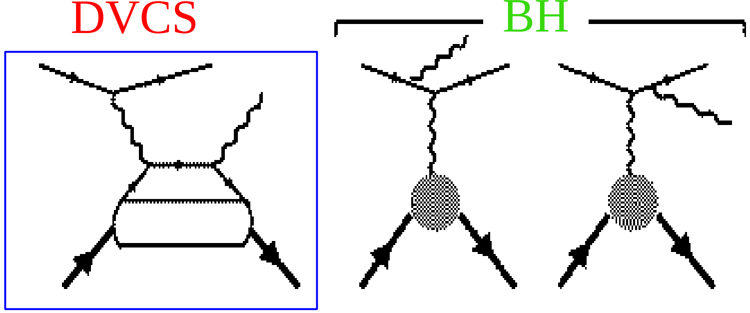
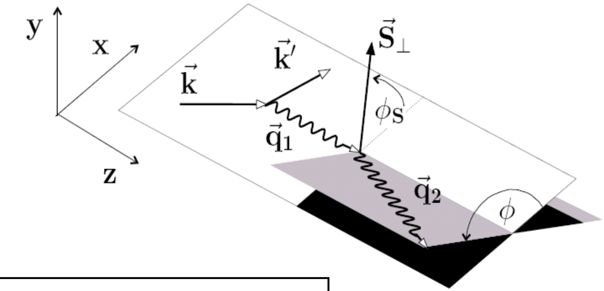




$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} \sim |T^{DVCS} + T^{BH}|^2$$

$$A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = \frac{\Delta\sigma}{2\sigma}$$

T^{BH}: real, given by elastic form factors

T^{DVCS}: complex, determined by GPDs

$\eta \sim x_B/(2-x_B)$

$k = t/4M^2$

Polarized beam, unpolarized target:

$$\Delta\sigma_{LU} \sim \sin\phi \, \{F_1\mathcal{H} + \eta(F_1+F_2)\tilde{\mathcal{H}} + kF_2\mathcal{E}\}d\phi$$

→ $H(x,\eta,t)$

Unpolarized beam, longitudinal target:

$$\Delta\sigma_{UL} \sim \sin\phi \, \{F_1\tilde{\mathcal{H}} + \eta(F_1+F_2)(\mathcal{H} + \eta/(1+\eta)\mathcal{E})\}d\phi$$

→ $\tilde{H}(x,\eta,t)$

Unpolarized beam, transverse target:

$$\Delta\sigma_{UT} \sim \cos\phi \, \sin(\phi_s-\phi)\{k(F_2\mathcal{H} - F_1\mathcal{E})\}d\phi$$

→ $E(x,\eta,t)$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

$$\left(t = \Delta^2 \to 0 , \quad \eta = \frac{-\Delta \cdot n}{2\bar{P} \cdot n} \to 0 \right)$$

$$H^q(x,0,0) = q(x) , \qquad \tilde{H}^q(x,0,0) = \Delta q(x).$$

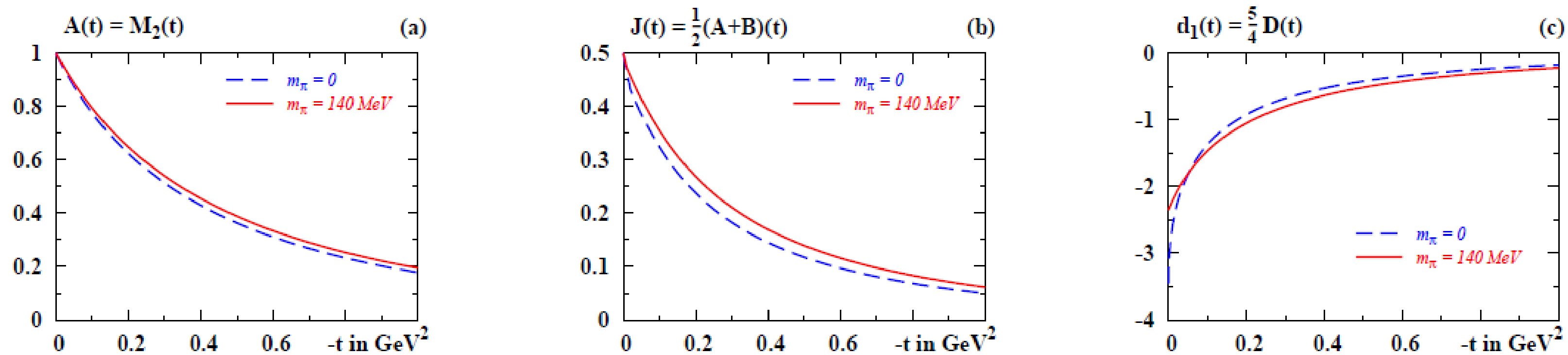
$$\int_{-1}^{1} dx x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t) , \qquad \int_{-1}^{1} dx x E^q(x,\eta,t) = B_q(t) - 4\eta^2 D_q(t)$$

$$\int_{-1}^{1} dx x q(x) = A_q(0)$$

$$\frac{1}{2} \int_{-1}^{1} dx x \left(H^q(x,\eta,t) + E^q(x,\eta,t) \right) = \frac{1}{2} \left(A_q(t) + B_q(t) \right)$$

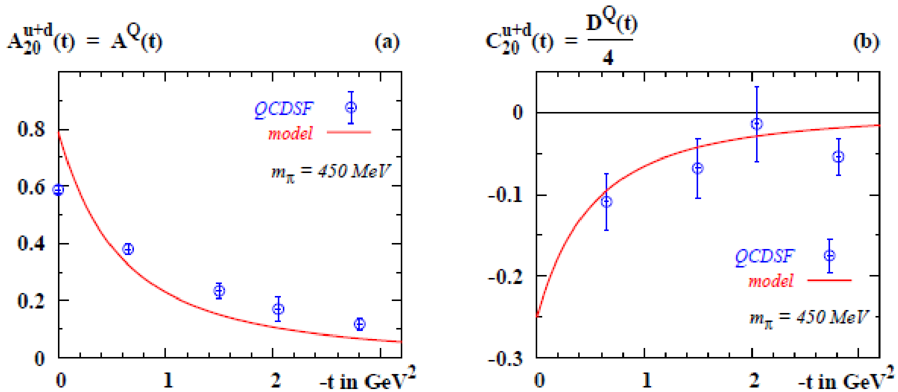
EMT form factors & D-term of nucleon:

- nature: **unknown!**
- model: e.g. chiral quark soliton model, Goeke et al, PRD75 (2007) 094021



well-tested model, many nucleon properties vs data within 30 % ✓

- lattice: QCDSF Collaboration, Göckeler et al, PRL92 (2004) 042002 & hep-ph/0312104 → test models

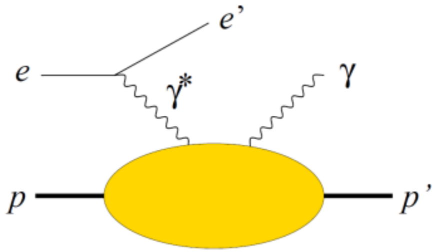


lattice QCD, bag model, Skyrme model, chiral quark soliton model: $D_{\text{nucleon}} < 0$

other particles:
nuclei, pions, photons, Q-balls, Q-cloud, Higgs(!) have also negative D-terms! (in theory!)

One day we will have this from **experiment!**

what will we learn?



