中国科学院高能物理研究所

ρ meson generalized parton distributions (GPDs) and its structure functions

Yubing Dong

Institute of High Energy Physics(IHEP)

Collaborator: (Baodong Sun)

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Narinder Kumar: Transverse Charge densities of ρ-meson in light-front quark model

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Outline

1, Introduction

2, Spin 1 particle and basic properties

3, Light-front constituent quark model

4, GPDs of ρ-meson (unpolarized and polarized)

4, Impact parameter space

5, Summary

1, Introduction

Electromagnetic probes

- Electric and magnetic proton form factors
- Proton and Neutron charge distributions
- Nucleon spin structure
- Nucleon-Delta transition (other resonances)
- Quark-hadron duality in structure functions
- Generalized parton distributions
- Pion and deuteron form factors

GPDs (generalized parton distributions)

GPDs $H_q(x,\xi,Q^2)$ naturally embody the information of both PDFs and FFs, and therefore display the unique properties to present a "3D" description for a system.

GPDs allow for a unified description of a number of hadronic properties; for example:

(1) In the forward limit they reduce to conventional PDFs

$$H_q(x,0,0) = q(x)\,,$$

$$\tilde{H}_q(x,0,0) = \Delta q(x)\,.$$

(2) When one integrates GPDs over x they reduce to the usual form factors, e.g. the Dirac form factors[a]

$$\sum_q e_q \int dx\, H_q(x,\xi,t) = F_1(t)\,,$$

$$\sum_q e_q \int dx\, E_q(x,\xi,t) = F_2(t)\,.$$

GPDs (generalized parton distributions)

GPDs for pion,

Broniowski, PLB 574, PRD78; Choi et al., PRD64; Fanelli, EPJC76; ……

for nucleon (proton and neutron)

Diehl et al., EPJC 73; Kroll, EPJA53; Pire et al., PRD79; Selyugin, PRD91;……

Light Nuclei: He-3

Rinaldi et al., PRC87……

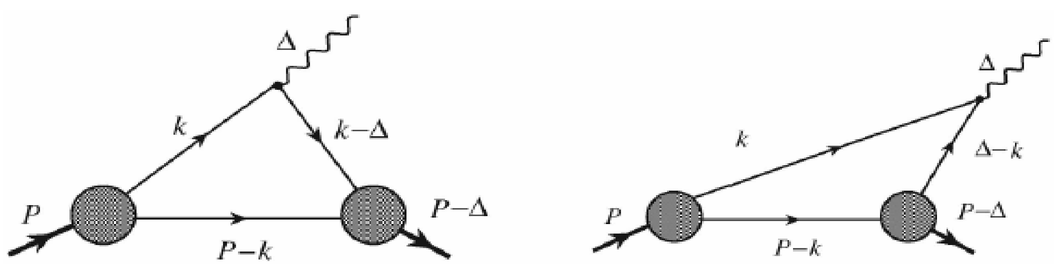
Deuteron

Cano et al., PRL87,Cosyn18……

ρ-meson: Narinder Kumar: Transverse Charge densities of ρ-meson…

Generalized Parton distributions for pion

Broniowski, PLB574,In the limit of ξ=0



Covariant amplitude with a reduced photon vertex for pion GPD (left diagram) and its nonvalence $x < \zeta$ part (right diagram).

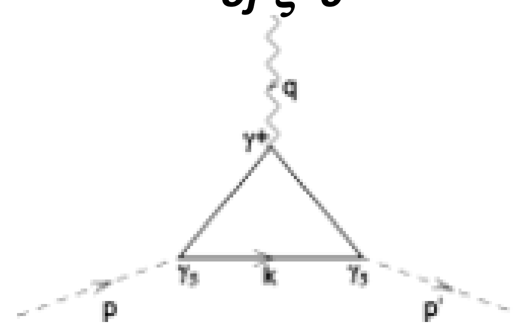
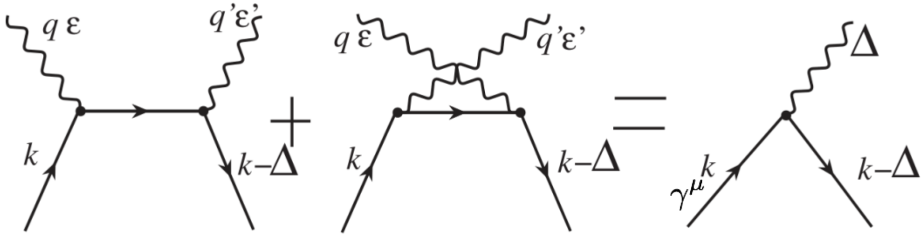


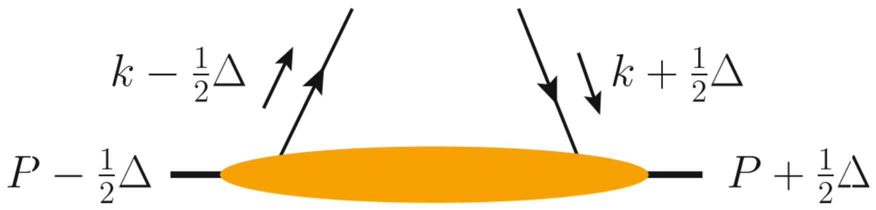
Fig. 1. The diagram for the evaluation of the generalized parton distribution of the pion in chiral quark models.

GPDs (generalized parton distributions)

Deep virtual Compton Scattering

[Chueng-Ryong Ji '06, Diehl '16]





A GPD factorization formula:

DVCS, TCS, meson production

$$\mathcal{A}(\xi, \Delta^2, Q^2) = \sum_i \int_{-1}^{1} \mathrm{d}x\, C_i\big(x, \xi; \log(Q/\mu)\big) H_i(x, \xi, \Delta^2; \mu)$$

It may be measured by
Deeply virtual Compton scattering
Or
Deeply virtual meson electro-productions

Parton correlation function:

flavor by flavor

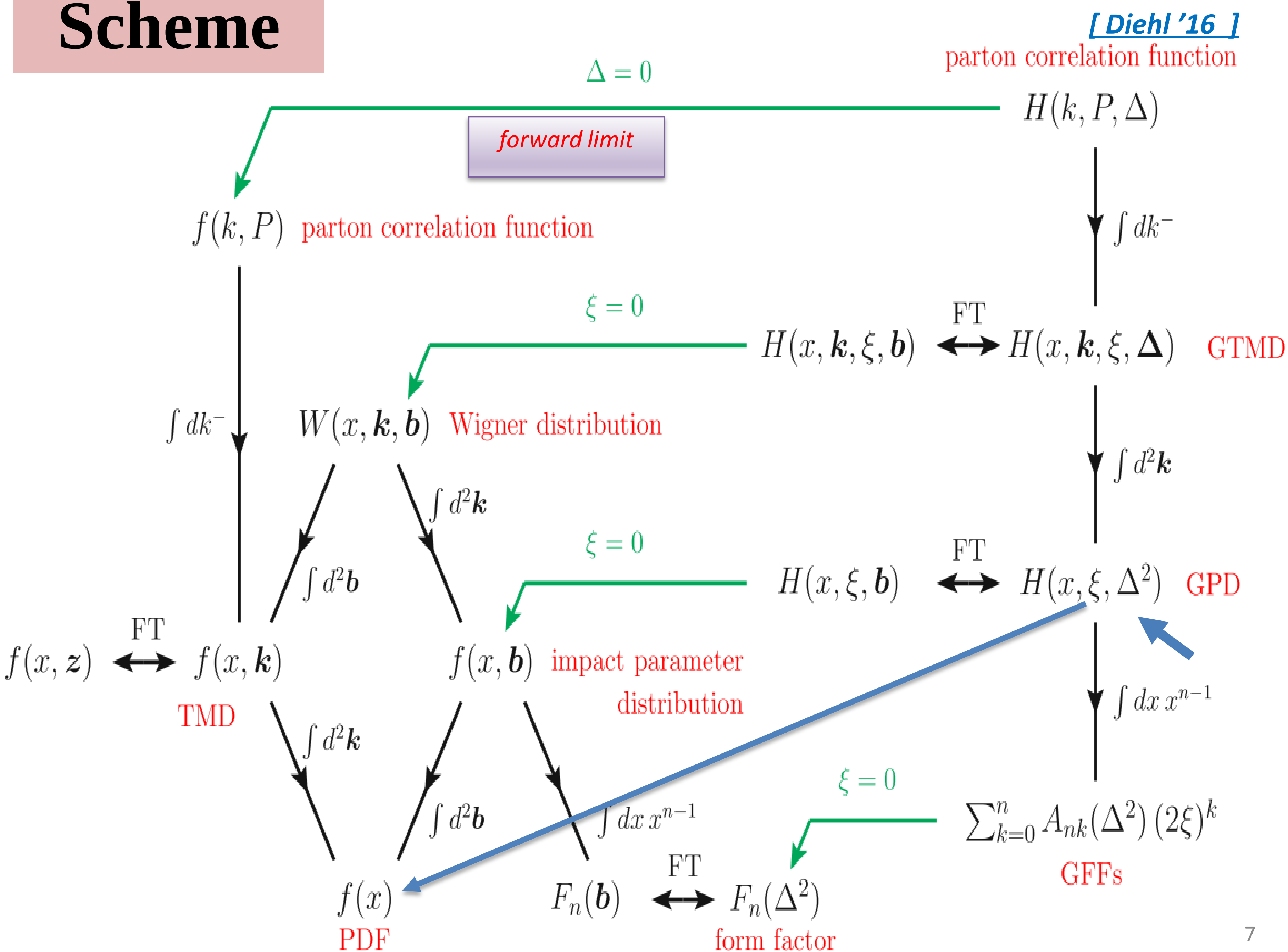
$$H(k, P, \Delta) = (2\pi)^{-4} \int d^4z\, e^{izk} \times \langle p(P+\tfrac{1}{2}\Delta)|\bar{q}(-\tfrac{1}{2}z)\Gamma q(\tfrac{1}{2}z)|p(P-\tfrac{1}{2}\Delta)\rangle$$

Gauge $A^+=0$

The Dirac matrix Γ selects the twist and the parton spin degrees of freedom.

