Pion and kaon parton distribution functions in a nuclear medium

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Outline

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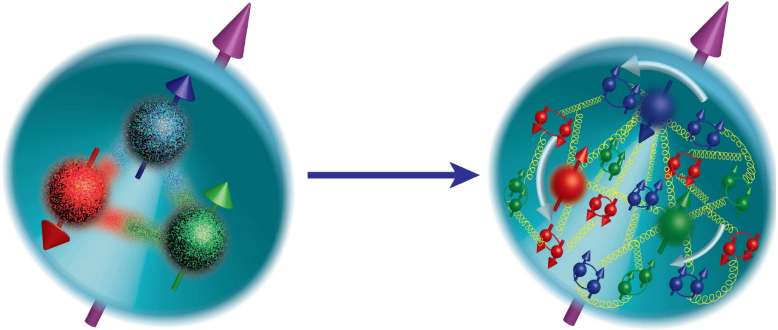
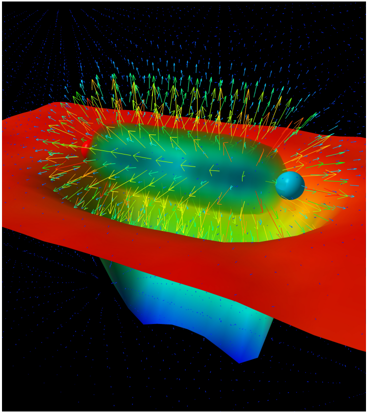
Proton

$\Rightarrow$ proton (qqq + *gluons* + *sea quarks* (*intrinsic*))



Meson

⇒ Focus on MESON ($\bar{q}q$ + *gluons* + *sea quarks* (*intrinsic*))



[1]

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Introduction: PDFs in the Pion and Kaon

- QCD, as underlying theory of strong interaction, is unable to directly predict structure of hadrons. The solution:
  - Lattice QCD: Large Momentum Effective Theory (LAMET)[2]
  - QCD inspired models (mimicking features of QCD) such as NJL model, DSE model, QCD Sum rules and Chiral effective models
- To understand the partonic dynamics in a hadron internal structure, PDF and FF are of fundamental importance and provide complementary information
- Pion and kaon structure is simpler than the nucleon, but not so simple. In fact, we do not really understand the structure of the pion and kaon. Also the pion and kaon are interesting due to the pion and kaon are both a dressed quark-antiquark bound state and Goldstone mode associated with DχSB in QCD. A great opportunity to gain useful information of the dynamics of quarks to use it to understand the quark dynamics in the nucleon, which is more complicated system
- From experimental side, next experimental data for the pion and kaon will be coming from JLAB, J-PARC as well as (CERN-SPS) COMPASS and EIC

Pion and Kaon in the BSE-NJL Model

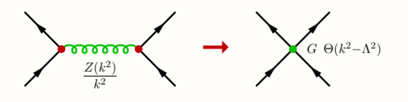
The three flavor NJL Lagrangian – containing only four fermion interactions

$$\mathscr{L}_{NJL} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}_q]\psi + G_\pi \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] - G_\rho \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] \quad (1)$$

- $\psi = (u, d, s)^T$ denotes the quark field with the flavor components
- G_π and G_ρ are four-fermion coupling constants
- $\lambda_1, \cdots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}\mathbb{1}$
- $\hat{m}_q = \mathrm{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

Pion and Kaon in the BSE-NJL Model

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction by integrating out the gluon field and absorbing into the coupling constant $\Longleftrightarrow$ *quark effective theory*



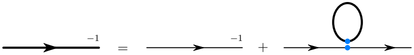
- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole). Therefore we regularize using the proper time regularization to simulate confinement (IC.Cloet, PRC**90** (2014), PTPH, PRC**94** (2016))

$$\frac{1}{\mathcal{X}^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\tau^{(n-1)}e^{-\tau\mathcal{X}} \rightarrow \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau\tau^{(n-1)}e^{-\tau\mathcal{X}} \quad (2)$$

where $\Lambda_{IR} \sim \Lambda_{QCD} \sim 0.24$ GeV and Λ_{UV} is determined.

