# Present status of the pion-nucleon sigma term (and strange content of nucleon)

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J-PARC

# Introduction

## Definition:

$$\sigma_{\pi N} \equiv \frac{m_u + m_d}{2} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

Pion-nucleon sigma term

$$\sigma_s \equiv m_s \langle N | \bar{s}s | N \rangle$$

Strange content of nucleon

## Fundamental property of QCD:

- Current quark mass contribution to the nucleon mass (trace anomaly relation)

$$m_N = \frac{\beta_{\rm QCD}}{2g_s} \langle N | G_a^{\mu\nu} G_{\mu\nu,a} | N \rangle + \sum_q m_q \langle N | \bar{q}q | N \rangle$$

- Quark scalar matrix element $\langle N | \bar{q}q | N \rangle$ :

  ⇒ Shift from 3 is contribution from relativistic quarks inside nucleon

  ⇒ Probe of quark-gluon interaction and confinement in nucleons

## Applications:

- Important input in the baryon chiral perturbation theory
- Gives the variation of chiral condensate in low density nuclear matter
- Useful input in BSM search (especially in dark matter direct detection)

# How to calculate?

- **Chiral perturbation theory (ChPT) based extractions:**

  Start from Cheng-Dashen (CD) low energy theorem (LET), relate $\sigma_{\pi N}$ and πN scattering by using analyticity

  M. Hoferichter et al., Phys. Rev. Lett. **115**, 092301 (2015)
  J. Ruiz de Elvira et al., J. Phys. G **45**, 024001 (2017)

  Fit $\sigma_{\pi N}$ by calculating πN scattering phase shift in ChPT

  J. M. Alarcon et al., , Phys. Rev. D **85**, 051503(R) (2012)

- **Lattice QCD:**

  Direct calculation of the nucleon 3-point correlator

  RQCD, chiQCD, ETMC, JLQCD, …

  Derivative of the nucleon mass (Feynman-Hellmann theorem)

  BMW, Lutz et al., Ling et al., …

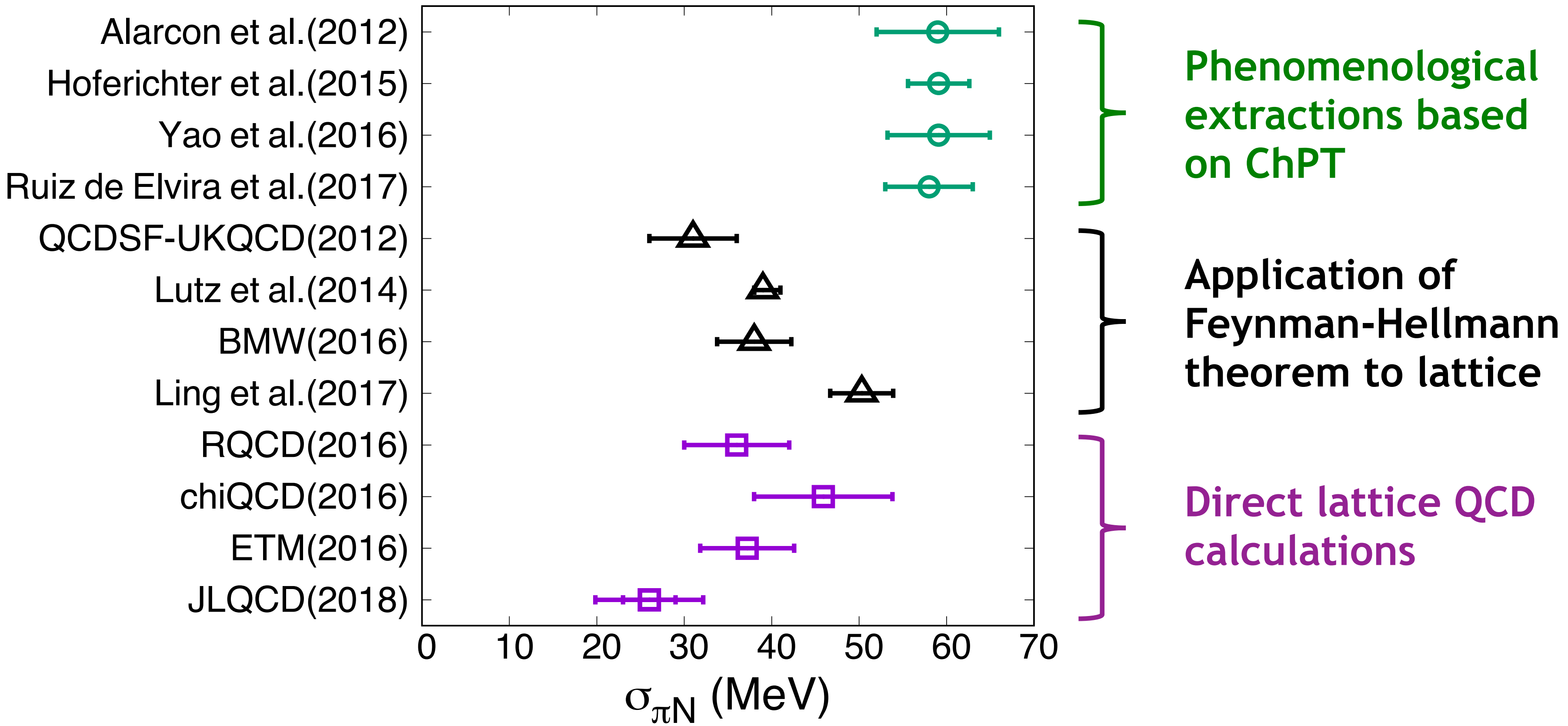
- **Perturbative QCD and fit from experimental data :**

