Hadron tomography studies in Japan at J-PARC, KEKB, and ILC

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Workshop on Progress on Hadron structure functions in 2018
November 18-19, 2018, Seminar-Hall, 4th Building, KEK Tsukuba Campus
http://j-parc-th.kek.jp/workshops/2018/11-18/

November 18, 2018

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  - → **Origin of nucleon mass:** Decomposition into internal mass components

  Hadron tomography: We study these topics through **3-dimensional structure functions.**
- **GPDs and GDAs for hadron tomography**

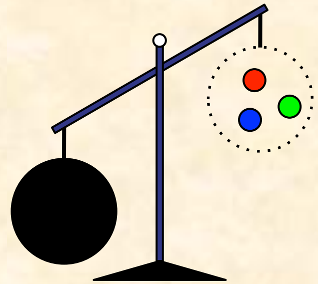
  GPD (Generalized Parton Distribution)

  GDA (Generalized Distribution Amplitude) = "Timelike GPDs"
- **Gravitational sources in hadrons**

  Gravitational-mass distributions from microscopic quark-gluon level
- **Summary**

Origins of nucleon spin and mass

Motivations: Unsolved mysteries in physics

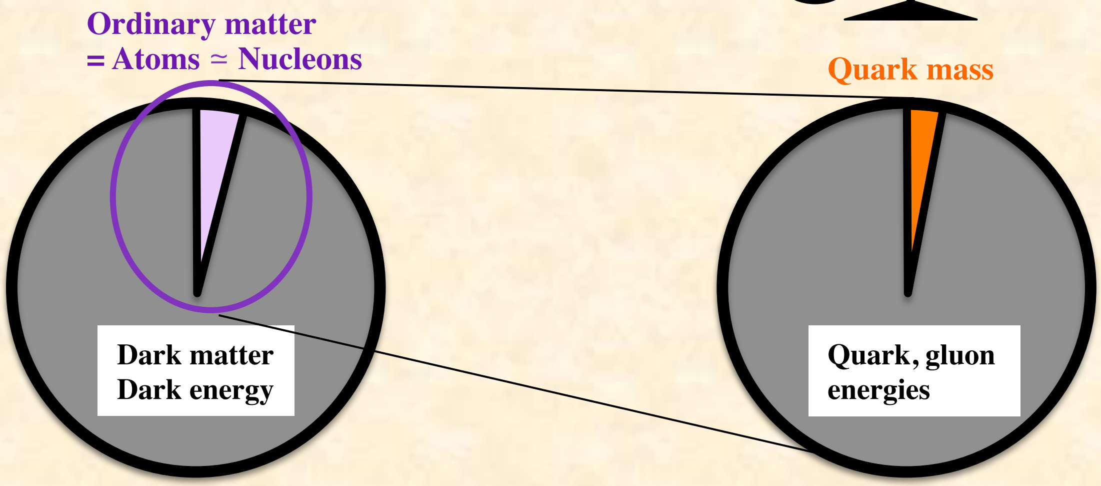


Mass and spin of the nucleon are two of fundamental quantities in physics.

Nucleon mass: $M = \left\langle p \middle| \int d^3x \, T^{00}(x) \middle| p \right\rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$

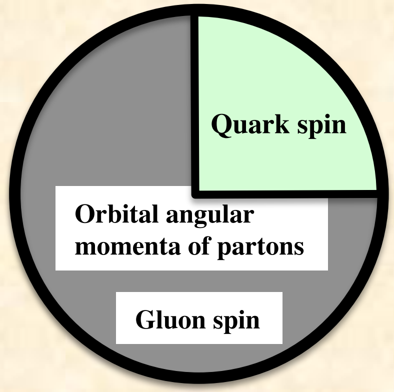


Dark matter — **Origin of nucleon mass**

Nucleon spin: $\frac{1}{2} = \left\langle p \middle| J^3 \middle| p \right\rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2}\varepsilon^{3jk} \int d^3x \, M^{3jk}(x)$

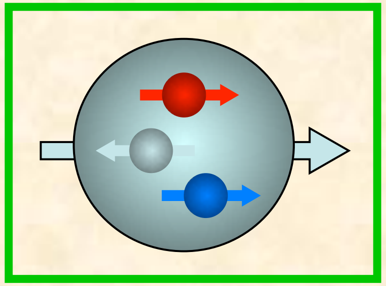
Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x) x^{\mu} - T^{\alpha\mu}(x) x^{\nu}$