Pion and Kaon in the BSE-NJL Model

- NJL Gap Equation is determined using quark propagator in momentum space $S_q^{-1}(p) = \not{p} - M_q + i\epsilon$



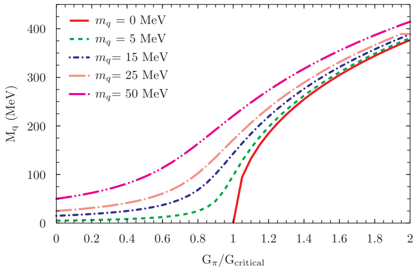
$$
\begin{aligned}
M_q &= m_q + M_q \frac{3G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} \\
&= m_q - 2G_\pi \langle \bar{\psi}\psi \rangle
\end{aligned} \tag{3}
$$

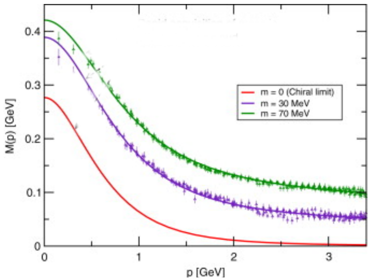
- Chiral quark condensates is defined by $\langle \bar{\psi}\psi \rangle = -\frac{3M_q}{2\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2}$
- Mass is generated through interaction vacuum $\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$

NJL Gap Equation

NJL and DSE gap equations PTPH *et al.*, PRC**94** (2016), C.D.Roberts, PPNP **61** (2008)

- The NJL constituent quark mass is a constant up to certain $p \sim 0.6$ GeV and it drops in higher p region

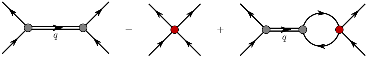




- The NJL model can be used for low momentum p and low energy E

Bethe Salpeter Equation for the pion and kaon

Mesons in the NJL model are quark-antiquark bound sates whose properties are determined by solving the BSE



- In the NJL model, $\mathscr{T}$-matrix is given by

$$\mathscr{T}(q) = \mathscr{K} + \int \frac{d^4k}{(2\pi)^4} \mathscr{K} S(q+k) \mathscr{T}(q) S(k)$$

- The solution to the BSE in the pion and kaon

$$\mathscr{T}_\alpha(q)_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab}\, t_\alpha(q) \left[\gamma_t \lambda_\alpha^\dagger\right] \tag{4}$$

- The reduced t-matrix in this channel take a form

$$t_\alpha(q) = \frac{-2iG_\pi}{1 + 2G_\pi \Pi_\pi(q^2)}$$

$$t_\beta^{\mu\nu}(q) = \frac{-2iG_\rho}{1 + 2G_\rho \Pi_\beta(q^2)} \left(g^{\mu\nu} + 2G_\rho \Pi_\beta(q^2) \frac{q^\mu q^\nu}{q^2} \right) \tag{5}$$

Bethe Salpeter Equation of the pion and kaon

- The bubble diagrams appearing read

$$\Pi_\pi(q^2) = 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_l(k+q)\right],$$
$$\Pi_K(q^2) = 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_s(k+q)\right],$$
$$\Pi_\nu^{aa}(q^2) = 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma^\mu S_a(k)\gamma^\nu S_a(k+q)\right] \quad (6)$$

- The kaon and pion masses is given by the pole of the t-matrix

$$1+2G_\pi\Pi_\pi(k^2=m_\pi^2) = 0$$
$$1+2G_\pi\Pi_K(k^2=m_K^2) = 0 \quad (7)$$

Pion and Kaon Masses

The meson masses are defined by the pole in the corresponding t-matrix and therefore the kaon and pion masses are given by

$$m_\pi^2 = \left[\frac{m}{M_l}\right] \frac{2}{G_\pi \mathcal{I}_{ll}(m_\pi^2)}$$
$$m_K^2 = \left[\frac{m_s}{M_s} + \frac{m}{M_l}\right] \frac{1}{G_\pi \mathcal{I}_{ls}(m_K^2)} + (M_s - M_l)^2 \quad (8)$$

where $\mathcal{I}_{ll}$ and $\mathcal{I}_{ls}$ in the proper time regularization scheme are defined by

$$\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \quad (9)$$

The meson-quark-quark coupling constants and Pion and Kaon Decay Constants

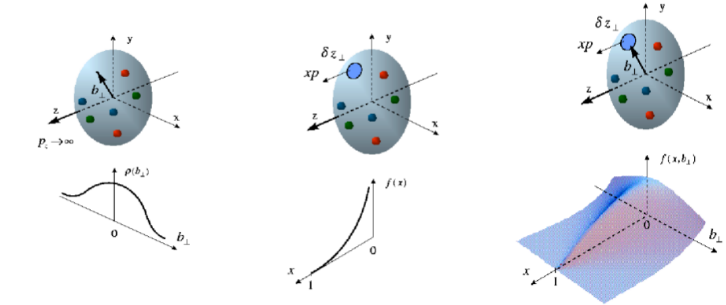
The residue at a pole in the $\bar{q}q$ $t-$matrix defines the effective meson-quark -quark coupling constants:

$$
\begin{aligned}
Z_\pi(q^2) &= -\frac{\partial \Pi_\pi(q^2)}{\partial q^2}\,\Big|_{q^2=m_\pi^2} \\
Z_K(q^2) &= -\frac{\partial \Pi_K(q^2)}{\partial q^2}\,\Big|_{q^2=m_K^2} \\
Z_\rho(q^2) &= -\frac{\partial \Pi_\rho(q^2)}{\partial q^2}\,\Big|_{q^2=m_\rho^2}
\end{aligned} \tag{10}
$$

Pion and kaon decay constant in the proper time regularization is given by

$$
\begin{aligned}
f_\pi &= \frac{N_C\sqrt{Z_\pi}M}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)} \\
f_K &= \frac{N_C\sqrt{Z_K}}{4\pi^2}[(1-x)M_2 + xM_1]\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)}
\end{aligned} \tag{11}
$$

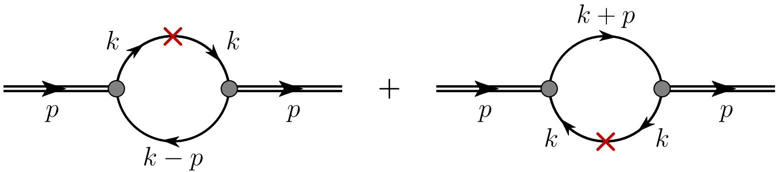
PARTON DISTRIBUTION FUNCTIONS IN THE BSE-NJL MODEL



Elastic Scattering	DIS	DES (GPDs)
transverse quark distribution in coordinate space	longitudinal quark distribution in momentum space	fully-correlated quark distribution in both coordinate and momentum space

Parton distribution functions

The valence quark distribution functions of the pion or kaon are given by the two Feynman diagrams



The operator insertion $\gamma^+\delta\left(k^+ - xp^+\right)\hat{P}_q$, where $\hat{P}_q$ is the projection operator for quarks of flavor q:

$$\hat{P}_{u/d} = \frac{1}{2}\left(\frac{2}{3}\mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right)$$
$$\hat{P}_s = \frac{1}{3}\mathbb{1} - \frac{1}{\sqrt{3}}\lambda_8 \qquad (12)$$

Parton distribution functions

The valence quark and anti-quark distributions in the pion or kaon are given by

$$\begin{aligned} q_\alpha(x) &= iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ - xp^+\right) \\ &\times Tr\left[\gamma_5 \lambda_\alpha^\dagger S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha S(k-p)\right] \\ \bar{q}_\alpha(x) &= -iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ + xp^+\right) \\ &\times Tr\left[\gamma_5 \lambda_\alpha S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha^\dagger S(k+p)\right] \end{aligned} \tag{13}$$

To evaluate these expression we first take the moments

$$\mathcal{A}_n = \int_0^1 dx x^{n-1} q(x) \tag{14}$$

where $n = 1, 2, 3, \cdots$ is an integer.

Parton distribution functions

Using the Ward-like identity $S(k)\gamma^+ S(k) = \frac{-\partial S(k)}{\partial k_+}$ and introducing the Feynman parameterization, the quark and anti-quark distributions can then be straightforwardly determined. For the valence quark and anti-quark distributions of the K^+ we find:

$$\begin{aligned} q_K(x) &= \frac{3Z_K}{4\pi^2} \int d\tau e^{-\tau\left[x(x-1)m_K^2 + xM_s^2 + (1-x)M_l^2\right]} \\ &\times \left[\frac{1}{\tau} x(1-x)\left[m_K^2 - (m_l - M_s)^2\right]\right] \\ \bar{q}_K(x) &= \frac{3Z_K}{4\pi^2} \int d\tau e^{-\tau\left[x(x-1)m_K^2 + xM_l^2 + (1-x)M_s^2\right]} \\ &\times \left[\frac{1}{\tau} x(1-x)\left[m_K^2 - (m_l - M_s)^2\right]\right] \end{aligned} \quad (15)$$