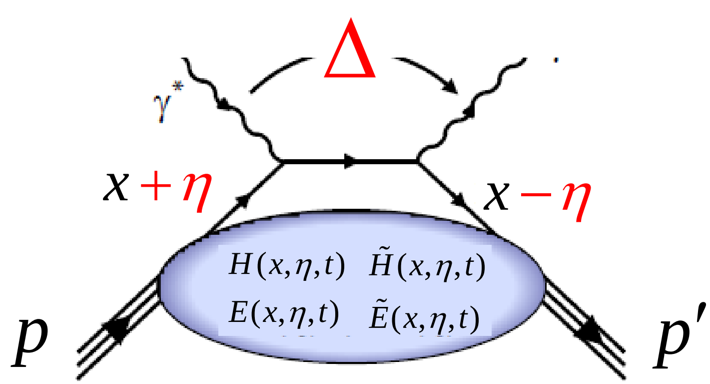
$$\bar{P} = \frac{p+p'}{2}$$

JLab, HERMES, COMPASS,...

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

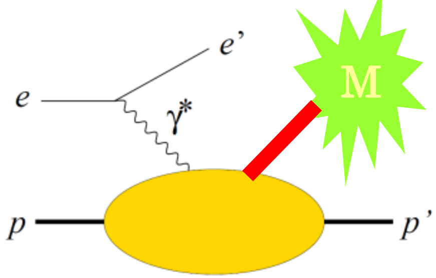
$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



$$\bar{P} = \frac{p + p'}{2}$$

JLab, HERMES, COMPASS,…

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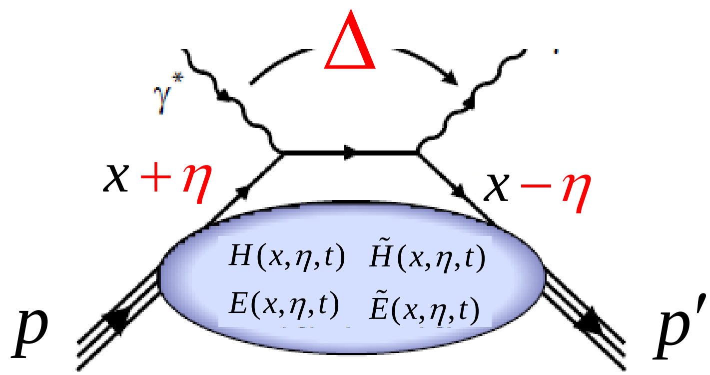
GPD



$$-2\eta\bar{P} = \Delta$$

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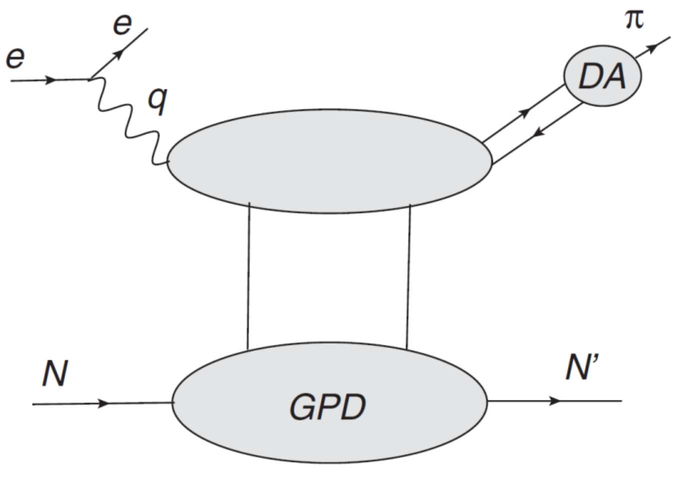
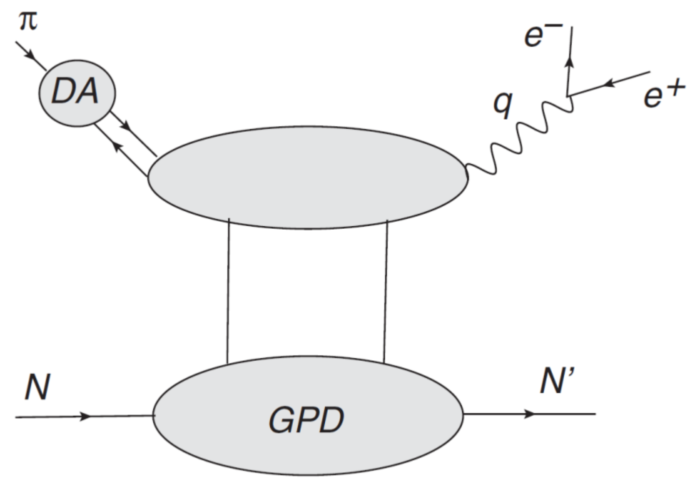


(c)



(d)

DVMP@HERA,JLab

exDY@J-PARC

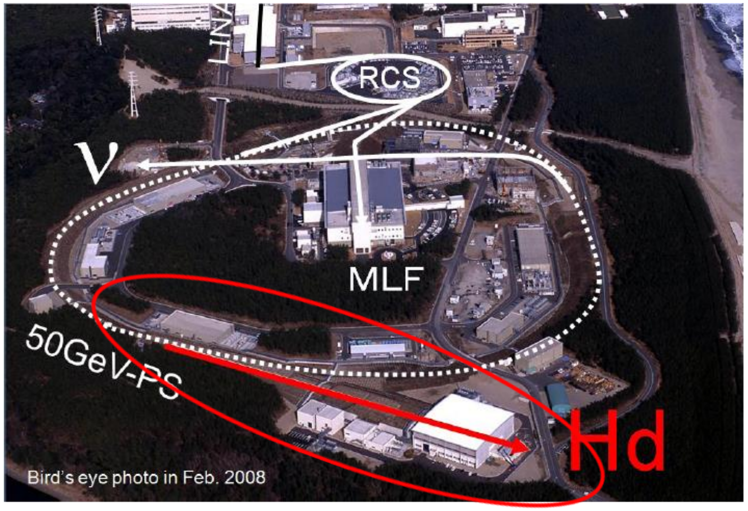
High momentum beam line at J-PARC

- Primary beam (proton)

$$E = 30\text{GeV} \ (\rightarrow 50\text{GeV?})$$

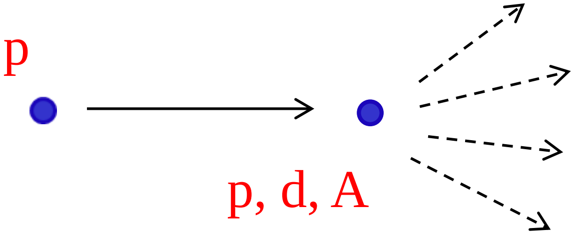
$$L = 10^{35}\text{cm}^{-2}\text{s}^{-1}$$

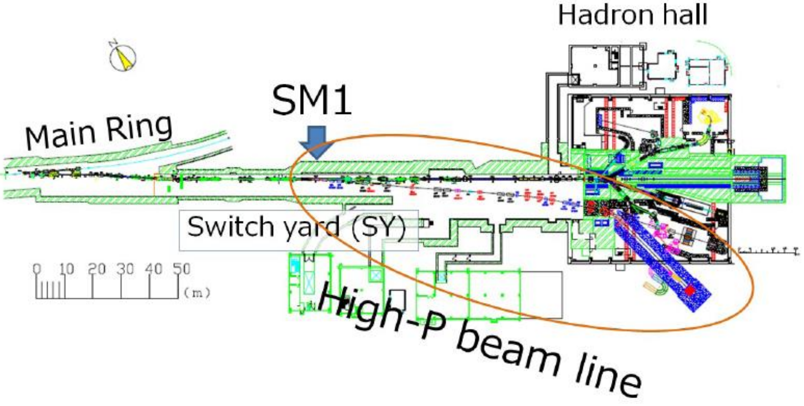
Hadron Facility at J-PARC



- Secondary beam (pion)