$$\Gamma^\mu \to \gamma^\mu$$

Scheme



[Diehl '16]
parton correlation function
$\Delta = 0$
$H(k, P, \Delta)$
forward limit
$f(k, P)$ parton correlation function
$\int dk^-$
$\xi = 0$
FT
$H(x, \boldsymbol{k}, \xi, \boldsymbol{b})$
$H(x, \boldsymbol{k}, \xi, \boldsymbol{\Delta})$ GTMD
$\int dk^-$
$W(x, \boldsymbol{k}, \boldsymbol{b})$ Wigner distribution
$\int d^2\boldsymbol{k}$
$\int d^2\boldsymbol{k}$
$\int d^2\boldsymbol{b}$
$\xi = 0$
FT
$H(x, \xi, \boldsymbol{b})$
$H(x, \xi, \Delta^2)$ GPD
FT
$f(x, \boldsymbol{z})$
$f(x, \boldsymbol{k})$
TMD
$f(x, \boldsymbol{b})$ impact parameter distribution
$\int dx\, x^{n-1}$
$\int d^2\boldsymbol{k}$
$\int d^2\boldsymbol{b}$
$\int dx\, x^{n-1}$
$\xi = 0$
$\sum_{k=0}^{n} A_{nk}(\Delta^2)\,(2\xi)^k$
GFFs
FT
$f(x)$
PDF
$F_n(\boldsymbol{b})$
$F_n(\Delta^2)$
form factor

2, Spin-1 particle and basic properties

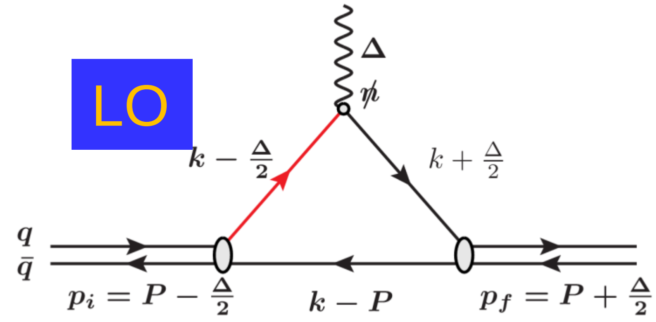
Definition of GPDs (spin--1)

- **Unpolarized**

[PRL: Berger '01 , for the deuteron]

$$V_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi}\, e^{ix(Pz)} \langle p', \lambda' |\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}\, q(\tfrac{1}{2}z)\, | p, \lambda\rangle \Big|_{z=\omega n}$$

$$= \sum_i \epsilon'^{*\nu} V^{(i)}_{\nu\mu} \epsilon^{\mu} H^q_i(x,\xi,t)$$



- **Polarized**

$$A_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi}\, e^{ix(Pz)} \langle p', \lambda' |\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}\gamma_5\, q(\tfrac{1}{2}z)\, | p, \lambda\rangle \Big|_{z=\omega n}$$

$$= \sum_i \epsilon'^{*\nu} A^{(i)}_{\nu\mu} \epsilon^{\mu} \tilde{H}^q_i(x,\xi,t)$$

$\mathbf{V_{\mu\nu}}$: $\{g_{\mu\nu}, P_\mu n_\nu, P_\nu n_\mu, P_\mu P_\nu, n_\mu n_\nu\}$

Unpolarized (5)
Polarized (4)

$$P = \frac{p'+p}{2}, \quad t = \Delta^2 = (p'-p)^2,$$

$$n^2 = 0, \text{ (lightlike four-vector)}$$

$$\xi = (n\cdot\Delta)/(n\cdot P), \quad \text{skewness parameter},$$

$$\epsilon = \epsilon(p,\lambda), \ \epsilon' = \epsilon'(p',\lambda'), \text{ polarizations},$$

Symmetry properties:

T

$$H_i(x,\xi,t) = H_i(x,-\xi,t) \quad (I = 1,2,3,5)$$

$$H_4(x,\xi,t) = -H_4(x,-\xi,t)$$

$$\tilde{H}_i(x,\xi,t) = \tilde{H}_i(x,-\xi,t) \quad (I = 1,2,4)$$

$$\tilde{H}_3(x,\xi,t) = -\tilde{H}_3(x,-\xi,t)$$

G

$$H^d_{\rho^+}(x,\xi,t) = -H^u_{\rho^+}(x,-\xi,t)$$

Sum rules, Spin—1 particle

[Frederico '97, Berger '01, Broniowski '08]

- **Form factor decomposition of Local current**

FFs in flavor

$$I^{\mu}_{\lambda'\lambda} = \langle p', \lambda' | \, \bar{q}(0) \, \gamma^{\mu} \, q(0) \, | p, \lambda \rangle$$
$$= \epsilon'^{*\beta} \epsilon^{\alpha} \Big[- \Big(G_1^q(t) g_{\beta\alpha} + G_3^q(t) \frac{P_\beta P_\alpha}{2M^2} \Big) P^{\mu} + G_2^q(t) \Big(g^{\mu}_{\alpha} P_\beta + g^{\mu}_{\beta} P_\alpha \Big) \Big]$$

- **Sum rules**

$$\int_{-1}^{1} dx H_i^q(x, \xi, t) = G_i^q(t) \quad (i = 1, 2, 3) \, ,$$
$$\int_{-1}^{1} dx H_i^q(x, \xi, t) = 0 \quad (i = 4, 5) \, .$$

- **Conventional Form factors**

$$G_C(t) = G_1(t) + \frac{2}{3} \eta G_Q(t) \, ,$$
$$G_M(t) = G_2(t) \, ,$$
$$G_Q(t) = G_1(t) - G_2(t) + (1 + \eta) G_3(t) \, ,$$

Forward limit

[Hoodbhoy '89, Berger '01 , Cosyn'17]

- **GPDs in forward limit**

$$H_1(x,0,0) = \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3},$$

$$H_5(x,0,0) = q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2},$$

for x > $\tilde{H}_1(x,0,0) = q^1_\uparrow(x) - q^{-1}_\uparrow(x)$

Quark densities:

$$q^\lambda(x) = q^\lambda_\uparrow(x) + q^\lambda_\downarrow(x)$$

$$q^\lambda_\uparrow = q^{-\lambda}_\downarrow$$

H1 --H5 for x < 0, antiquark

- DIS structure functions

$$F_1(x) = \frac{1}{2}\sum_q e_q^2 \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3} + \{q \to \bar{q}\},$$

$$b_1(x) = \frac{1}{2}\sum_q e_q^2\left[q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2}\right] + \{q \to \bar{q}\}$$

$$g_1(x) = \frac{1}{2}\sum_q e_q^2[q^1_\uparrow(x) - q^{-1}_\uparrow(x)] + \{q \to \bar{q}\}.$$

- Single-flavor $F_1^{q\uparrow(\downarrow)}$, $b_1^{q\uparrow(\downarrow)}$

Callan-Gross relation

$$F_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_1^u(x,0,0)$$

$$b_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_5^u(x,0,0)$$

$W^{\mu\nu} \sim F_1, F_2, g_1, g_2$

b_1, b_2, b_3, b_4

3, Light-front constituent quark model

Isospin combinations

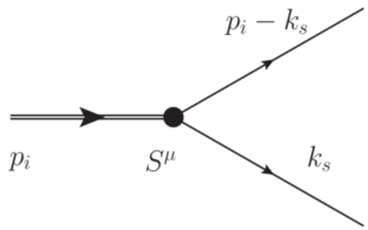
[Berger '01, Frederico '09 , Bronioski'03]

- **Effective Chiral Lagrangian:**

$$\mathcal{L}_{\rho\to q\bar{q}} = -\imath (M/f_\rho)\bar{q} S^\mu \tau q \cdot \rho_\mu = -\imath (M/f_\rho)\left[\bar{u} S^\mu u \rho^0_\mu + \sqrt{2}\bar{u} S^\mu d \rho^+_\mu + \sqrt{2}\bar{d} S^\mu u \rho^-_\mu + \bar{d} S^\mu d \rho^0_\mu\right]$$

- **Quark field doublets:**

$$q(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}, \quad \tau_3 q(x) = \begin{pmatrix} u(x) \\ -d(x) \end{pmatrix}$$



- **5 un-polarized GPDs: Isospin combinations**

$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix(Pz)} \langle \rho^b(p',\lambda')|\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}\, \tau_3 q(\tfrac{1}{2}z)\, |\rho^a(p,\lambda)\rangle\Big|_{z=\lambda n} = \imath \epsilon_{3ab}\Big\{ -(\epsilon'^* \cdot \epsilon)\, H^{I=1}_{1,\rho^b}$$
$$+ \frac{(\epsilon\cdot n)(\epsilon'^*\cdot P) + (\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n} H^{I=1}_{2,\rho^b} - \frac{2(\epsilon\cdot P)(\epsilon'^*\cdot P)}{m^2} H^{I=1}_{3,\rho^b}$$
$$+ \frac{(\epsilon\cdot n)(\epsilon'^*\cdot P) - (\epsilon'^*\cdot n)(\epsilon\cdot P)}{P\cdot n} H^{I=1}_{4,\rho^b} + \left[m^2 \frac{(\epsilon\cdot n)(\epsilon'^*\cdot n)}{(P\cdot n)^2} + \frac{1}{3}(\epsilon'^*\cdot\epsilon)\right] H^{I=1}_{5,\rho^b} \Big\}$$

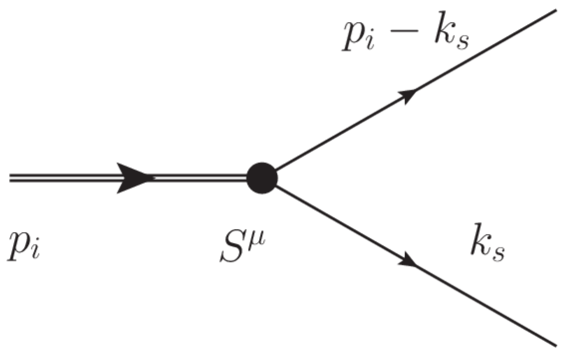
$H_1 \to F_1$
$H_5 \to b_1$
$\widetilde{H}_1 \to g_1$

Isospin combinations: $$H^{I=1}_{i,\rho^\pm}(x,\xi,t) = \frac{1}{2}[H^u_{i,\rho^\pm}(x,\xi,t) - H^d_{i,\rho^\pm}(x,\xi,t)]$$

G parity: $$H^d_{\rho^+}(x,\xi,t) = -H^u_{\rho^+}(x,-\xi,t)$$

Phenomenological vertex ρ meson

[Choi '04, Frederico '09]



$$x' = \frac{-k_s^+}{p_i^+}$$

$$\kappa_\perp = k_{s\perp} - \frac{k_s^+}{p_i^+} p_{i\perp}$$

Phenomenal vertex:

$$S^\mu = \Gamma^\mu \Lambda(k_s, p)$$

Bethe-Salpeter amplitude(BSA):

$$\Lambda(k_s, p) = \frac{c}{[k_s^2 - m_R^2 + \imath\epsilon][(p-k_s)^2 - m_R^2 + \imath\epsilon]}$$

