# Gravitational waves from binary neutron stars

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# Plan of the talk

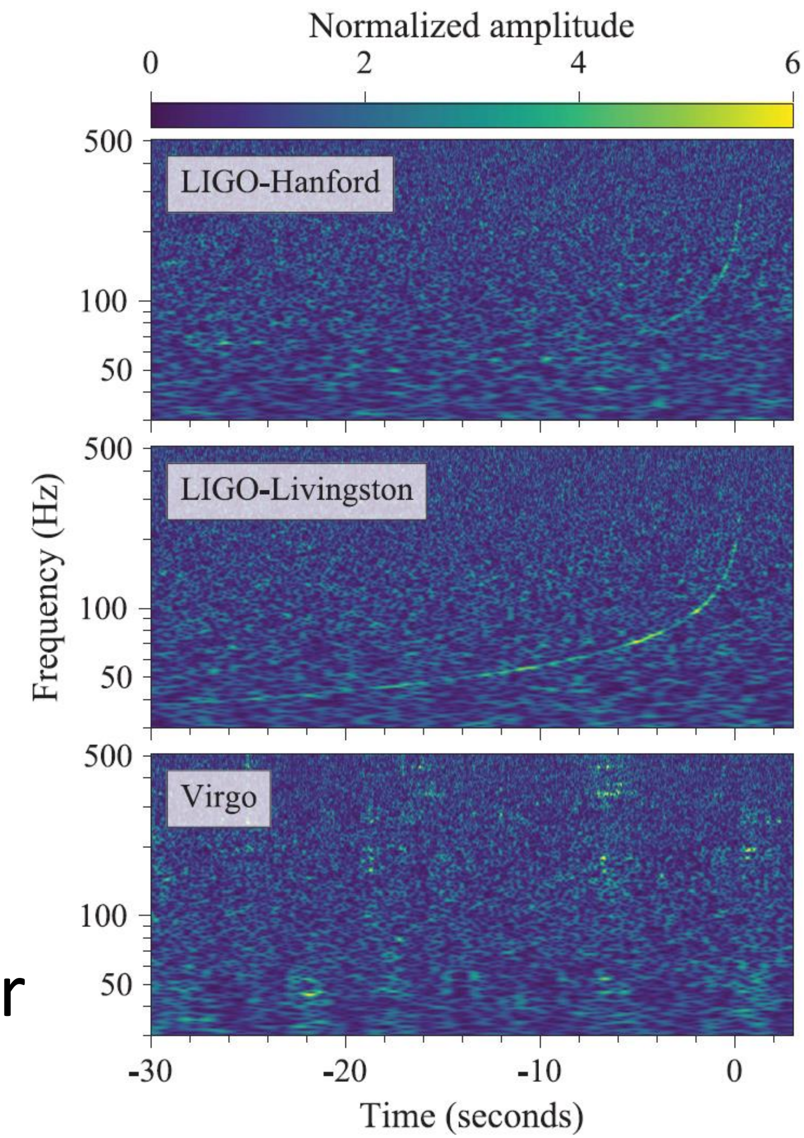
1. Introduction
2. Gravitational waves and tidal deformability
3. Reanalysis of GW170817
4. Summary

# 1. Introduction

# GW170817

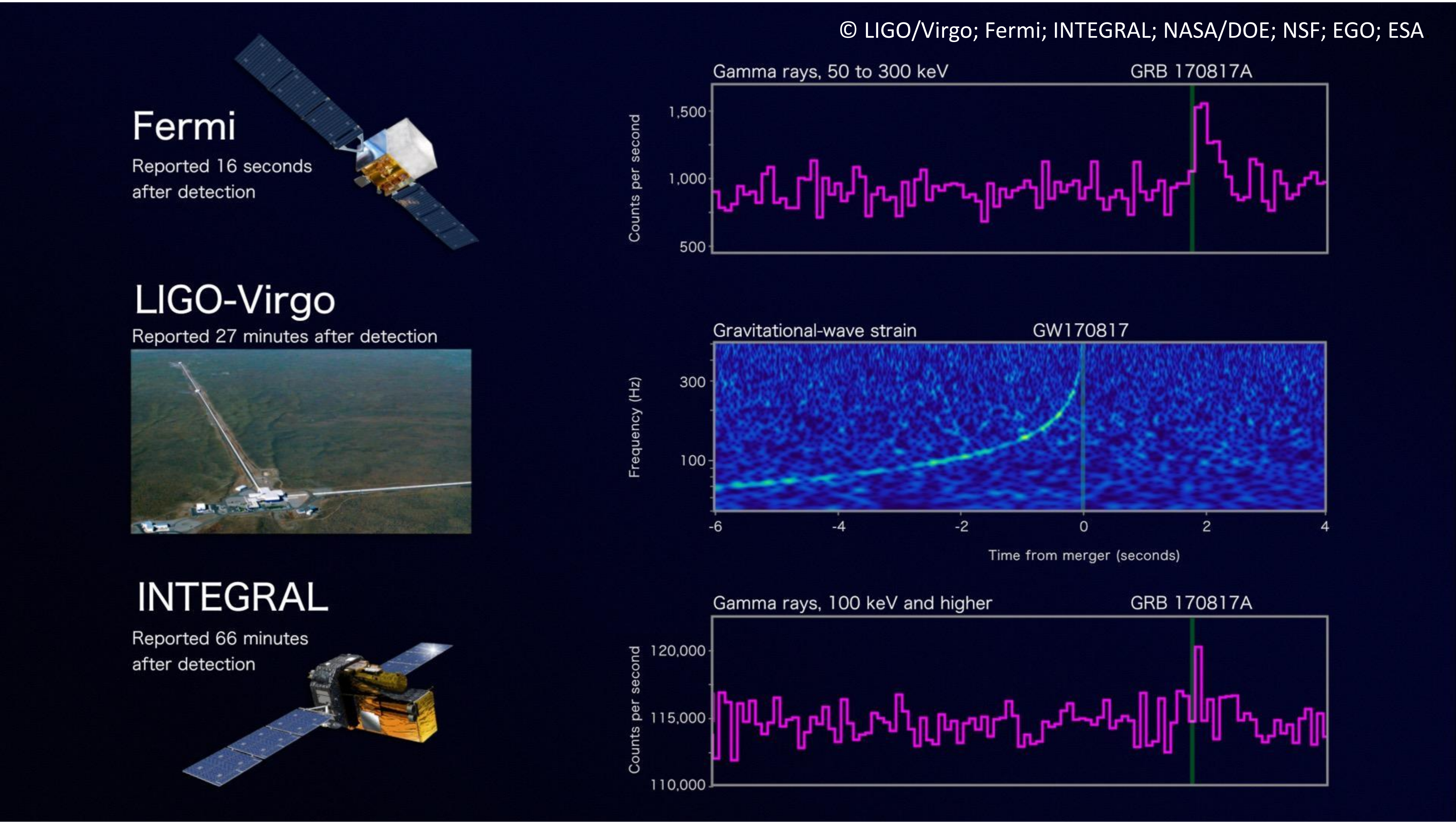
The LIGO twins observed
clear “chirp” signals, i.e., gravitational waves with increasing frequency and amplitude in time

But Virgo did not see…
not useful for estimating binary’s intrinsic parameter




LIGO&Virgo (2017)

# Gamma-ray burst: GRB 170817A



# 1.7s timing difference

Gravitational waves should propagate with $\approx c$

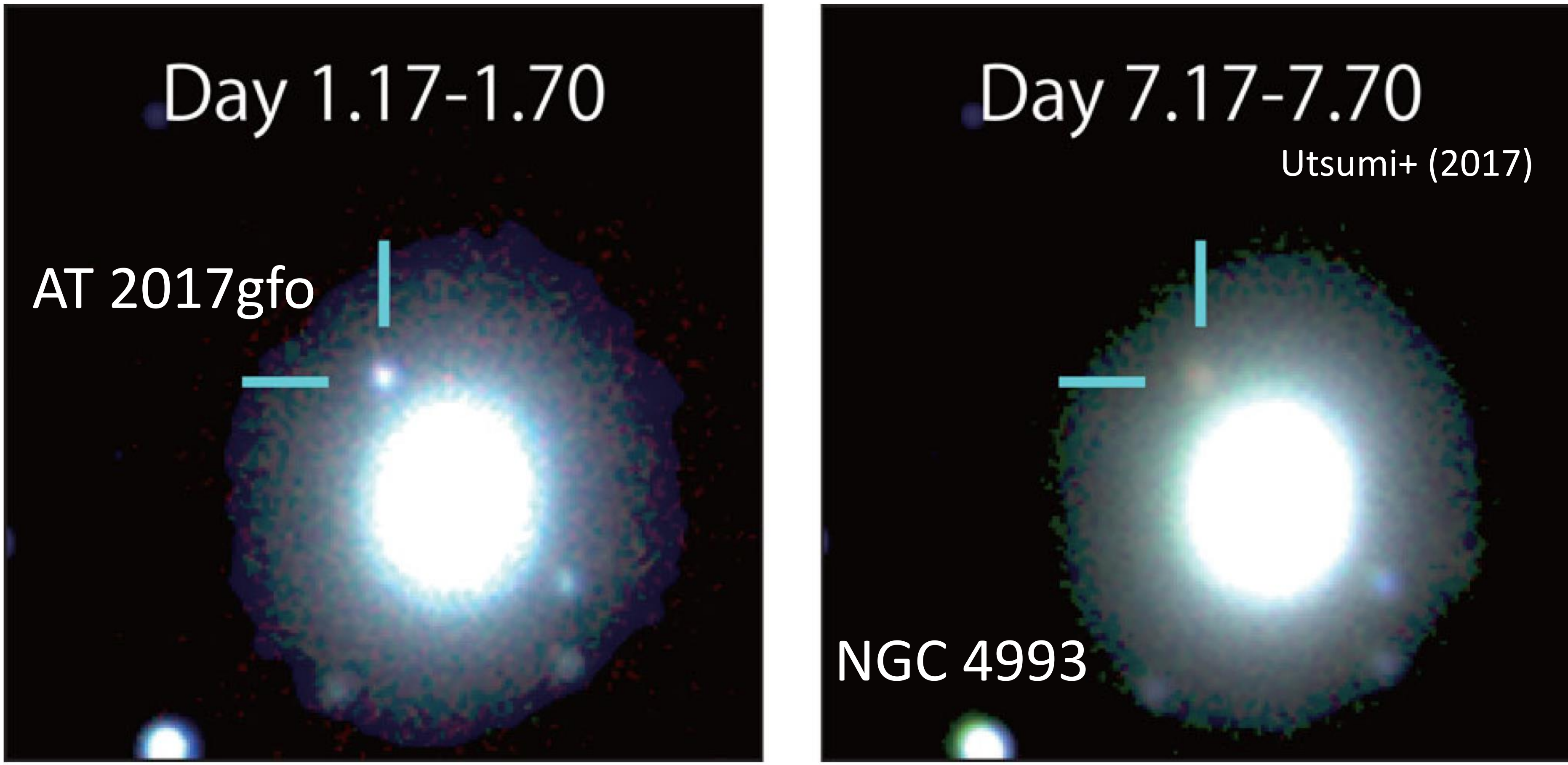
Many theories of modified gravity are rejected

Ezquiaga-Zumalacarregui (2017)

| | $c_g = c$ | $c_g \neq c$ |
|---|---|---|
| Horndeski | General Relativity<br>quintessence/k-essence [46]<br>Brans-Dicke/$f(R)$ [47, 48]<br>Kinetic Gravity Braiding [50] | quartic/quintic Galileons [13, 14]<br>Fab Four [15]<br>de Sitter Horndeski [49]<br>$G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [51], $f(\phi)\cdot$Gauss-Bonnet [52] |
| beyond H. | Derivative Conformal (19) [17]<br>Disformal Tuning (21)<br>quadratic DHOST with $A_1 = 0$ | quartic/quintic GLPV [18]<br>quadratic DHOST [20] with $A_1 \neq 0$<br>cubic DHOST [23] |
| | Viable after GW170817 | Non-viable after GW170817 |