E = 15-20GeV



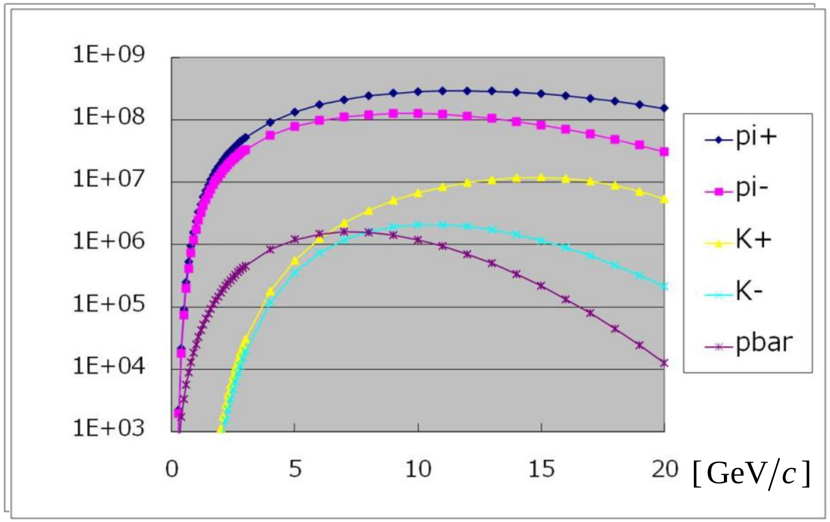


High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

Unseparated Secondary Beam Intensity

beam loss limit @ SM1:15kW
(limited by the thickness of the tunnel wall)



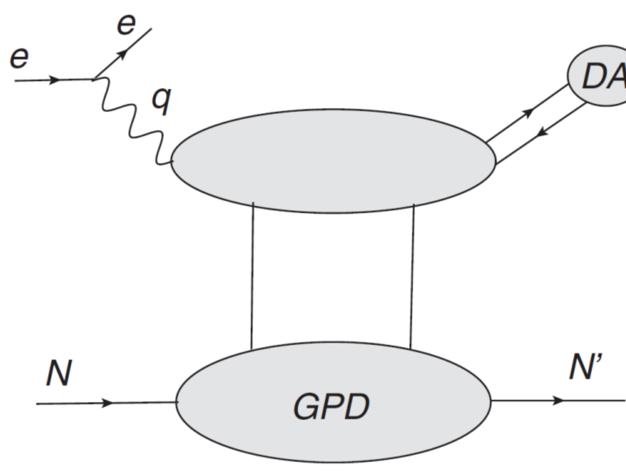
0° extraction angle

high intensity

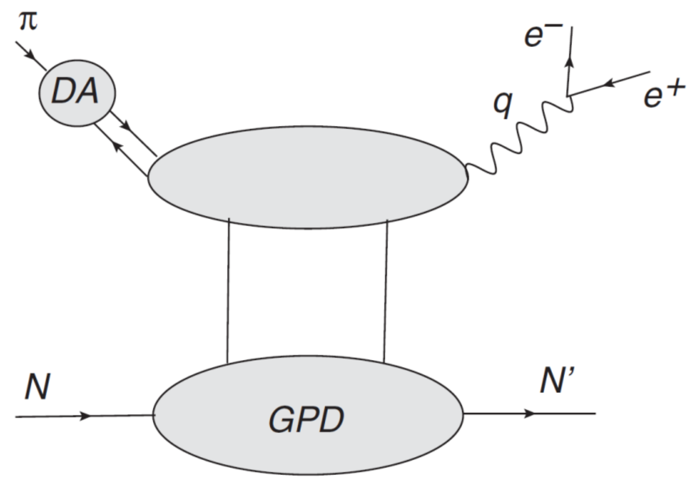
not too high energy $d\sigma \sim 1/s^a$

best suited to study meson-induced hard exclusive processes

$$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$$



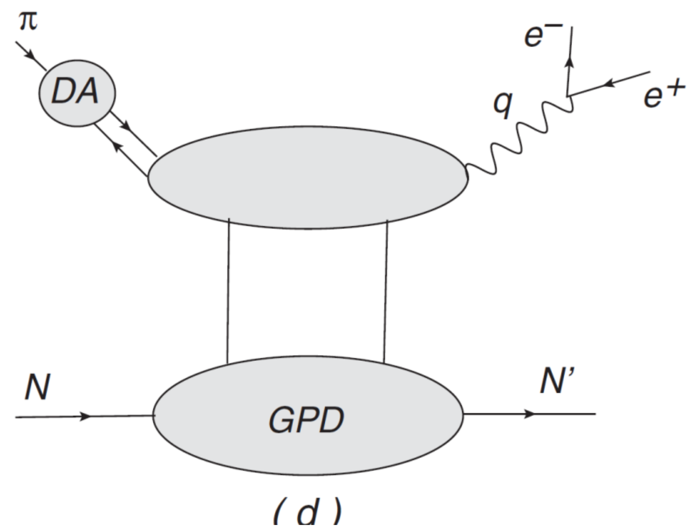
(c)



(d)

DVMP@HERA,JLab

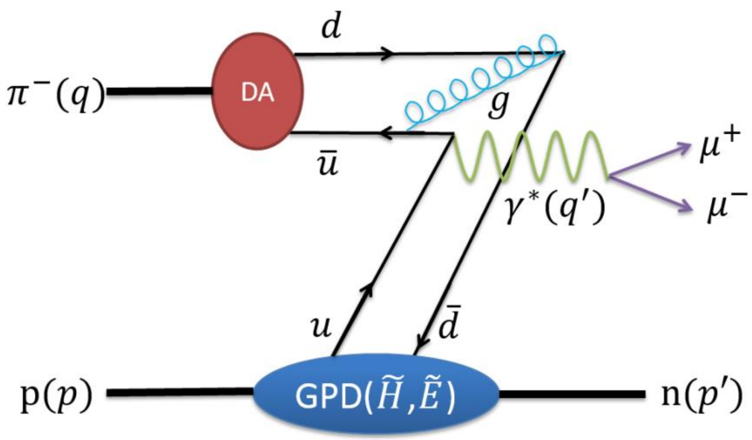
exDY@J-PARC



exDY@J-PARC

Pion-induced Drell-Yan process

$$\pi N \to \mu^+ \mu^- N$$

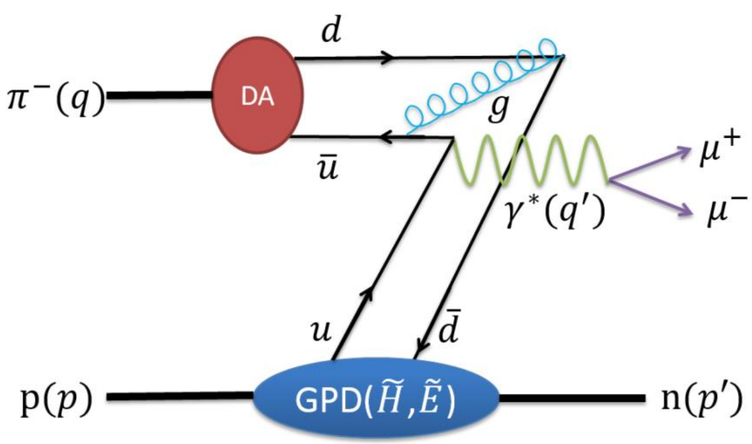


exclusive

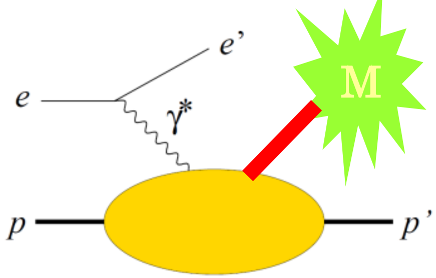
Pion-induced Drell-Yan process

QCD factorization formula

$$\pi N \to \mu^+ \mu^- N$$



exclusive



$$\bar{P} = \frac{p+p'}{2}$$

JLab, HERMES, COMPASS,…

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2}) | p\rangle = \frac{1}{\bar{P}^+}\left[H(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

