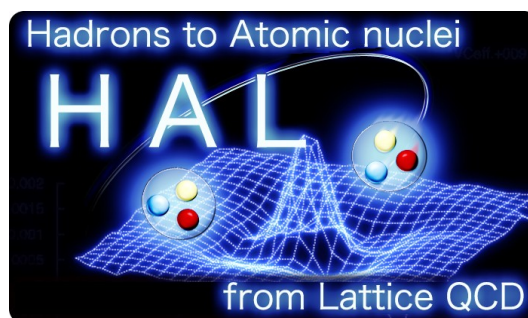


Lattice QCD study on YN / YY interactions

Kenji Sasaki (*YITP, Kyoto University*)

for HAL QCD Collaboration



HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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Contents

- Introduction
- Baryon interaction from LQCD
 - HAL QCD method
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- Summary

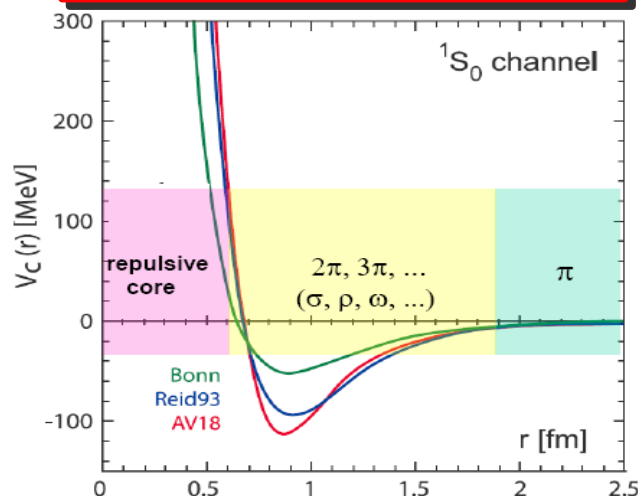
Introduction

Introduction

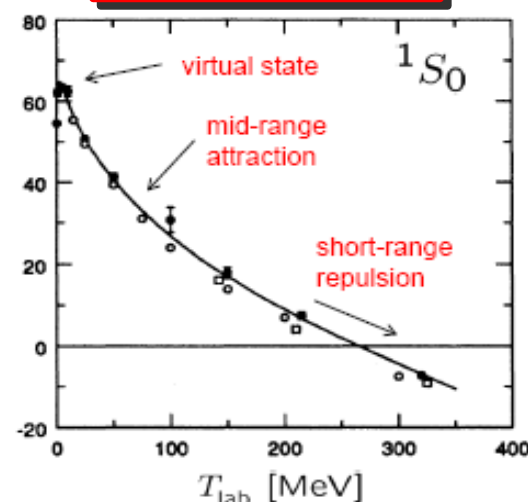
Baryon-baryon interactions are key to understand nuclear structures and astrophysical phenomena

Traditional way to research the BB interaction / potential

BB interaction (potential)



BB phase shift



NN interaction

Large amount of scattering data to determine theoretical parameters

YN / YY interaction

More strangeness, more difficult to access it experimentally. Experimental data are scarce.

Lattice QCD results for YN and YY interactions are highly awaited

Baryon interaction from QCD

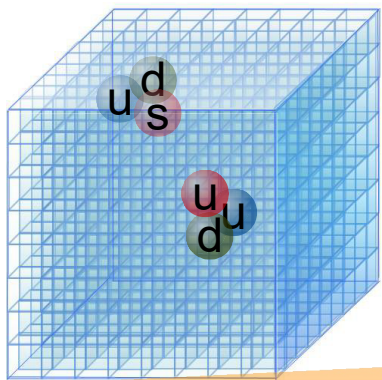
QCD is the fundamental theory of strong interactions

QCD Lagrangian

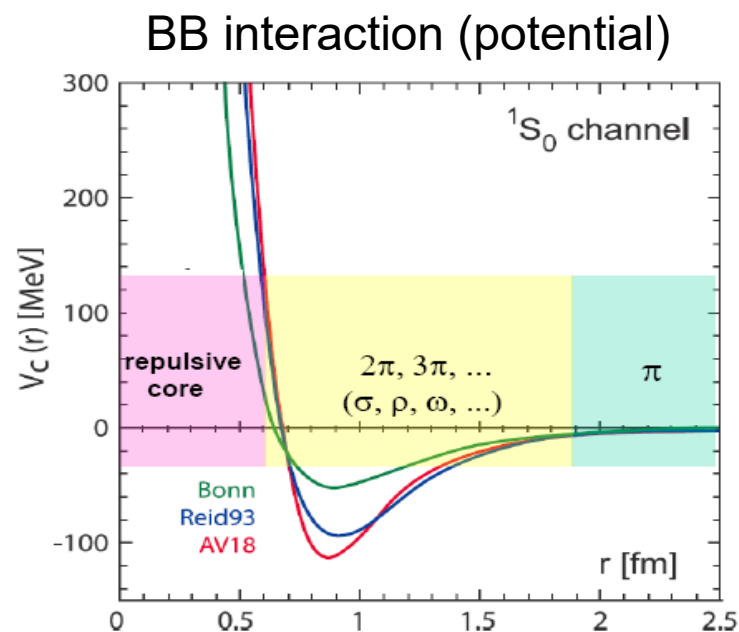
$$L_{QCD} = \bar{q}(i \gamma_\mu D^\mu - m)q + \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

it is difficult to solve the dynamics of quarks and gluons because of its non-perturbative nature at low-energies.

Lattice QCD simulation



- Non-perturbative calculation
- High computer resource is necessary
- Independent of experimental situation



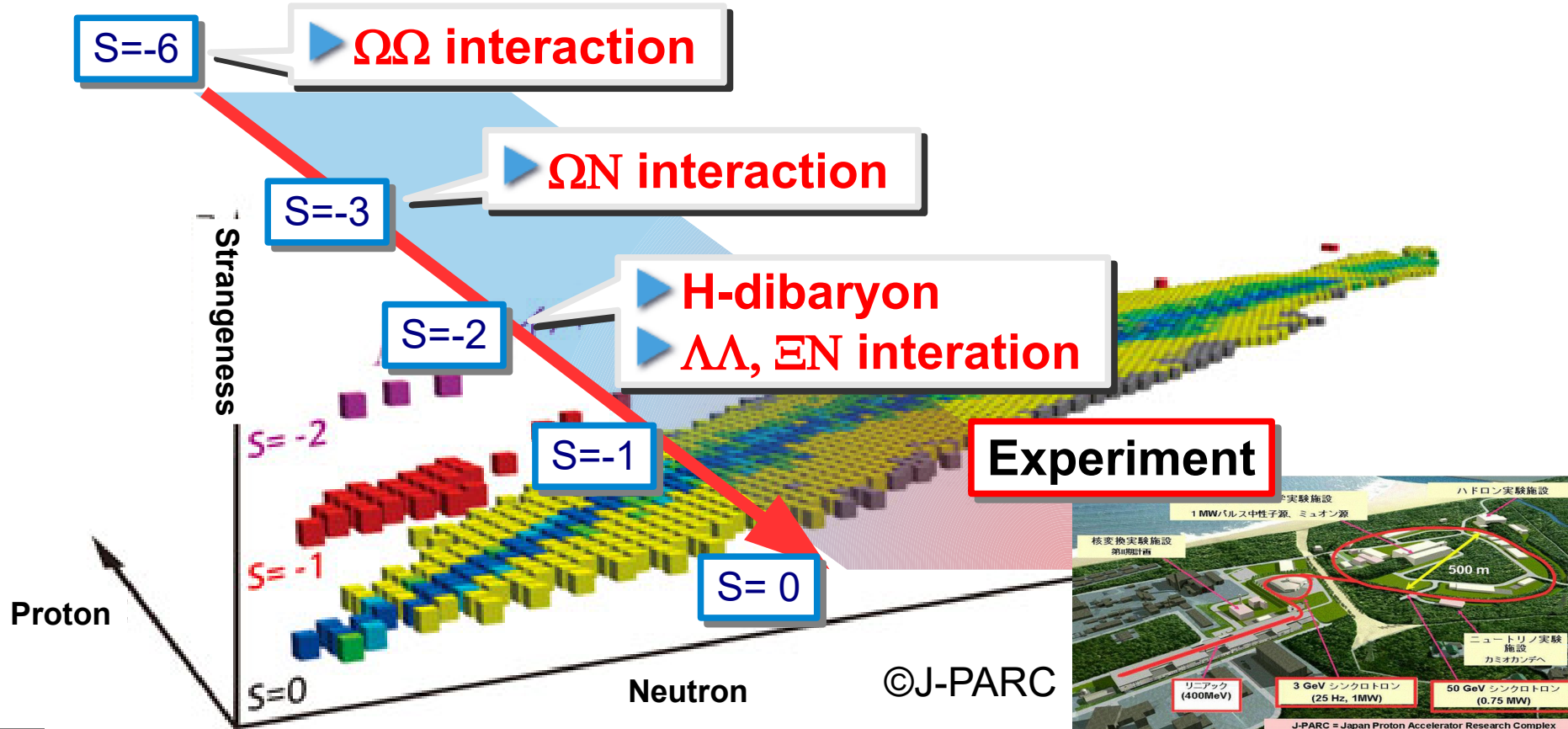
Introduction

Aim : Nuclear structures and exotic states from QCD

Lattice QCD simulation



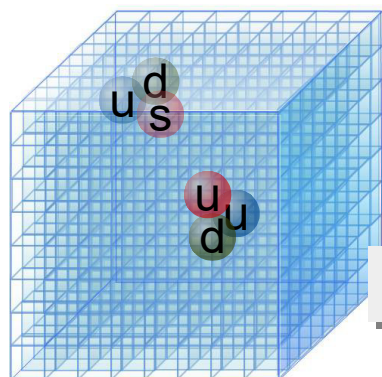
- Advantageous for **more strange quarks**
- Noises are getting larger as increasing the number of light quarks.
- **Complementary role to experiment.**