  Integrate twist-3 parton distribution function $e_q(x)$ (uncertainty is large)

- **Other model approaches:**

  QCD sum rules, P. Gubler and K. Ohtani, Phys. Rev. D **90**, 094002 (2014)
  Dyson-Schwinger,
  etc.

# Present status of the calculations of πN sigma term

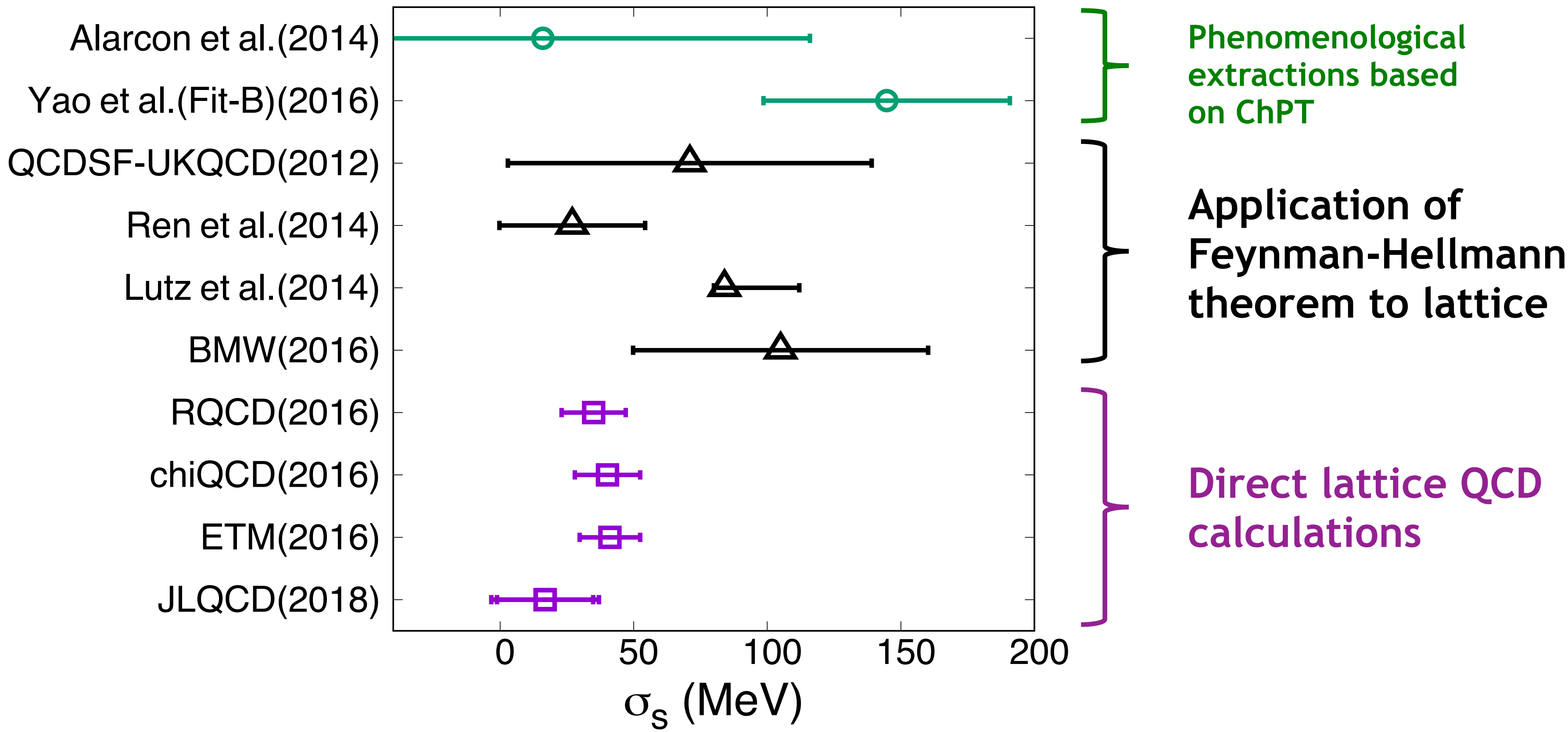



Phenomenological extractions : $\sigma_{\pi N} \sim 60$ MeV

Lattice QCD calculations : $\sigma_{\pi N} \sim 30$ MeV

**⇒ Visible disagreement between lattice QCD results and phenomenological extractions**

# Present status of the calculations of strange content




Lattice QCD calculations : $\sigma_s \sim 40$ MeV

Other approaches have large uncertainties

# Scope of this talk

The ChPT based phenomenological and lattice QCD predictions of the pion-nucleon sigma term differ significantly

⇒ A puzzle in QCD which has not yet convincing answer

Unfortunately, this is one of the argument used by one community (either ChPT or lattice) to claim that the other approach does not work

(´∀`;)

In this talk,
I will review the two approaches,
and let the audience consider it
in the hope to resolve it one day

# Chiral perturbation based approaches

# *Chiral perturbation based approaches*

- **Extraction from πN scattering experimental data (Cheng-Dashen low-energy theorem approach)**

- **Full calculation in chiral perturbation theory**

# Principle of $\sigma_{\pi N}$ extraction from Cheng-Dashen LET

**Consider the πN scattering :** $\pi^a(q) + N(p) \to \pi^b(q') + N(p')$

**Low-energy theorem (LET):** T. P. Cheng and R. F. Dashen, Phys. Rev. Lett. **26**, 594 (1971)

$$f_\pi^2 D^+(\nu = 0, t = 2m_\pi^2) - (\text{Born term}) - \Delta_R = \sigma(t = 2m_\pi^2)$$

$\nu \equiv \frac{s-u}{4m_N}$ $\sigma(t) \equiv \langle N(p')|m_{ud}(\bar{u}u + \bar{d}d)|N(p)\rangle$