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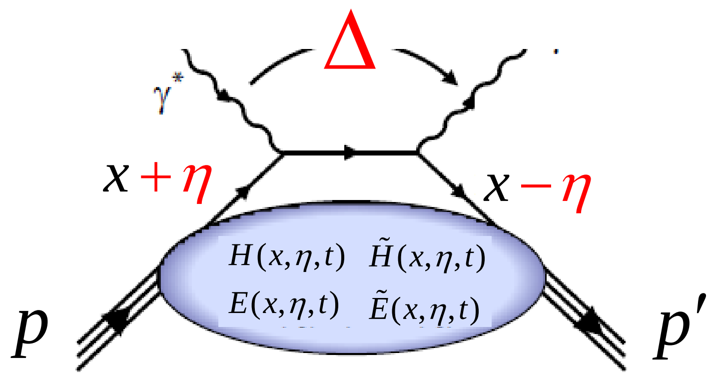
GPD



$$-2\eta\bar{P} = \Delta$$

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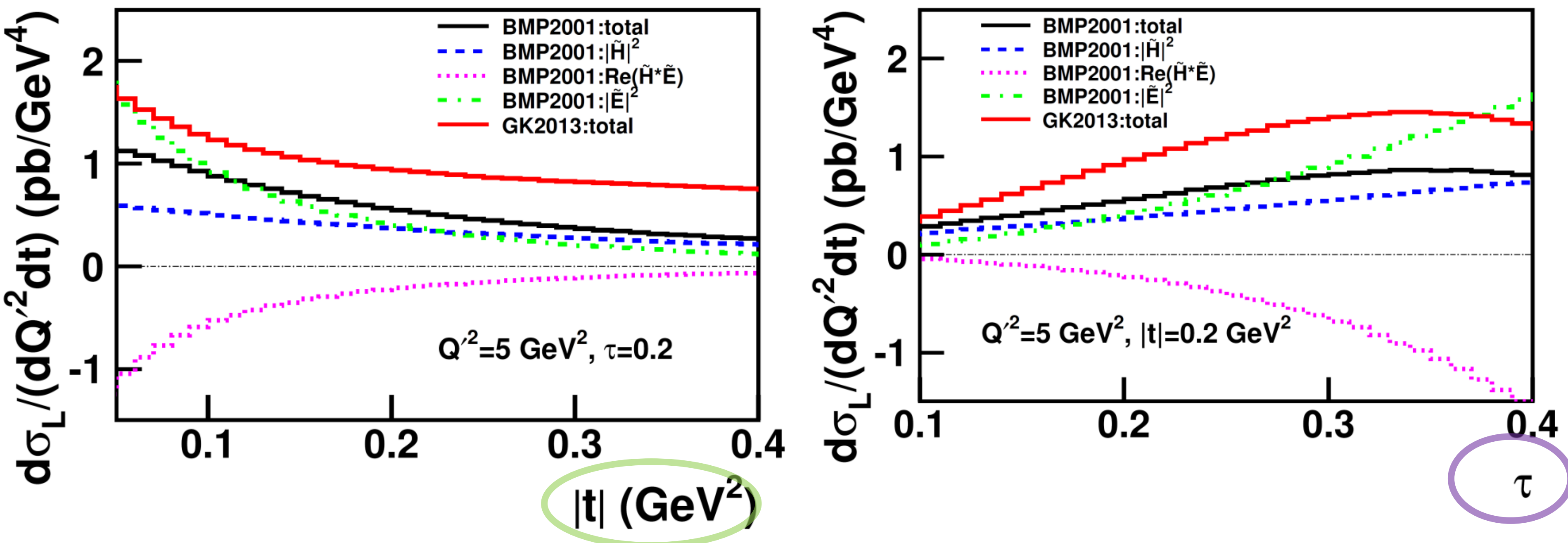
PRD93, 114034

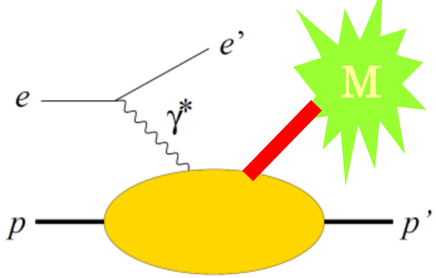
Bjorken variable $\tau = \frac{Q'^2}{s - M^2}$

$$Q'^2 = 5\ \mathrm{GeV}^2$$



$$\frac{d\sigma}{dQ'^2\,dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{\rm em}^2}{27}\frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2)|\mathscr{H}^{du}|^2 - 2\eta^2 \mathrm{Re}\left(\mathscr{H}^{du*}\mathscr{E}^{du}\right) - \eta^2 \frac{t}{4M^2}|\mathscr{E}^{du}|^2 \right]$$

$$\mathscr{H}^{du} = \frac{8\alpha_s}{3}\int_0^1 du \frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx \left(\frac{e_d}{-\eta - x - i\epsilon} - \frac{e_u}{-\eta + x - i\epsilon} \right)\left(\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t) \right)$$



$$\bar{P} = \frac{p+p'}{2}$$

JLab, HERMES, COMPASS,...

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

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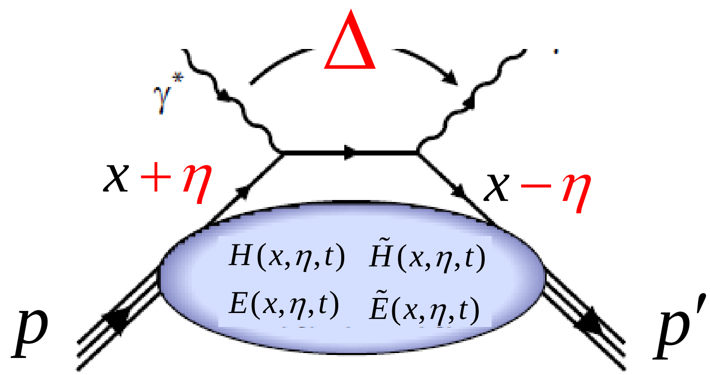
GPD



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$$\left(t = \Delta^2 \to 0 , \quad \eta = \frac{-\Delta \cdot n}{2\bar{P} \cdot n} \to 0 \right)$$

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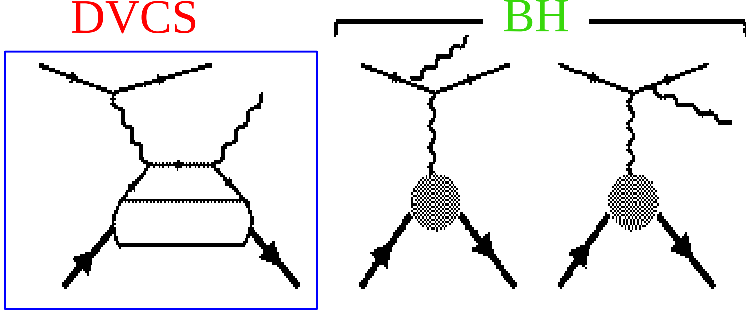