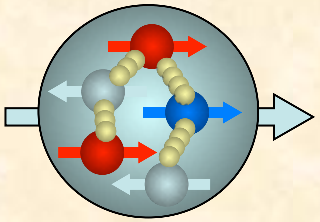
Origin of nucleon spin
("Dark spin")

Recent progress on origin of nucleon spin

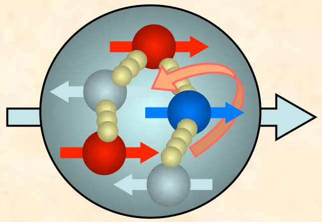
“old” standard model



gluon spin



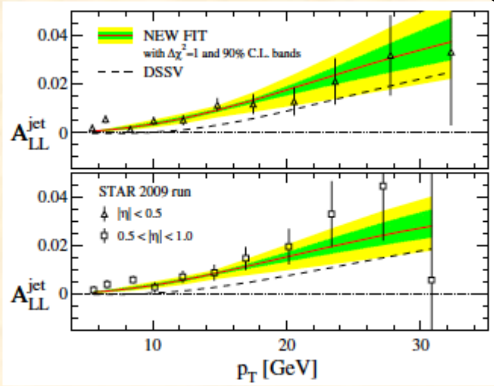
angular momentum

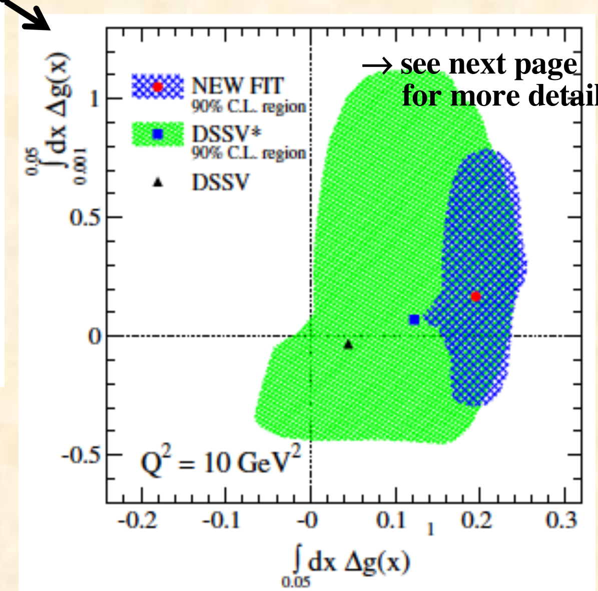


$$p_{\uparrow} = \frac{1}{3\sqrt{2}}\left(uud\left[2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right] + \text{permutations} \right)$$

$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

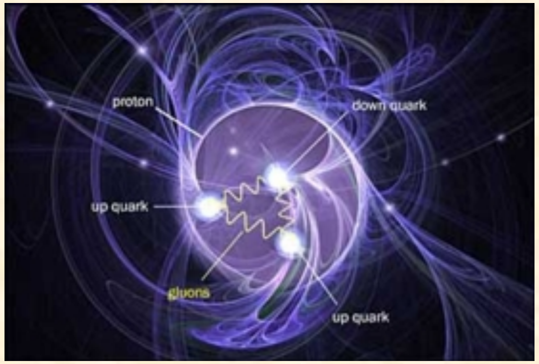
$$\Delta\Sigma = \sum_i \int dx \left[\Delta q_i(x) + \Delta \bar{q}_i(x) \right] \to 1 \ (100\%)$$