***D***$^+$ **:** **Isoscalar, momentum-symmetric πN amplitude**

**Why subtract Born amplitude?** 

**⇒ Remove the massless QCD contribution** $\langle N(p')|\mathcal{L}_{\text{QCD}}(m_{ud} = 0)|N(p)\rangle$

$\Delta_R$ **:** **Higher order SU(2) ChPT correction to** ***D***$^+$

**Estimated to be small,** $O(\sigma_{\pi N} m_\pi^2 / m_N^2)$

V. Bernard et al., Phys. Lett. B **389**, 144 (1996)

**Important point :**

**πN sigma term is** $\sigma(t = 0)$**, not** $\sigma(t = 2m_\pi^2)$ **!!**

$D^+(\nu = 0, t = 0)$ **corresponds to an off-shell, unphysical point!**

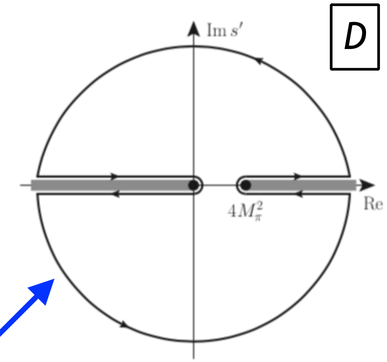
**We must extrapolate the amplitude to *t* = 0 !**

# Roy-Steiner equation

We can analytically extrapolate $\sigma_{\pi N}$ to off-shell region

We consider a system of “subtracted“ dispersion relations (DRs) with channel couplings : $\pi N \to \pi N$, $\pi\pi \to K\bar{K}$, $\pi\pi \to N\bar{N}$,...

$$D(s,t) = c(t) + \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \left[\frac{s^2}{s'^2(s'-s)} + \frac{u^2}{s'^2(s'-u)}\right] \mathrm{Im}\, D(s',t). + \ldots$$




DRs respects analyticity, unitarity, crossing symmetry,

⇒ To make converge the integral (get rid of the contour),

we set additional polynomial in the denominator (“subtraction”)

⇒ Due to Cauchy’s theorem, additional terms ($c(t)$) appear

⇒ If we can identify them with known quantities,

we can constrain the analytic form of $D^+$

(e.g. use scattering length for s-wave $D^+$ at $t$=0)

⇒ The analytic continuation has limits, but the error is quantifiable

by partial wave decomposition, changing subtraction, considering resonances, etc


J. Gasser et al., PLB **253**, 252 (1991);
M. Hoferichter et al., Phys. Rep. **625**, 1 (2016).

# Overall : result of CD LET approach

**Synopsis of the pion-nucleon sigma term:**

$$\sigma_{\pi N} \equiv \sigma(0) = \Sigma_d + \Delta_D - \Delta_\sigma - \Delta_R$$