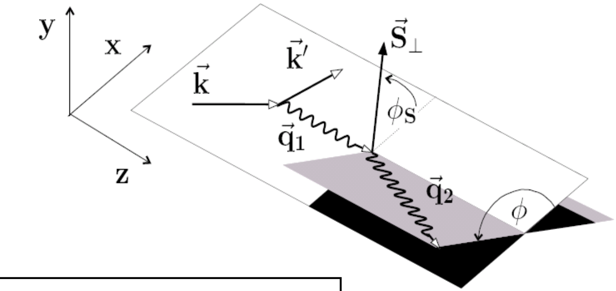

$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} \sim |T^{DVCS} + T^{BH}|^2$$

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Polarized beam, unpolarized target:

$$\Delta\sigma_{LU} \sim \sin\phi \, \{F_1\mathcal{H} + \eta(F_1+F_2)\tilde{\mathcal{H}} + kF_2\mathcal{E}\}d\phi$$

$\Rightarrow$ $H(x,\eta,t)$

Unpolarized beam, longitudinal target:

$$\Delta\sigma_{UL} \sim \sin\phi \, \{F_1\tilde{\mathcal{H}} + \eta(F_1+F_2)(\mathcal{H} + \eta/(1+\eta)\mathcal{E})\}d\phi$$

$\Rightarrow$ $\tilde{H}(x,\eta,t)$

Unpolarized beam, transverse target:

$$\Delta\sigma_{UT} \sim \cos\phi \, \sin(\phi_s-\phi)\{k(F_2\mathcal{H} - F_1\mathcal{E})\}d\phi$$

$\Rightarrow$ $E(x,\eta,t)$

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Bjorken variable $\tau = \frac{Q'^2}{s - M^2}$

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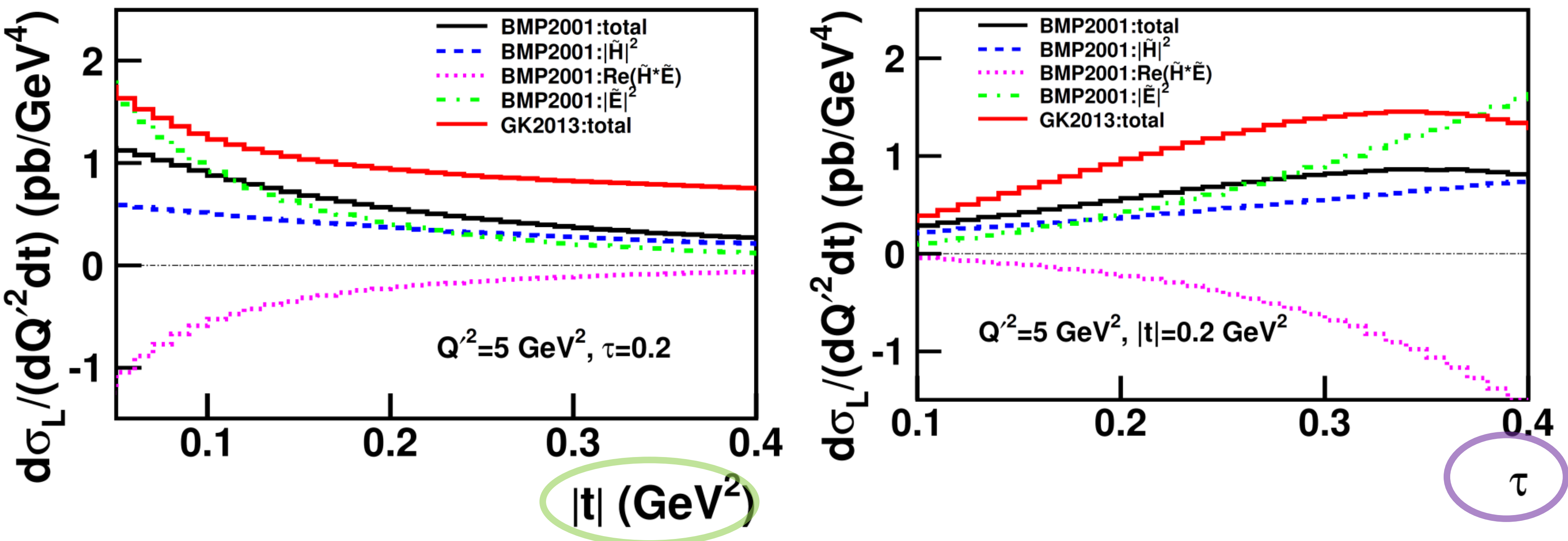

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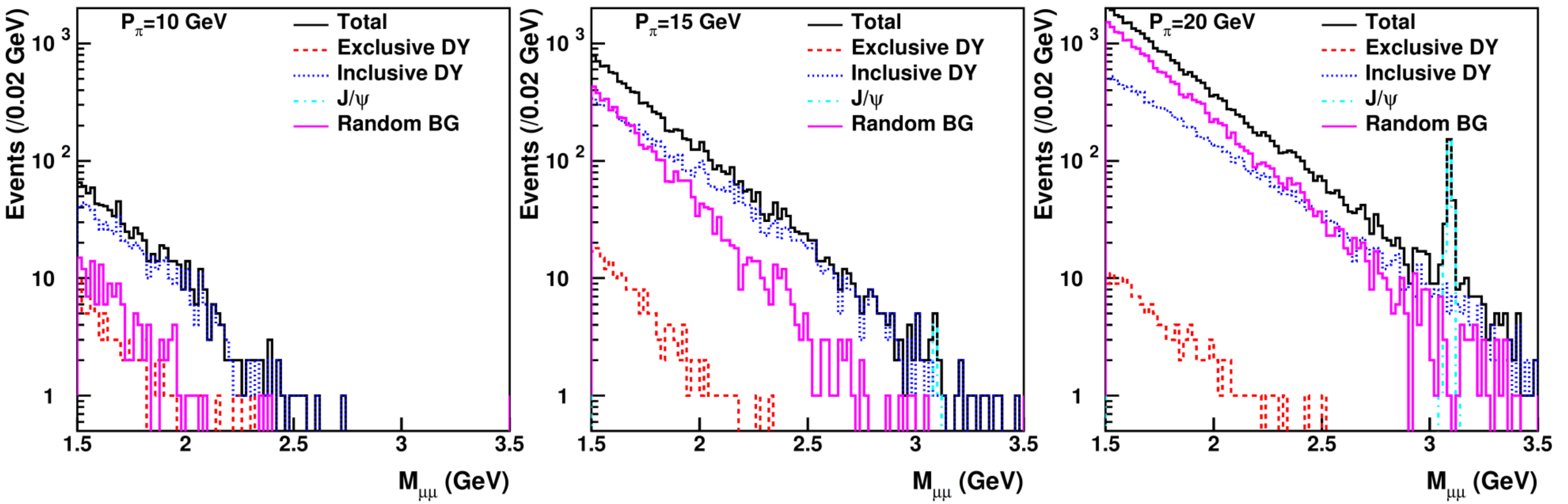
$$\mathscr{H}^{du} = \frac{8\alpha_s}{3}\int_0^1 du \frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx \left(\frac{e_d}{-\eta - x - i\epsilon} - \frac{e_u}{-\eta + x - i\epsilon} \right)\left(\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t) \right)$$