Baryon interaction from LQCD

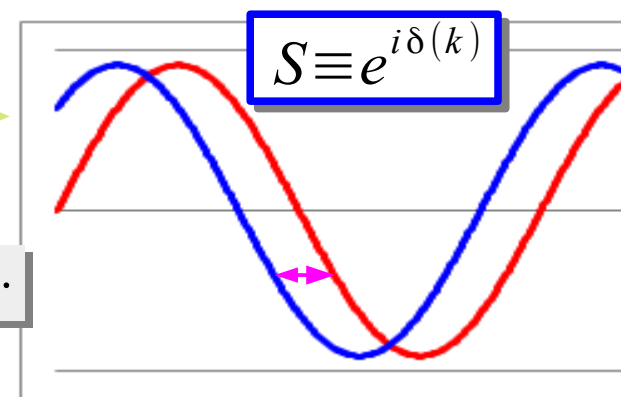
Hadron interaction from Lattice QCD

Lattice QCD simulation



$$\langle 0 | B_1 B_2(t, \vec{r}) \bar{I}(t_0) | 0 \rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \dots$$

Scattering S-matrix



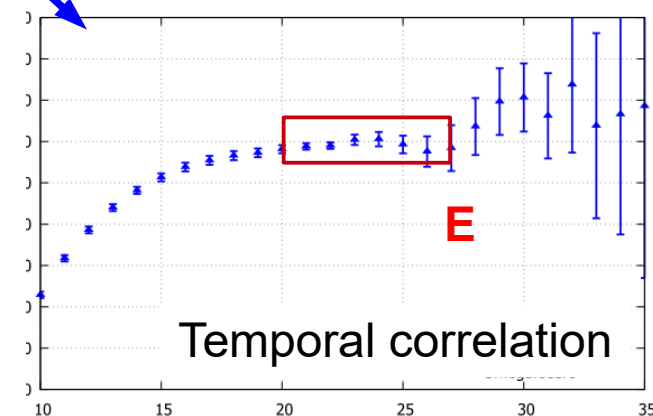
Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

1. Measure the discrete energy spectrum, **E**
2. Put the **k** from E into the formula which connects **k** and **δ**

$$k_n \cot \delta(k_n) = \frac{4\pi}{L^3} \sum_{m \in \mathbb{Z}^3} \frac{1}{\vec{p}_m^2 - k_n^2}$$

Scattering phase shift



Eigen energy is extracted from plateau region

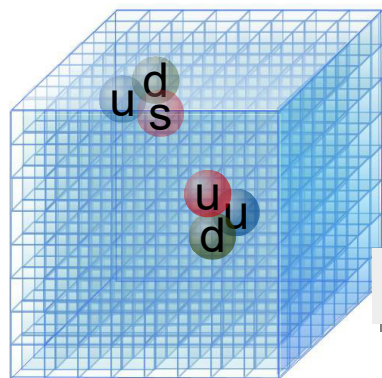
Issue of mirage plateau

T. Iritani et al. (HAL), JHEP1610(2016)101

HAL QCD method

Hadron interaction from LQCD

Lattice QCD simulation

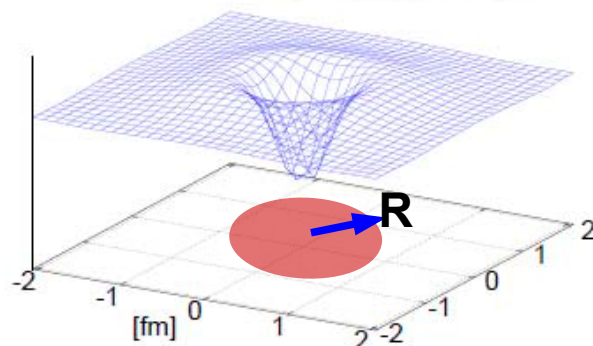


HAL QCD method

Ishii, Aoki, Hatsuda, PRL⁹⁹ (2007) 022001

$$\langle 0 | B_1 B_2(t, \vec{r}) \bar{I}(t_0) | 0 \rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \dots$$

NBS wave function

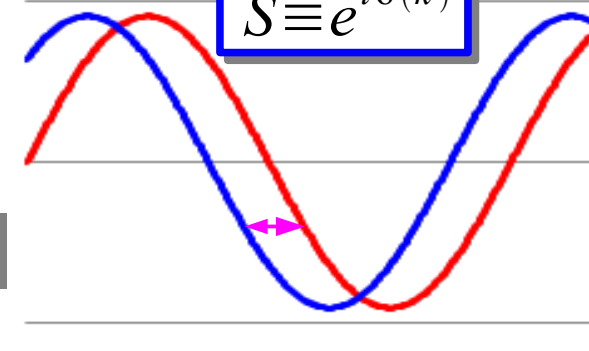


$$(p^2 + \nabla^2) \Psi(\vec{r}, E) = 0, (r > R)$$

$$\Psi(\vec{r}, E) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$

Scattering S-matrix

$$S \equiv e^{i\delta(k)}$$



$$\Psi(\vec{r}, E) e^{-Et} = \sum_{\vec{x}} \langle 0 | B_1(t, \vec{x} + \vec{r}) B_2(t, \vec{x}) | E \rangle$$

● Inside of “interacting region”

$$(p^2 + \nabla^2) \Psi^\alpha(E, \vec{x}) \equiv \int d^3y U_\alpha^\alpha(\vec{x}, \vec{y}) \Psi^\alpha(E, \vec{y})$$

- $U(\mathbf{x}, \mathbf{y})$ is faithful to the S-matrix.
- $U(\mathbf{x}, \mathbf{y})$ is not an observable.
- $U(\mathbf{x}, \mathbf{y})$ is energy independent but non-local.

Phase shift is embedded in NBS w.f.

Time-dependent Schrödinger like equation

Start with the normalized four-point correlator.

$$R_I^{B_1 B_2}(t, \vec{r}) = F^{B_1 B_2}_I(t, \vec{r}) e^{(m_1 + m_2)t}$$

$$= A_0 \Psi(\vec{r}, E_0) e^{-(E_0 - m_1 - m_2)t} + A_1 \Psi(\vec{r}, E_1) e^{-(E_1 - m_1 - m_2)t} + \dots$$

$$\left(\frac{p_0^2}{2\mu} + \frac{\nabla^2}{2\mu} \right) \Psi(\vec{r}, E_0) = \int U(\vec{r}, \vec{r}') \Psi(\vec{r}', E_0) d^3 r'$$

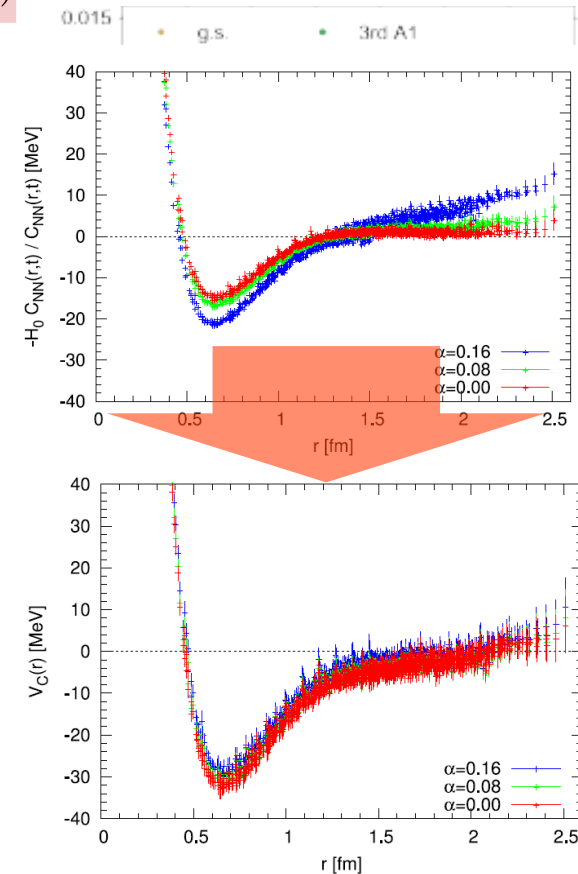
$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu} \quad \left(\frac{p_1^2}{2\mu} + \frac{\nabla^2}{2\mu} \right) \Psi(\vec{r}, E_1) = \int U(\vec{r}, \vec{r}') \Psi(\vec{r}', E_1) d^3 r'$$