→ see next page for more details

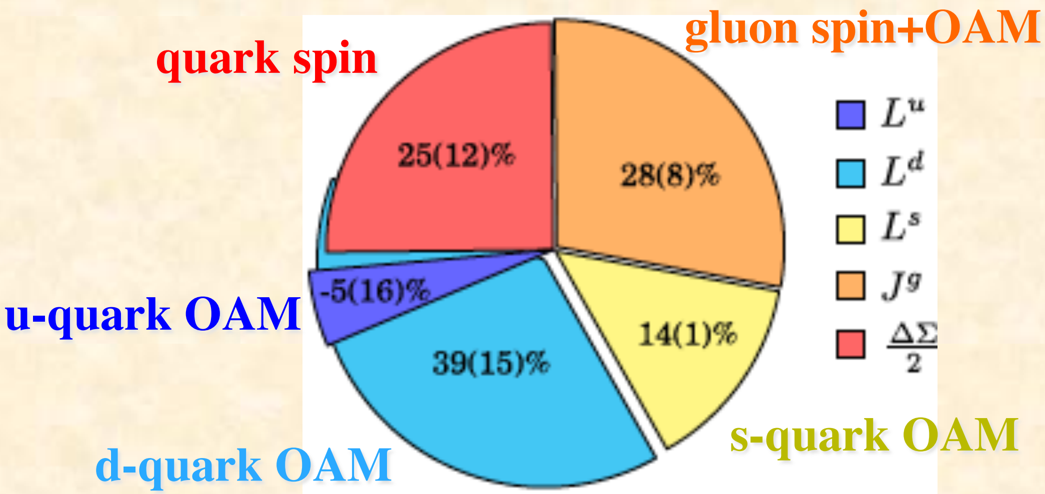
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_{q,g}$$



Scientific American (2014)

Origin of nucleon spin: decomposition

$$\frac{1}{2} = \langle p | J^3 | p \rangle = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g, \quad J^3 = \frac{1}{2}\varepsilon^{3jk}\int d^3x\, M^{3jk}(x), \;\; M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x)x^\mu - T^{\alpha\mu}(x)x^\nu$$



Lattice QCD estimate in M. Deke *et al*., PRD 91 (2015) 0145505

Spin decomposion

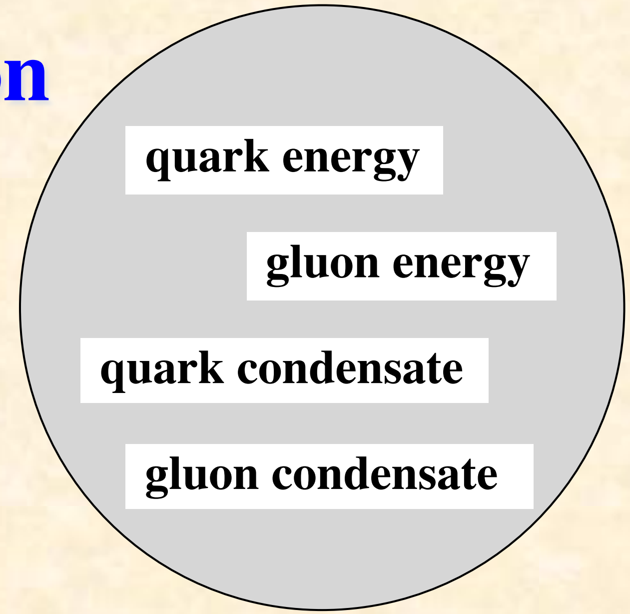
- quark spin 25%
- quark OAM 45%
- gluon spin + OAM 30%

Origin of nucleon mass: decomposition

Nucleon mass: $M = \langle p | H | p \rangle, \quad H = \int d^3x\, T^{00}(x)$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$



We need theoretical and experimental efforts to decompose nucleon mass for finding its origin.

Hadron tomography
(3D structure functions)

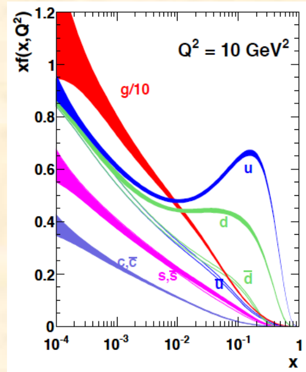
Wigner distribution and various structure functions

Form factor

$\int dx\, d^2k_T$

PDF (Parton Distribution Function)

$\int d^2k_T,\ \Delta \to 0$



GTMD $W(x, \vec{k}_T, \Delta) \xrightarrow{\Delta^+ \to 0}$ **Wigner** $W(x, \vec{k}_T, \vec{r}_T)$