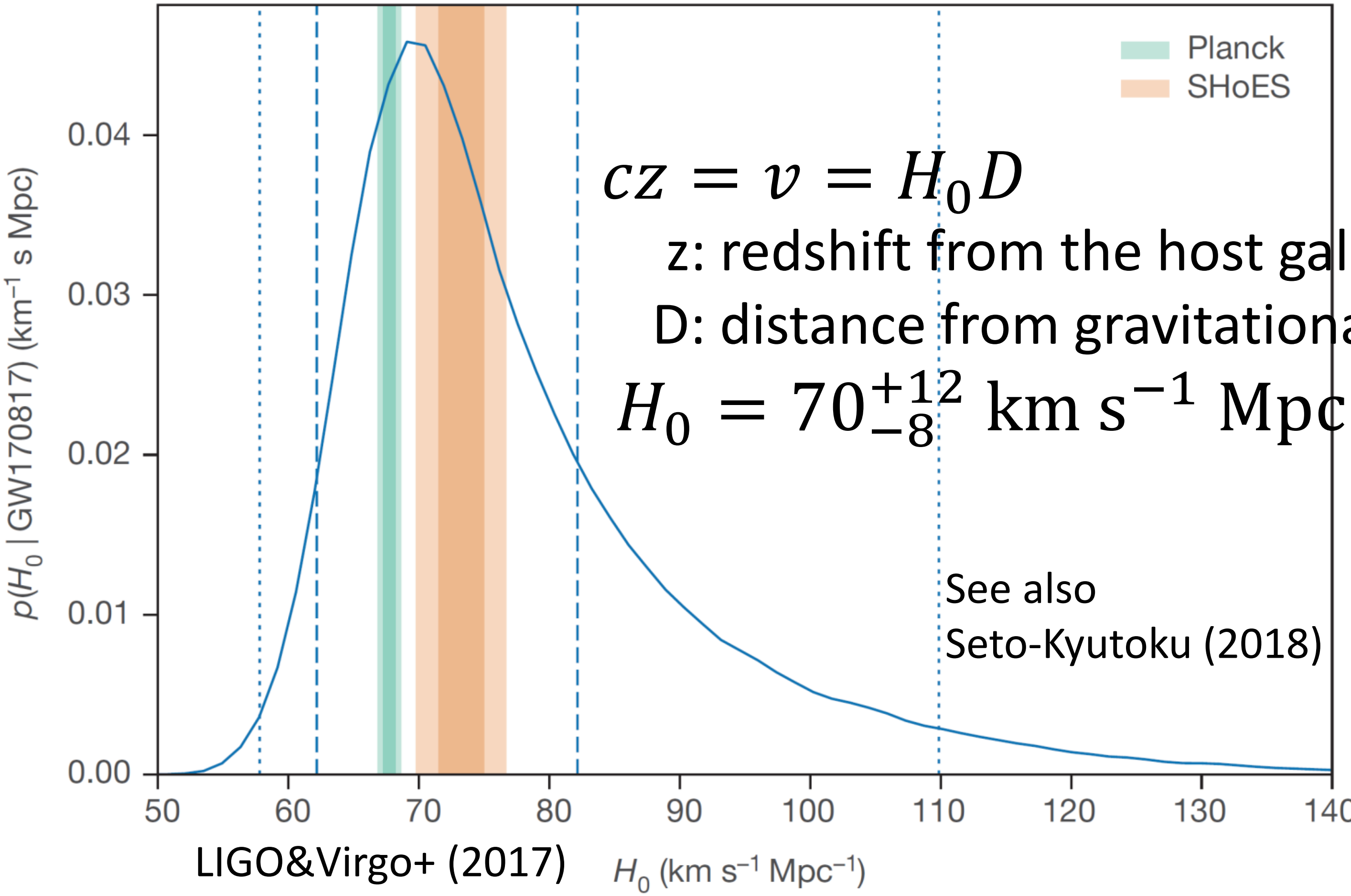
# Kilonova/macronova: AT 2017gfo

UV/optical/IR transient, determined the host galaxy



# Distance vs redshift

## Hubble's constant is determined in a novel manner




CMB

Supernova

$$cz = v = H_0 D$$

z: redshift from the host galaxy

D: distance from gravitational waves

$$H_0 = 70^{+12}_{-8}\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

See also

Seto-Kyutoku (2018)

LIGO&Virgo+ (2017)

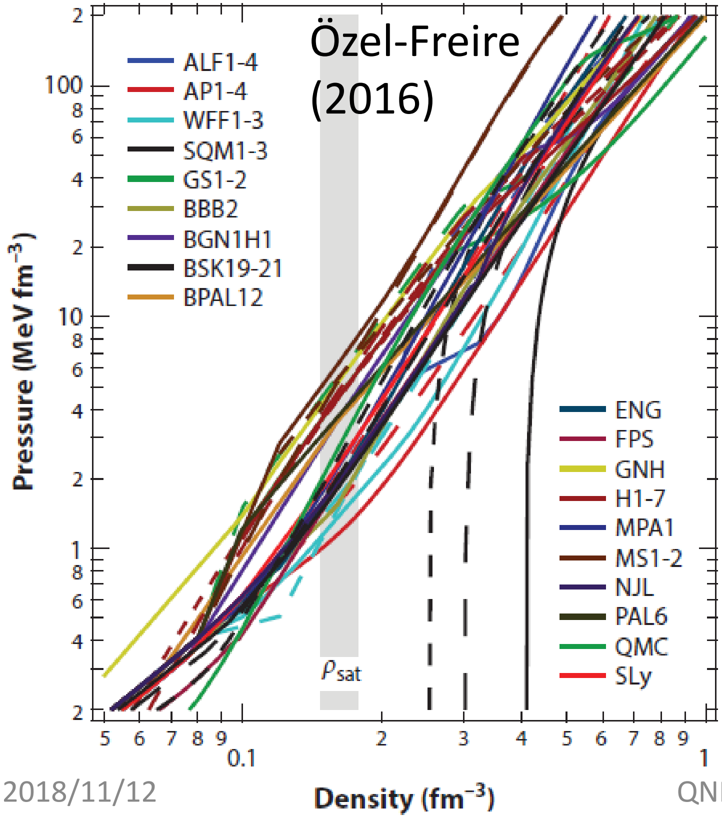
# 2. Gravitational waves and tidal deformability

# Neutron star equation of state

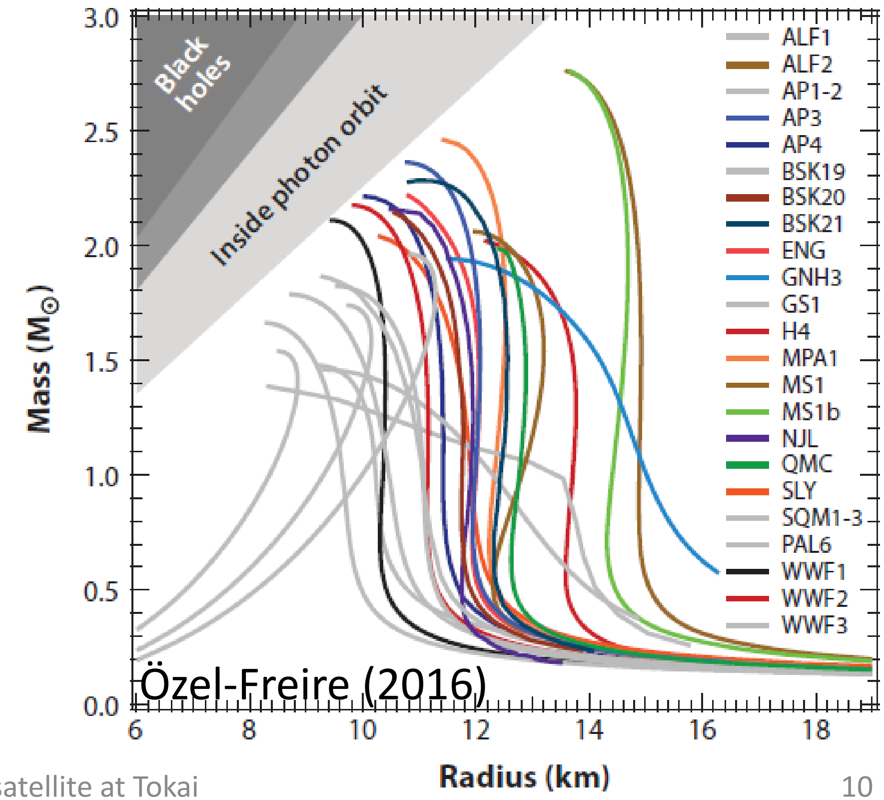
Note: not need to observe the radius, and other quantities may be fine

We want to know the realistic equation of state, that uniquely determines the mass-radius relation

Equation of state: Nuclear physics




Mass-Radius relation: Astrophysics

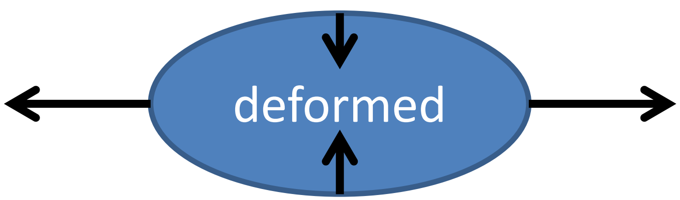


# Quadrupolar tidal deformability

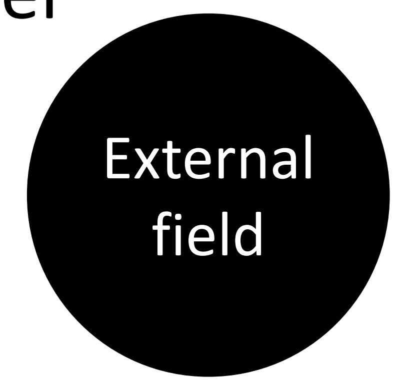
Leading-order finite-size effect on orbital evolution (strongly correlated with the neutron-star radius)

$$\Lambda = G\lambda\left(\frac{c^2}{GM}\right)^5 = \frac{2}{3}k\left(\frac{c^2 R}{GM}\right)^5 \propto R^5$$

$k \sim 0.1$: (second/electric) tidal Love number



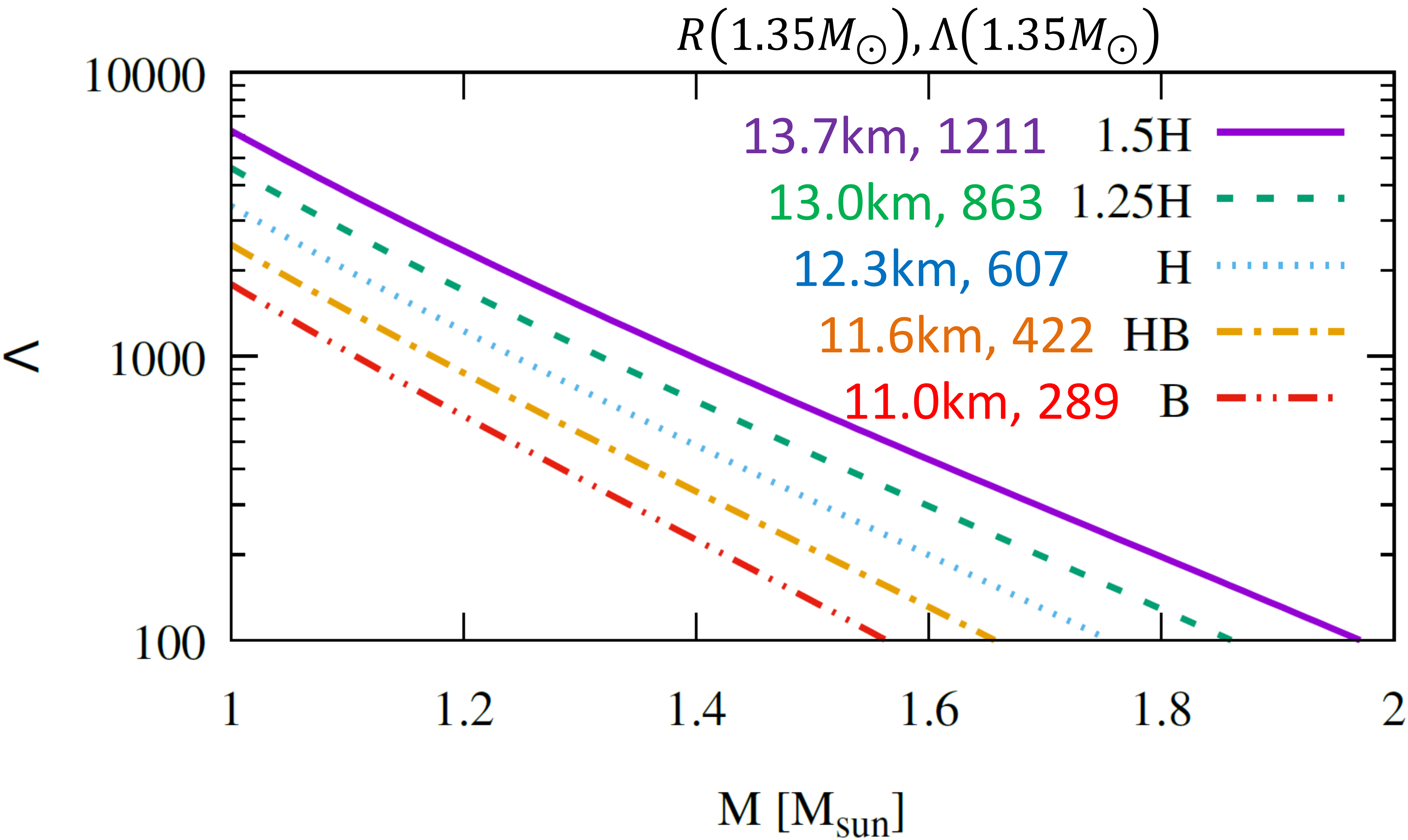

$$\mathcal{Q}_{ij} = -\lambda\mathcal{E}_{ij}$$




$$\mathcal{Q}_{ij} \equiv \int \rho\left(x_i x_j - \frac{1}{3}x^2\delta_{ij}\right) d^3x$$

$$\mathcal{E}_{ij} \equiv \frac{\partial^2 \Phi_{\text{ext}}}{\partial x^i \partial x^j}$$