⇒ Results for the π^+ are obtained by $M_s \to M_l$ and $Z_K \to Z_\pi$, giving the result $u_\pi(x) = \bar{d}_\pi(x)$

Parton distribution functions

The quark distributions satisfy the baryon number and momentum sum rules, which for the K^+ read:

$$\int_0^1 dx \left[u_{K^+}(x) - \bar{u}_{K^+}(x)\right] = \int_0^1 \left[\bar{s}_{K^+}(x) - s_{k^+}(x)\right] = 1 \qquad (16)$$

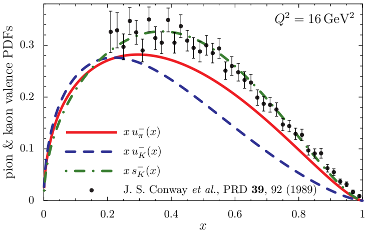
for the number sum rules and at the model scale the momentum sum rules is given by

$$\int_0^1 dxx \left[u_{K^+}(x) + \bar{u}_{K^+}(x) + \bar{s}_{K^+}(x) + s_{k^+}(x)\right] = 1 \qquad (17)$$

Analogous results holds for the remaining kaons and the pions.

Kaon PDFs Results

Results for the valence quark distributions of the π^+ and K^+ evolved from model scale to $Q^2 = 16\ GeV^2$ using NLO DGLAP equations[3] and compared empirical data for the pion valence PDF.



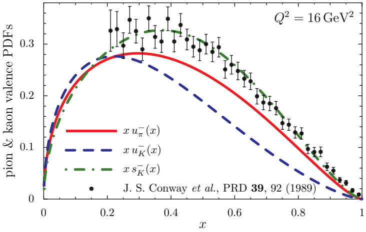
⇒ At model scale, the momentum fraction by the u and s quarks in the K^+, $< xu >= 0.42$ and $< xs >= 0.58$

⇒ The flavor breaking effects of $[< xs > - < xu >]/[< xs > + < xu >] \sim$ 16% which is similar to that seen in masses $[M_s - M_u]/[M_s + M_u] \sim 21\%$.

[3] M. Miyama and S. Kumano, Comput.Pys.Commun. **94**, 185

Kaon PDFs Results

Results for the valence quark distributions of the π^+ and K^+, evolved from the model scale using NLO DGLAP equations.

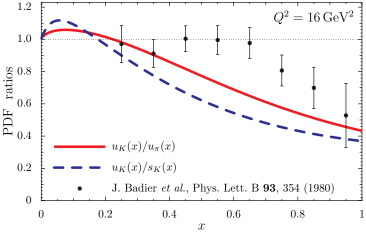


→ $SU(3)$ flavor breaking at the model scale $u_K(x)$ peaks at $x_u = 0.237$ and $\bar{s}_K$ peaks at the $x_s = 1 - x_u = 0.763$

→ This implies flavor breaking effects of around $[x_s - x_u]/[x_s + x_u] \sim 53\%$. For the pion the peak at $x = 0.5$ when $m_u = m_d$.

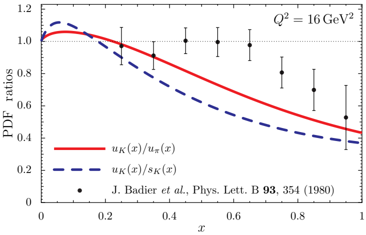
Kaon PDFs Results

The ratio of the u quark distribution in the K^+ to the u quark distribution in the π^+, after NLO evolution to $Q^2 = 16$ GeV^2



$\rightarrow$ The ratio of $u_K/u_\pi \rightarrow 0.434 \sim M_u/M_s$ as $x \rightarrow 1$, which is in a good agreement with existing data.
$\rightarrow$ The x-dependence differs from much of data in the valence region. This may lie with the absence of the momentum dependence in the NJL Bethe Salpeter vertices, or with data itself.

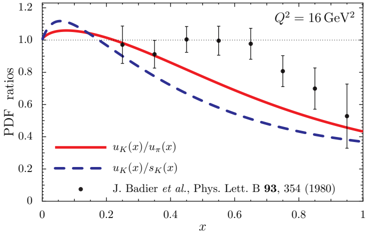
Kaon PDFs Results



$\rightarrow$ The ratio $u_K(x)/s_K(x)$ approaches 0.37 as $x \rightarrow 1$. It is evident that the flavor breaking effects have a sizable x dependence, being maximal at large x and becoming negligible at small x where the perturbative effects from DGLAP evolution dominate.