S-wave

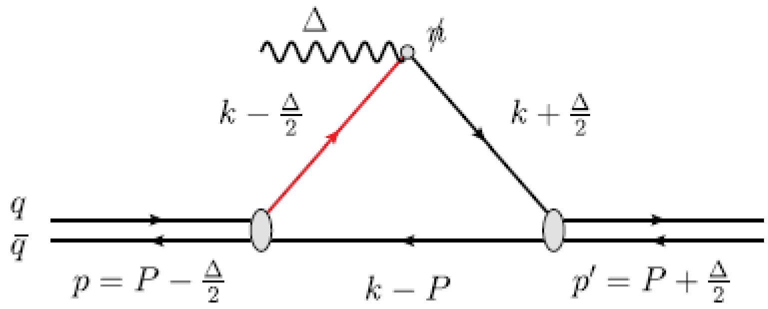
Meson vertex:

$$\Gamma^\mu = \gamma^\mu - \frac{(k_q + k_{\bar{q}})^\mu}{M_0 + 2m}$$

Dispersion relation

Kinematic invariant mass:

$$M_0^2 = \frac{\kappa_\perp^2 + m^2}{1 - x'} + \frac{\kappa_\perp^2 + m^2}{x'}$$



$$V_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi}\, e^{ix(Pz)} \langle p', \lambda' |\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}\, q(\tfrac{1}{2}z)\, | p, \lambda \rangle \Big|_{z=\omega n}$$
$$= \sum_i \epsilon'^{*\nu} V^{(i)}_{\nu\mu} \epsilon^{\mu} H^q_i(x, \xi, t)$$

$$V^u(x, \xi, t) = N_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \delta[n\cdot(xP - k)](-)\mathrm{Tr}\left[\frac{\imath(\not{k} - \not{P} + m)}{(k-P)^2 - m^2 + \imath\epsilon}\gamma^{\nu}\frac{\imath(\not{k} + \frac{\not{\Delta}}{2} + m)}{(k+\frac{\Delta}{2})^2 - m^2 + \imath\epsilon}\not{n}\right.$$
$$\left.\times \frac{\imath(\not{k} - \frac{\not{\Delta}}{2} + m)}{(k-\frac{\Delta}{2})^2 - m^2 + \imath\epsilon}\gamma^{\mu}\right]\Lambda(k-P, p')\Lambda(k-P, p),$$

where m is the constituent quark mass and

$$N_{\mu\nu} = \frac{M^2}{f_\rho^2}\frac{\epsilon'^{*}_{\nu}(p', \lambda')\epsilon_{\mu}(p, \lambda)}{2(2\pi)^3\sqrt{\omega_{p'}\omega_p}}$$

Residuals

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP)
Efremov-Radyushkin-Brodsky-Lepage (ERBL)

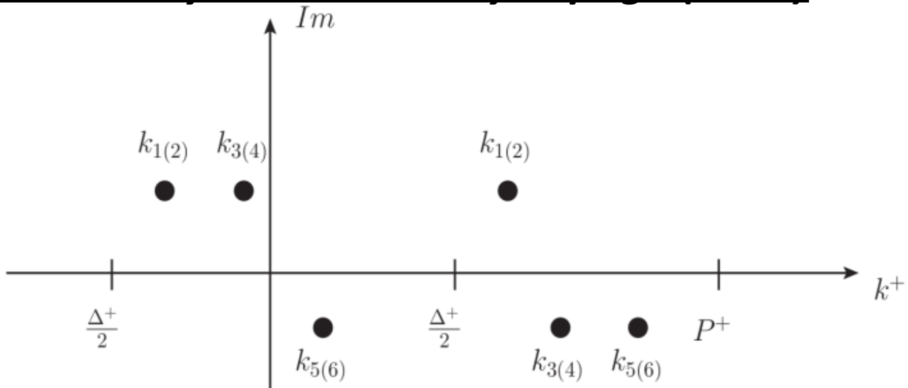
[Choi '04, Frederico '09, Miller'09]

- **Six pole (Valence)**

$$k^-_{1(2)} = P^- + (k-P)^-_{on(R)} - i\frac{\epsilon}{k^+ - P^+},$$

$$k^-_{3(4)} = \frac{\Delta^-}{2} + (k - \frac{\Delta}{2})^-_{on(R)} - i\frac{\epsilon}{k^+ - \frac{\Delta^+}{2}},$$

$$k^-_{5(6)} = -\frac{\Delta^-}{2} + (k + \frac{\Delta}{2})^-_{on(R)} - i\frac{\epsilon}{k^+ + \frac{\Delta^+}{2}}.$$



Nonvalance ($|x|<|\xi|$, ERBL)

Valance ($|x|>|\xi|$, DGLAP)

- **Nonvalance kinematic invariant mass**

$$M^2_{0i(v)} = \frac{\kappa_\perp^2 + m^2}{1-x'} + \frac{\kappa_\perp^2 + m^2}{x'}$$

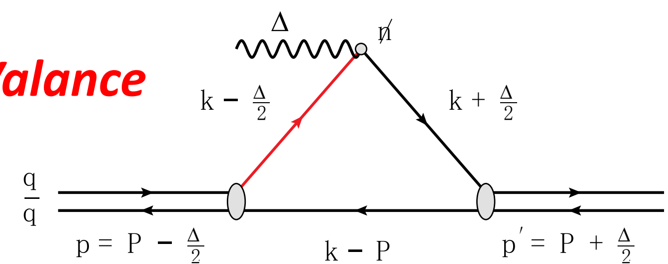
$$\rightarrow \frac{\kappa_\perp^2 + m^2}{x'-1} + \frac{\kappa_\perp^2 + m^2}{x'} = M^2_{0i(nv)}$$

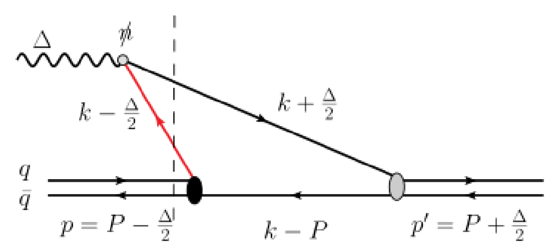
$$x = \frac{n\cdot k}{n\cdot P} = \frac{k^+}{P^+}$$

$$x' = \frac{1-x}{1-|\xi|}$$

$$-|\xi| < x < |\xi|$$

Valance





The struck u quark in the nonvalence regime, yielded by the off-diagonal terms in the Fock space. The black blob represents the non-wave-function vertex. The red line has the negative sign in this regime.

Non-valance

$x \rightarrow 0,1$ intrinsic momentum go infinite!

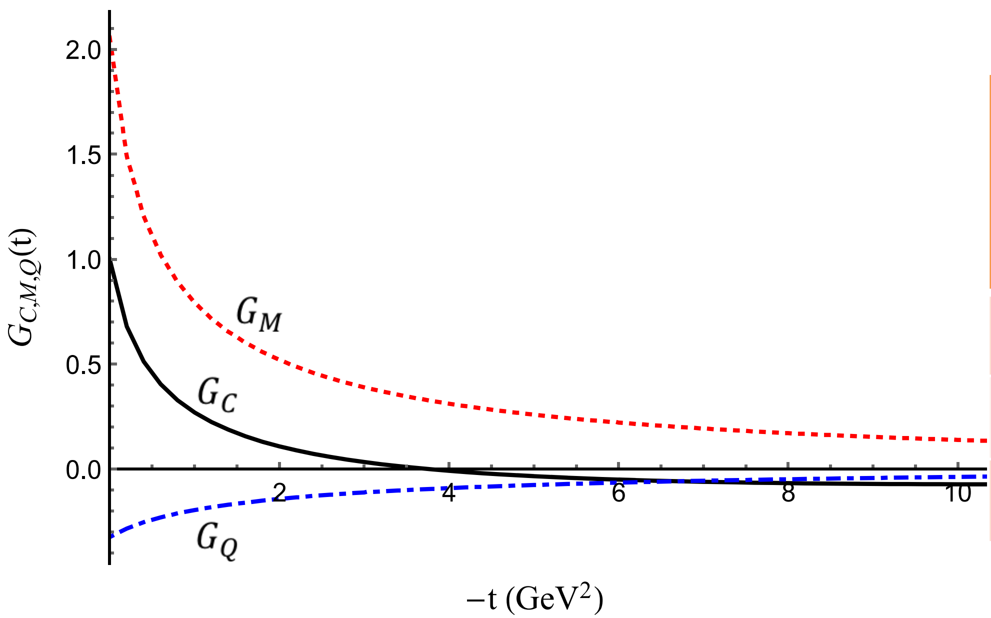
pair production

4, GPDs of ρ-meson (unpolarized and polarized)

Results for the Form factors $G_{C,M,Q}$

- **Form factors**

$$G_C(t) = G_1(t) + \frac{2}{3}\eta G_Q(t) ,$$
$$G_M(t) = G_2(t) ,$$
$$G_Q(t) = G_1(t) - G_2(t) + (1+\eta) G_3(t) ,$$



- **low-energy observables**

$$G_C(0) = 1 ,$$
$$G_M(0) = 2M\mu ,$$
$$G_Q(0) = M^2 Q_\rho ,$$
$$< r^2 > = \lim_{t\to 0} \frac{6\,[G_C(t) - 1]}{t} .$$

	This work	Melo1997	Exp. [Gudino2014]
$\langle r^2 \rangle (\mathrm{fm}^2)$	0.52	0.37	--
μ	2.06	2.19	2.1(5)
$Q_2 (\mathrm{fm}^2)$	0.021	0.050	--

m (constituent mass)	mR (regulator mass)
0.403GeV	1.61GeV

TABLE I. The comparison of the results for the magnetic moment μ_ρ (in natural magnetons $e/2M_\rho$) in different approaches.