A single state saturation is not required!!

$$\left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu} \right) R_I^{B_1 B_2}(t, \vec{r}) = \int U(\vec{r}, \vec{r}') R_I^{B_1 B_2}(t, \vec{r}') d^3 r'$$

Derivative (velocity) expansion of U

$$U(\vec{r}, \vec{r}') = [V_C(r) + S_{12} V_T(r)] + [\vec{L} \cdot \vec{S}_s V_{LS}(r) + \vec{L} \cdot \vec{S}_a V_{ALS}(r)] + O(\nabla^2)$$



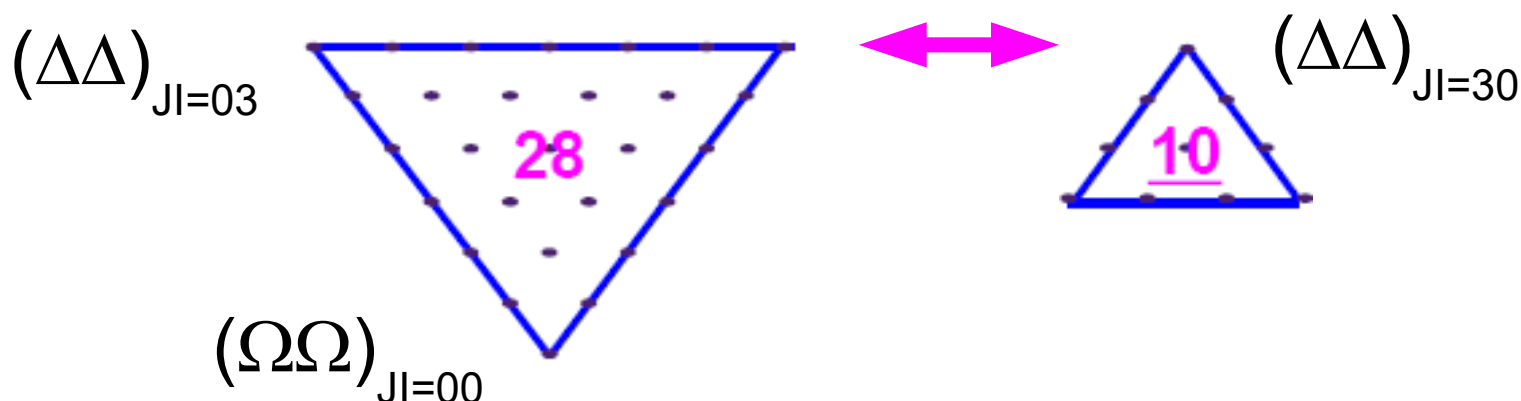
$\Omega\Omega$ interaction

Decuplet-Decuplet interaction

● Flavor symmetry aspect

Decuplet-Decuplet interaction can be classified as

$$10 \otimes 10 = 28 \oplus 27 \oplus 35 \oplus \bar{10}$$

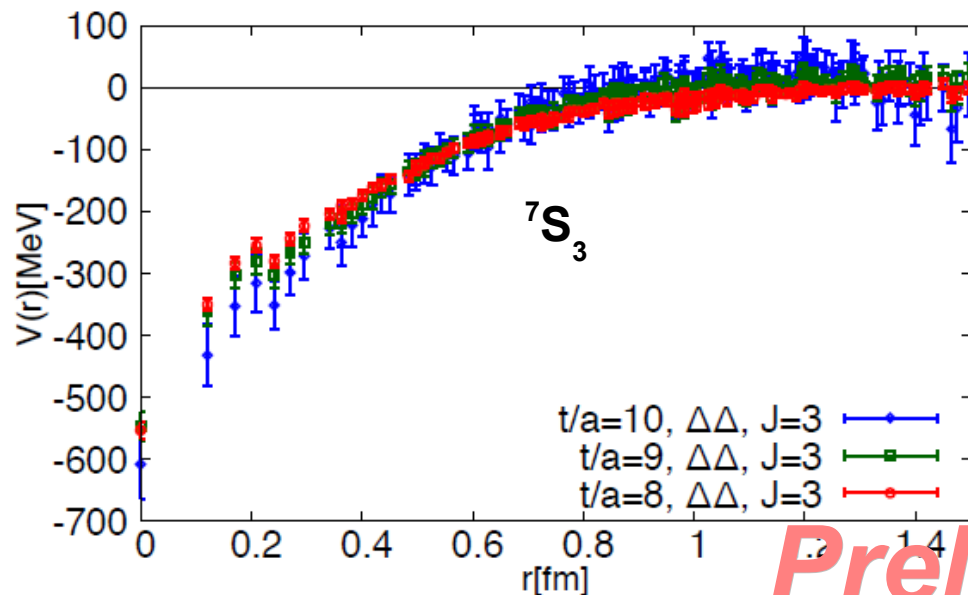


	28plet (0^+)	10*plet (3^+)
Pauli	allowed	allowed
CMI	repulsive	No

- $\Delta-\Delta(J=3)$: **Bound (resonance) state was found in experiment.**
- $\Delta-\Delta(J=0)$ [and $\Omega-\Omega(J=0)$] : **Mirror of $\Delta-\Delta(J=3)$ state**

Decuplet-Decuplet interaction in $SU(3)$ limit

► $N_f = 3$ full QCD with $L = 1.93\text{fm}$, $m_\pi = 1015\text{ MeV}$ $m_\Delta = 2225\text{ MeV}$



$\Delta-\Delta(J=3)$

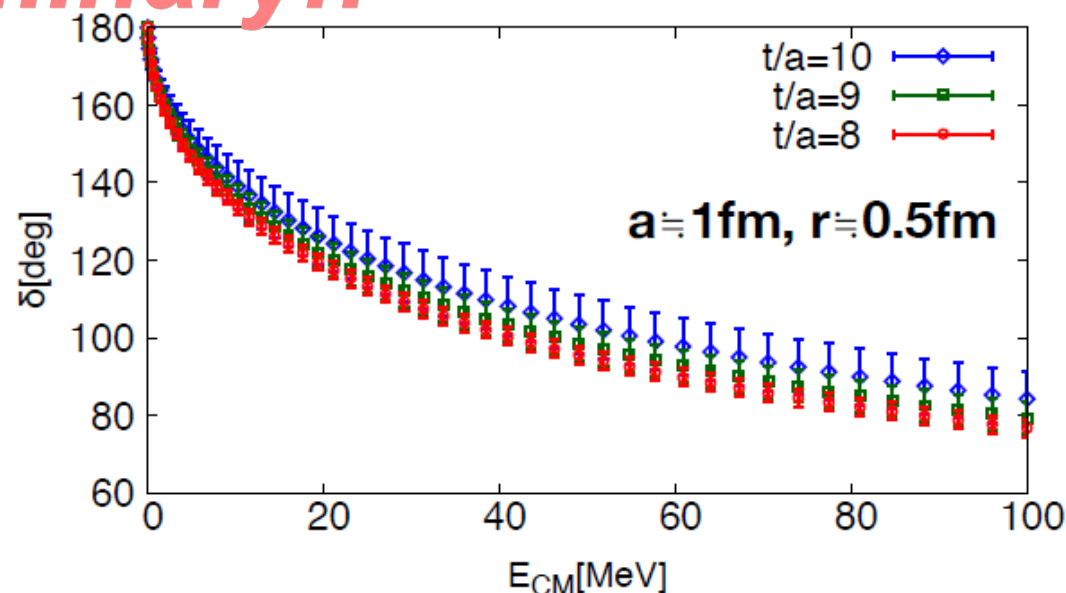
10^*plet

- Interaction in 10^*plet [$J^P(I)=3^+(0)$] is strongly attractive.
- There is no repulsive core at short distances

Preliminary!!

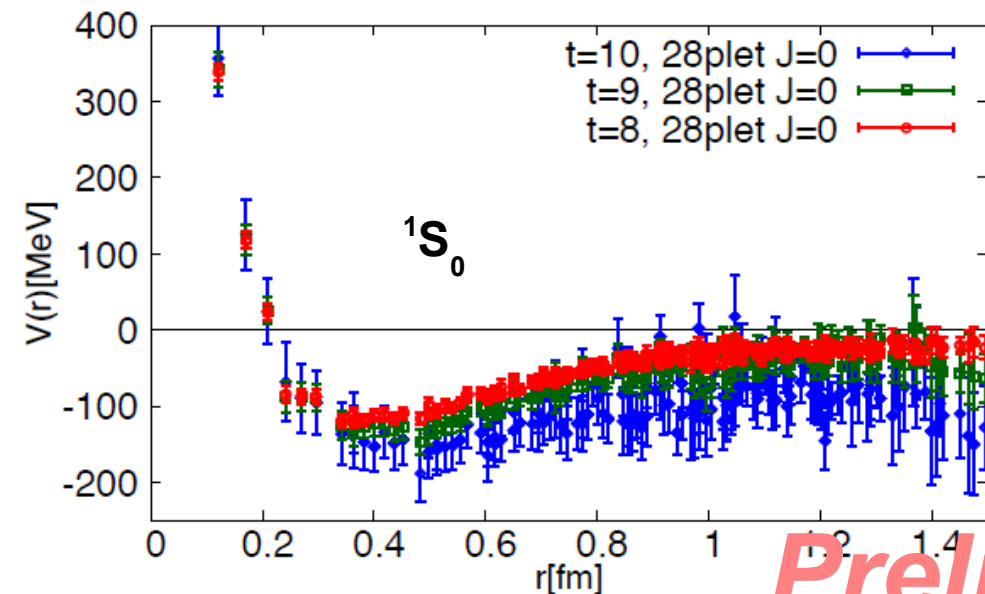
Bound $\Delta\Delta$ state is found.