$$\Sigma_d \equiv f_\pi^2 (d_{00}^+ + 2m_\pi^2 d_{01}^+)$$

**Lowest terms the Taylor and partial wave expansion** $\bar{D}^+(\nu, t) = \sum_{n,m=0}^{\infty} d_{mn}^+ \nu^{2m} t^n$

$$\Delta_D \equiv \bar{D}^+(0, 2m_\pi^2) - \Sigma_d$$

**Determined by DR analysis (Roy-Steiner eq.)**

$$\Delta_\sigma \equiv \sigma(2m_\pi^2) - \sigma_{\pi N}$$

**Can be determined from DR analysis as well**

J. Gasser et al., PLB **253**, 260 (1991)

**Small due to the cancellation (both are ∼15 MeV):**

$$\Delta_D - \Delta_\sigma = (-1.8 \pm 0.2)\,\mathrm{MeV}$$

**Determined from Roy-Steiner analysis**

$\Delta_R$ **(Uncertainty of ChPT)**

**< 2 MeV**

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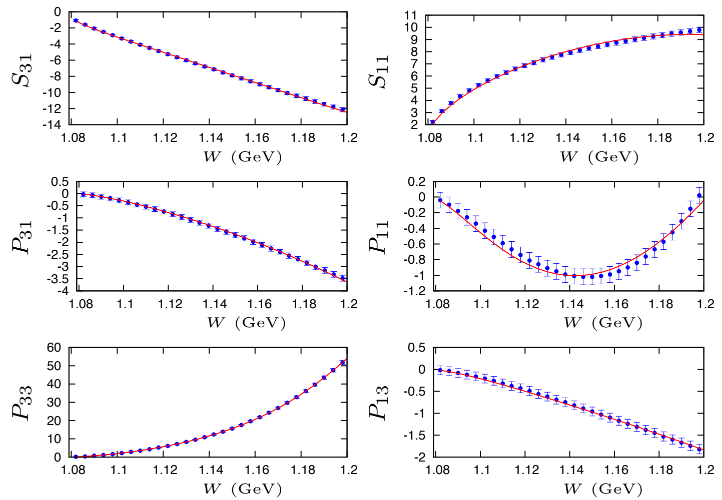
V. Bernard et al., Phys. Lett. B **389**, 144 (1996)

**Total :** $\sigma_{\pi N} = (59.1 \pm 3.5)\,\mathrm{MeV}$

M. Hoferichter et al., PRL **115**, 092301 (2015).

# Full calculation in chiral perturbation theory




G. Washington Group data set and O(p³) ChPT fit of πN scattering phase shift

Extract low energy constants ($c_1$, $c_2$, $c_3$, $c_4$) from $O(p^2)$ Lorentz covariant ChPT

Fits for several πN scattering data sets :

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| KH | −0.80(6) | 1.12(13) | −2.96(15) | 2.00(7) |
| GW | −1.00(4) | 1.01(4) | −3.04(2) | 2.02(1) |
| EM | −1.00(1) | 0.58(3) | −2.51(4) | 1.77(2) |

(unit: $GeV^{-1}$)

$\sigma_{\pi N}$ at $O(p^3)$ in covariant ChPT at in extended on-shell mass scheme :

$$\sigma_{\pi N} = -4c_1 m_\pi^2 - \frac{3g_A^2 m_\pi^3}{16\pi^2 f_\pi^2 M_N}\left(\frac{3M_N^2 - m_\pi^2}{\sqrt{4M_N^2 - m_\pi^2}}\arccos\frac{m_\pi}{2M_N} + m_\pi \log\frac{m_\pi}{M_N}\right)$$

Data sets of G. Washington Group :

➡ $\sigma_{\pi N}$ = 59(7)MeV

Consistent with the result of CD LET approach!

J. M. Alarcon et al., Phys. Rev. D **85**, 051503 (2012)

# Lattice QCD approaches

# *Lattice QCD approaches*

- **Direct calculation of πN sigma term**

- **Feynman-Hellmann theorem calculation**

## Formulation:

$$\langle N \mid \bar{q}q \mid N \rangle = Z_S \lim_{\substack{|t_{snk}-t_{src}|,\\ |t_{vtx}-t_{src}| \to \infty}} 2m_N \frac{\langle 0 \mid N(y,t_{snk})\, (\bar{q}q)(x,t_{vtx})\, N^\dagger(0,t_{src}) \mid 0 \rangle}{\langle 0 \mid N(y,t_{snk})\, N^\dagger(0,t_{src}) \mid 0 \rangle}$$

$Z_S$ : nonperturbative renormalization ($\mu$ = 2 GeV)

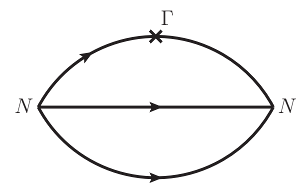
(need careful treatment, see next page)

⇒ Ratio of 3pt and 2pt functions

## 3-point function on lattice:

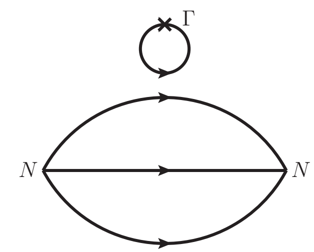
Mandatory techniques:

- Operator improvement
  (smearing, etc)
- Noise method
  (for Dirac high modes)
- Truncated solver method
  (effectively ↑ statistics)
- ...