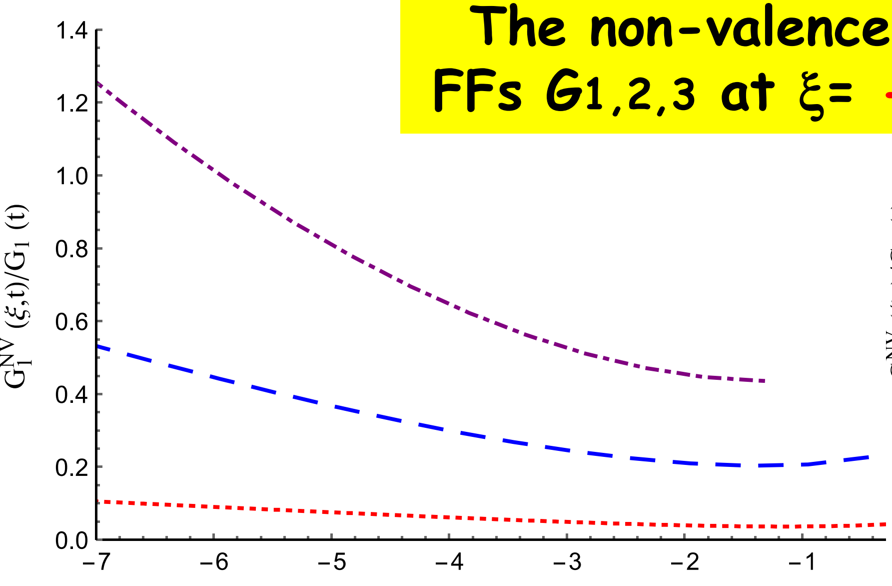
Model	μ_ρ
This work, mIF RHD	2.16 ± 0.03
Cardarelly, LF RHD [1]	2.26
Melo, LF RHD [2]	2.14
Bakker, LF RHD [3]	2.1
Jaus, LF RHD [4]	1.83
Choi, LF RHD [5]	1.92
He, LF, IF RHD [6]	1.5
He, PF RHD [6]	0.9
Biernat, PF RHD [7]	2.20
Sun, LF CQM [8]	2.06
Hawes, Dyson-Schwinger equation (DSE) [9]	2.69
Ivanov, DSE [10]	2.44
Bhagwat, DSE [11]	2.01
Roberts, DSE [12]	2.11
Pitschmann, DSE [13]	2.11
Carrillo-Serrano, Nambu–Jona-Lasinio model (NJL) [14]	2.59
Luan, NJL [15]	2.1
Samsonov, QCD sum rules [16]	2.0 ± 0.3
Aliev, QCD sum rules [17]	2.4 ± 0.4
Melikhov, LF triangle [18]	2.35
Šimonis, bag model [19]	2.06
Bagdasaryan, relativistic CQM [20]	2.3
Badalian, relativistic Hamiltonian [21]	1.96
Djukanovic, effective field theory [22]	2.24
Andersen, lattice [23]	2.25 ± 0.34
Hedditch, lattice [24]	2.02
Lee, lattice [25]	2.39 ± 0.01
Owen, lattice [26]	2.21 ± 0.08
Lushevskaya, lattice [27]	2.11 ± 0.10
Gudinõ, experiment [28]	2.1 ± 0.5

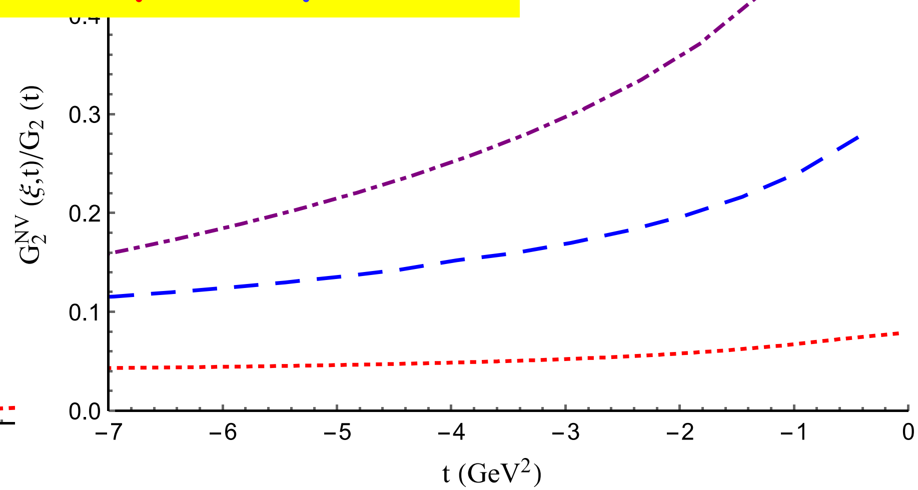
[Krutov, Polezhaev, and Troitsky, PRD97, 033007]

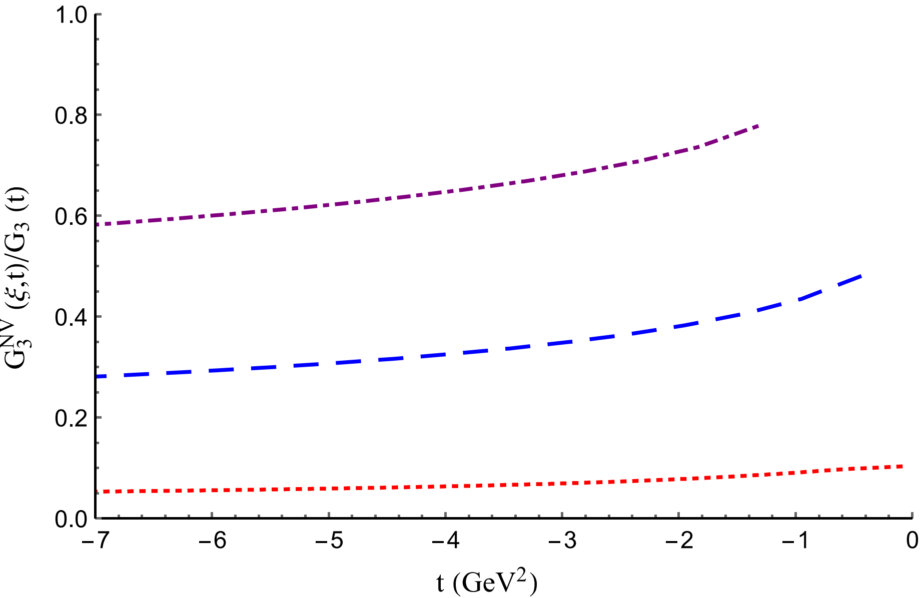
Form factors $G_{1,2,3}$: Nonvalence contributions

The non-valence contributions to FFs $G_{1,2,3}$ at ξ= -0.2, -0.4, -0.6.

$$|\xi| < \frac{1}{\sqrt{1-4M^2/t}}$$







$$\int_{-1}^{1} dx H_i^q(x,\xi,t) = G_i^q(t) \quad (i=1,2,3)\ ,$$

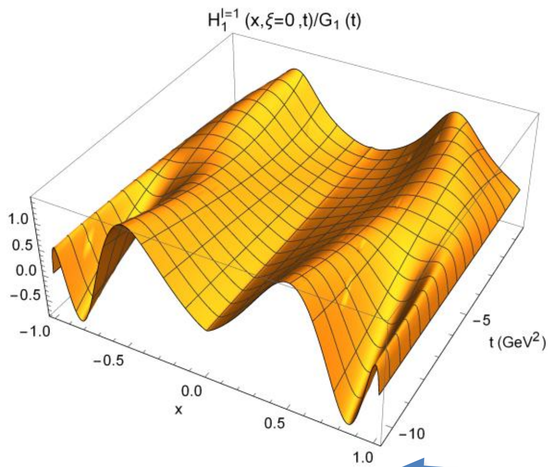
$$\int_{-1}^{1} dx H_i^q(x,\xi,t) = 0 \quad (i=4,5)\ .$$

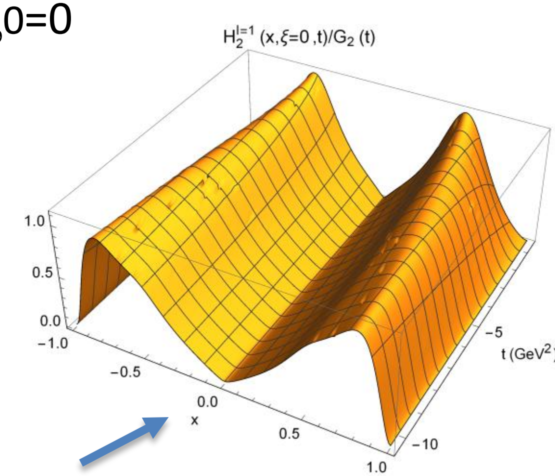
All Three FFs, the sum of the numerical result of the valence and nonvalence contribution only has a small variation over ξ

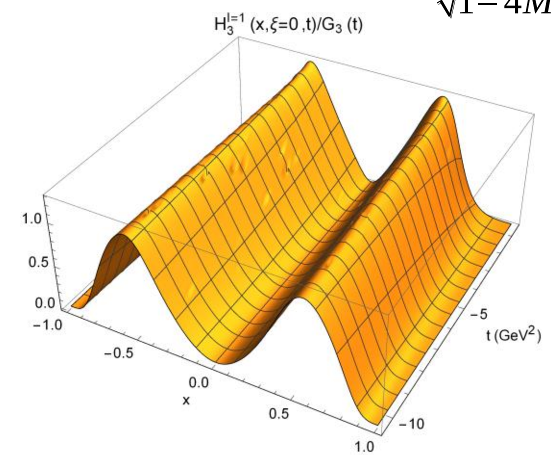
Results: unpolarized GPDs $H_{1,2,3}(x,\xi_0,t)$