- Decay to $NN(^3D_3)$ is neglected.
- Δ baryon can not decay into $N+\pi$ in this lattice setup



Decuplet-Decuplet interaction in $SU(3)$ limit

► $N_f = 3$ full QCD with $L = 1.93\text{fm}$, $m_\pi = 1015\text{ MeV}$ $m_\Delta = 2225\text{ MeV}$



$\Delta-\Delta(J=0)$

28plet

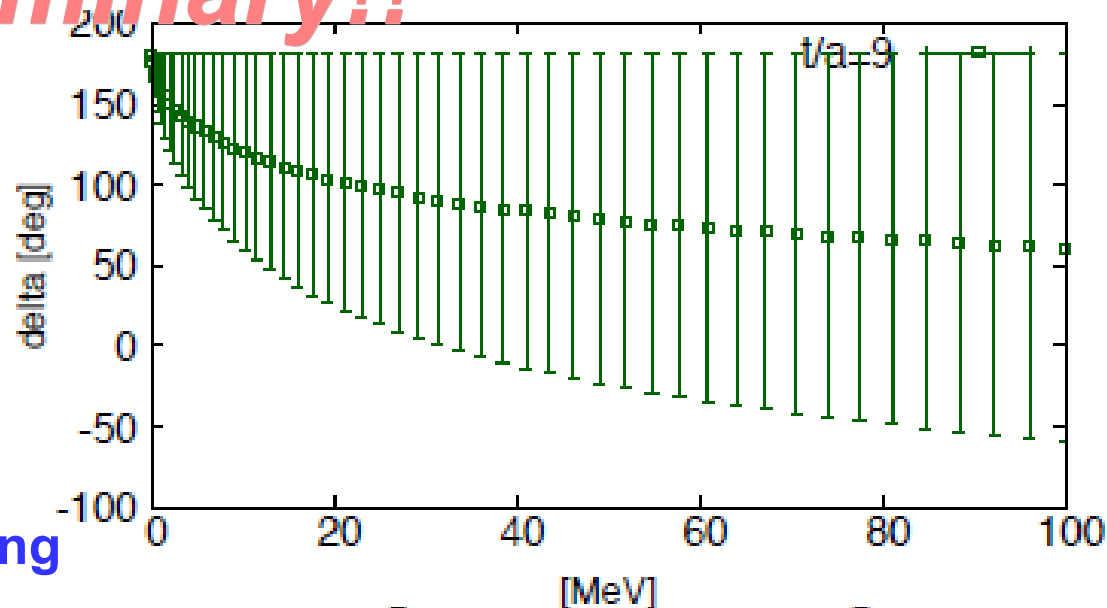
- Repulsive core is surrounded by attractive pocket in 28plet
- $[\Delta\Delta J^P(I)=0^+(3)]$ and $[\Omega\Omega J^P(I)=0^+(0)]$.

Preliminary!!

Phase shift shows that
the system is in the unitary limit.

- Δ baryon can not decay into $N+\pi$ in this lattice setup

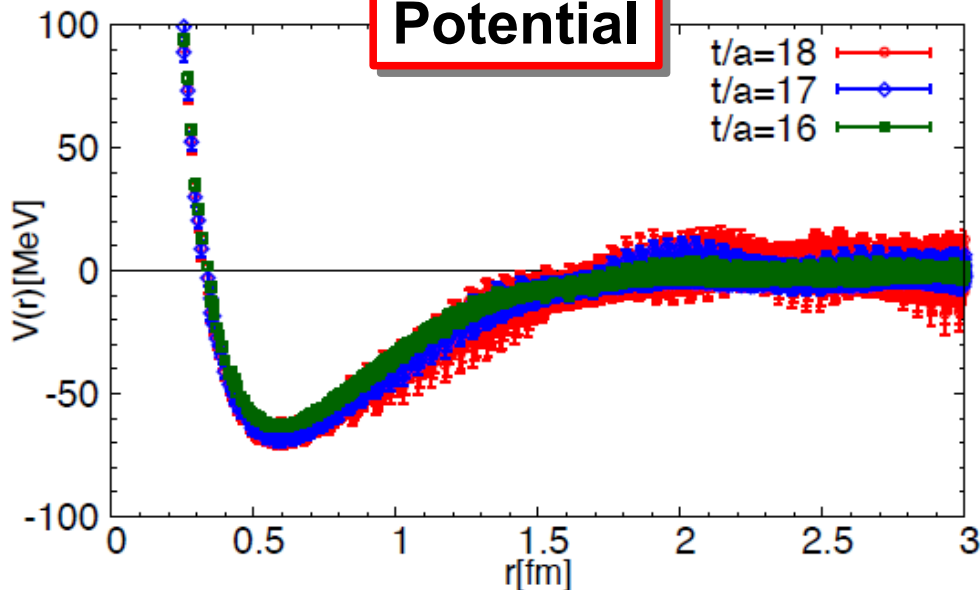
Go to the lighter quark mass region with consideration of $SU(3)$ breaking



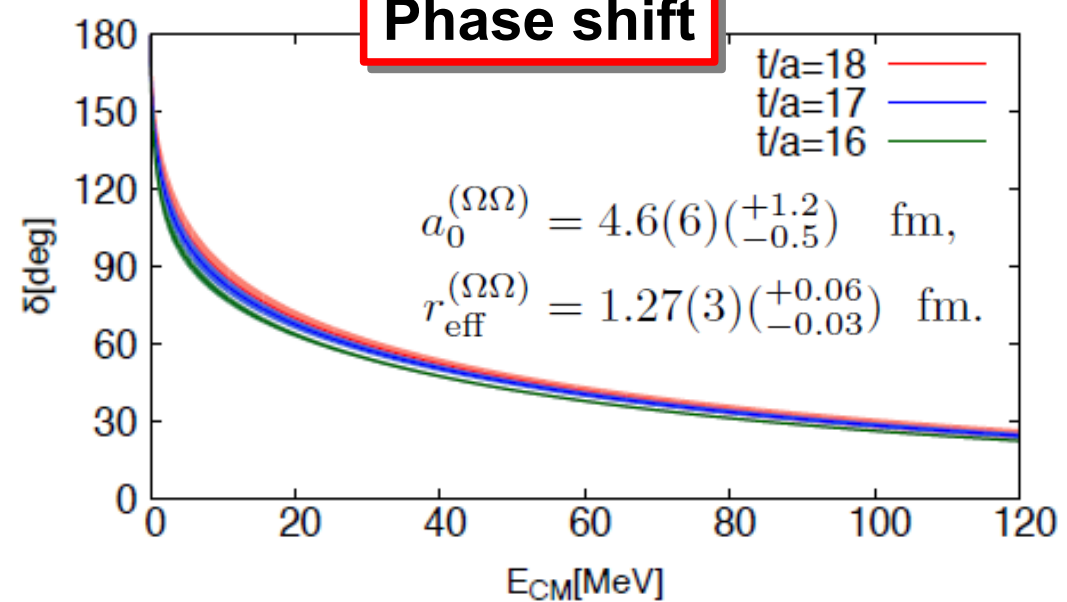
$\Omega\Omega J^P(I) = 0^+(0)$ state near the physical point

► $N_f = 2+1$ full QCD with $L = 8\text{fm}$, $m_\pi = 146\text{ MeV}$ $m_\Omega = 1712\text{ MeV}$