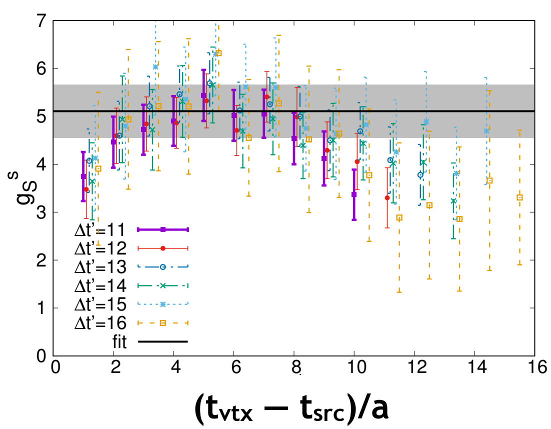

Connected diagrams

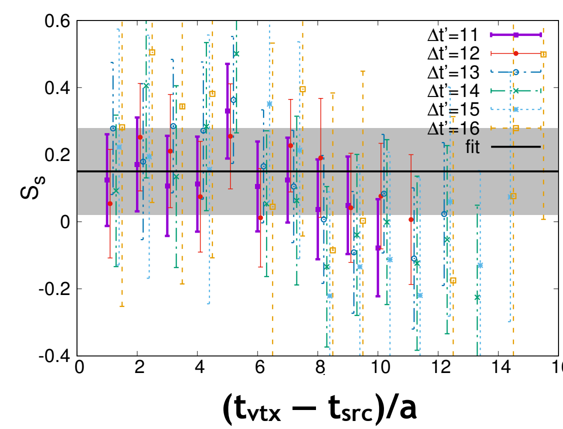
(Leading contribution of $\sigma_{\pi N}$)




Disconnected diagram

(strange content)



# Renormalization of $\bar{q}q$ and operator mixing

**Renormalizations of flavor singlet and nonsinglet are in general different:**

$$\begin{cases} (\bar{q}q)_{\rm phys} = Z_0(\bar{q}q) \\ (\bar{u}u + \bar{d}d - 2\bar{s}s)_{\rm phys} = Z_8(\bar{u}u + \bar{d}d - 2\bar{s}s) \end{cases}$$

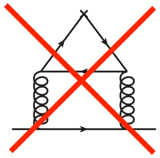
**Renormalizations of ūu+d̄d:** $(\bar{u}u + \bar{d}d)_{\rm phys} = \frac{2}{3}\left[\left(Z_0 + \frac{1}{2}Z_8\right)\underline{(\bar{u}u + \bar{d}d)} + (Z_0 - Z_8)\underline{(\bar{s}s)} + \underline{\frac{b_0}{a^3}} + \cdots\right]$

**Renormalizations of s̄s:** $(\bar{s}s)_{\rm phys} = \frac{1}{3}\left[(Z_0 + 2Z_8)\underline{(\bar{s}s)} + (Z_0 - Z_8)\underline{(\bar{u}u + \bar{d}d)} + \underline{\frac{b_0}{a^3}} + \cdots\right]$

Disconnected diagram (Small) — Connected diagram (Large) — Divergence due to quark loop (mixing with identity operator)

- **In Wilson-type fermion, $Z_0 - Z_8 \neq 0$ :**
  - **s̄s operator mixes with the large connected diagram and identity operator!**
    - **⇒ $(\bar{s}s)_{\rm phys}$ affected by the increase of statistical error due to subtraction**
  - **ūu+d̄d operator is less affected since connected diagram is the largest**