$$|\xi| < \frac{1}{\sqrt{1-4M^2/t}}$$

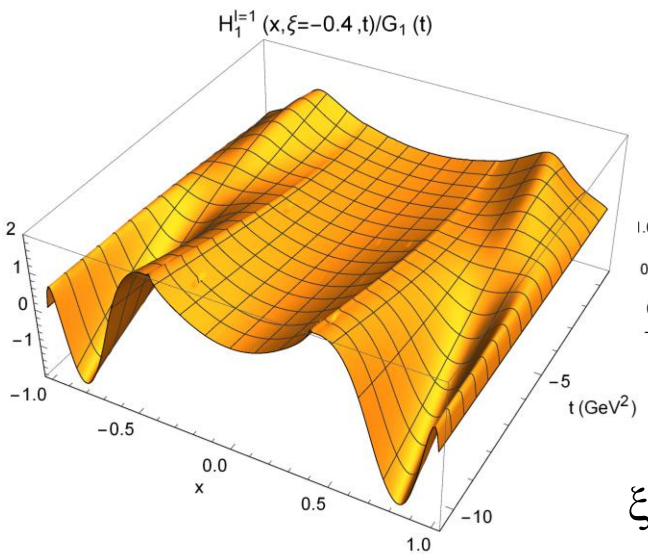
ξ0=0

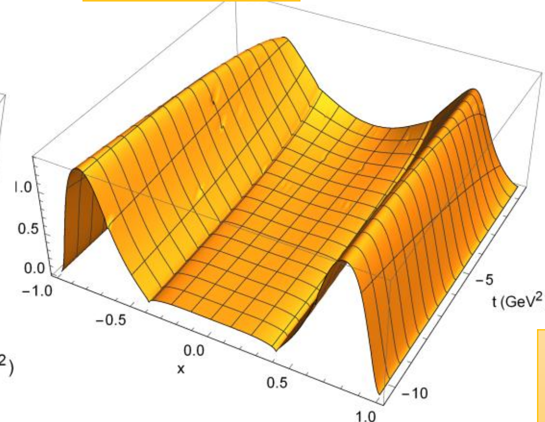


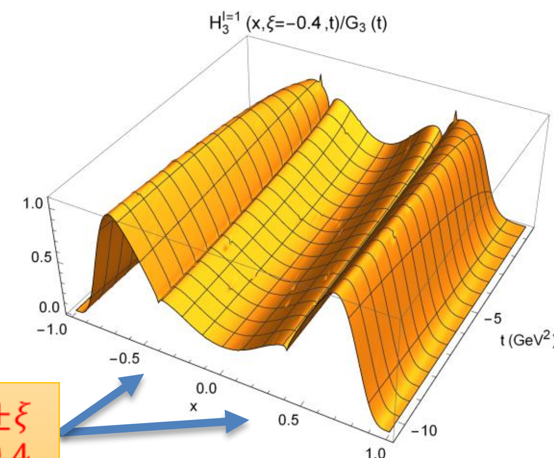




$x = 0, 1$





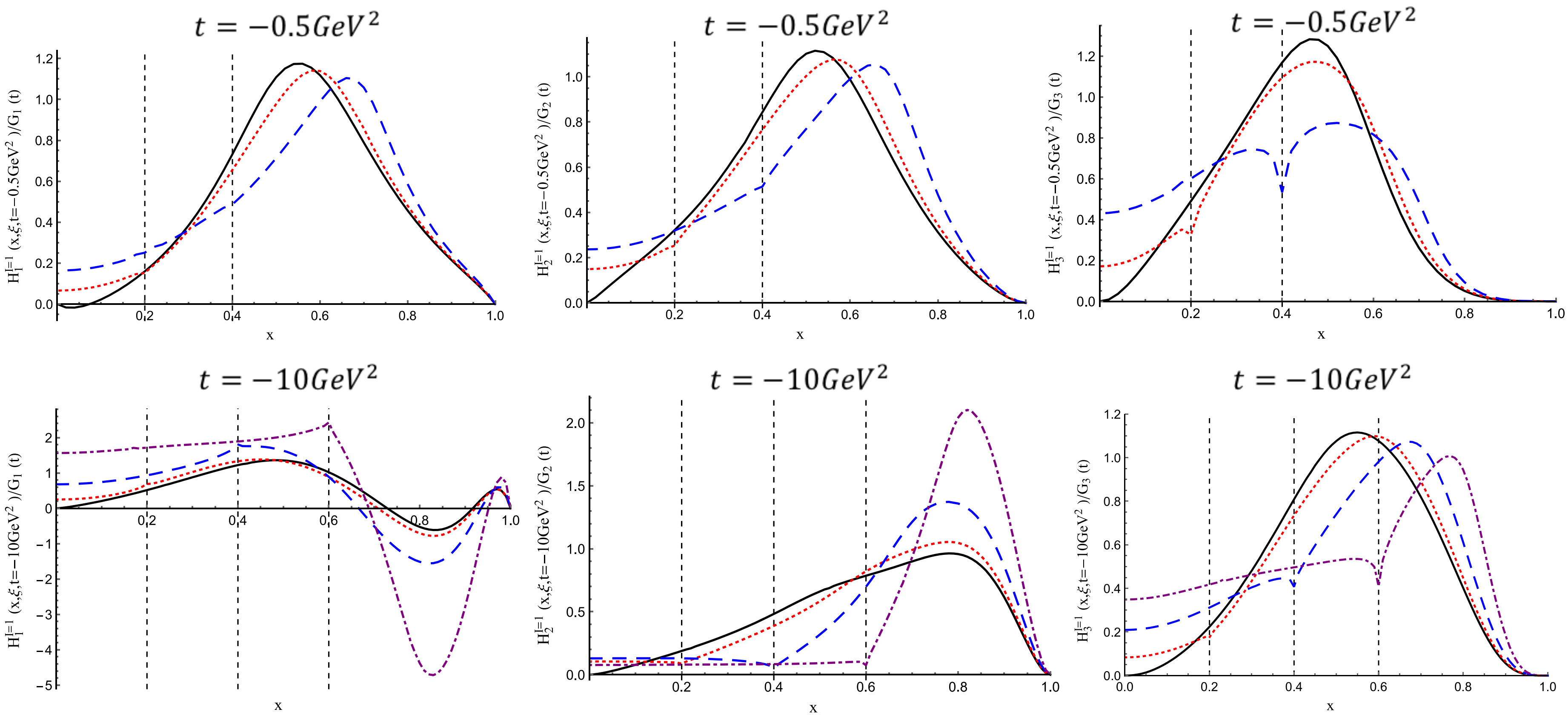


$x = \pm\xi$
$= \mp 0.4$

ξ0=-0.4

Results: unpolarized GPDs $H_{1,2,3}(x, \xi_0, t_0)$

$$|\xi| < \frac{1}{\sqrt{1 - 4M^2/t}}$$



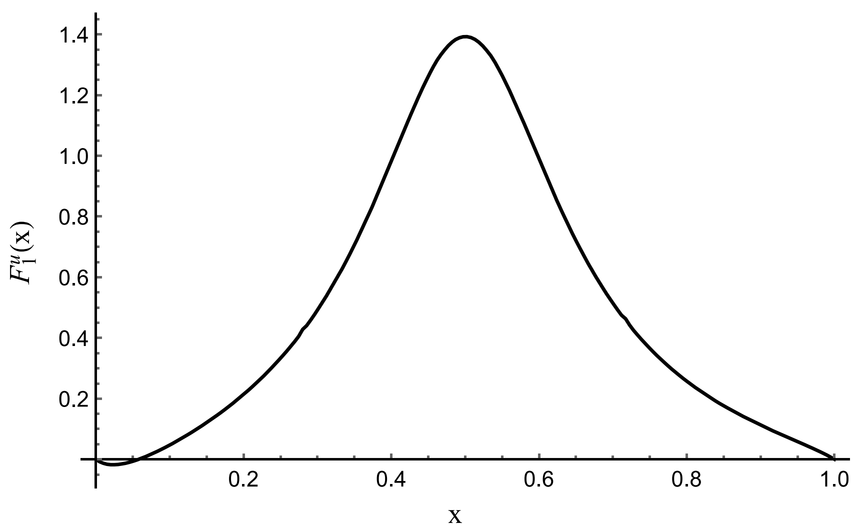
$\xi = 0$ (solid black line), -0.2 (dotted red line), -0.4 (dashed blue line), -0.6 (dot-dashed purple line)

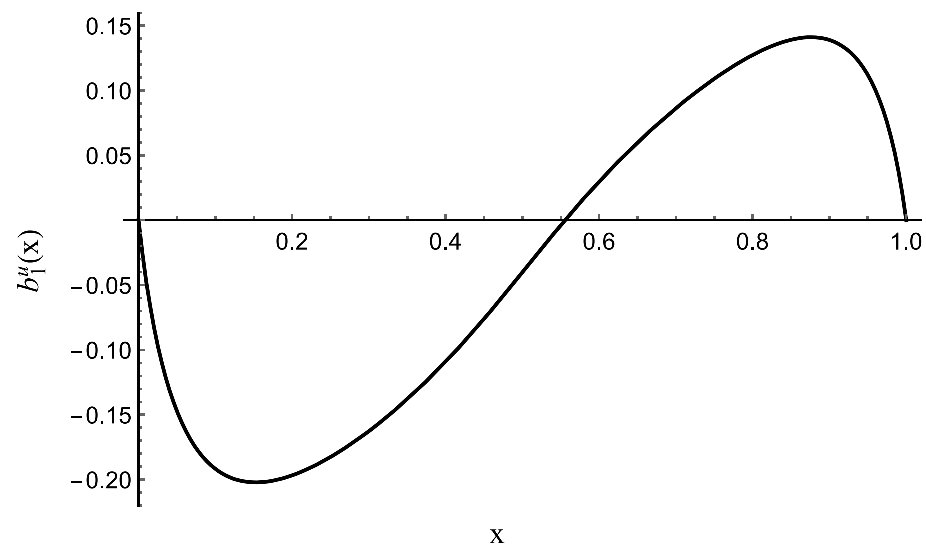
Forward limit: Single-flavor F_1^q, b_1^q

$$F_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_1^u(x,0,0)$$

$$b_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_5^u(x,0,0)$$

$$u_{\rho^+}(x) = \bar{d}_{\rho^+}(1-x)$$





Polarized GPDs of ρ-meson

$$A_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi}\, e^{ix(Pz)} \langle p', \lambda' |\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}\gamma_5\, q(\tfrac{1}{2}z)\ |p, \lambda\rangle \Big|_{z=\omega n}$$
$$= \sum_i \epsilon'^{*\nu} A^{(i)}_{\nu\mu} \epsilon^{\mu} \tilde{H}^q_i(x,\xi,t)$$

$$\int_{-1}^{1} dx\, \tilde{H}^q_i(x,\xi,t) = \tilde{G}^q_i(t) \qquad (i=1,2),$$

with matrix elements of

$$\langle p' |\, \bar{q}(0)\, \gamma^{\mu}\gamma_5\, q(0)\, | p\rangle = -2i\, \epsilon^{\mu}{}_{\alpha\beta\gamma}\, \epsilon'^{*\alpha} \epsilon^{\beta} P^{\gamma}\ \tilde{G}^q_1(t)$$
$$+\, 4i\, \epsilon^{\mu}{}_{\alpha\beta\gamma}\, \Delta^{\alpha} P^{\beta}\, \frac{\epsilon^{\gamma}(\epsilon'^{*}P) + \epsilon'^{*\gamma}(\epsilon P)}{M^2}\ \tilde{G}^q_2(t).$$