feasibility with E50 spectrometer at J-PARC

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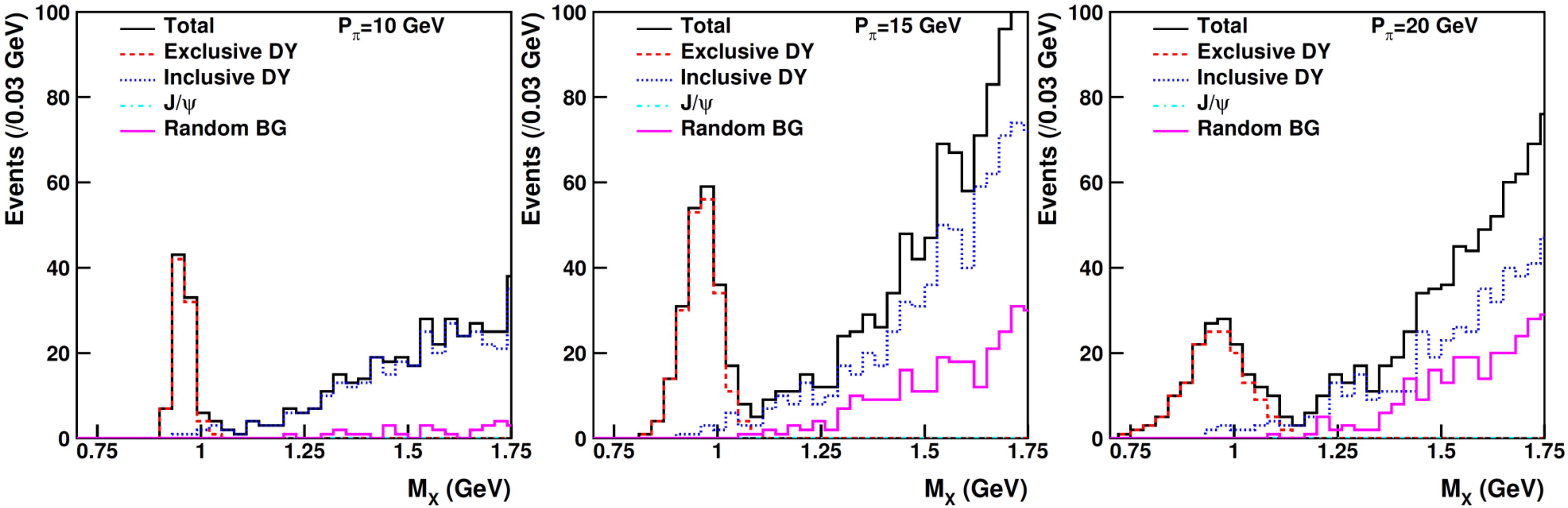

feasibility with E50 spectrometer at J-PARC

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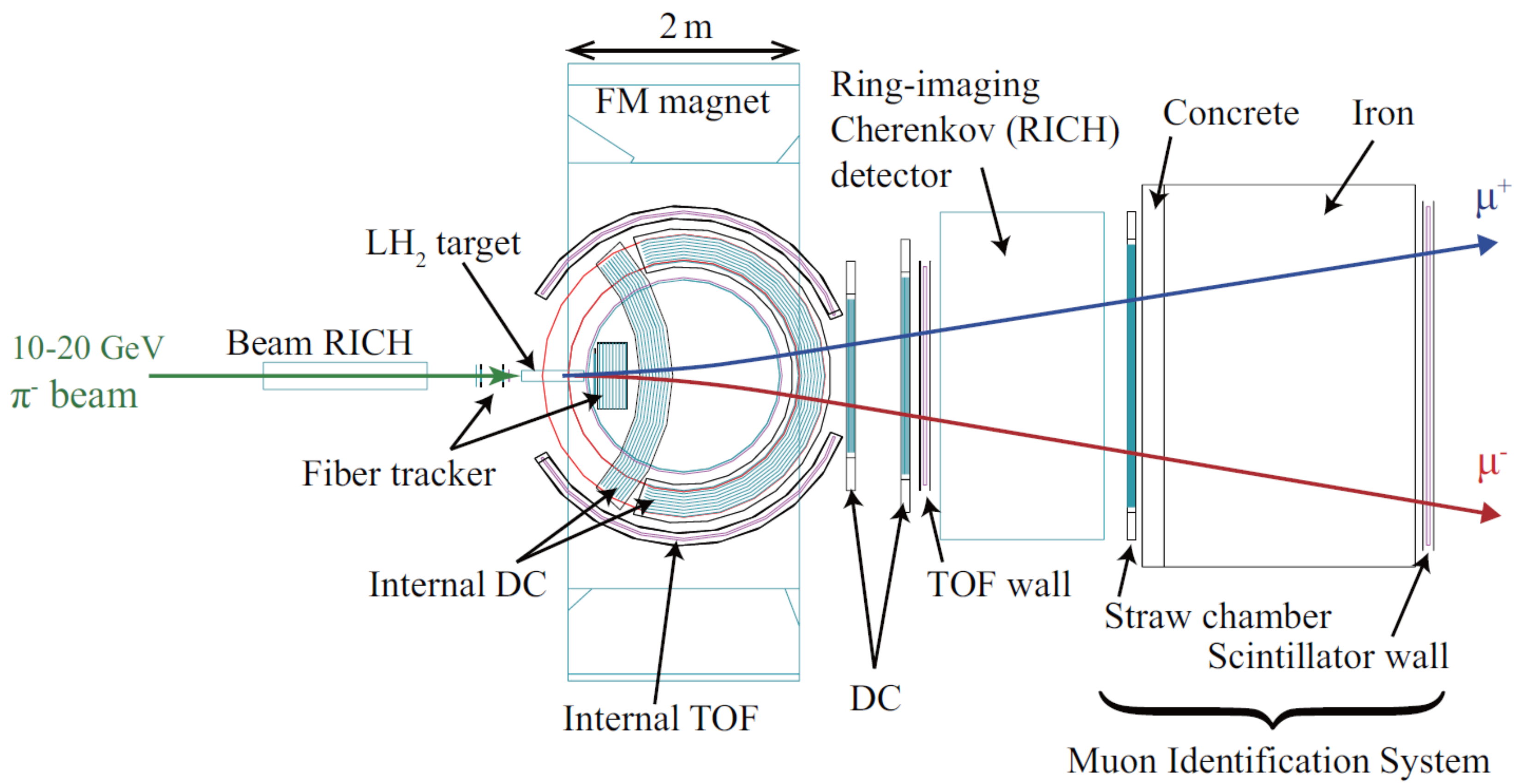

FIG. 9: Conceptual design of J-PARC E50 spectrometer with muon identification system.