- **In chiral fermion (DW,overlap) with massless renormalization scheme, the trace of quark loop (disconnected diagram) vanishes ⇒ $Z_0 = Z_8$**
  - **⇒ No need of subtraction for $\bar{q}q$ operators!**



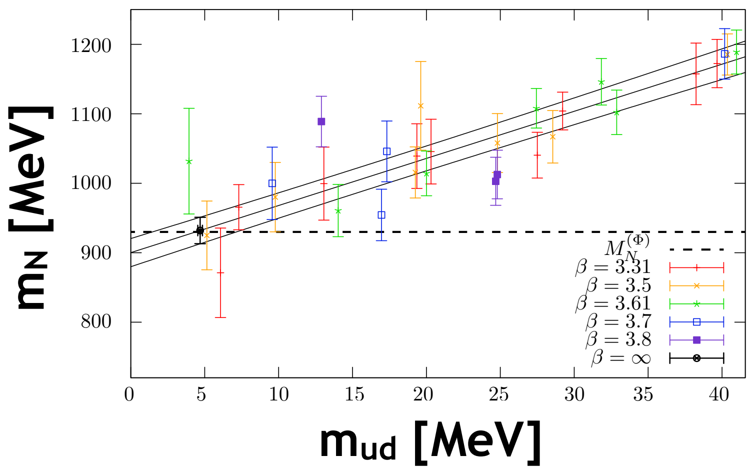
JLQCD Collaboration, PRD **83**, 114506 (2011); PRD **98**, 054516 (2018)

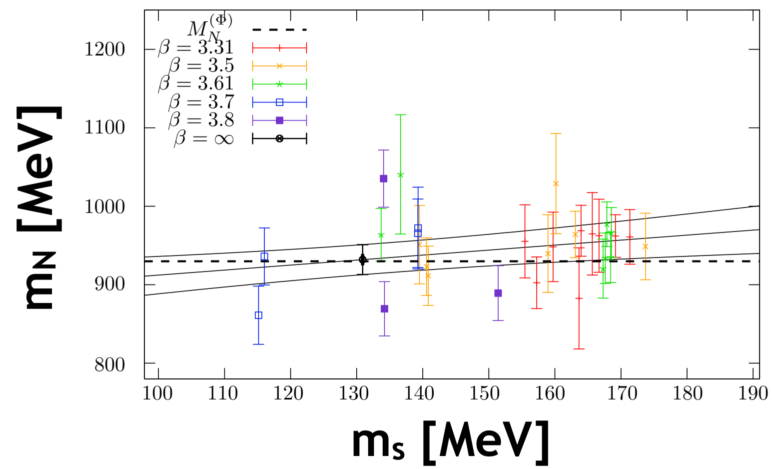
# Lattice QCD : Feynman-Hellmann theorem approach

On lattice, one can artificially change the current quark mass ($m_q$)

**Feynman-Hellmann theorem:**

$$\frac{\partial m_N}{\partial m_q} = \langle N|\bar{q}q|N\rangle$$




BMW Collaboration, PRL **116**, 172001 (2016)

- Feynman-Hellmann theorem may be combined with ChPT :

  Fit $m_{ud}$ dependence of $m_N$ in ChPT with lattice data and take derivative

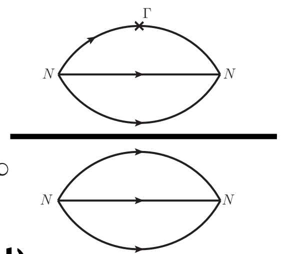
  (see Lutz (2014), Ren (2015), Ling (2017))

- Correct renormalization needed for Wilson-type quark (see next page)