For other two GPDs, time reversal invariance gives

$$\int_{-1}^{1} dx\, \tilde{H}^q_3(x,\xi,t) = 0\ ,$$

and the Lorenz invariance constraints

$$\int_{-1}^{1} dx\, \tilde{H}^q_4(x,\xi,t) = 0\ .$$

$$\Delta q \equiv \int_0^1 [g_1^u(x) + g_1^d(x)]\, dx = \int_0^1 \Delta u(x)\, dx$$

$$\int_0^1 g_2(x)\, dx = 0\ .$$

$$\Delta q = 0.86$$

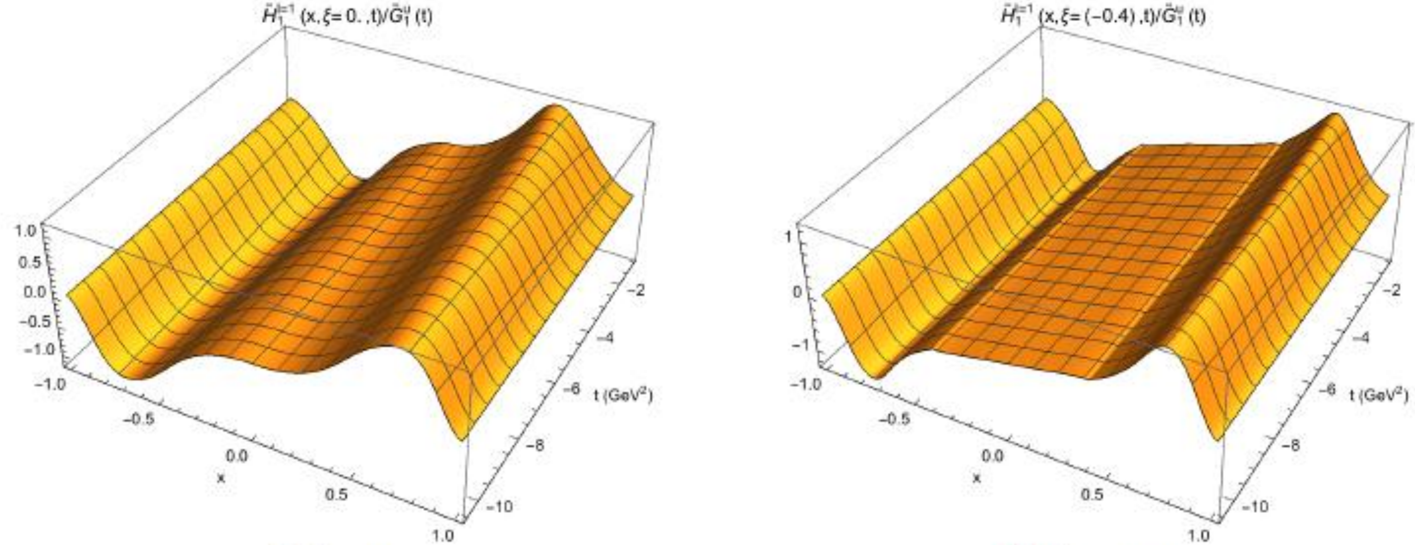
Wandzura-Wilcze relation

$$g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y).$$

$$\int_0^1 g_2(x)\, dx = 0\ .$$

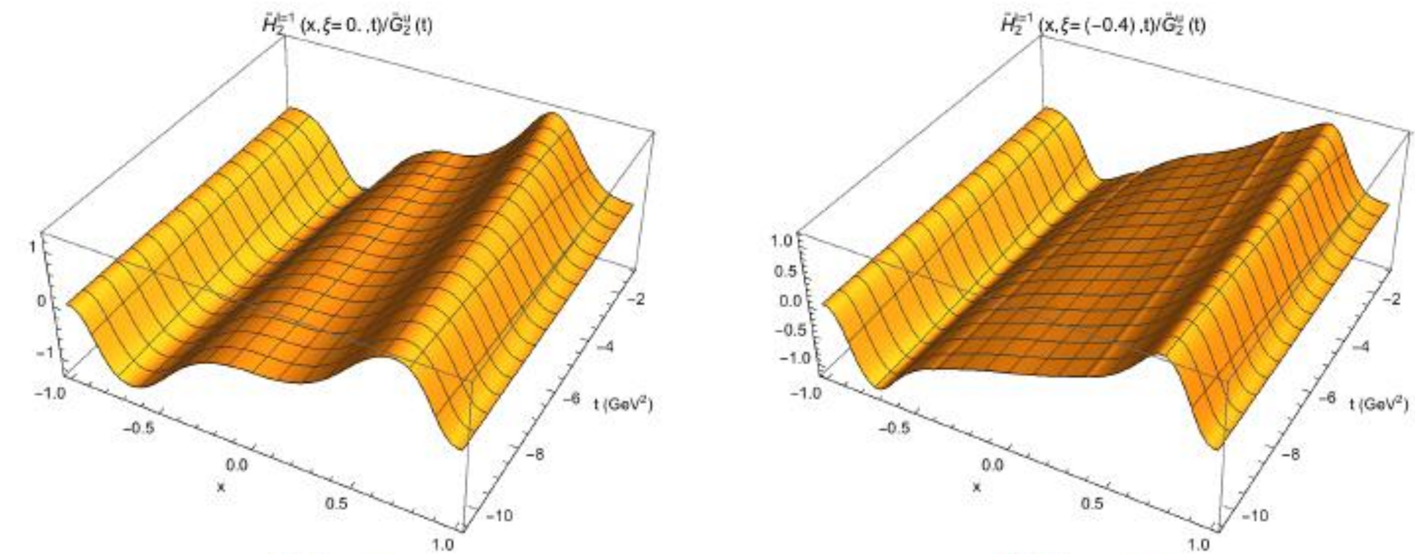
Transverse spin density

$$g_T(x) = g_1(x) + g_2(x) \sim \int_x^1 \frac{dy}{y} g_1(y).$$



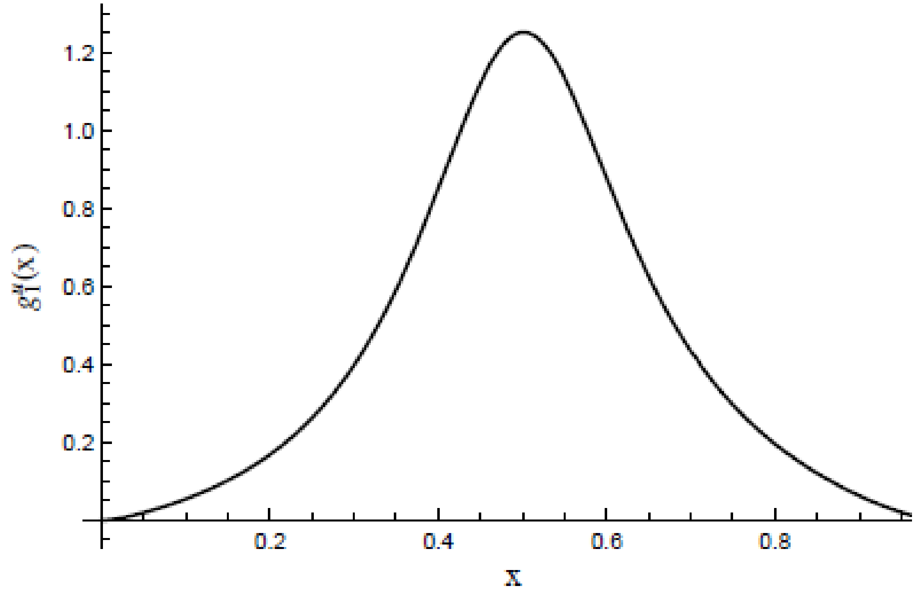
(a) $\xi = 0$ (b) $\xi = -0.4$

Fig. 3. ρ^+ GPD $\tilde{H}_1$ with $\xi = 0$ and -0.4.

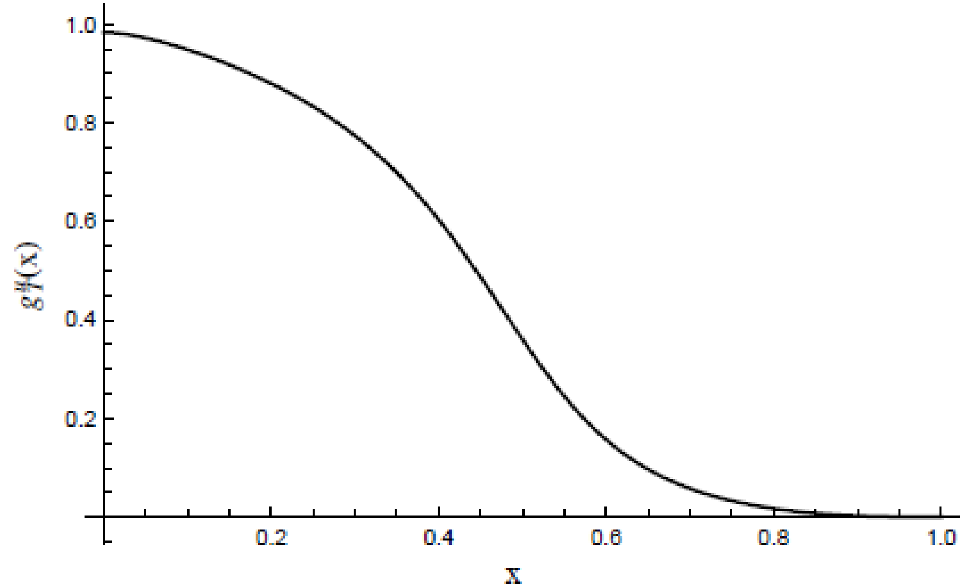


(a) $\xi = 0$ (b) $\xi = -0.4$

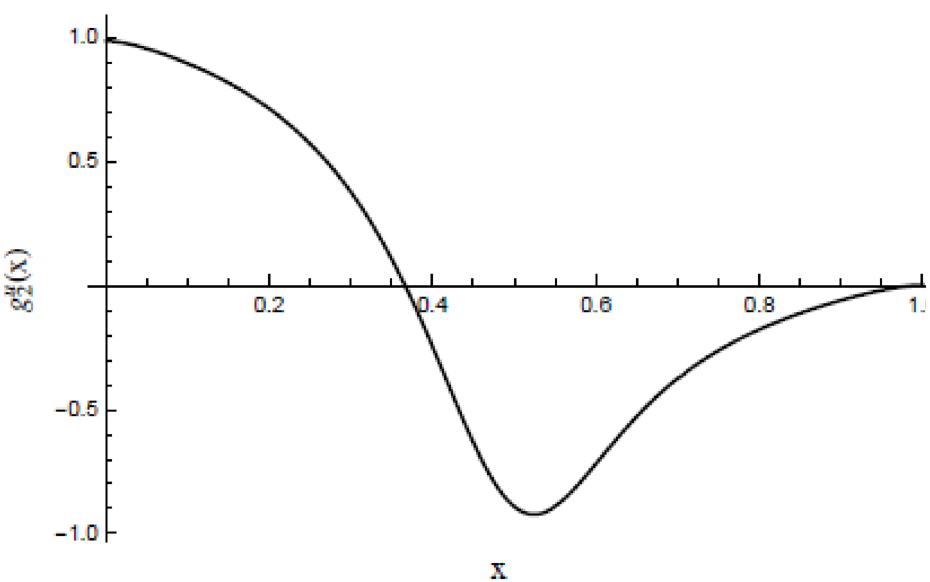
ρ^+ GPD $\tilde{H}_2$ with $\xi = 0$ and -0.4.