# $M - \Lambda$ relation



# How $\Lambda$ affects gravitational waves

Primarily via the Newtonian orbital evolution

For point particles (binary black holes):

- Keplerian motion due to potential $\Phi \propto 1/r$

For finite-size objects (binary neutron stars):

- The tidal field is given via $\nabla\nabla\Phi \propto 1/r^3$
- This induces quadrupole deformation $Q \propto \Lambda/r^3$
- The quadrupole generate potential $\delta\Phi \propto Q/r^3$
- $\Phi$ is enhanced by an amount $\delta\Phi/\Phi \sim \Lambda(R/r)^5$

# Particularly important parameters

Chirp mass $\mathcal{M} = \mu^{3/5} M^{2/5}$: accurately measured

- Total mass $M = m_1 + m_2$
- Reduced mass $\mu = m_1 m_2 / M$

Symmetric mass ratio $\eta = \mu / M$: not very accurate...

Binary tidal deformability ($m_1 \geq m_2$)

$$\tilde{\Lambda} = \frac{8}{13}[(1 + 7\eta - 31\eta^2)(\Lambda_1 + \Lambda_2) + \sqrt{1 - 4\eta}(1 + 9\eta - 11\eta^2)(\Lambda_1 - \Lambda_2)]$$

# 90% credible interval ~ 100-800

| Low-spin prior, $\chi_i \leq 0.05$ | TaylorF2 | SEOBNRT | PhenomDNRT |
|---|---|---|---|
| Binary inclination $\theta_{JN}$ | $146^{+24}_{-28}$ deg | $146^{+24}_{-28}$ deg | $147^{+24}_{-28}$ deg |
| Binary inclination $\theta_{JN}$ using EM distance constraint [104] | $149^{+13}_{-10}$ deg | $152^{+14}_{-11}$ deg | $151^{+14}_{-10}$ deg |
| Detector frame chirp mass $\mathcal{M}^{det}$ | $1.1975^{+0.0001}_{-0.0001} M_\odot$ | $1.1976^{+0.0001}_{-0.0001} M_\odot$ | $1.1975^{+0.0001}_{-0.0001} M_\odot$ |
| Chirp mass $\mathcal{M}$ | $1.186^{+0.001}_{-0.001} M_\odot$ | $1.186^{+0.001}_{-0.001} M_\odot$ | $1.186^{+0.001}_{-0.001}$ |
| Primary mass $m_1$ | (1.36, 1.61) $M_\odot$ | (1.36, 1.59) $M_\odot$ | (1.36, 1.60) $M_\odot$ |
| Secondary mass $m_2$ | (1.16, 1.36) $M_\odot$ | (1.17, 1.36) $M_\odot$ | (1.17, 1.36) $M_\odot$ |
| Total mass $m$ | $2.73^{+0.05}_{-0.01} M_\odot$ | $2.73^{+0.04}_{-0.01} M_\odot$ | $2.73^{+0.04}_{-0.01} M_\odot$ |
| Mass ratio $q$ | (0.72, 1.00) | (0.74, 1.00) | (0.73, 1.00) |
| Effective spin $\chi_{eff}$ | $0.00^{+0.02}_{-0.01}$ | $0.00^{+0.02}_{-0.01}$ | $0.00^{+0.02}_{-0.01}$ |
| Primary dimensionless spin $\chi_1$ | (0.00, 0.02) | (0.00, 0.02) | (0.00, 0.02) |
| Secondary dimensionless spin $\chi_2$ | (0.00, 0.02) | (0.00, 0.02) | (0.00, 0.02) |
| Tidal deformability $\tilde{\Lambda}$ with flat prior (symmetric/HPD) | $340^{+580}_{-240}/340^{+490}_{-290}$ | $280^{+490}_{-190}/\ 280^{+410}_{-230}$ | $300^{+520}_{-190}/\ 300^{+430}_{-230}$ |

Top: LIGO-Virgo (2018) / Bottom: De+ (2018)

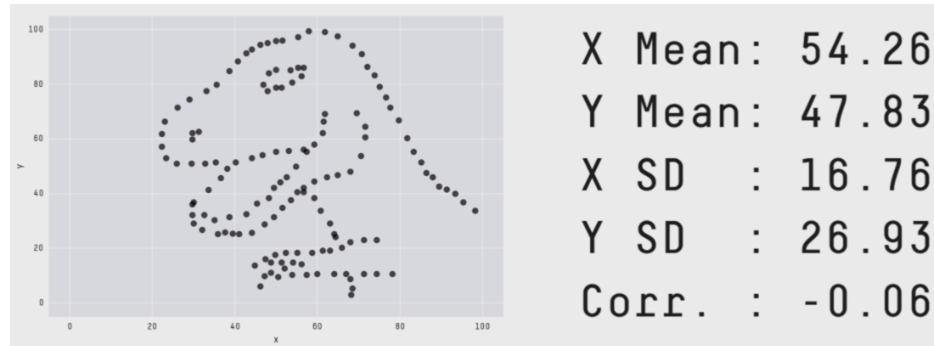
TaylorF2
Assume the EOS as common to both binary members

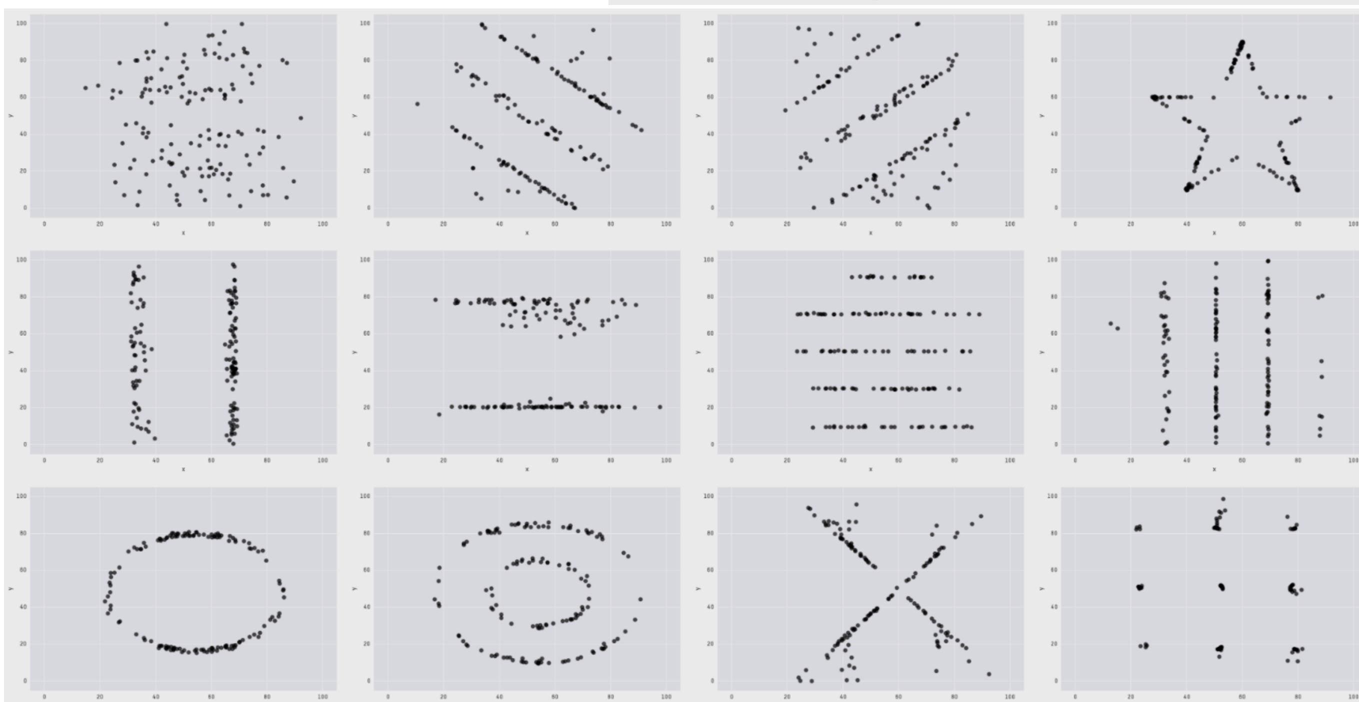
| Mass prior | $\tilde{\Lambda}$ | $\hat{R}$ (km) | $\mathcal{B}$ | $\tilde{\Lambda}_{90\%}$ |
|---|---|---|---|---|
| Uniform | $222^{+420}_{-138}$ | $10.7^{+2.1}_{-1.6} \pm 0.2$ | 369 | $< 485$ |
| Double neutron star | $245^{+453}_{-151}$ | $10.9^{+2.1}_{-1.6} \pm 0.2$ | 125 | $< 521$ |
| Galactic neutron star | $233^{+448}_{-144}$ | $10.8^{+2.1}_{-1.6} \pm 0.2$ | 612 | $< 516$ |