# Issue of renormalization in Feynman-Hellmann approach

**From Feynman-Hellmann theorem, we have**

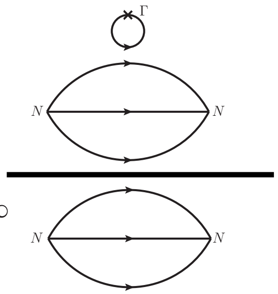
$$\frac{\partial(am_N)}{\partial m^b_{\rm val}} = \lim_{\substack{|t_{\rm snk}-t_{\rm src}| \\ |t_{\rm xyz}-t_{\rm src}|} \to \infty}$$




**(connected)**

**and**

$$\frac{\partial(am_N)}{\partial m^b_{\rm sea}} = -N_f \lim_{\substack{|t_{\rm snk}-t_{\rm src}| \\ |t_{\rm xyz}-t_{\rm src}|} \to \infty}$$




**(disconnected)**

## Renormalization of quark mass:

$$\begin{cases} m^b_{\rm val} = m_A + am_{\rm val} \\ m^b_{\rm sea} = m_A + am_{\rm sea} \end{cases}$$

(bare mass)

***$m_A$*** **: additive mass renormalization (in Wilson-type fermion)**
**(must be determined for each gauge ensemble)**

- **Connected : simple derivative of valence q mass** $\frac{\partial m_N}{\partial m_{\rm val}} = \frac{\partial(am_N)}{\partial m^b_{\rm val}}$
- **Disconnected : derivative of sea q mass have several terms!**

$$\frac{\partial m_N}{\partial m_{\rm sea}}(1 - am_{\rm sea}B - X) = \frac{\partial(am_N)}{\partial m^b_{\rm sea}} + (am_{\rm sea}B + X)\underline{\frac{\partial(am_N)}{\partial m^b_{\rm val}}} - m_N B$$

$$B \equiv \frac{\partial(\ln a)}{\partial m^b_{\rm sea}} \qquad X \equiv \frac{\partial m_A}{\partial m^b_{\rm sea}}$$

**⇒ Mixing with valence quark contribution**
**(consequence of additive mass renormalization)**

**(Note that lattice spacing depends also on $m^b_{\rm sea}$ )**

C. Michael et al. (UKQCD Collaboration), NPB Proc. Suppl. **106**, 293 (2002).

# Both approaches seem robust…
# Then what to do??

# *Road to the resolution of the puzzle*

- Calculation of πN (ππ, NN, …) scattering phase shift(s) in lattice QCD

  Input of Cheng-Dashen LET approach with Roy-Steiner analysis

  Check the consistency with the direct calculation

  ⇒ HAL QCD method (see talks of T. Hatsuda, K. Sasaki)

  What if consistent ?
  - If yes : Lattice QCD has trouble with exp. data
  - If no : ChPT based extractions have problems

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  ⇒ Did we overlook contribution beyond SU(3) ChPT?

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- Direct measurement in high-energy QCD experiments?

  The integration of the twist-3 quark distribution $e_q(x)$ gives the sigma term

  ⇒ Measure $e_{u,d}(x)$ in experiments to determine $\sigma_{\pi N}$

  ⇒ Measurement of $e_s(x)$ may provide the first experimental $\langle N|\bar{s}s|N\rangle$

  (Compete with the QCD sum rules based extraction from J-PARC exp.,
  ask P. Gubler for detail)

# Summary

- A review of πN sigma term and strange content.
- An important puzzle in the calculations of $\sigma_{\pi N}$ :
  ChPT based phenomenological approach vs lattice QCD
  ⇒ Do not agree, but both seem OK…
- The evaluation of $\langle N|\bar{s}s|N\rangle$ on lattice requires a careful renormalization : consistent among modern lattice QCD works, with relatively small absolute value.

## Future prospects:

- Lattice calculation of ππ, πN, NN, πK, … phase shifts will give a strong hint whether ChPT approach is OK or not.
- The high-energy experimental measurement of the twist-3 PDF $e_q(x)$ will unveil the true value of $\sigma_{\pi N}$.
- Could you give any other ideas?