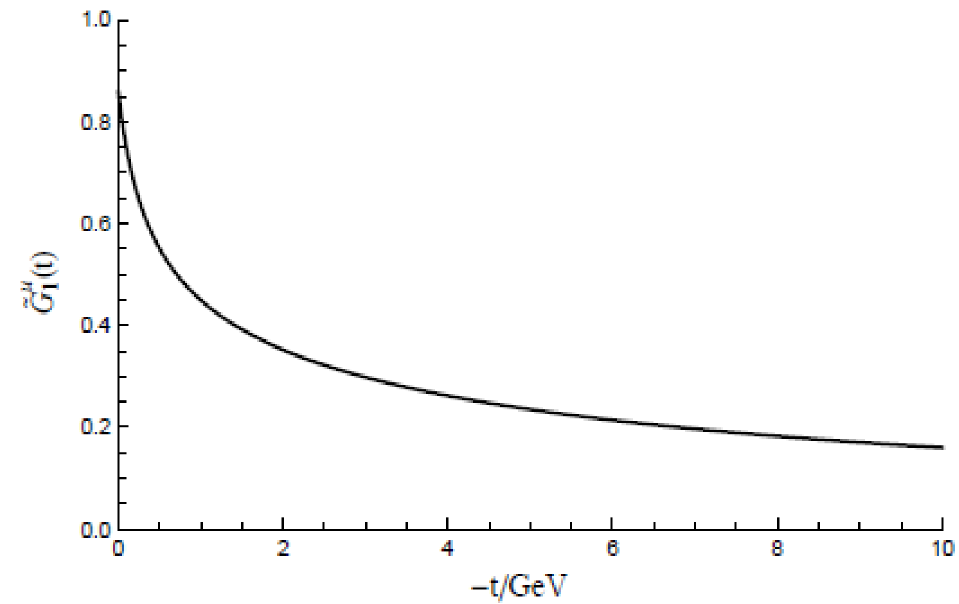
The u quark structure function $g_1^u(x)$



$g_T^u(x)$



The u quark structure function $g_2^u(x)$



The u quark axial form factor $\tilde{G}_1^u(t)$.

5, Impact Parameter Space

[Burkardt '03, Hoodbhoy '89]

- **Spin ½**

$W_{1/2}^{\mu\nu} \sim F_1, F_2, g_1, g_2$

$$
\begin{aligned}
q_N(x,\mathbf{b}) &= |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} \\
&\quad \times \langle p^+, \mathbf{p}'_\perp, \lambda | \left[\int \frac{dz^-}{4\pi} \bar{q}(-\frac{z^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{z^-}{2}, \mathbf{b}_\perp) e^{-\imath x p^+ z^-} \right] | p^+, \mathbf{p}_\perp, \lambda \rangle \\
&= |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, \xi = 0, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)} \\
&= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H_q(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp},
\end{aligned}
$$

⇨ **Fourier transformation**
Density interpretation

- **Spin 1**

$$
\begin{aligned}
q(x,\mathbf{b}) &= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H_1(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp} \\
&= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H_1(x, 0, -\boldsymbol{\Delta}_\perp^2)
\end{aligned}
$$

$W_1^{\mu\nu} \sim F_1, F_2, g_1, g_2$

b_1, b_2, b_3, b_4

Impact Parameter Distributions & Gaussian Package (Cut off)

$$\int \frac{d^2\mathbf{p}_\perp dp^+}{(2\pi)^2 p^+} p^+ \delta(p^+ - p_0^+) G(\mathbf{p}_\perp, \frac{1}{\sigma^2}) |p, \lambda\rangle \sim \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \exp\left(-\frac{\mathbf{p}_\perp^2 \sigma^2}{2}\right) |p^+, \mathbf{p}_\perp, \lambda\rangle$$

[Diehl '02]

$$q_\sigma(x,b) = \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) e^{-\Delta_\perp^2 \sigma^2/4} H_1(x, 0, -\Delta_\perp^2)$$

$$q_\sigma(b) = \int_0^1 dx\, q_\sigma(x,b)$$

$$q(x,\mathbf{b}) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H_1(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

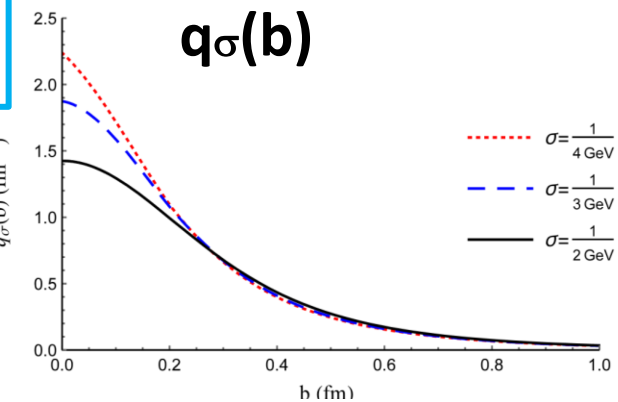
$$= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H_1(x,0,-\mathbf{\Delta}_\perp^2)$$

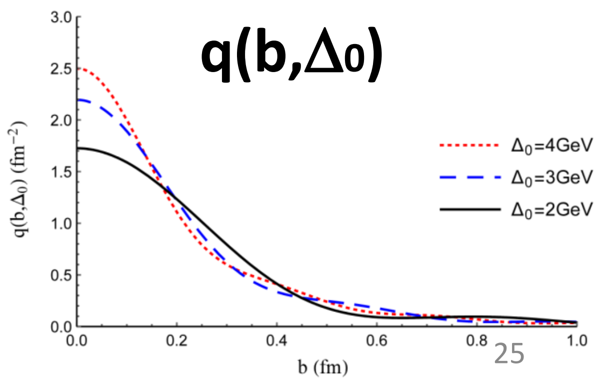
Only limit value of "t" can be measured

$$q(x,\mathbf{b},\Delta_0) = \int_0^{\Delta_0} \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H(x,0,-\mathbf{\Delta}_\perp^2)$$

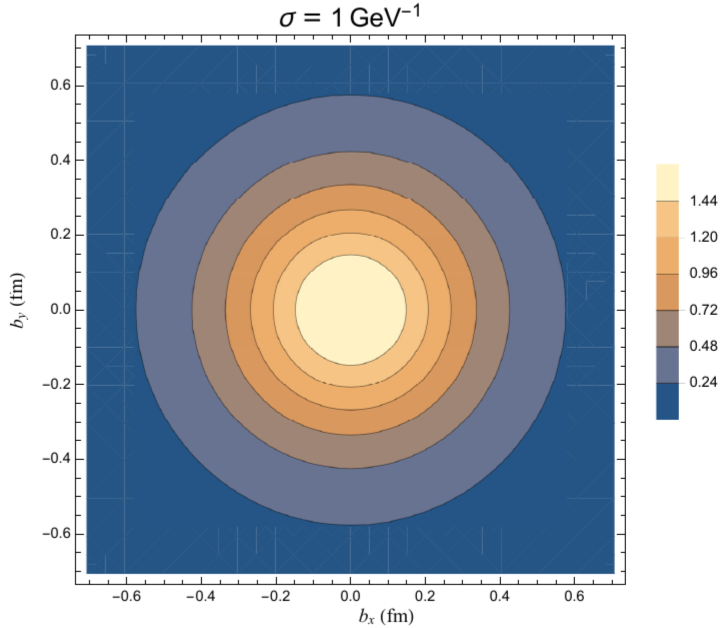
$$q(\mathbf{b},\Delta_0) = \int_0^1 dx\, q(x,\mathbf{b},\Delta_0)$$

Gaussian Package V.S. Cut off

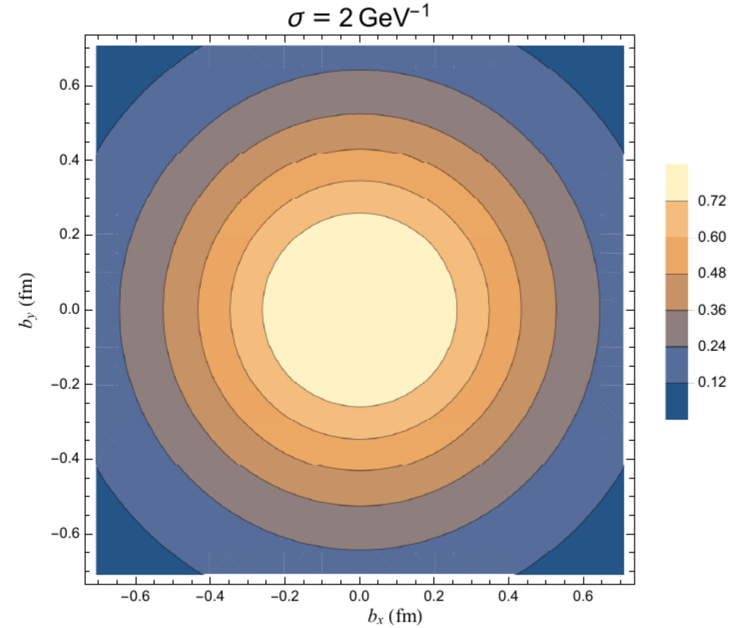




Gaussian Package $q_\sigma(b)$

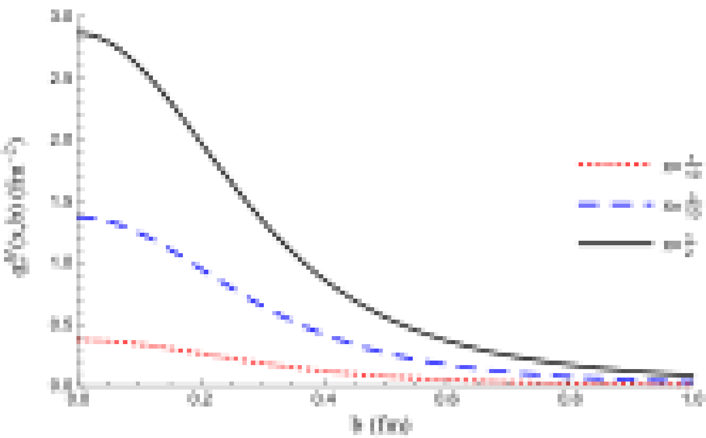


(a) $q_\sigma(b)$ (fm^{-2}) with packet width $\sigma = 1\ \text{GeV}^{-1}$.