# Never skip looking at the distribution

See also

Anscombe's quartet



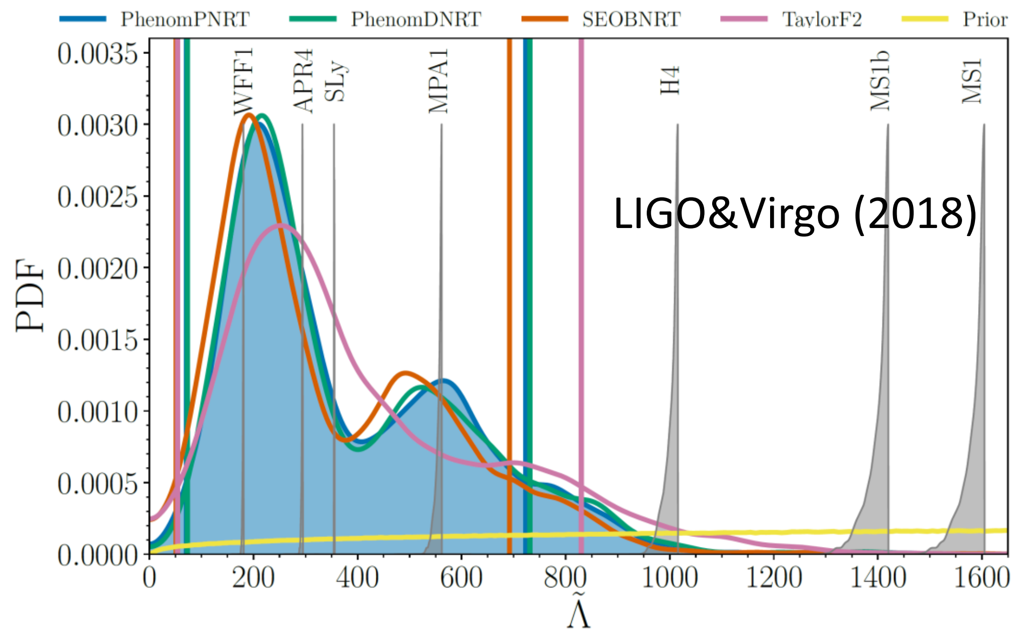



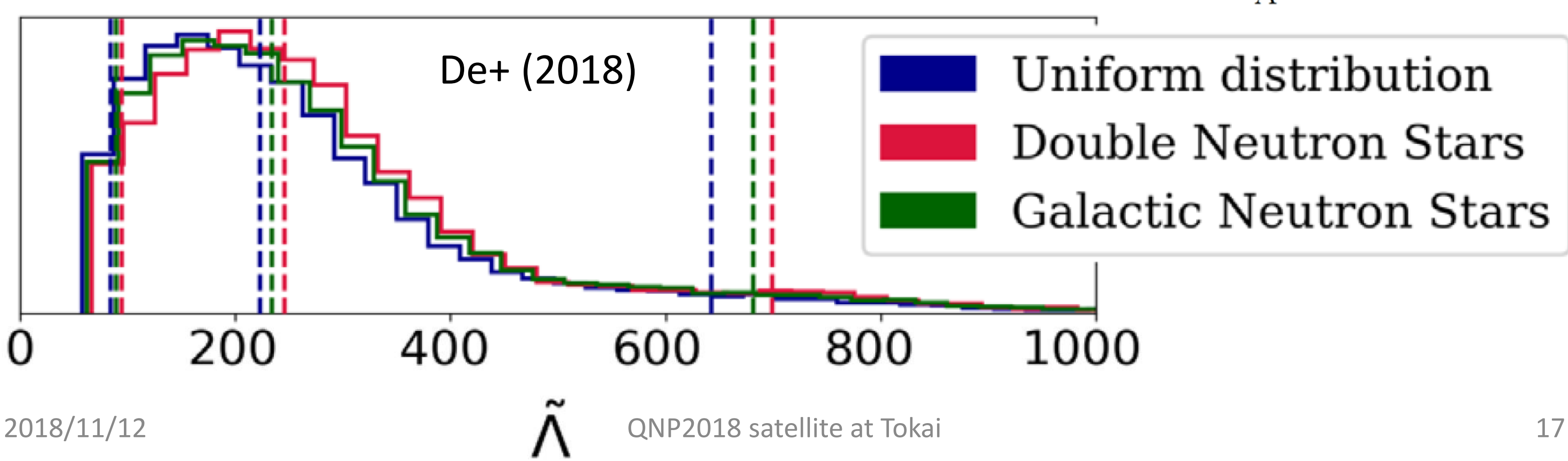
https://www.autodeskresearch.com/publications/samestats

# Double peak/high-$\tilde{\Lambda}$ tail

Posterior distribution is far from Gaussian in LVC/non-LVC analysis

Moderate dependence on waveform models



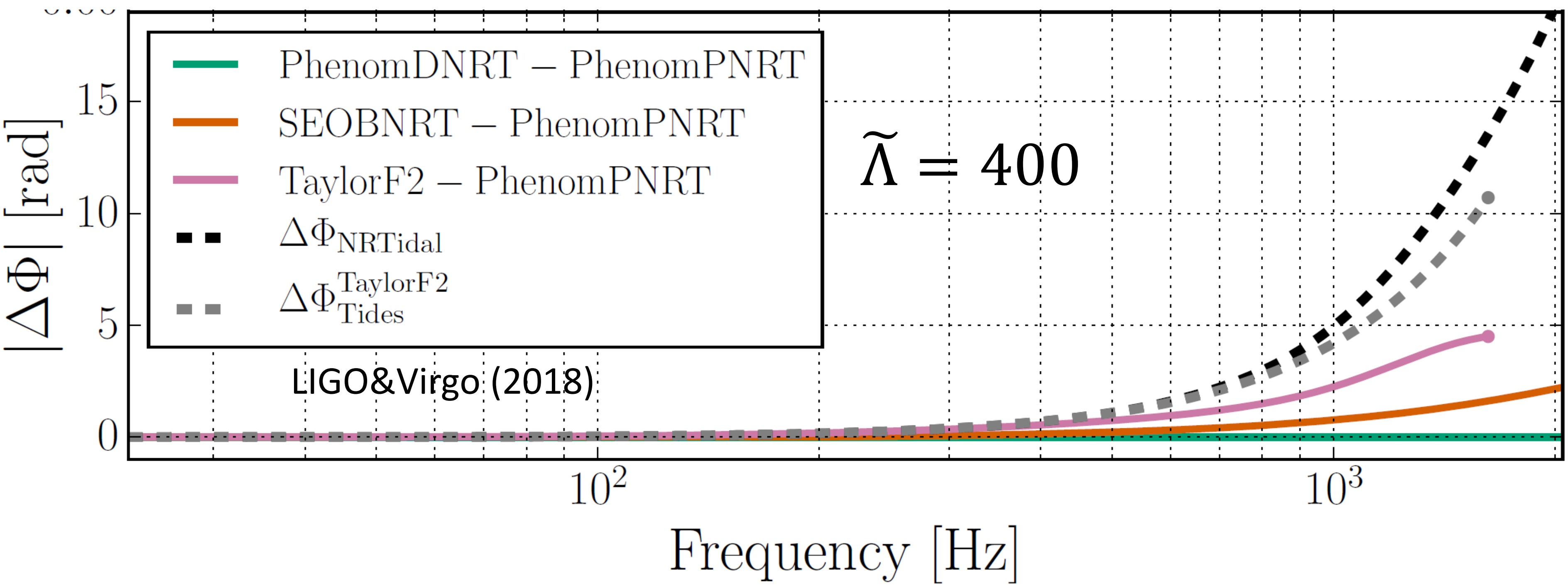
# 3. Reanalysis of GW170817

Led by T. Narikawa (Kyoto U) and N. Uchikata (Niigata U)

# Waveform model dependence

O(1)rad phase differences are not very comfortable

Deviations are small at low frequency (<100Hz) but become large at high frequency due to nonlinearity

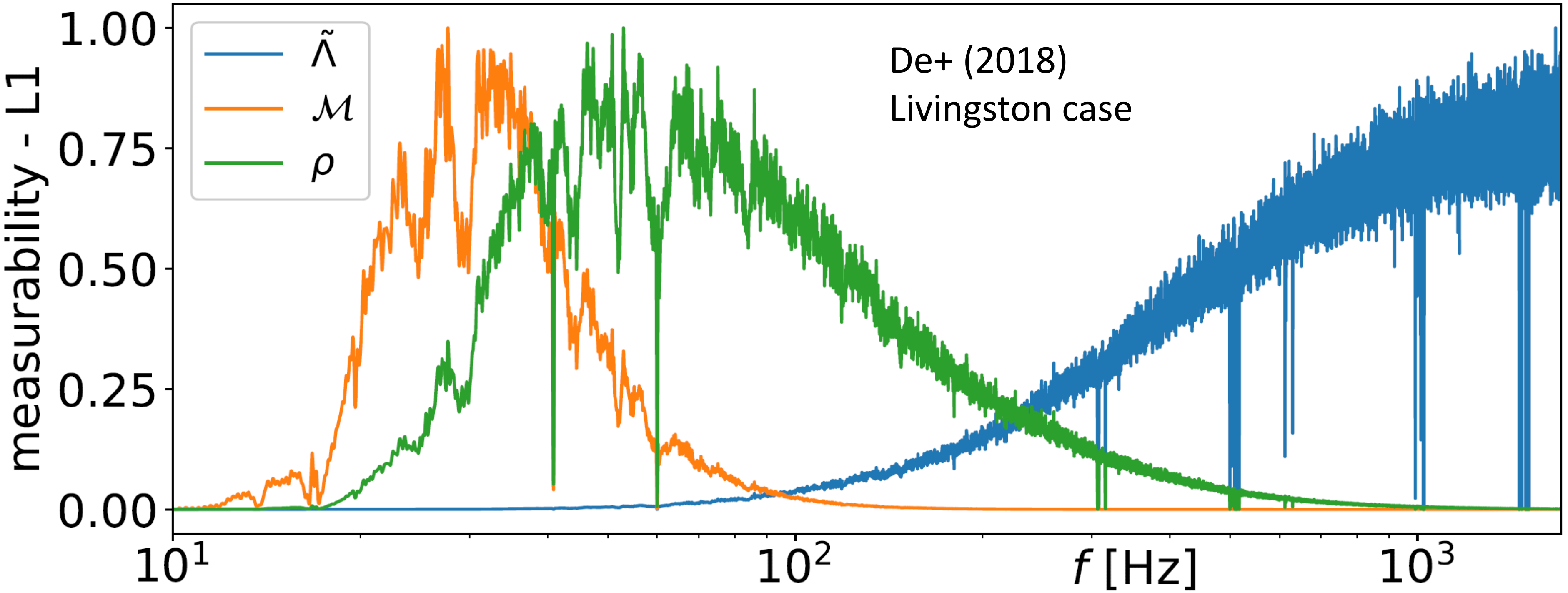


# Important frequency range

$\mathcal{M}$: low frequency (many gravitational-wave cycles)

$\rho$ = signal-to-noise ratio: inverse of the sensitivity

$\tilde{\Lambda}$: high frequency (closer orbit->large deformation)

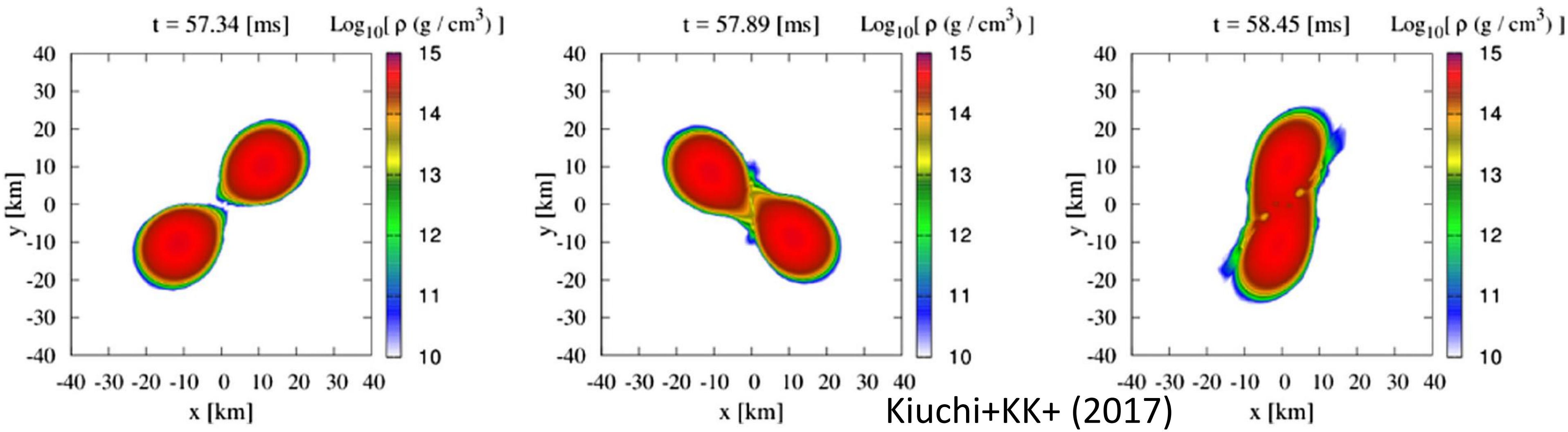


# Need for numerical simulations

The wave amplitude peaks after the contact of binary neutron stars (the right panel)

Nonlinearity of gravity and tidal deformation makes this stage beyond the reach of analytic calculation

But we have to struggle with finite resolutions




Kiuchi+KK+ (2017)