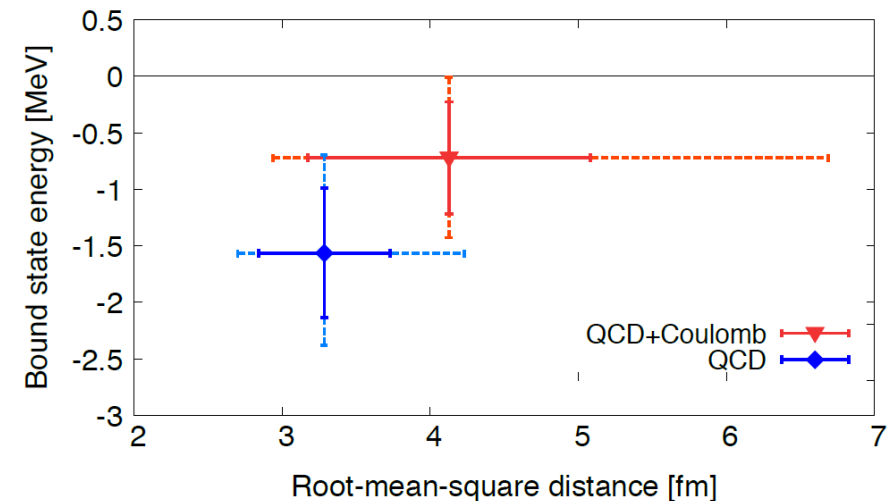
Potential



Phase shift



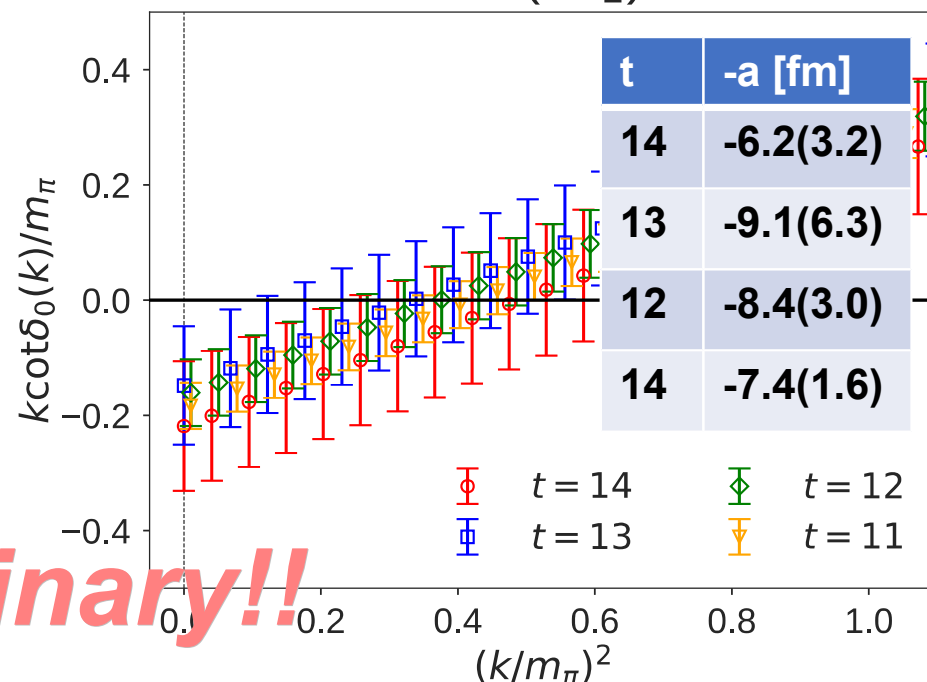
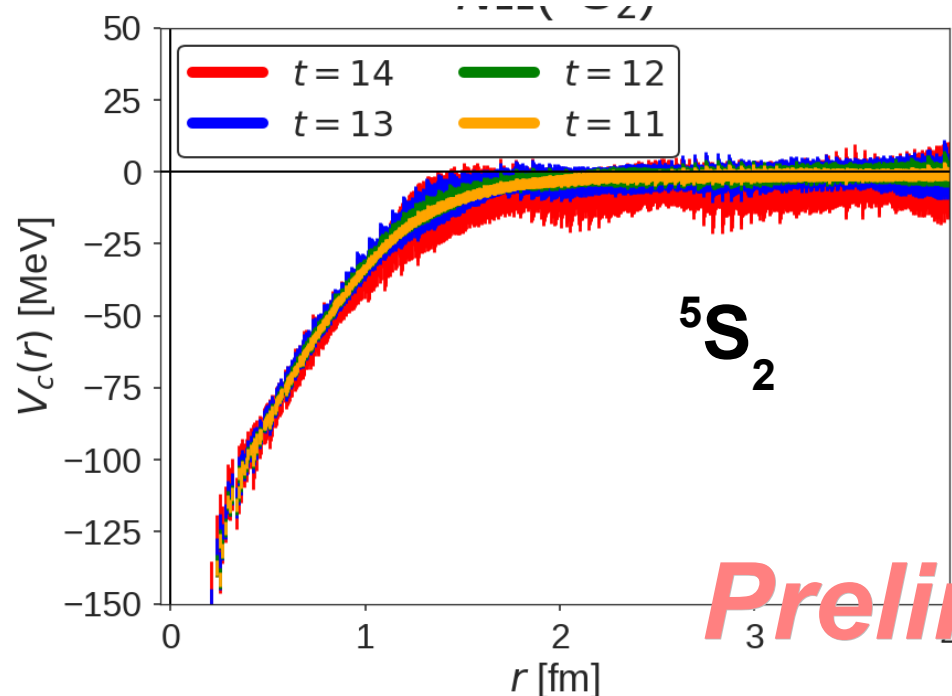
- Short range repulsion and attractive pocket is found.
- Calculated phase shift indicates a bound $\Omega\Omega$ state
[Most strange dibaryon].
- Physical $\Omega\Omega$ state in $J^P(I) = 0^+(0)$ is very close to unitary region.



$N\Omega$ interaction

$N\Omega$ system $J^P=2^+$ near the physical point

► $N_f = 2+1$ full QCD with $L = 8\text{fm}$, $m_\pi = 146\text{ MeV}$



$N\Omega$ state decay into $\Lambda\Xi$ (D-wave) state is suppressed.

- The system is bound (compared to the $N\Omega$ threshold) within the errors because of the strongly attractive potential.
→ **The doorway to the $S=-3$ nuclear system.**
- Measurement of strong $N\Omega$ attraction at RHIC and LHC is expected.

K.Morita et al PRC94(2016)031901

S=-2 baryon-baryon potentials

Interests of $S=-2$ multi-baryon system

H-dibaryon

- The flavor singlet state with $J=0$ predicted by R.L. Jaffe.
 - Strongly attractive color magnetic interaction.
 - No quark Pauli principle for flavor singlet state.

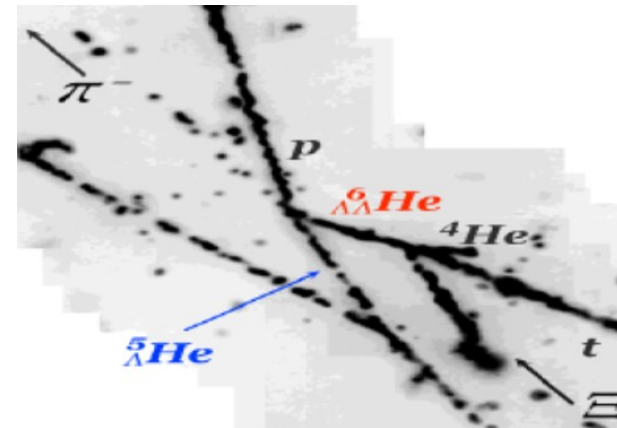
Double- Λ hypernucleus

- Conclusions of the “NAGARA Event”

K.Nakazawa and KEK-E176 & E373 Collaborators

Λ – Λ weak attraction

$$m_H \geq 2m_\Lambda - 6.9\text{MeV}$$

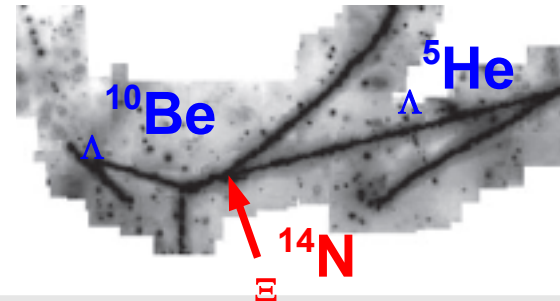


Ξ hypernucleus

- Conclusions of the “KISO Event”

K.Nakazawa and KEK-E373 Collaborators

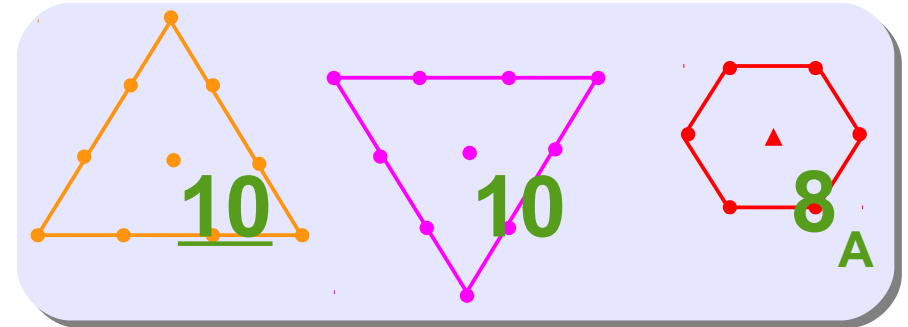
Ξ –N attraction



SU(3) feature of BB interaction

SU(3) classification

$$8 \times 8 = 27 + 8_s + 1 + 10 + 10 + 8_A$$



In view of quark degrees of freedom

- Short range repulsion in BB interaction could be a result of **Pauli principle** and **color-magnetic interaction** for the quarks.