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PRD93, 114034

Letter of Intent on an experiment for generalized parton distributions and pion distribution amplitudes with exclusive pion-induced Drell-Yan process at J-PARC

Takahiro Sawada,[a] **Wen-Chen Chang,**[b] **Shunzo Kumano,**[c,d] **Jen-Chieh Peng,**[e] **Shinya Sawada,**[f] **and Kazuhiro Tanaka**[g,d]

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[b] *Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*

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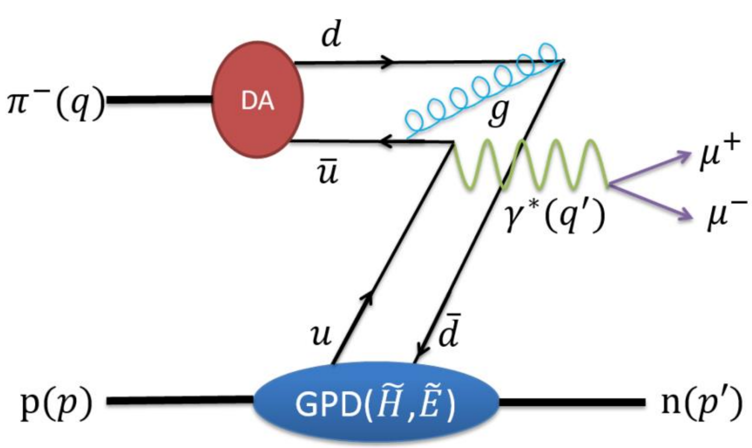
Pion-induced Drell-Yan process

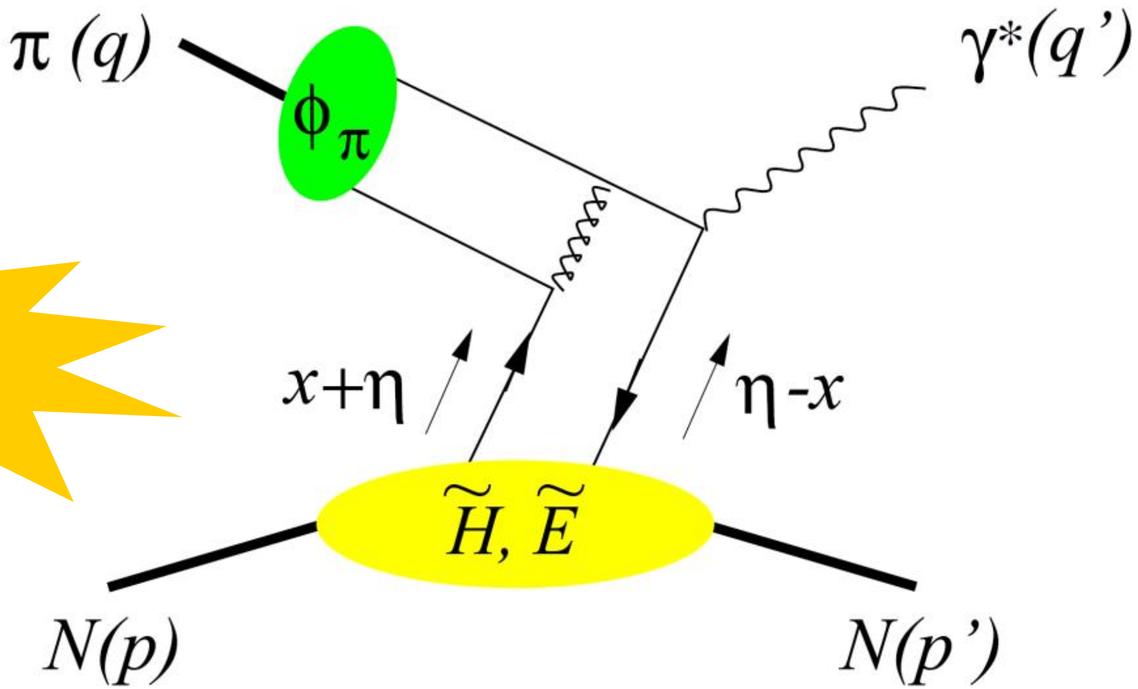
QCD factorization formula

$$\pi N \rightarrow \mu^+ \mu^- N$$

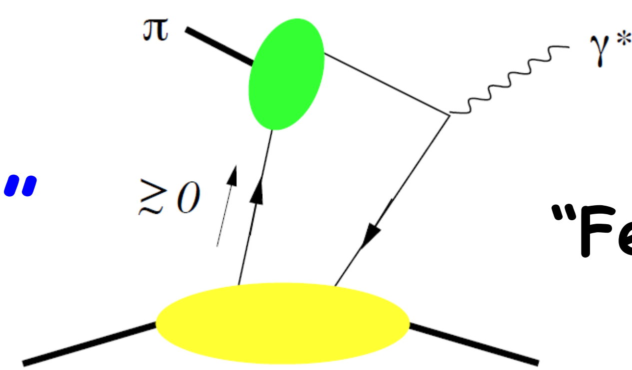


exclusive

LO in QCD factorization



"nonfactorizable"

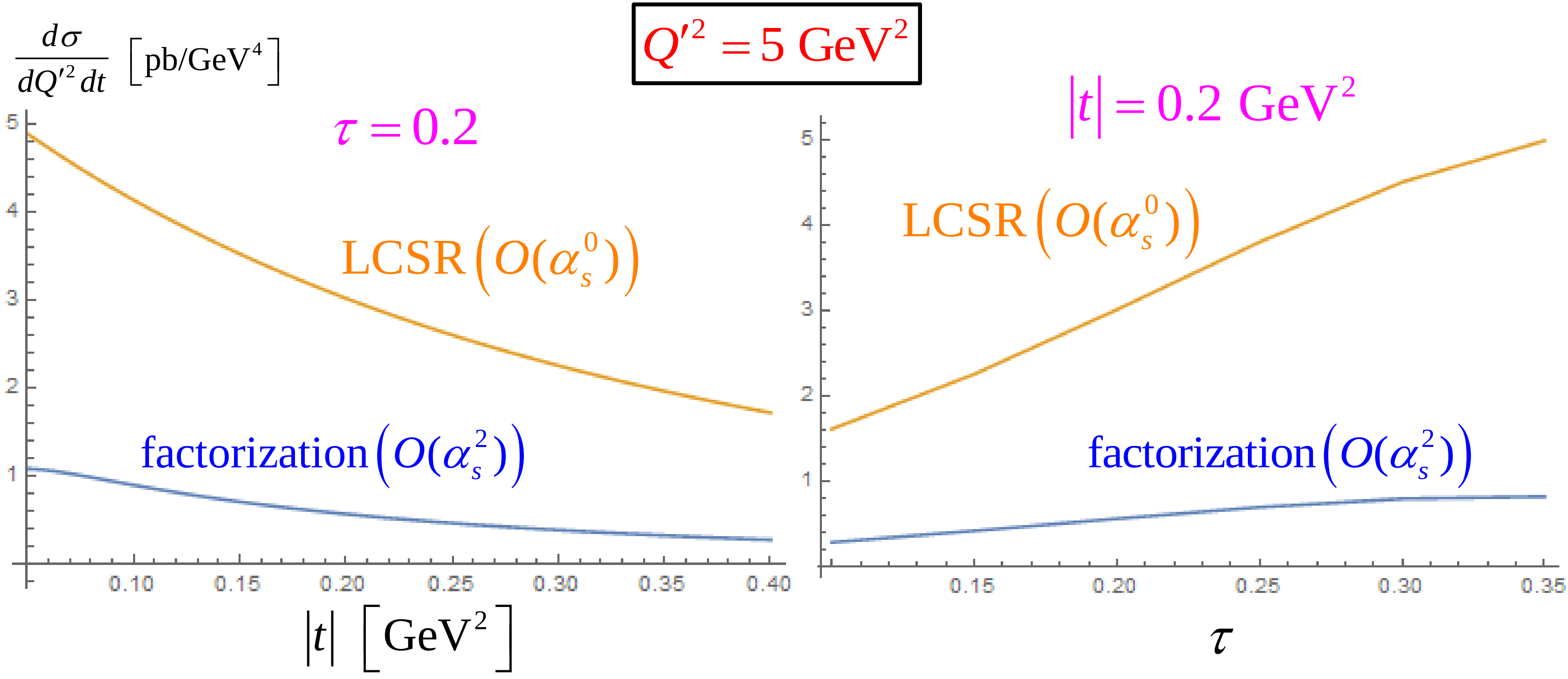


"Feynman mechanism"