$\Delta = p' - p$

$\int d^2k_T$

3D world

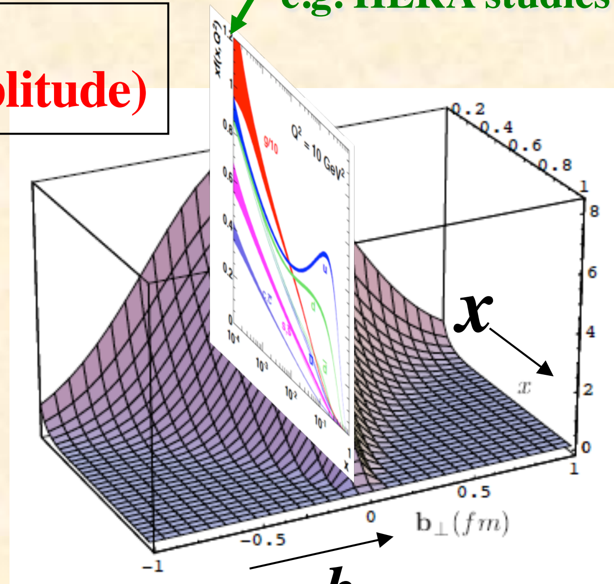
GPD (Generalized Parton Distribution)

$\xrightarrow[s \leftrightarrow t]{}$ **GDA (Generalized Distribution Amplitude)**

e.g. HERA studies

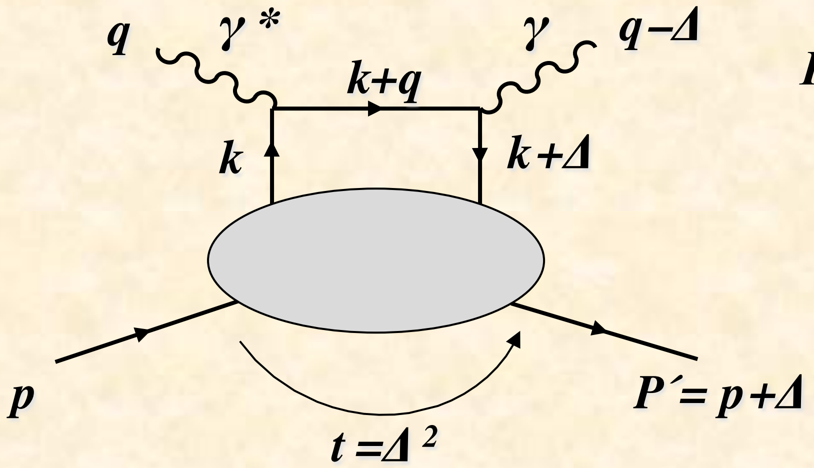
$\Delta \to 0$

TMD (Transverse Momentum Dependent) parton distribution



Our work by the two-photon process $\gamma\gamma \to h\bar{h}$.

Generalized Parton Distributions (GPDs)



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $\quad x = \frac{Q^2}{2p\cdot q}$

Momentum transfer squared $\quad t = \Delta^2$

Skewdness parameter $\quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

Forward limit: PDFs

$$H(x,\xi,t)\Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\Big|_{\xi=t=0} = \Delta f(x),$$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\quad \int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

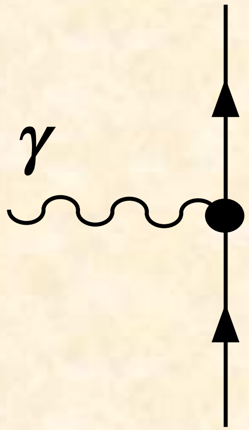
Axial and Pseudoscalar form factors G_A, G_P $\quad \int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

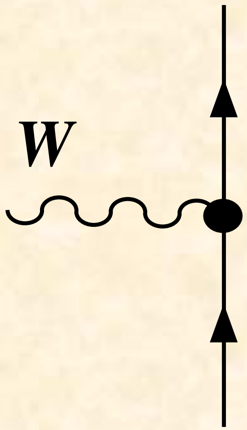
Sum rule: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \frac{1}{2}\Delta q + L_q$

⇒ probe L_q, key quantity to solve the spin puzzle!