$\rightarrow$ The Drell-Yan-West (DYW) relation, $F(Q^2) \sim \frac{1}{Q^{2n}} \leftrightarrow q(x) \sim (1-x)^{2n-1}$. For the pion, $F_\pi \sim 1/Q^2$ and the DYW relation implies $q_\pi(x) \sim (1-x)$. Kaon PDF do behave as the pion.

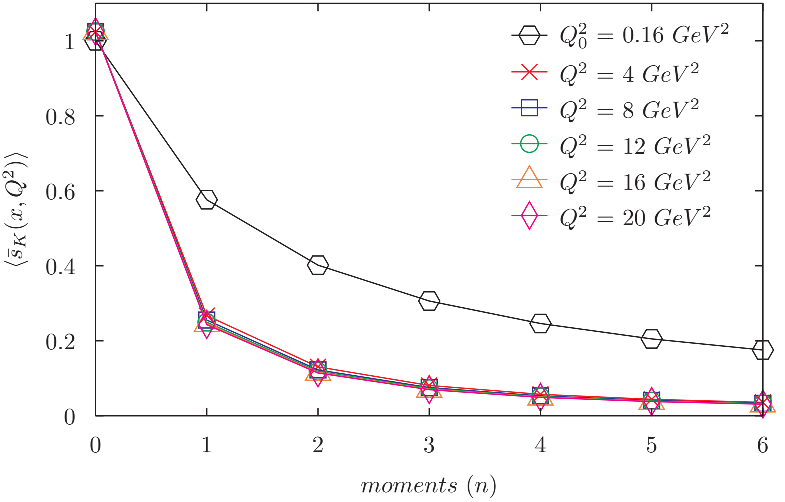
Kaon PDFs Results



→ As reflection of the expectations of may be expected by DYW like relations, $u_K/s_K < 1$ as $x \to 1$ and $\mid F_K^u/F_K^s \mid < 1$ for $Q^2 >> \Lambda_{QCD}$.

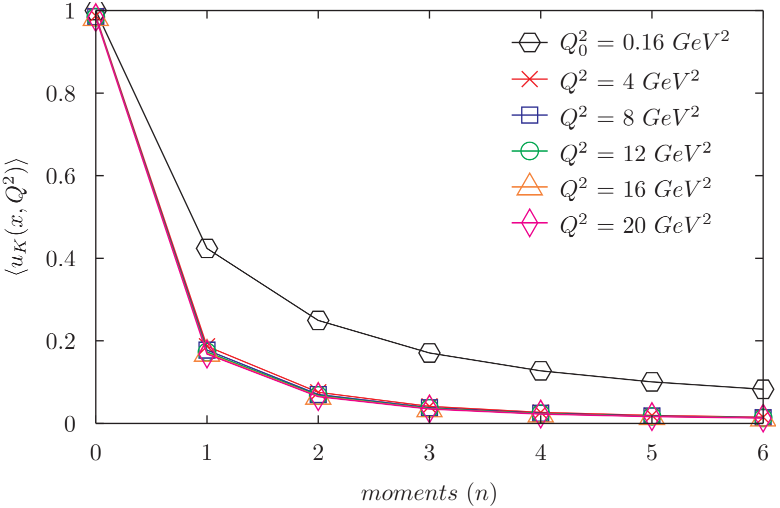
Kaon PDFs Results

Moments PDFs of the s quark in the kaon



KAON PDFs RESULTS

Moment PDFs of the *u* quark in the kaon



PION AND KAON PDFs IN A NUCLEAR MEDIUM

In collaboration with:

- J. J. Cobos-Martinez
  Catedra CONACyt, Departmento de Fisica, Mexico
- Kazuo Tsushima
  Laboratorio de Fisica Teorica e Computacional-LFTC, Universidade Cruzeiro do Sul
- Yongseok Oh
  Asia Pacific Center for Theoretical Physics (APCTP)
  Nuclear Physics Group, Kyungpook National University

Lagrangian QMC model

The effective Lagrangian for a symmetric nuclear matter in the QMC model:

$$\mathscr{L}_{\mathrm{QMC}} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M_N^*(\sigma) - g_\omega\gamma_\mu\omega^\mu\right]\psi + \mathscr{L}_m, \tag{18}$$