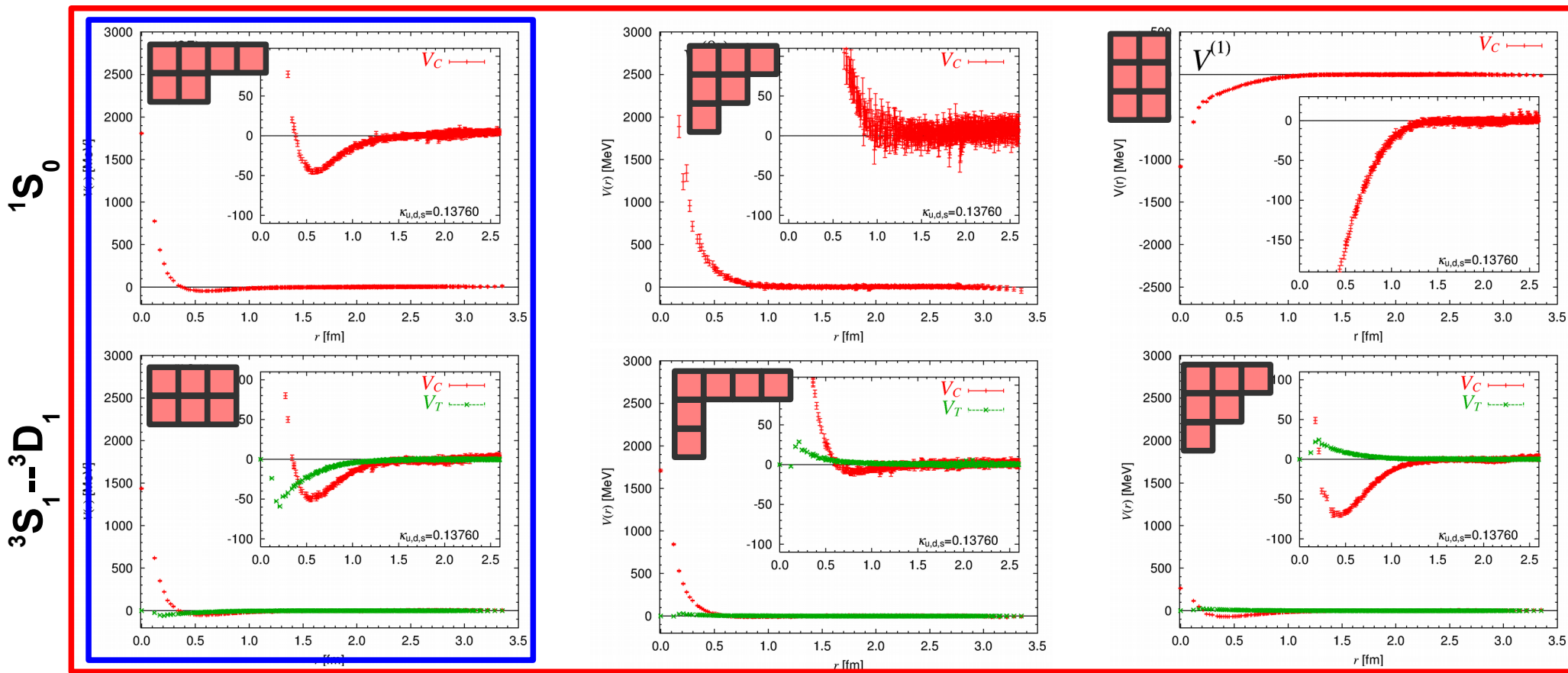
$$V_{OGE}^{CMI} \propto \frac{1}{m_{q1} m_{q2}} \langle \lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2 \rangle f(r_{ij})$$

- For the s-wave BB system, **no repulsive core** is predicted in **flavor singlet state** which is known as **H-dibaryon** channel.

	Flavor symmetric states			Flavor anti-symmetric states		
	27	8	1	10	10	8
Pauli	mixed	forbidden	allowed	mixed	forbidden	mixed
CMI	repulsive	repulsive	attractive	repulsive	repulsive	repulsive

B-B potentials in SU(3) limit

$m_\pi = 469 \text{ MeV}$



Two-flavors

Three-flavors

- Quark Pauli principle can be seen at around short distances
 - No repulsive core in flavor singlet state
 - Strongest repulsion in flavor 8s state
- Possibility of bound H-dibaryon in flavor singlet channel.**

H-dibaryon channel

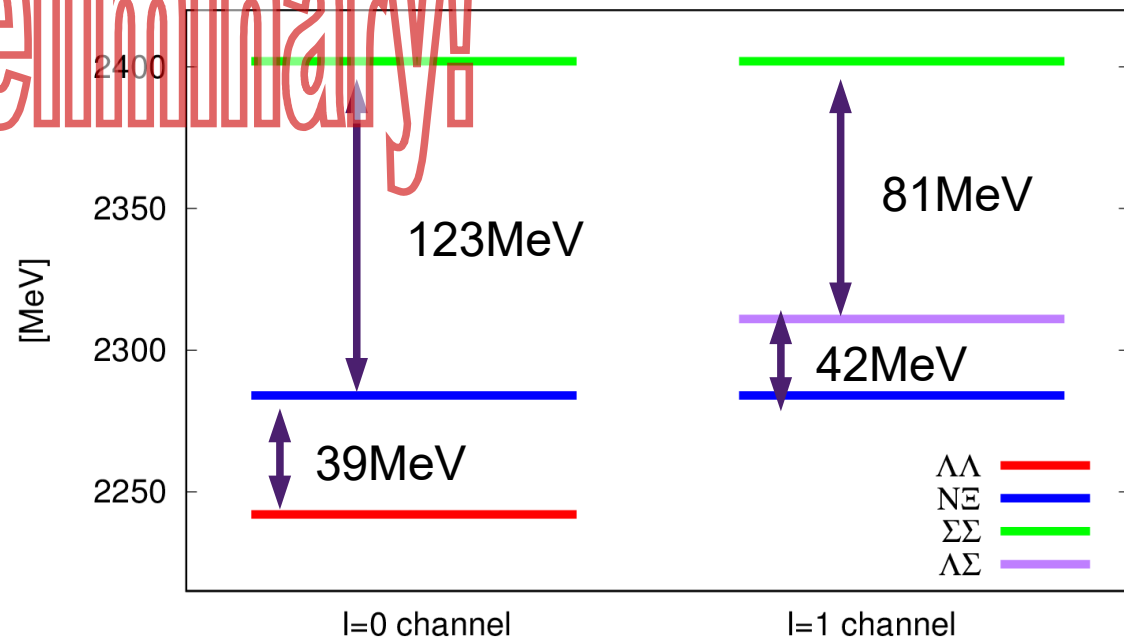
Numerical setup

- ▶ **2+1 flavor** gauge configurations.
 - Iwasaki gauge action & $O(a)$ improved Wilson quark action
 - $a = 0.085$ [fm], $a^{-1} = 2.300$ GeV.
 - $96^3 \times 96$ lattice, $L = 8.21$ [fm].
 - 414 confs x 84 sources x 4 rotations.
- ▶ **Wall source** is considered to produce S-wave B-B state.



	Mass [MeV]
π	146
K	525
m_π / m_K	0.28
N	953 ± 7
Λ	1123 ± 3
Σ	1204 ± 1
Ξ	1332 ± 2

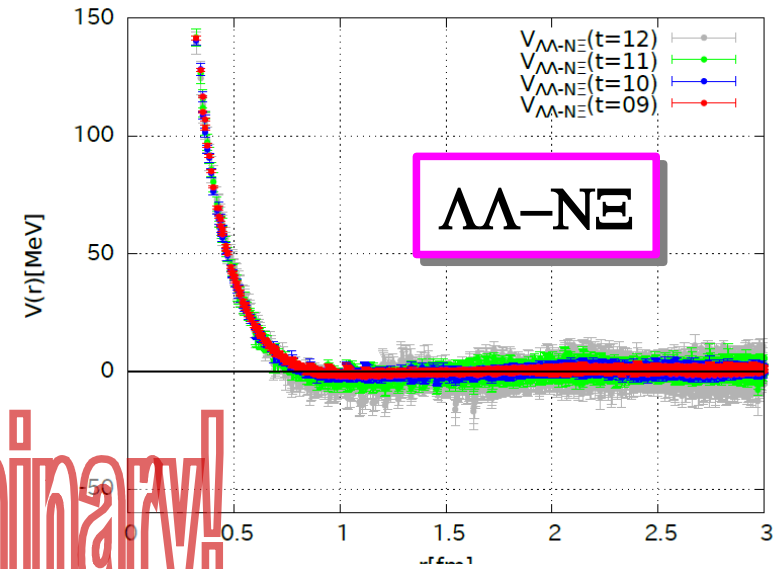
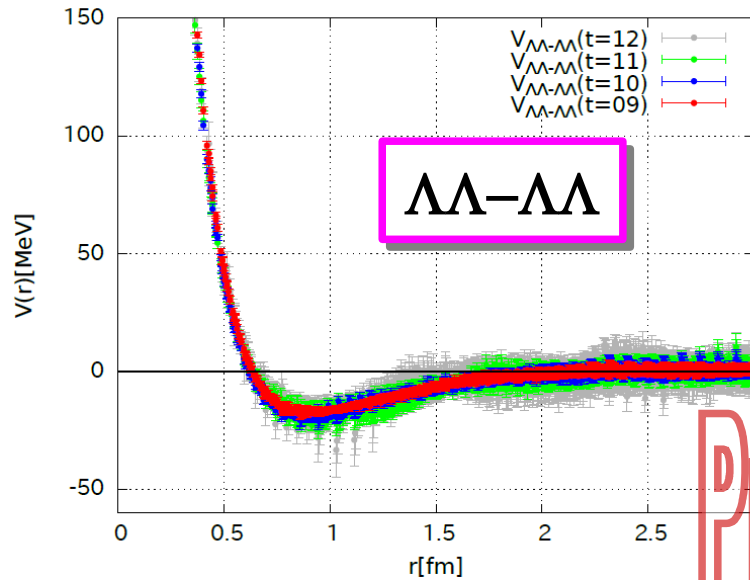
Preliminary!