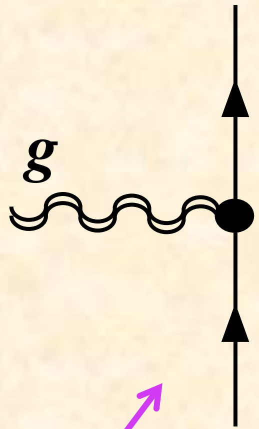
Why gravitational interactions with hadrons ?



vector
$\bar{q}\gamma^{\mu}q$



vector – axial-vector
$\bar{q}\gamma^{\mu}(1-\gamma^{5})q$



tensor
$\bar{q}\gamma^{\mu}\partial^{\nu}q$

How about gravitational radius?

Electron-proton elastic scattering cross section:

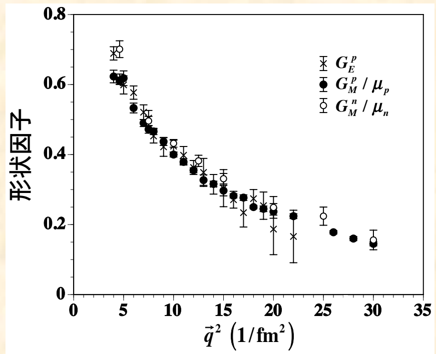
$$\frac{d\sigma}{d\Omega}=\frac{\alpha^{2}E_{f}\cos^{2}\dfrac{\theta}{2}}{4E_{i}^{3}\sin^{4}(\theta/2)}\left[\frac{G_{E}^{2}+\tau G_{M}^{2}}{1+\tau}+2\tau G_{M}^{2}\tan^{2}\frac{\theta}{2}\right],\quad \tau=-\frac{q^{2}}{4M^{2}}$$

$$F(\vec{q})=\int d^{3}x\, e^{i\vec{q}\cdot\vec{x}}\rho(\vec{x})=\int d^{3}x\left[1-\frac{1}{2}(\vec{q}\cdot\vec{x})^{2}+\cdots\right]\rho(\vec{x})$$

$$\left\langle r^{2}\right\rangle=\int d^{3}x\, r^{2}\rho(\vec{x}),\quad r=|\vec{x}|$$

$\sqrt{\left\langle r^{2}\right\rangle}$ = root-mean-square (rms) radius

$$F(\vec{q})=1-\frac{1}{6}\vec{q}^{\,2}\left\langle r^{2}\right\rangle+\cdots,\qquad \left\langle r^{2}\right\rangle=-6\left.\frac{dF(\vec{q})}{d\vec{q}^{\,2}}\right|_{\vec{q}^{2}\to 0}$$

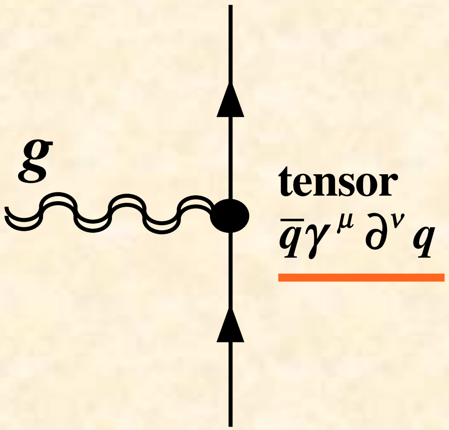


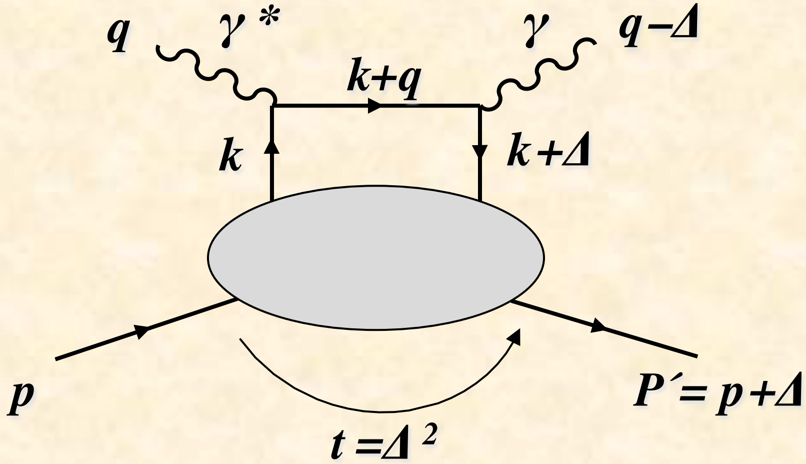
$$\rho(r)=\frac{\Lambda^{3}}{8\pi}e^{-\Lambda r}\;\Leftrightarrow\;\text{Dipole form: }F(q)=\frac{1}{\left(1+|\vec{q}|^{2}/\Lambda^{2}\right)^{2}},\quad \Lambda^{2}\simeq 0.71\ \text{GeV}^{2}$$