The free meson Lagrangian density:

$$\mathscr{L}_m = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{2}\partial_\mu\omega_\nu\left(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu\right) + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu$$

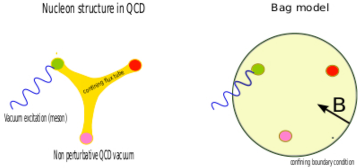


FIGURE: The QCD picture of the nucleon and the bag model [4]

[4] J.Stone *et al.*, Prog.Part.Nucl.Phys. **100** (2018)

Lagrangian QMC model

- In the QMC model, the nuclear matter is treated as a collection of the nucleons that are assumed to be non-overlapping MIT bags
- The Dirac equation for the light quarks inside the bags are given by

$$\left[i\gamma\cdot\partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q + \frac{1}{2}V_\rho^q\right)\right]\begin{pmatrix}\psi_u(x)\\ \psi_{\bar{u}}(x)\end{pmatrix} = 0$$

$$\left[i\gamma\cdot\partial_x - (m_q - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q - \frac{1}{2}V_\rho^q\right)\right]\begin{pmatrix}\psi_d(x)\\ \psi_{\bar{d}}(x)\end{pmatrix} = 0$$

$$[i\gamma\cdot\partial_x - m_s]\begin{pmatrix}\psi_s(x)\\ \psi_{\bar{s}}(x)\end{pmatrix} = 0, \qquad (19)$$

where the effective current quark mass m_q^* is defined as

$$m_q^* \equiv m_q - V_\sigma^q, \qquad (20)$$

where m_q is the current quark mass, where $q = (u, d, s)$ and V_σ^q is the scalar potential.

LAGRANGIAN QMC MODEL

The effective nucleon mass:

$$M_N^*(\sigma) \equiv M_N - g_\sigma(\sigma)\,\sigma, \tag{21}$$

Total energy per nucleon:

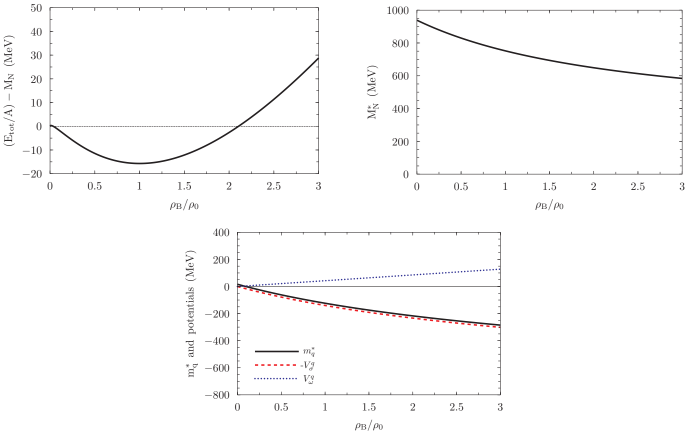
$$E^{\rm tot}/A = \frac{4}{(2\pi)^3\rho_B}\int d\mathbf{k}\,\theta(k_F - |\mathbf{k}|)\sqrt{M_N^{*2}(\sigma) + \mathbf{k}^2} + \frac{m_\sigma^2\sigma^2}{2\rho_B} + \frac{g_\omega^2\rho_B}{2m_\omega^2}. \tag{22}$$

TABLE: Parameters of the QMC model and the obtained nucleon properties at saturation density $\rho_0 = 0.15$ fm^{-3} for two quark mass values in free space, $m_q = 5.0$, and 16.4 MeV. The m_q, M_N^*, and K are given in units of MeV. The parameters are fitted to the free space nucleon mass $M_N = 939$ MeV with $R_N = 0.8$ fm (input), and the nuclear matter saturation properties.

m_q	$g_\sigma^2/4\pi$	$g_\omega^2/4\pi$	$B^{1/4}$	z_N	M_N^*	K
5	5.393	5.304	170.0	3.295	754.6	279.3
16.4	5.438	5.412	169.2	3.334	752.0	281.5

Lagrangian QMC model

Energy per nucleon ($E^{\rm tot}/A - M_N$), effective nucleon mass M_N^* and effective quark mass (m_q^*) and the quark potentials (V_σ^q and V_ω^q) for symmetric nuclear matter in the QMC model for the current quark mass $m_q = 16.4$ MeV (PTPH, Yongseok Oh and K. Tsushima, in preparation (2018))