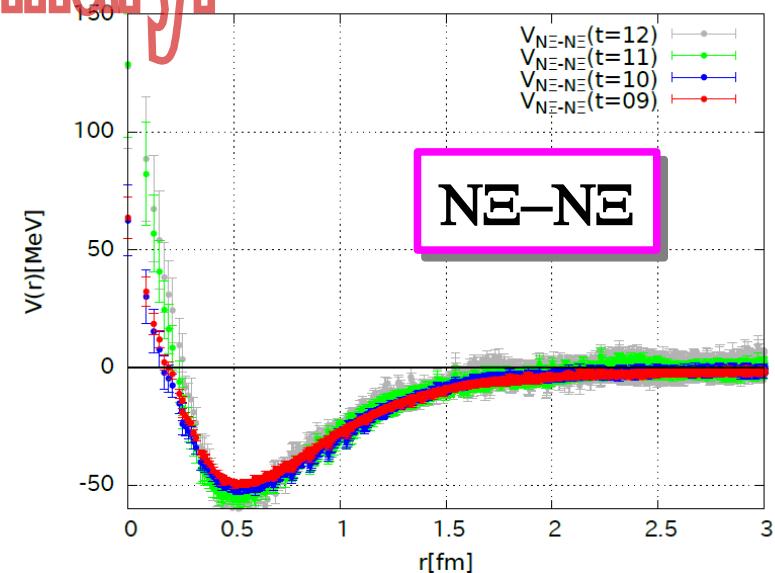
$\Lambda\Lambda, N\Xi (I=0) {}^1S_0$ potential

$t=09$
 $t=10$
 $t=11$
 $t=12$

► $N_f = 2+1$ full QCD with $L = 8.1\text{fm}$, $m_\pi = 146\text{ MeV}$



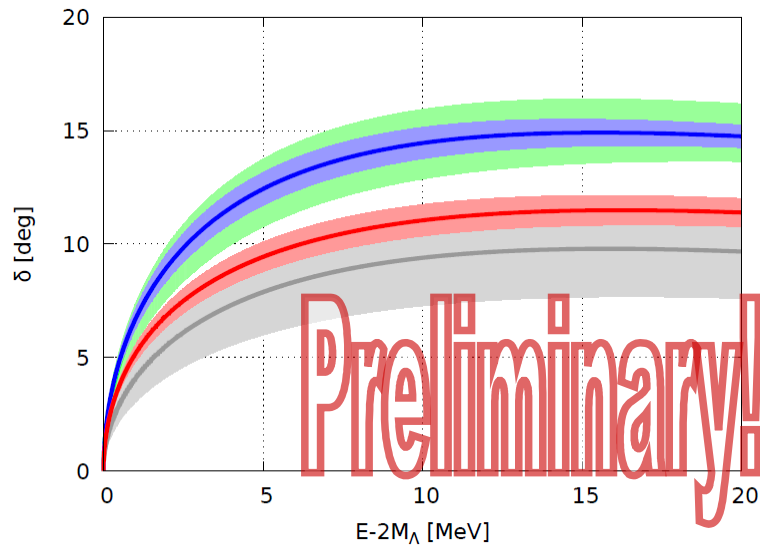
- $\Lambda\Lambda$ and $N\Xi$ potential have attractive pocket and repulsive core.
- Short range part of $N\Xi$ potential shows a “t” dependence.
- $\Lambda\Lambda - N\Xi$ transition potential is quite small in $r > 0.7\text{fm}$ region



$\Lambda\Lambda$ scattering length

$t=09$
 $t=10$
 $t=11$
 $t=12$

Phase shift



Λ		NLO			
		500	550	600	650
$\Lambda\Lambda$	a_{1S0}	-0.62	-0.61	-0.66	-0.70
	r_{1S0}	6.95	6.06	5.05	4.56

J. Haidenbauer et al, NPA954(2016)273

$$a_{\Lambda\Lambda} = -0.821 \text{ fm}$$

Y.Fujiwara et al, PPNP58(2007)439

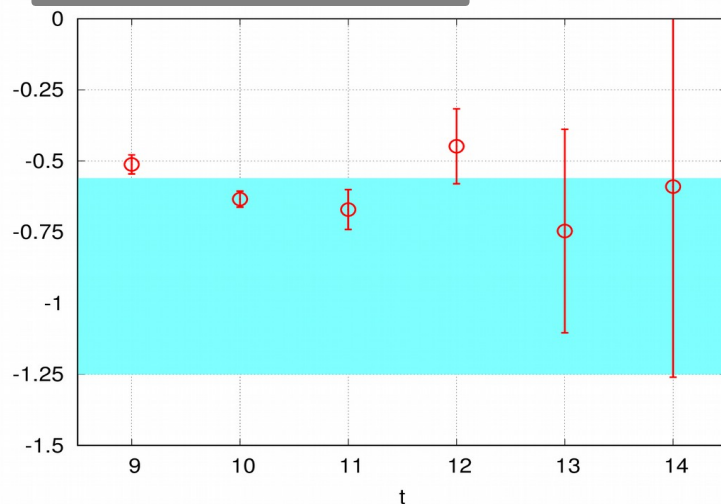
$$a_{\Lambda\Lambda} = -0.97 \text{ fm}$$

Th.A.Rijken et al, Few-Body Syst 54(2013)801

$$-1.25 < a_{\Lambda\Lambda} < -0.56 \text{ fm (or 0 fm)}$$

K.Morita et al, PRC91 (2015)024916
K.Morita, private communication

Scattering length



● Scattering length in $\Lambda\Lambda$ ($l=0$) is almost saturated.

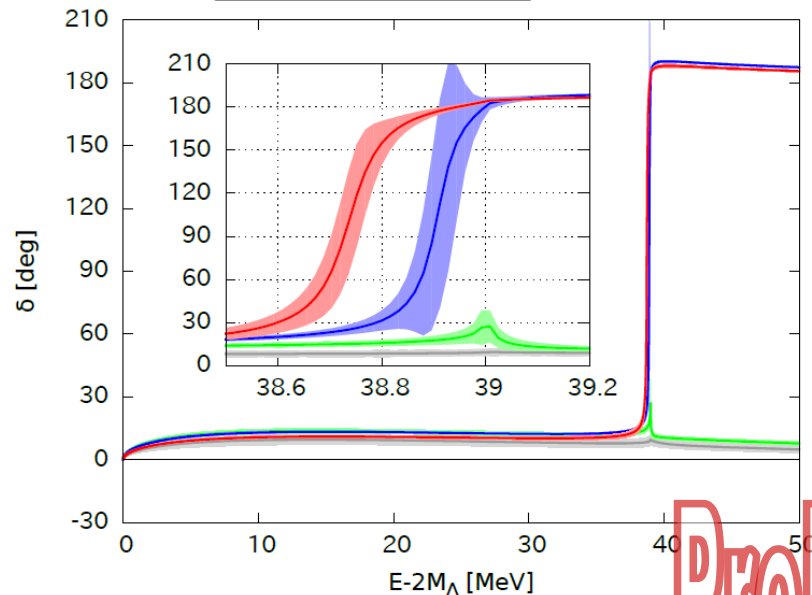
● Attraction is a little bit weaker

than the phenomenological values.

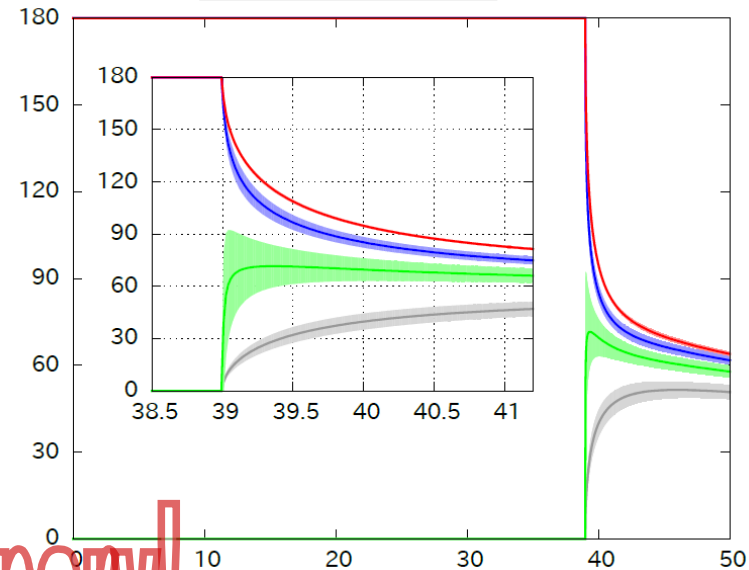
$\Lambda\Lambda$ and $N\Xi$ phase shift and inelasticity