Proton-charge-radius puzzle:

$R_{\text{electron scattering}}$ = 0.8775 fm
⇕
$R_{\text{muonic atom}}$ = 0.8418 fm

Gravitational sources and 3D structure functions





GPDs: $\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2)\gamma^+ q(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$

Angular momentum: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x \left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0) \right], \quad J_q = \frac{1}{2}\Delta q + L_q$

Non-local operator of GPDs/GDAs:

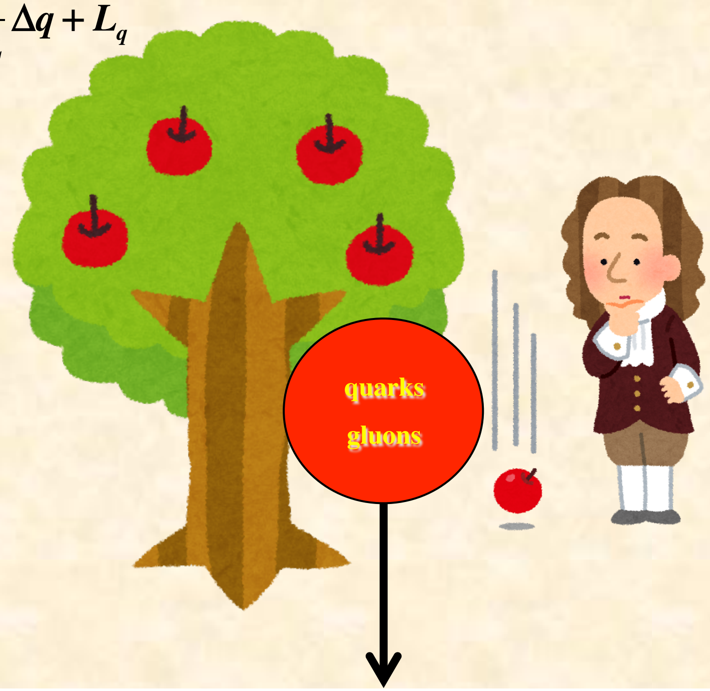
$$\left(P^+\right)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z^+=0,\vec{z}_\perp=0}$$

$$= \left(i\frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z=0}$$

$$= \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+ \right)^{n-1} q(0)$$

= energy-momentum tensor of a quark for $n=2$
(electromagnetic for $n=1$)