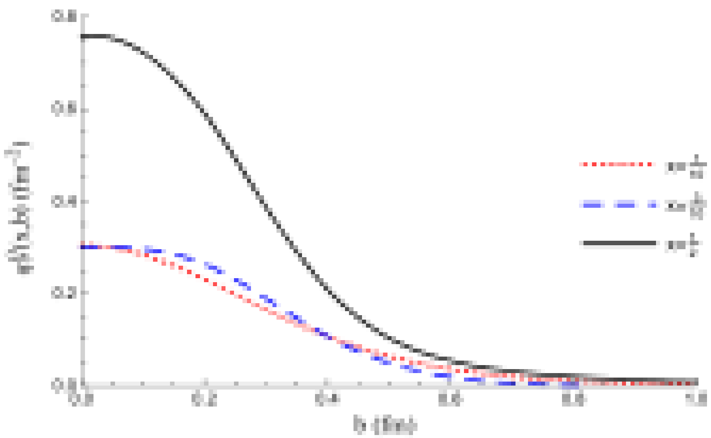


(b) $q_\sigma(b)$ (fm^{-2}) with packet width $\sigma = 2\ \text{GeV}^{-1}$.

Gaussian package, $q_\sigma(x,b)$



(a) $q_\sigma^M(x,b)$ with $\sigma = 1\ \text{GeV}^{-1}$ and $x = 1/10,\ 3/10$ and $1/2$.



(b) $q_\sigma^Q(x,b)$ with $\sigma = 1\ \text{GeV}^{-1}$ and $x = 1/10,\ 3/10$ and $1/2$.

(color online) The impact parameter dependent FFs $q_\sigma^{M,Q,QC}(x,b)$ with $\sigma = 1\ \text{GeV}^{-1}$ and $x = 1/10,\ 3/10$ and $1/2$.

6, Summary and outlook

- GPDs for ρ meson (spin-1)
- Phenomenological approach for ρ meson
- ρ meson FFs / GPDs
- Impact parameter Distributions

○ GDAs & $\rho\rho$ production

L3 Collaboration

Exp:

PLUTO/ TASSO/ CELLO/ ARGUS
@ DESY, ’82-’91
L3 @ LEP, ’03-’06
STAR @ RHIC, ’07-’09
Babar @ PEP-II, ‘08
LHCb, ‘12 (TeV, double charm)

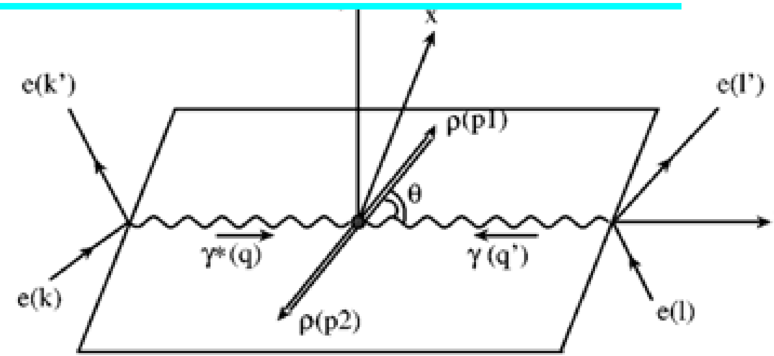
ARGUS Collaboration etc.

[Albrecht ’90, ’91]

$$\sigma(e^+e^- \to \rho^+\rho^-) = 8.3 \pm 0.7(\text{stat}) \pm 0.8(\text{syst})\ \text{fb}$$

$$\gamma^*\gamma \to \rho\rho$$

- Full reaction: **[Anikin ’04, ‘05]**



$$2e \to 2e + \rho^0\rho^0(\rho^+\rho^-)$$

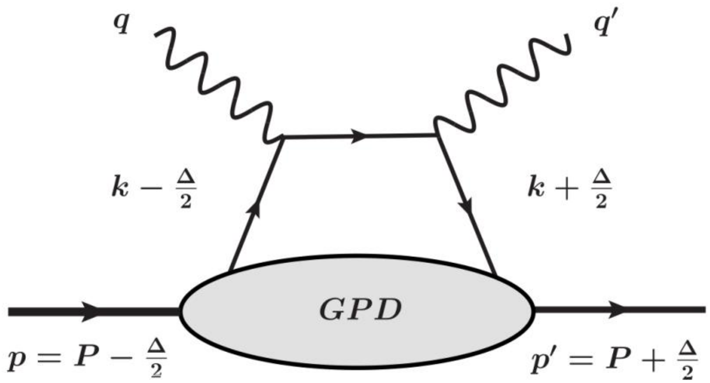
  - @LO (twsit-2), $I = 0$
  - charged/neutral cross sec. NOT independent (CG coefs)
  - but charged has bremsstrahlung

- Also related to: **[García ’15, Kłusek-Gawenda ’17, Kumano ’17, ‘18]**

$$2e \to 2e + \rho^0 + 2\pi \qquad \to AA + \pi^+\pi^-\pi^+\pi^-$$
$$\to 4\pi \qquad \to AA + \pi^+\pi^- 2\pi^0$$

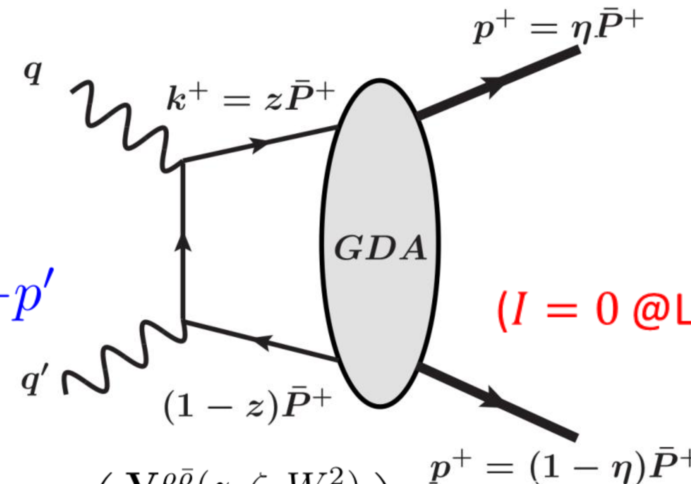
GDA (Generalized Distribution Amplitude)

[PRL: Diehl '98, '03, Kumano '17]



$t \leftrightarrow s$

$p', p \leftrightarrow p, -p'$



$(I = 0$ @LO)

$$\Phi_q^{\rho\bar{\rho}}(z,\zeta,W^2) = \int \frac{dx^-}{2\pi} e^{-iz\bar{P}^+x^-} \langle \rho(p)\rho(p') |\, \bar{q}(x^-)\gamma^+ \begin{pmatrix} 1 \\ \gamma_5 \end{pmatrix} q(0)\, |0\rangle = \begin{pmatrix} \mathbf{V}_q^{\rho\bar{\rho}}(z,\zeta,W^2) \\ \mathbf{A}_q^{\rho\bar{\rho}}(z,\zeta,W^2) \end{pmatrix}$$

$$\Phi_q^{\rho\bar{\rho}}(z,\zeta,W^2) \longleftrightarrow H_\rho^h\left(x = \frac{1-2z}{1-2\zeta}, \xi = \frac{1}{1-2\zeta}, t = W^2\right)$$

[Kawamura '13, Kumano '17, '18]

- Other observable
- Double parton distributions (DPDs)
- Deuteron

Thanks!

QCD evolution of the structure functions

The moments of the structure functions at different scale

$$\frac{\tilde{V}_n^u(\mu)}{\tilde{V}_n^u(\mu_0)} = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_n^{(0)}/(2\beta_0)} ,$$

where the single quark spin fractions

$$\tilde{V}_n^u = 2M_{n+1}\left[g_1^u(x)\right] \sim r_{n+1}$$

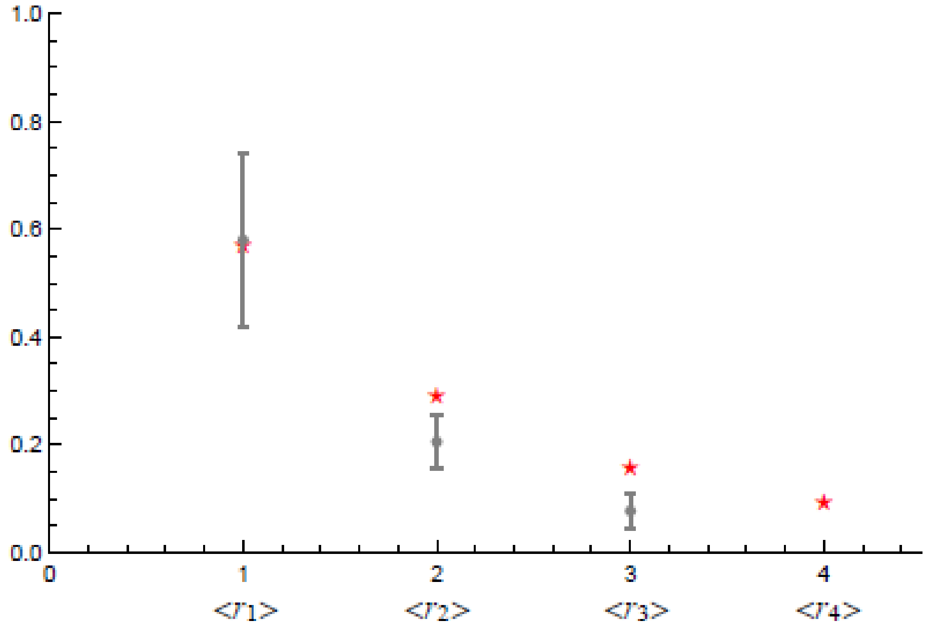
and the running coupling constant is

$$\alpha(\mu) = \frac{4\pi}{\beta_0 \log(\mu^2/\Lambda_{QCD}^2)} ,$$

where $\beta_0 = 11N_c/3 - 2N_f/3$ with $N_c = N_f = 3$ and

$$\Lambda_{QCD} = 0.226\ GeV$$

For the polarized structure function



r_n for u quark. The red stars are our results and the gray ones with errors are the Lattice QCD results [14].

Possible Lattice calculation with quench approximation at μ=2.4 GeV, *Best'97, PRD56, 2743*