# Numerical relativity

The Einstein equation

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad (G = c = 1)$$

Local energy-momentum conservation equation

$$\nabla_\nu T^{\mu\nu} = 0$$

Rest-mass (or particle number) continuity equation

$$\nabla_\mu(\rho u^\mu) = 0$$

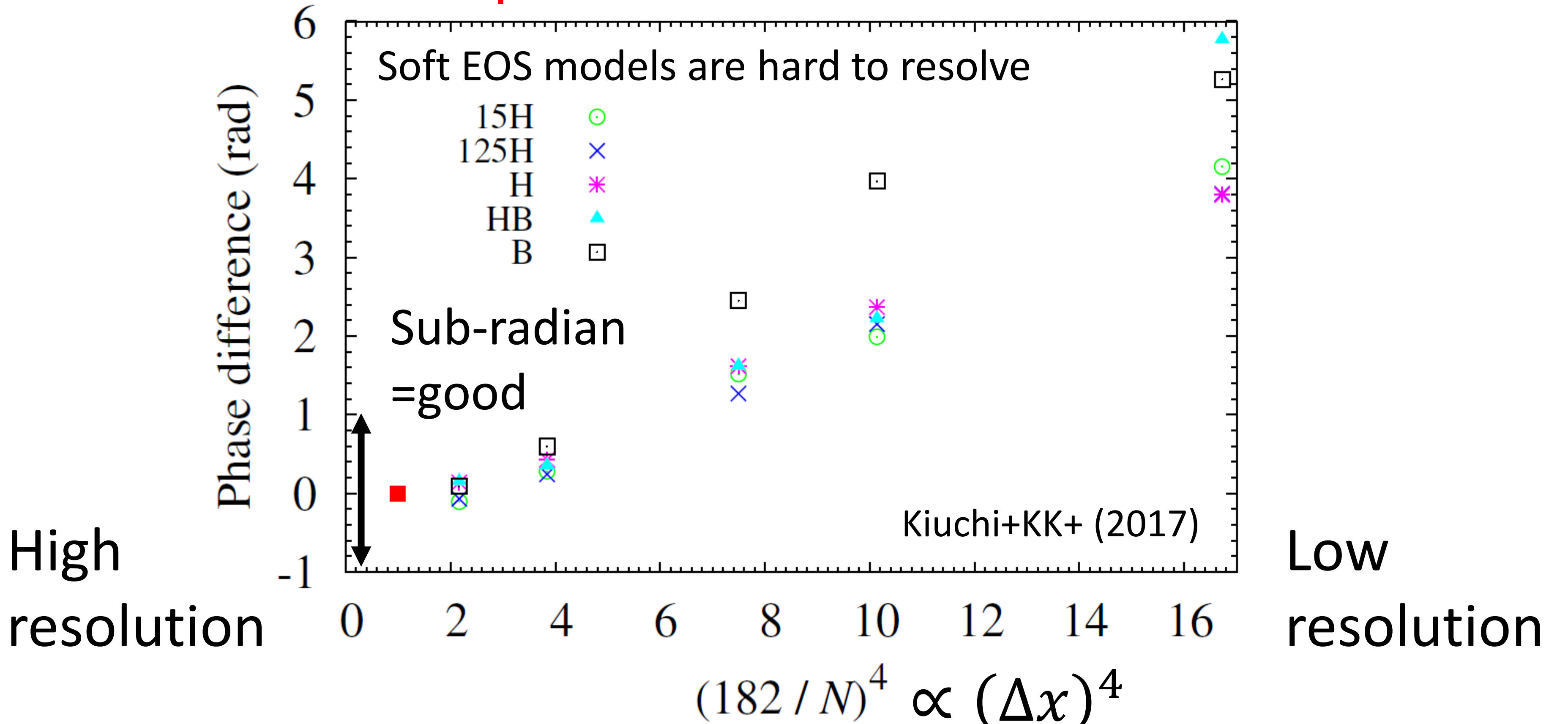
\+ equation of state, e.g., $P = P(\rho), P(\rho, T, Y_e)$ ...

Magneto/Radiation-HD Eqs. are not required here

# Intensive convergence study

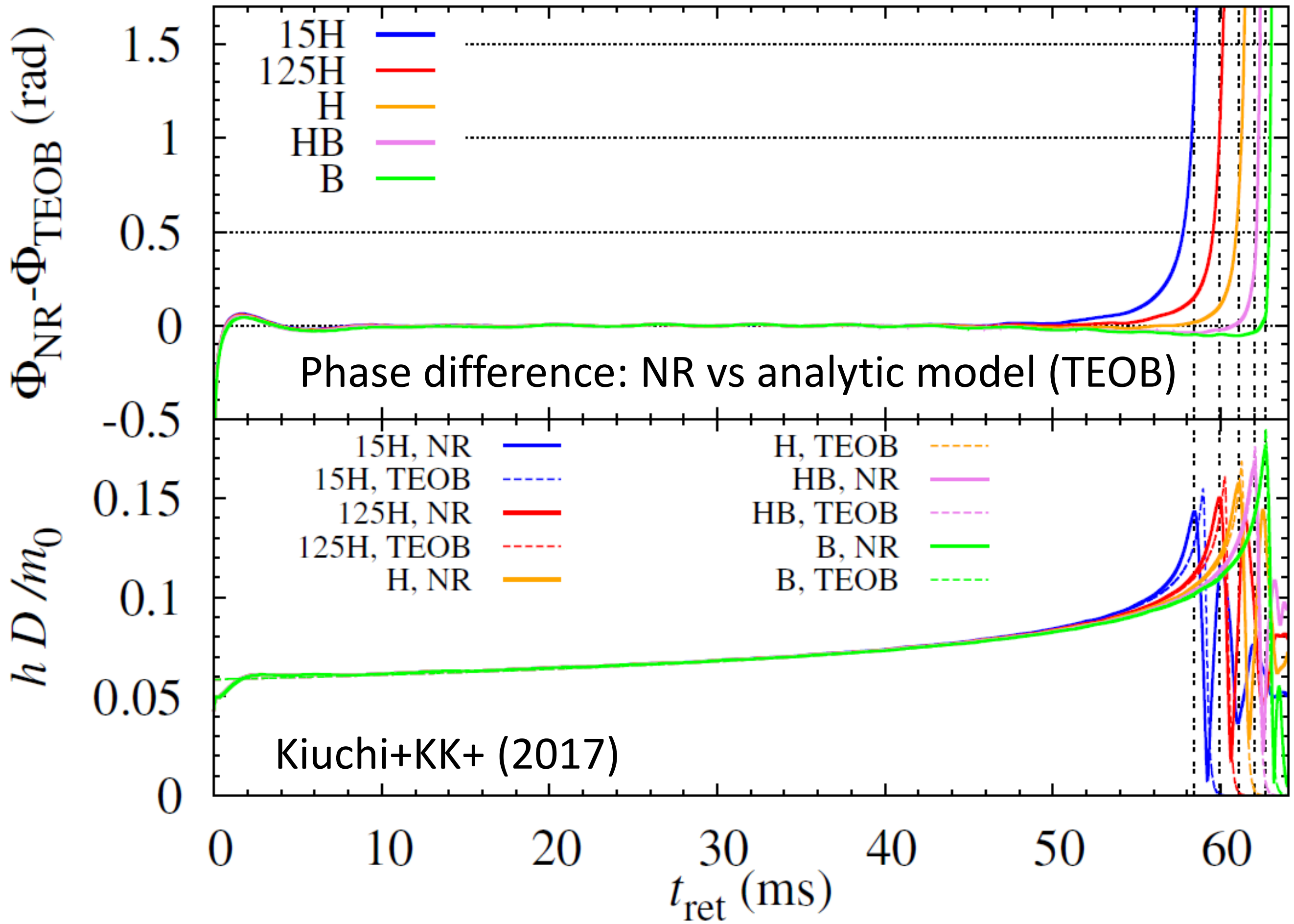
Approximate 4th order convergence before merger

The sub-radian phase error seems to be achieved



# Comparison with an analytic model

Analytic models sometimes exhibit ~1 radian error



# Our model (Kyoto model)

TaylorF2: Post-Newton phase $\left(x \propto f^{2/3}\right)$

$$\Psi_{\rm tidal}^{\rm 2.5PN} = \frac{3}{128\eta}\left(-\frac{39}{2}\tilde{\Lambda}\right)x^{5/2}\left[1+\frac{3115}{1248}x-\pi x^{3/2}+\frac{28024205}{3302208}x^2-\frac{4283}{1092}\pi x^{5/2}\right]$$

+ insignificant correction terms associated with $\eta$

We introduce a nonlinear-in-$\tilde{\Lambda}$ term (empirically)

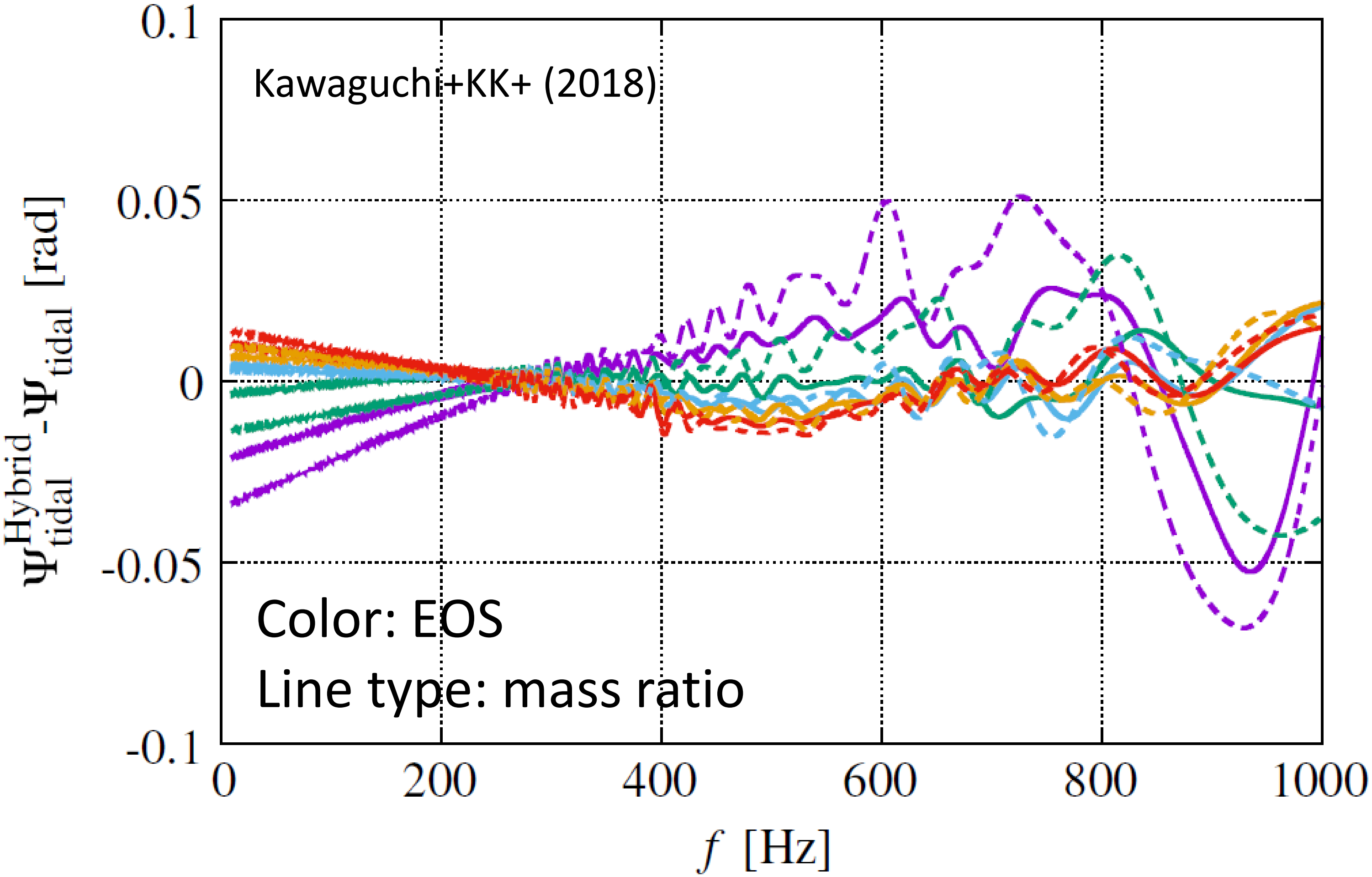
$$-\frac{39}{2}\tilde{\Lambda}(1+12.55\tilde{\Lambda}^{2/3}x^{4.240})$$

Another model Pade-resums the post-Newton part

$$\frac{1+\tilde{n}_1 x+\tilde{n}_{3/2}x^{3/2}+\tilde{n}_2 x^2+\tilde{n}_{5/2}x^{5/2}}{1+\tilde{d}_1 x+\tilde{d}_{3/2}x^{3/2}}$$