= source of gravitiy



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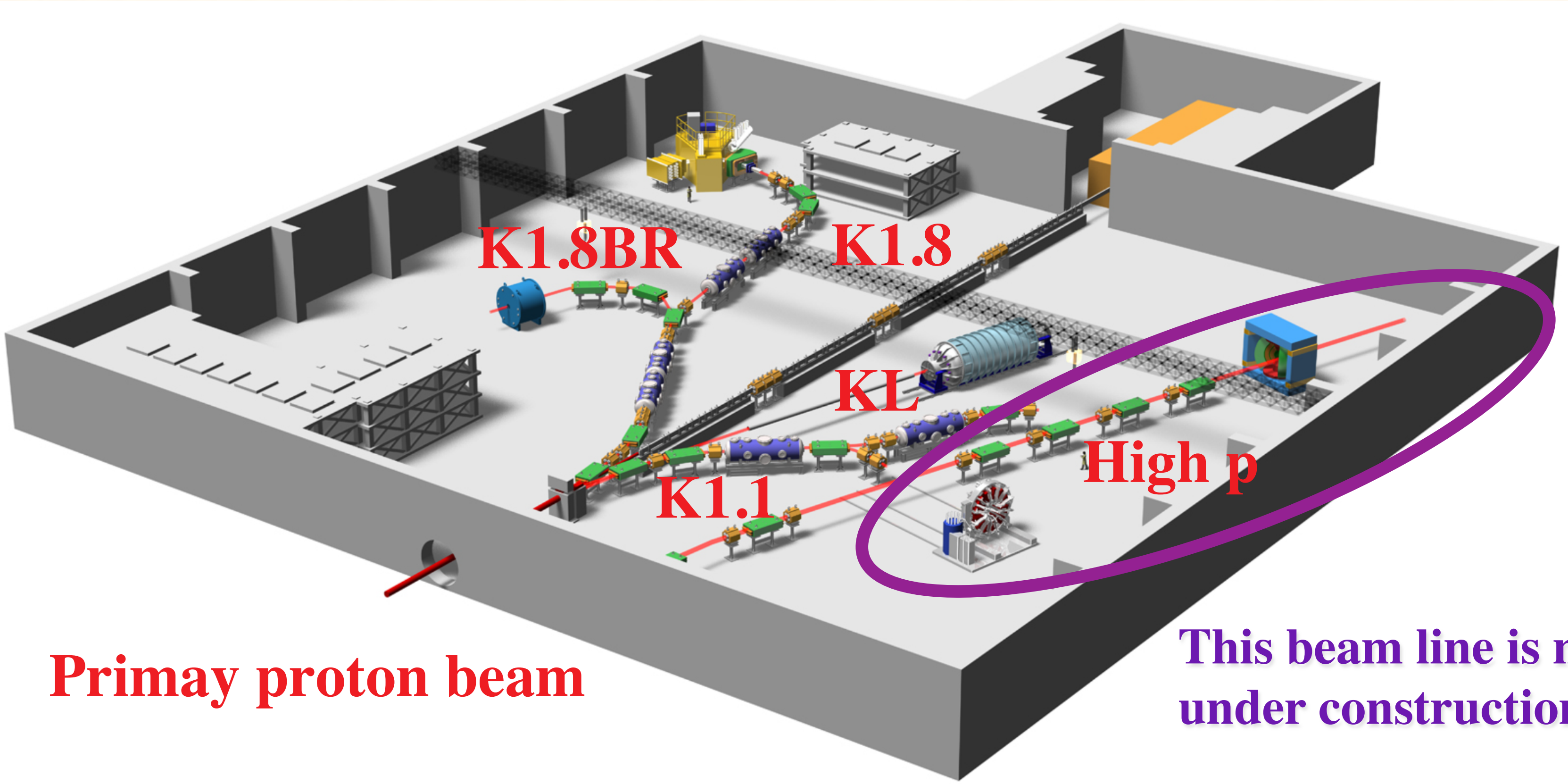
Generalized Parton Distributions (GPDs)

and J-PARC project

Hadron facility

Workshops on high-momentum beamline physics,
http://www-conf.kek.jp/hadron1/j-parc-hm-2013/
http://research.kek.jp/group/hadron10/j-parc-hm-2015/.



- Proton beam up to 30 GeV
- Unseparated hadron (pion, …) beam up to 15~20 GeV

Exclusive Drell-Yan $\pi^- + p \to \mu^+\mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \,\mathrm{Re}\left\{ \tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t) \right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$

$$Q'^2 = q'^2,\ \ t = (p-p')^2,\ \tau = \frac{Q'^2}{2p\cdot q_\pi} \simeq \frac{Q'^2}{s-m_N^2}$$

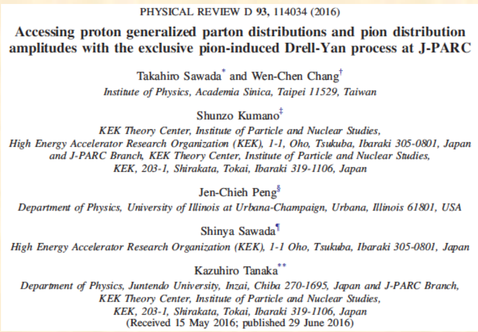
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p(p')|\bar{q}(-z/2)\gamma^+\gamma_5 q(z/2)|p(p)\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_p^q(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_p^q(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle n(p')|\bar{q}_d(-z/2)\gamma^+\gamma_5 q_u(z/2)|p(p)\rangle\Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}_{p\to n}^{du}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

$$\tilde{H}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon} \right]\left[\tilde{H}^d(x',\xi,t) - \tilde{H}^u(x',\xi,t) \right]$$

$$\tilde{E}^{du}(x,\xi,t) = \frac{8}{3}\alpha_s \int_{-1}^{1} dz \frac{\phi_\pi(z)}{1-z^2}\int_{-1}^{1} dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon} \right]\left[\tilde{E}^d(x',\xi,t) - \tilde{E}^u(x',\xi,t) \right]$$

T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng, S. Sawada, and K. Tanaka, PRD93 (2016) 114034.