In-medium pion properties (NJL+QMC)

- Using the in-medium properties corresponding to $m_q = 16.4$ MeV calculated in the QMC model,
- we calculate the effective quark mass M_u^*, in-medium pion decay constant, in-medium quark condensate, and in-medium πqq coupling constant using the NJL model.
- The in-medium dressed quark propagator:

$$S_q^*(k^*) = \frac{\not{k} + V^0 + M_q^*}{(k + V^0)^2 - M_q^* + i\epsilon}, \tag{23}$$

where the medium modification enter as the shift of the quark momentum through $(k^*)\mu \to k^\mu + V^\mu$ where vector potential, $V^\mu = (\delta_0^\mu V^0, \vec{0})$[5]. The asterisk denotes the in-medium quantity

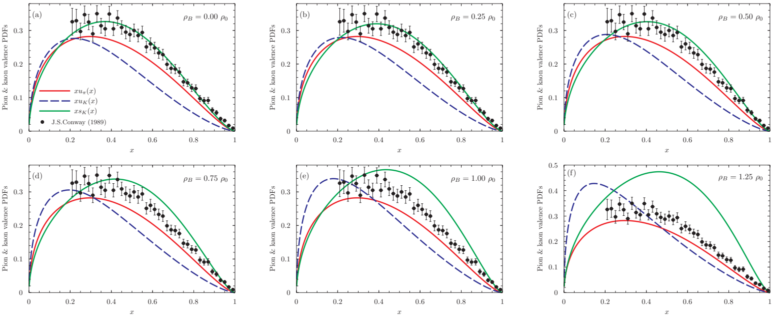
- The in-medium NJL gap mass in the proper-time regularization scheme:

$$M_q^* = m_q^* + \frac{3G_\pi M_q^*}{\pi^2} \int_{\frac{1}{\Lambda_{UV}^2}}^{\infty} \frac{d\tau}{\tau^2} e^{\left(-\tau (M_q^*)^2\right)} \tag{24}$$

[5] Miller, Phys. Rev. Lett. **103** (2009)

Numerical results of PDFs of the pion and kaon in a nuclear medium

PDFs of the pion and kaon in nuclear medium (*PRELIMINARY RESULT*)



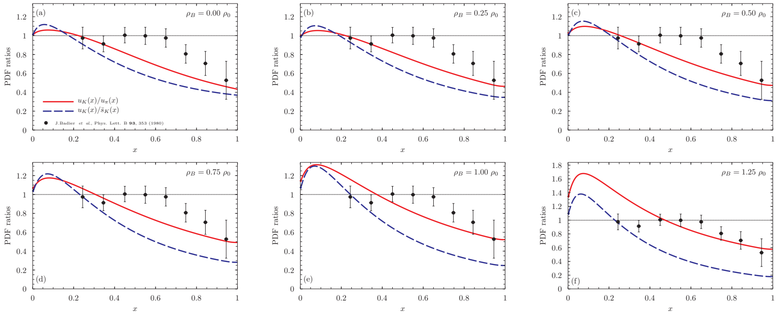
The effect of the vector field is then incorporated through scaling the quark distribution and shifting the Bjorken variable

$$q_{K^+}(x_a) = \frac{\epsilon_F}{E_F} q_{K^+}\left(\tilde{x_a} = \frac{\epsilon_F}{E_F} x_a - \frac{V_0}{E_F}\right) \tag{25}$$

where $\epsilon_F = \sqrt{k_F^q + M_q^2} \pm V_0 \equiv E_F \pm V_0$.

Numerical results of PDFs of the pion and kaon in a nuclear medium

Ratio of $u_{K^+}(x)/u_{\pi^+}(x)$ in nuclear medium (*PRELIMINARY RESULT*)



Conclusion and outlook

- We have studied pion and kaon PDFs in vacuum as well as in medium.
- Our prediction on pion and kaon PDFs are in good agreement with existed data
- Strange quark distribution in the kaon affects significantly the quark distribution of the kaon
- We have extend our study on the in-medium modifications PDFs of the pion and kaon in order to understand the feature of PDFs of the pion and kaon in the medium. The result looks very interesting and promising.
- It would be interesting to extend the calculation to the generalized parton distributions (GPDs) of the pion, kaon, and ρ meson in medium

THANK YOU VERY MUCH FOR ATTENTION !!