# Accuracy of our waveform model

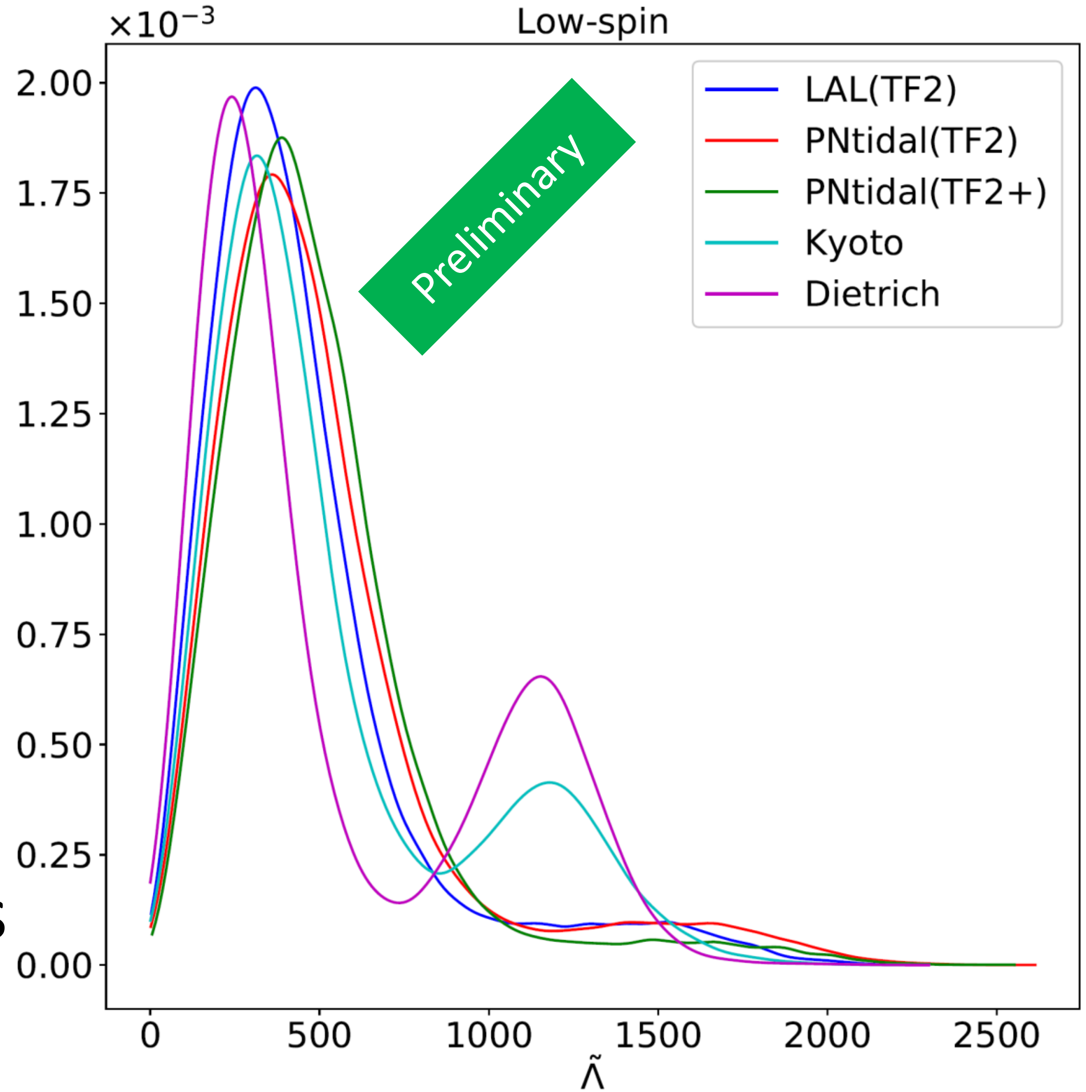
<0.1 radian for non-calibration models up to 1kHz



# Our independent analysis

So far, differences associated with waveform models may be minor

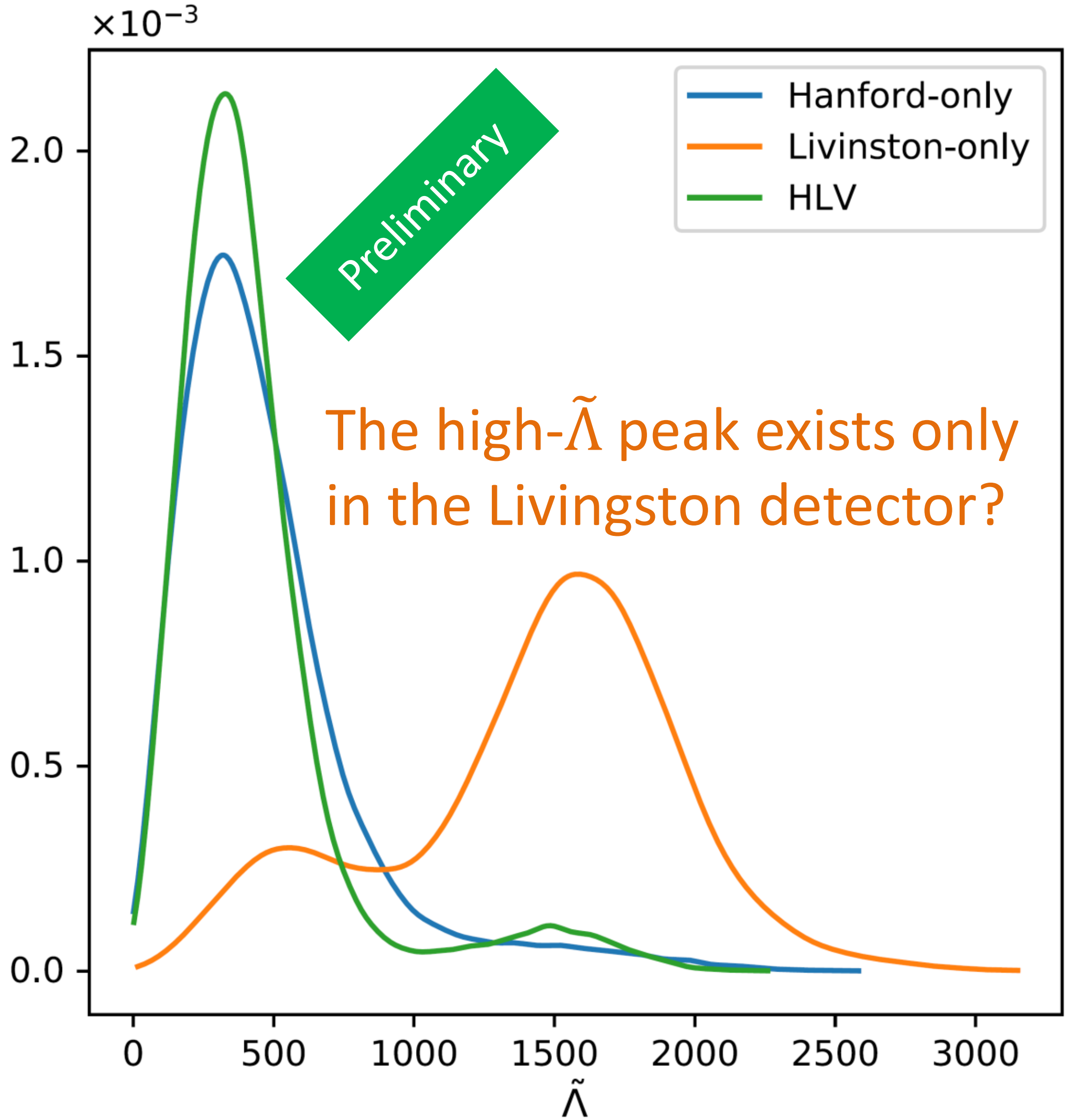
Double peaks remain particularly for sophisticated models



# Discrepancy of the LIGO twins

The 90% credible interval (5%-95%) of LIGO-Virgo is wider than that of the Hanford-only…

combination is not always helpful!



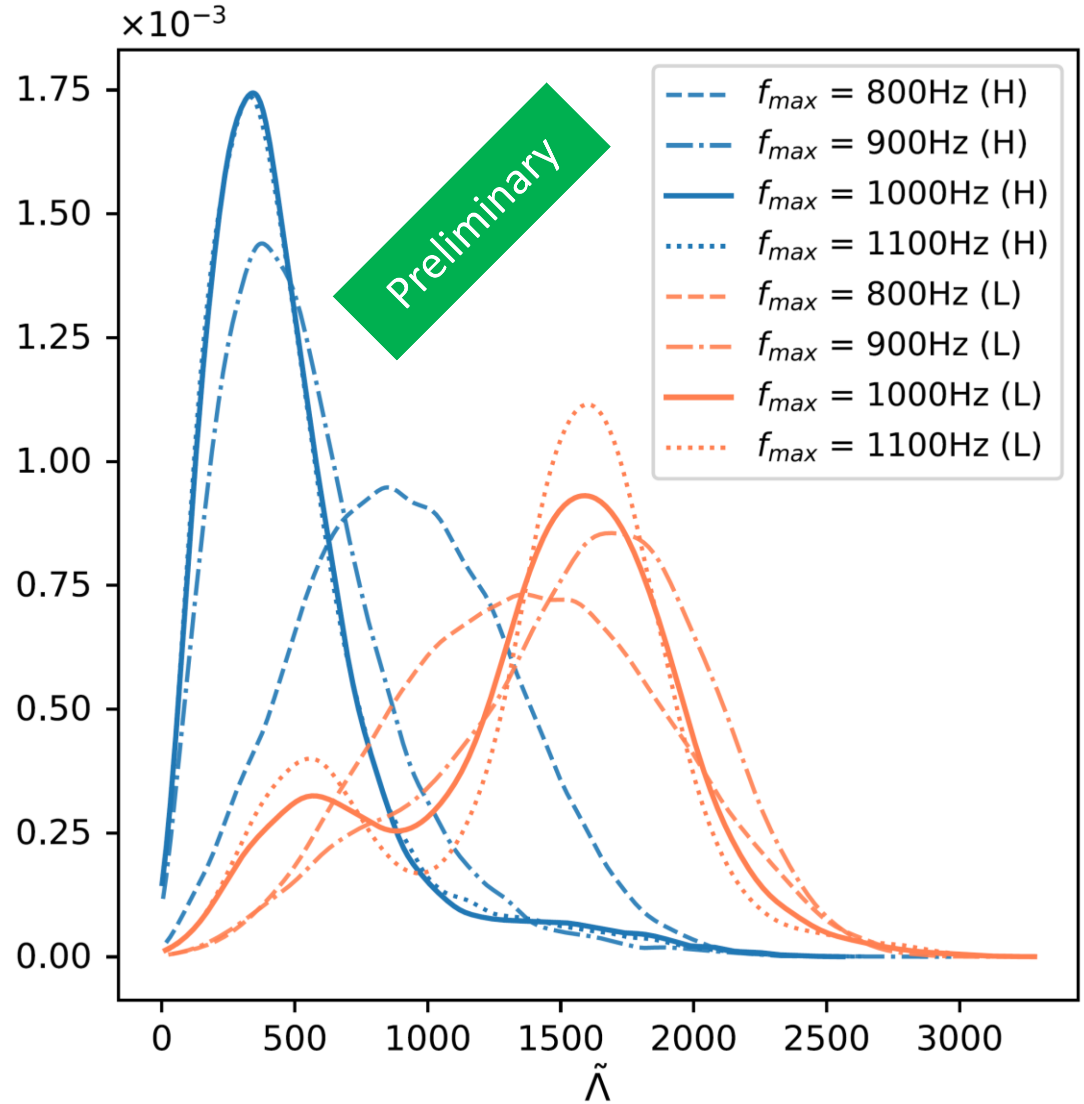
# Dependence on high-frequency cutoff

Hanford detector:

- single (low) peak
- converge smoothly w.r.t $f_{\max}$ change

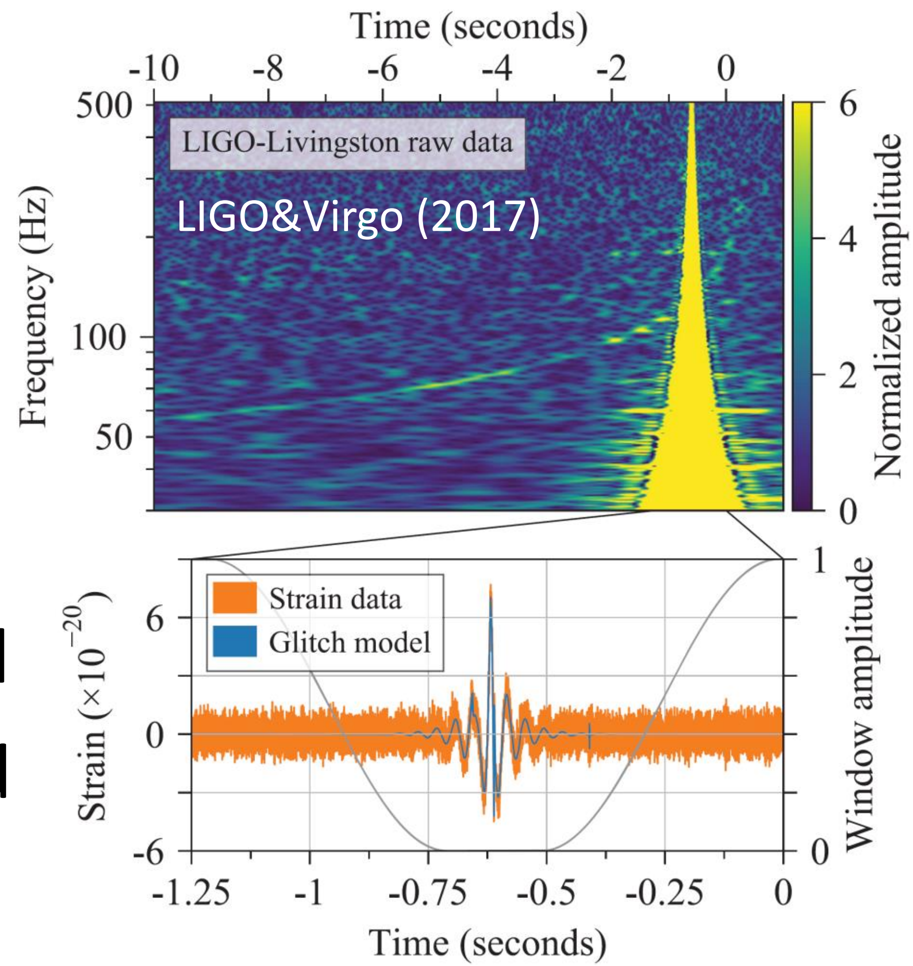
Livingston detector:

- double peak
- irregular variation w.r.t $f_{\max}$ change



# Random noise or specific component?

E.g., a glitch and incomplete subtraction thereof
(this is just an example!)




If the "second" peak is
associated with noises
that do not average out,
future results will be biased
-> noise hunting warranted

# More than 3 detectors preferable

Advanced LIGO (Hanford, USA)
another at Livingston

https://www.advancedligo.mit.edu/graphics/summary01.jpg


http://gwcenter.icrr.u-tokyo.ac.jp/wp-content/themes/lcgt/images/img_abt_lcgt.jpg

KAGRA (Kamioka, Japan)

Advanced Virgo
(Pisa, Italy)

http://virgopisa.df.unipi.it/sites/virgopisa.df.unipi.it.virgopisa/files/banner/virgo.jpg

# 4. Summary

# Summary

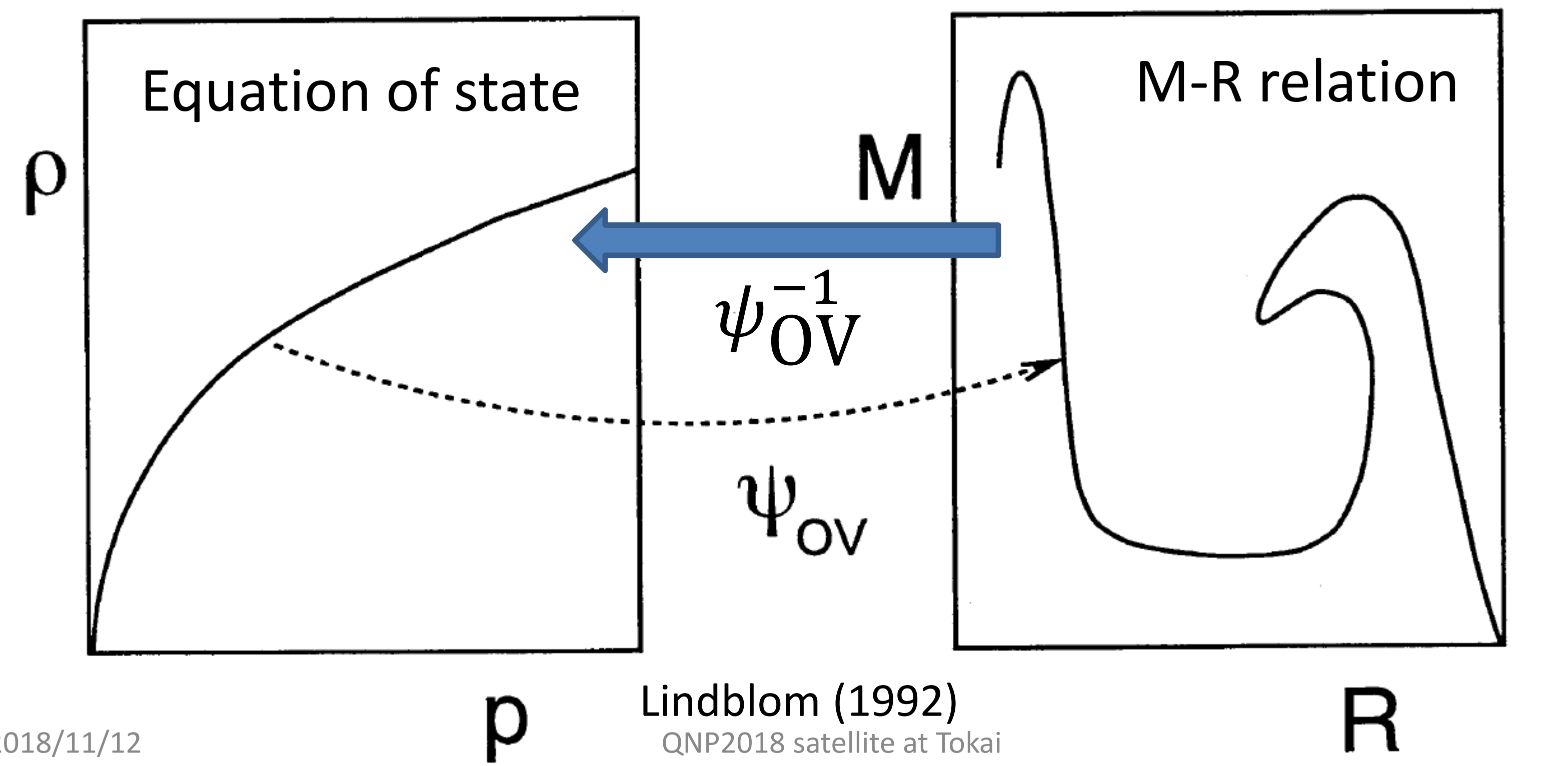
- Binary tidal deformability of GW170817 is constrained to $100 \leq \tilde{\Lambda} \leq 800$ depending on the method of analysis and waveform models.
- We have independently analyzed LIGO-Virgo data of GW170817 using our waveform models and the constraint is consistent with others.
- The second peak exists only for Livingston and behaves irregularly with respect to changes of the high-frequency cutoff (<-> Hanford).

# Appendix

# One-to-one correspondence

Via Tolman-Oppenheimer-Volkoff equation of GR

$$\frac{dP}{dr} = -\frac{(e+P)(m+4\pi P r^3)}{r(r-2m)} \qquad \left(\to -\frac{\rho m}{r^2}\right)$$




Lindblom (1992)

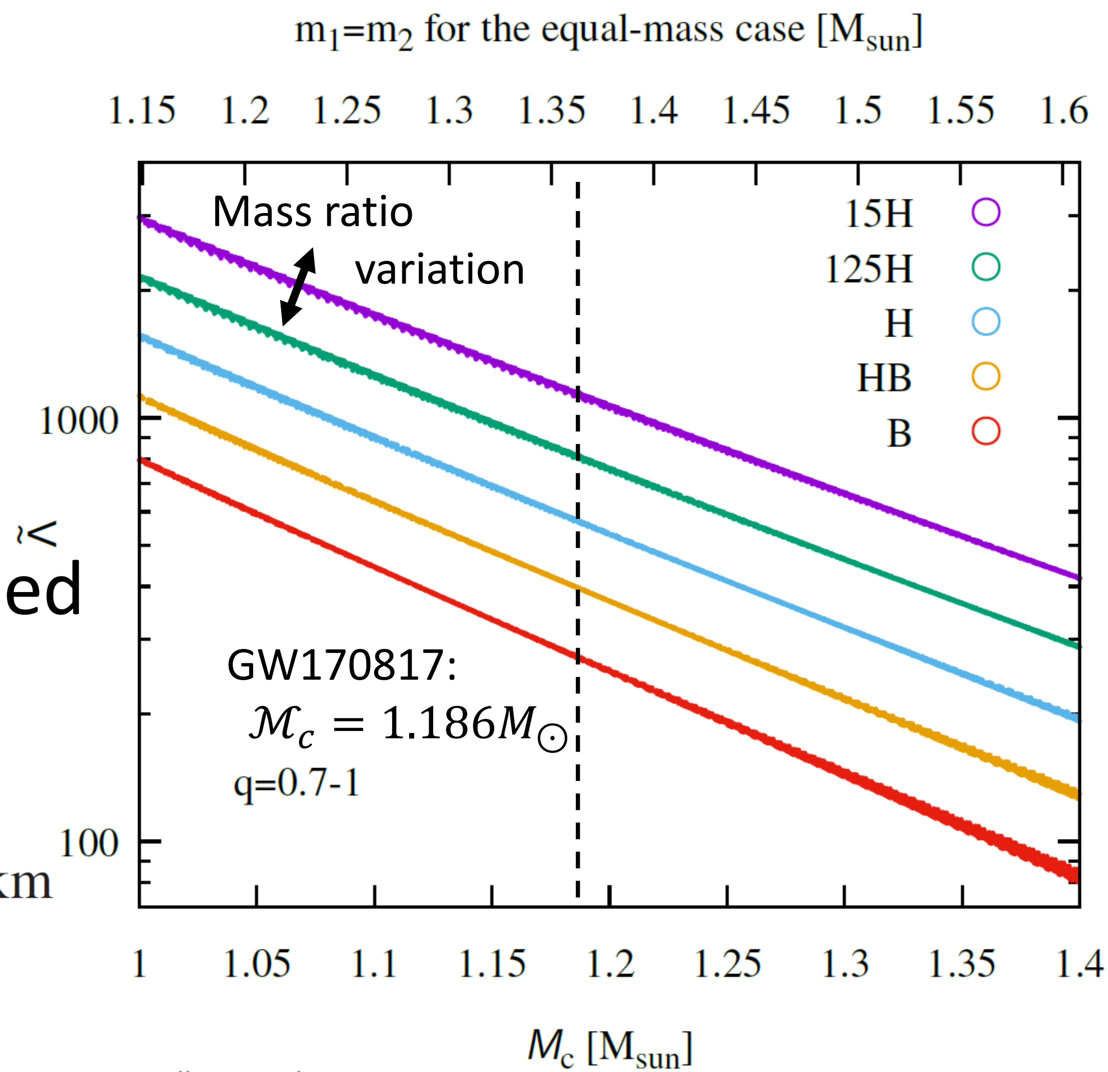
# Tight correlation of $\tilde{\Lambda} - \mathcal{M}_c$