$t=09$
 $t=10$
 $t=11$
 $t=12$

$\Lambda\Lambda$ phase shift



$N\Xi$ phase shift

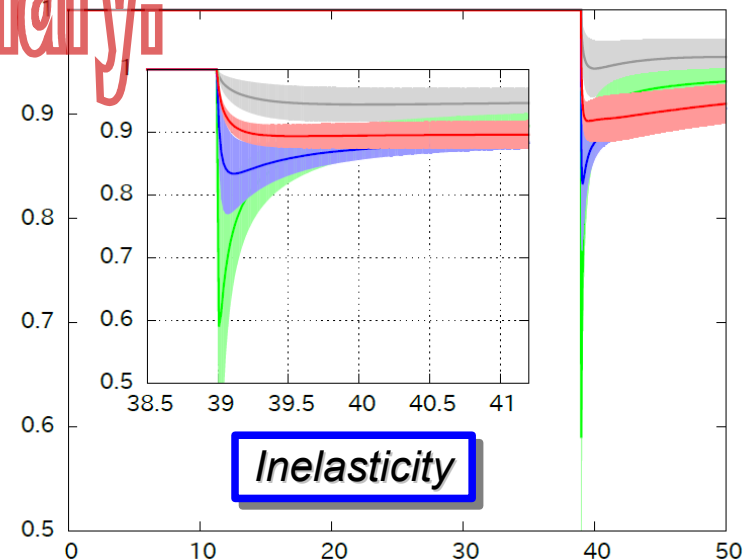


Preliminary!

- A sharp resonance is found below the $N\Xi$ threshold for $t=9$ and 10 .



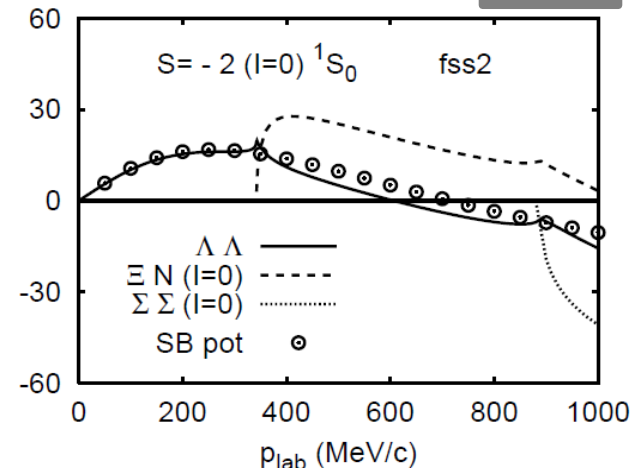
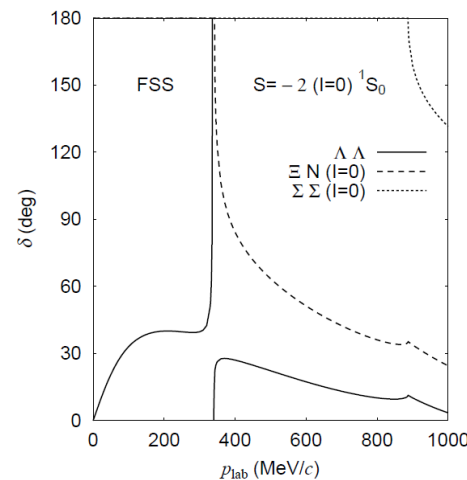
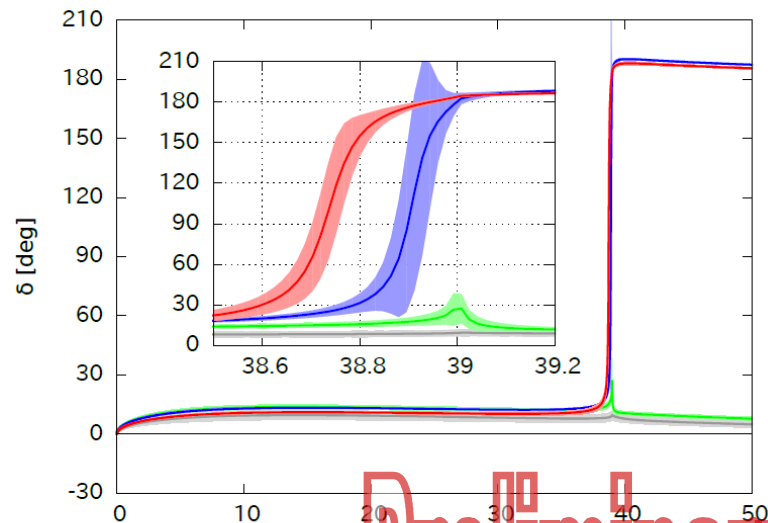
- Strength of $N\Xi$ attraction could be key to understand the H-dibaryon at the physical point.



$\Lambda\Lambda$ and $N\Xi$ phase shift –comparison–

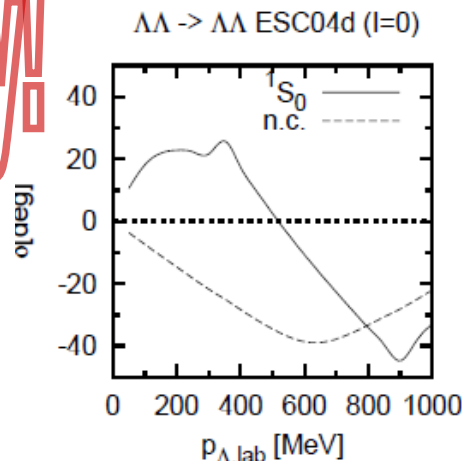
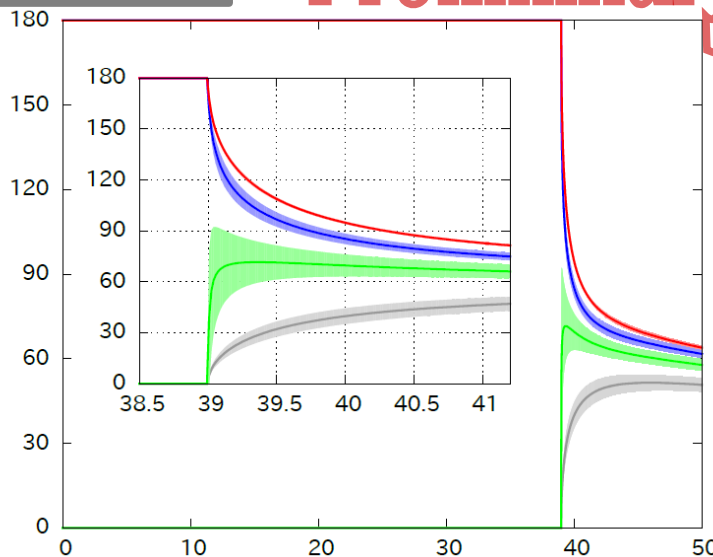
$t=09$
 $t=10$
 $t=11$
 $t=12$

$\Lambda\Lambda$ phase shift

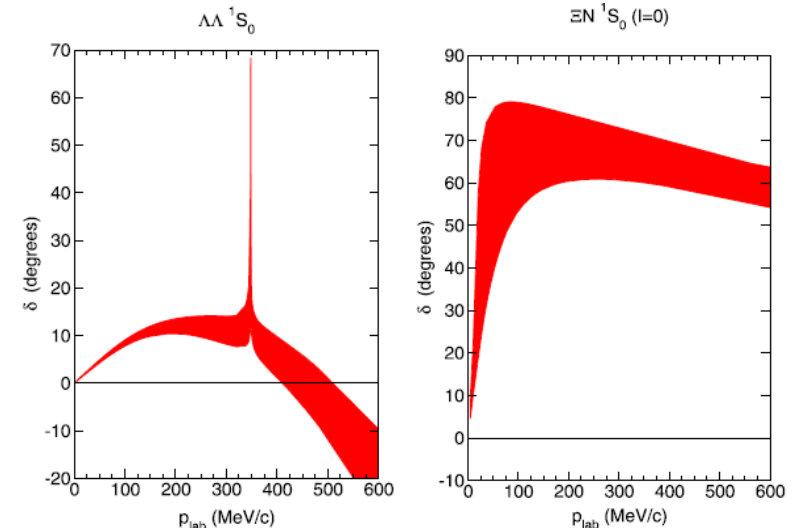


Y.Fujiwara et al, PPNP58(2007)439

$N\Xi$ phase shift



Th.A. Rijken, nucl-th/060874



J. Haidenbauer et al, NPA954(2016)273

● Our results are compatible
with the phenomenological ones.

Summary

► We have investigated YN / YY interaction from LQCD

- $\Omega\Omega$ (and $\Delta\Delta$) state
 - $\Delta\Delta(I=0)$ have strongly attractive potential.
 - $\Delta\Delta(I=3)$ and $\Omega\Omega$ potential have repulsive core and attractive pocket.
 - Physical $\Omega\Omega$ system in $J=0$ forms the most strange dibaryon (or unitary region...)
- $N\Omega$ state with $J^P=2^+$
 - Interaction is strongly attractive and no short range repulsion.
 - It forms a bound state with about a few MeV B.E..
- H-dibaryon channel
 - We found a strong attraction in $N\Xi$ $J=0$ with $I=0$.
 - It is still difficult to conclude the fate of H-dibayon.