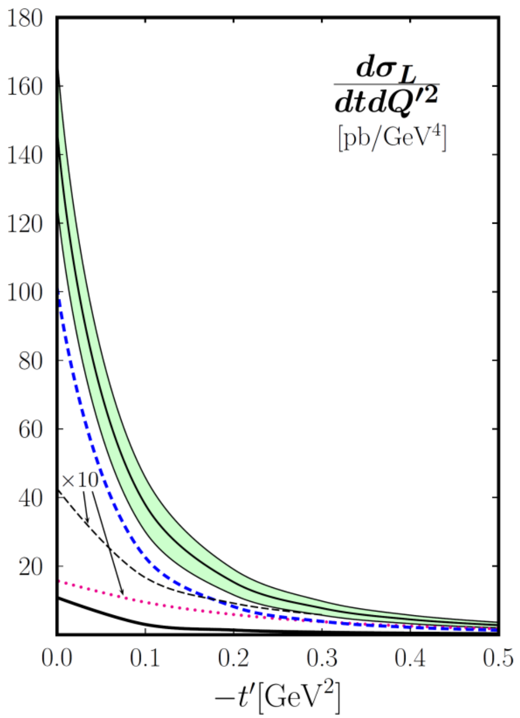
KT, arXiv:1703.02190, 1709.01063

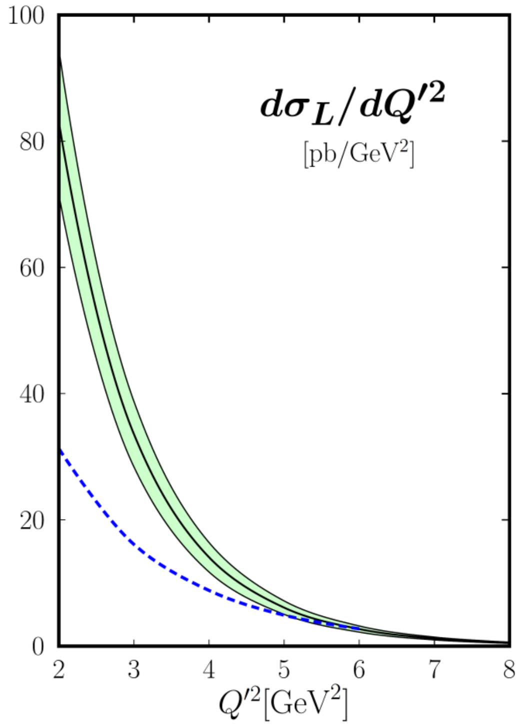
$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \to \gamma^* n)$$

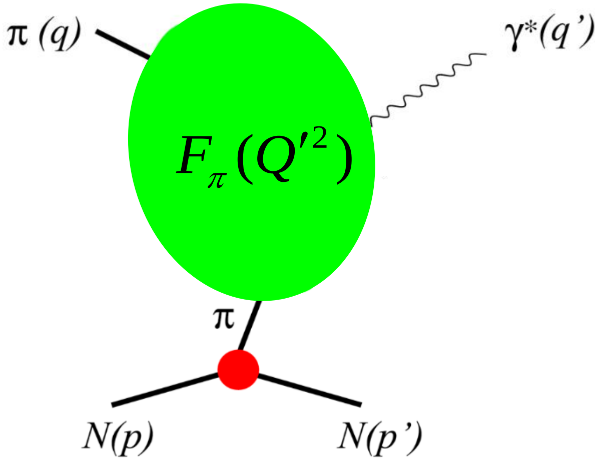
$Q'^2 = 5\ \mathrm{GeV}^2$

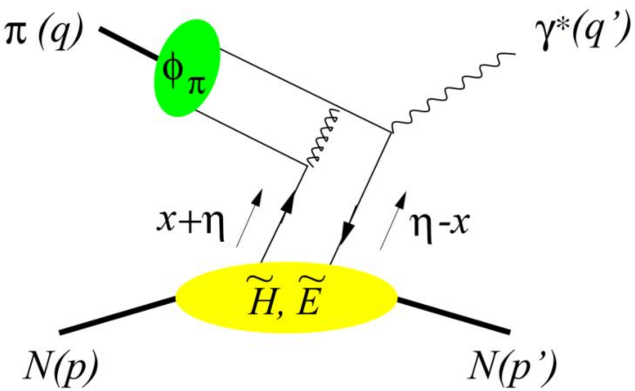


Goloskokov, Kroll ('15)

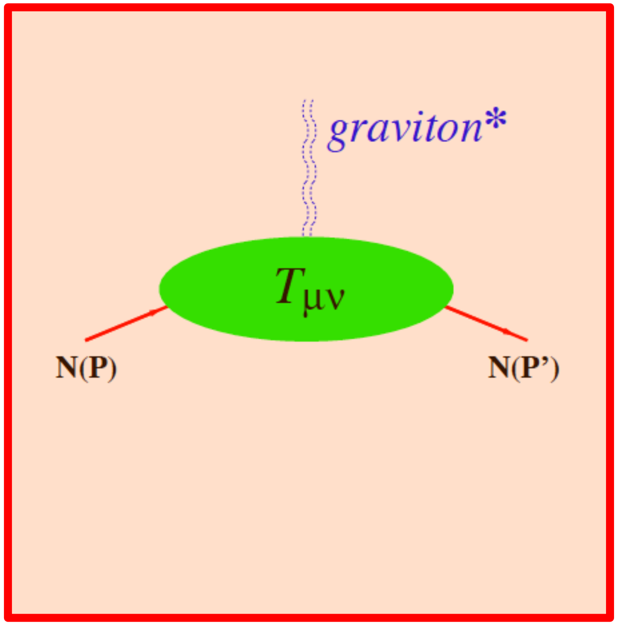








Summary



mass & energy distribution

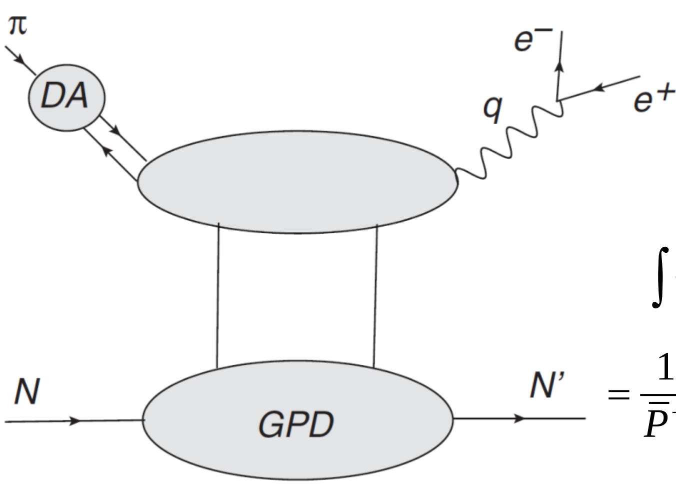
$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t)$$

$$\int_{-1}^{1} dx\, x E^q(x,\eta,t) = B_q(t) - 4\eta^2 D_q(t)$$

angular momentum distribution

force & pressure distribution

Gravitational form factors can be accessed through GPDs



exDY@J-PARC

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p'|\bar{q}(-\frac{z^-}{2})\gamma^+\gamma_5 q(\frac{z^-}{2})|p\rangle$$

$$= \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$