GW-measured $\tilde{\Lambda}$ is tightly correlated w/ the chirp mass

$\Lambda\left(M = 2^{1/5}\mathcal{M}_c\right)$ is effectively constrained

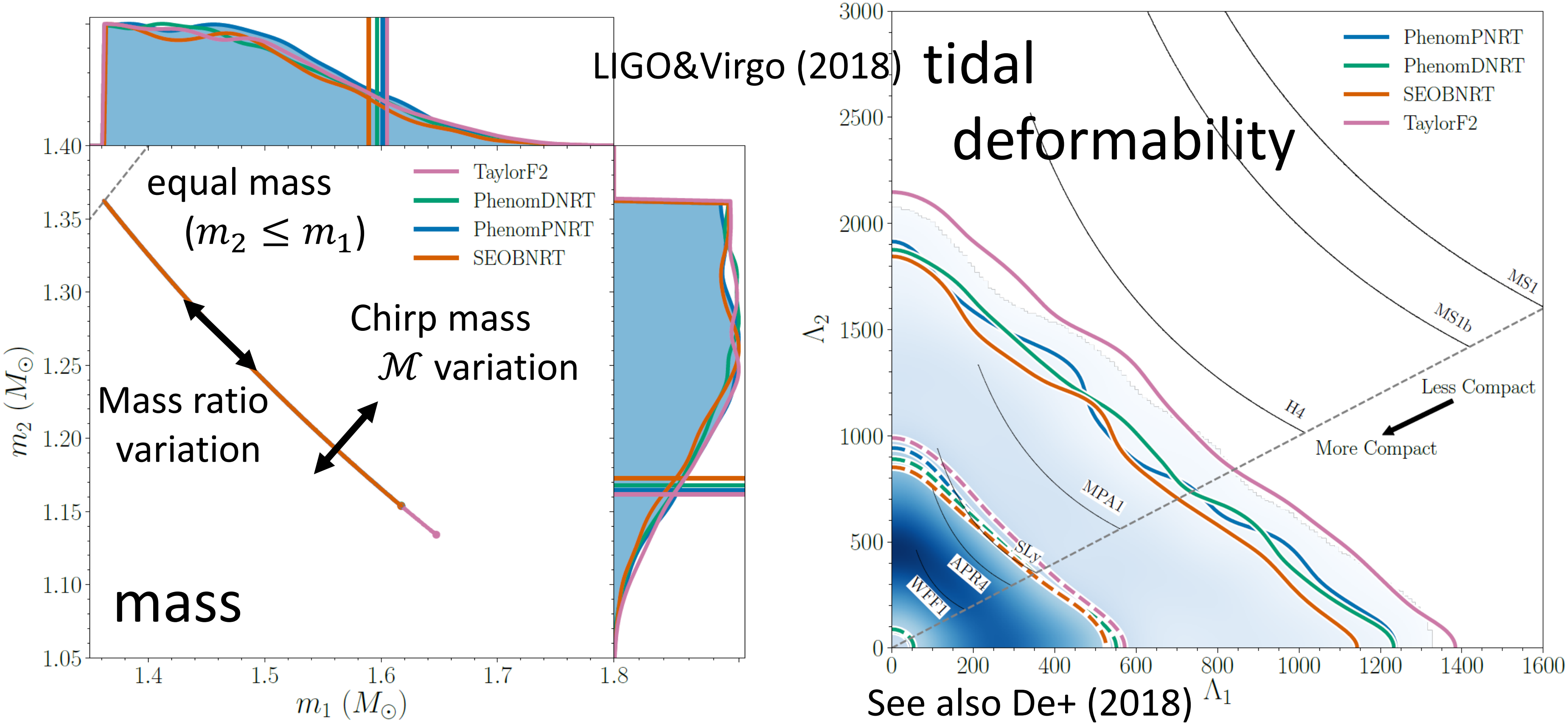
Approximately

$$R_{1.4} \simeq (11.2 \pm 0.2)\frac{\mathcal{M}}{M_\odot}\left(\frac{\tilde{\Lambda}}{800}\right)^{1/6}\ \mathrm{km}$$



# Constraints from GW170817

$100 < \tilde{\Lambda} < 800$ depending on waveform models, and the neutron star radius is about 10.5-13.5km




See also De+ (2018)

# Shape of mass constraints

Gravitational waves tightly constrain the chirp mass

$$\mathcal{M} = \frac{m_1^{3/5} m_2^{3/5}}{(m_1 + m_2)^{1/5}} = \mu^{3/5} M^{2/5}$$

But the mass ratio (e.g., $q = m_2/m_1 < 1$) tends to be degenerated with the spin of components,

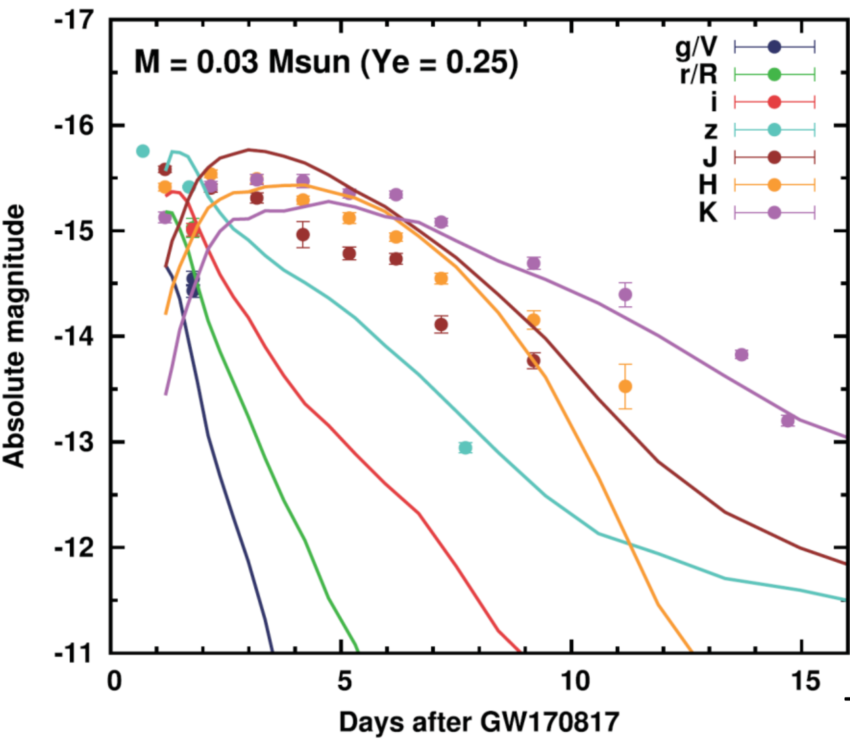
$$\chi_i = \frac{cS_i}{Gm_i^2} \quad (i = 1,2)$$

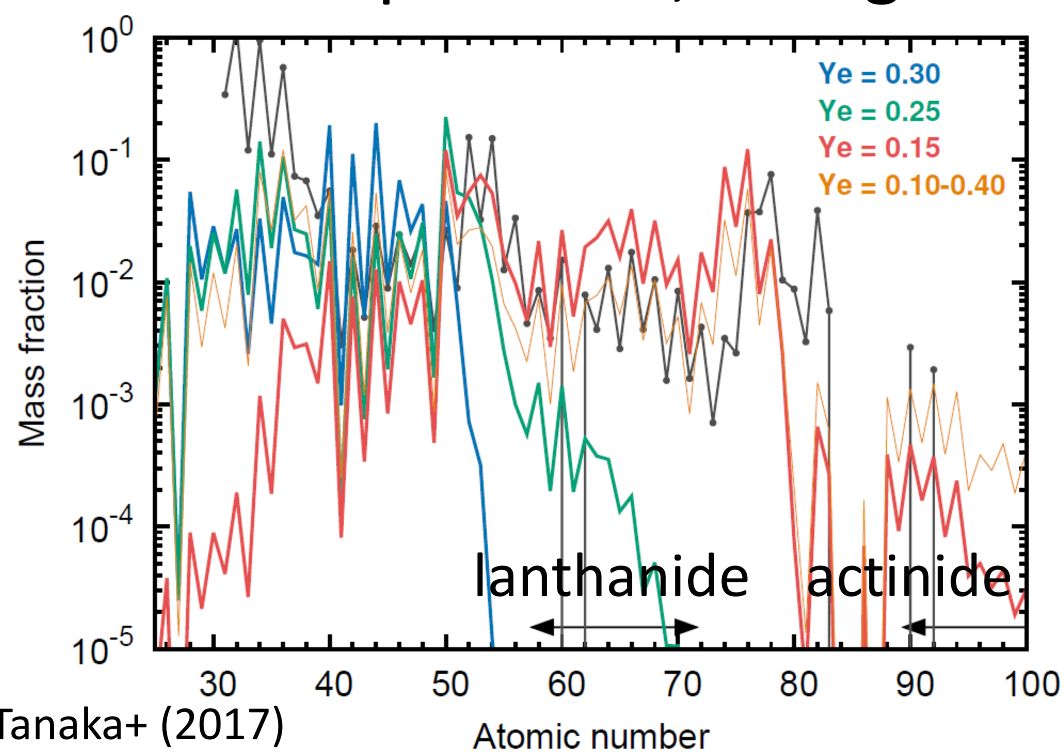
The error in $q$ appears large particularly for nearly equal-mass systems like binary neutron stars

# No indication of ultraheavy elements

A moderate amount of lanthanide is required but 3rd peak or actinides are not concretely detected

- it is simply hard to confirm their presence, though




Tanaka+ (2017)

# Maximum mass from GW170817

Upper limits are proposed based on assumptions

- Optical emission rejects magnetar models

Margalit-Metzger: $\leq 2.17 M_{\odot}$

Shibata+KK+: $2.15 - 2.25 M_{\odot}$

- A GRB jet launch calls for gravitational collapse

Rezzolla+, Ruiz+: $\leq 2.16 M_{\odot}$

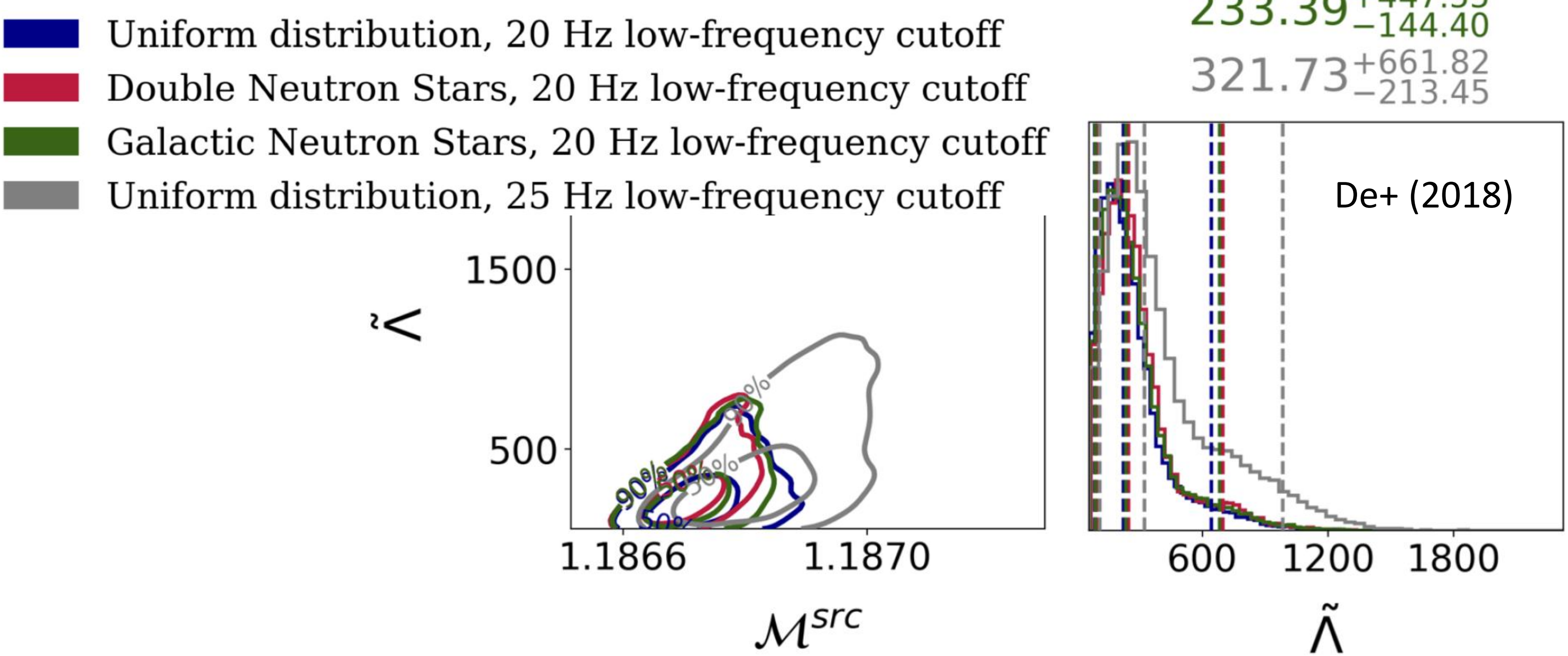
I do not think any argument is strongly convincing, but similar values are inferred anyway

# Low-frequency cutoff is also imporant

Degeneracy can be solved

and constraints become tight

$$\tilde{\Lambda} = 222.29^{+419.83}_{-138.48}$$
$$245.39^{+453.12}_{-151.53}$$
$$233.39^{+447.55}_{-144.40}$$
$$321.73^{+661.82}_{-213.45}$$



# Stacking estimation

~tidal deformability