PHYSICAL REVIEW D 93, 114034 (2016)

Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC

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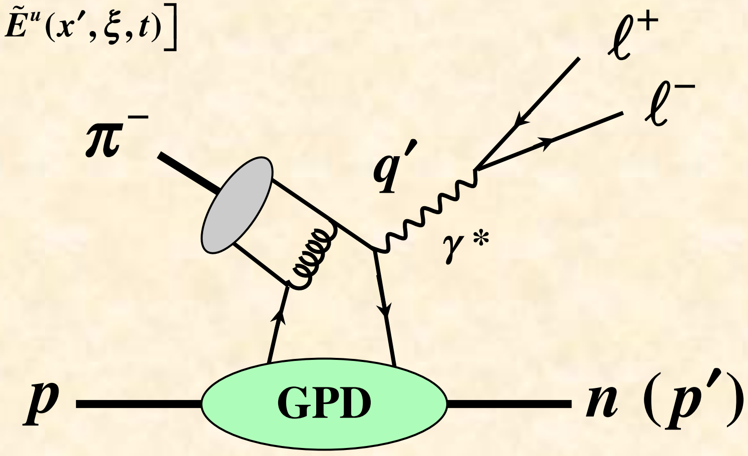
(Received 15 May 2016; published 29 June 2016)

LoI under consideration for a J-PARC experiment

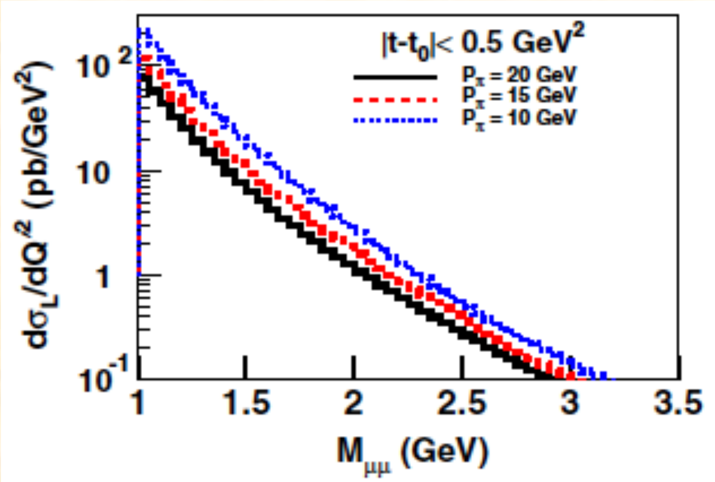


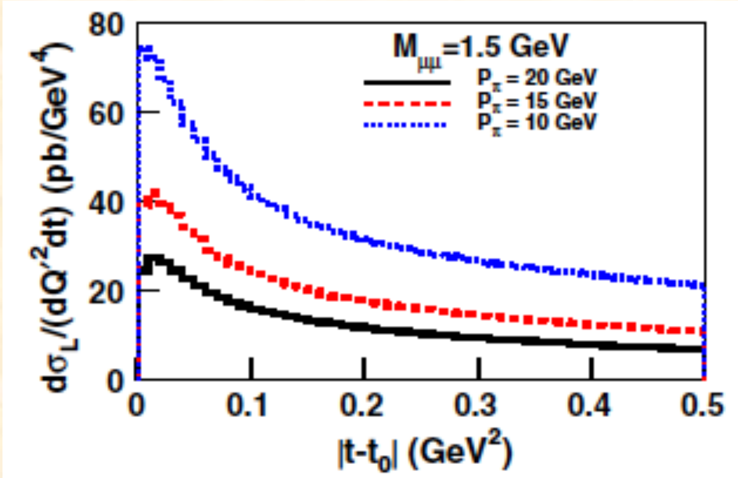
$$\pi^-(\bar{u}d) + p(uud) \to n(udd) + \gamma^*(\to \ell^+\ell^-)$$

Expected Drell-Yan events at J-PARC

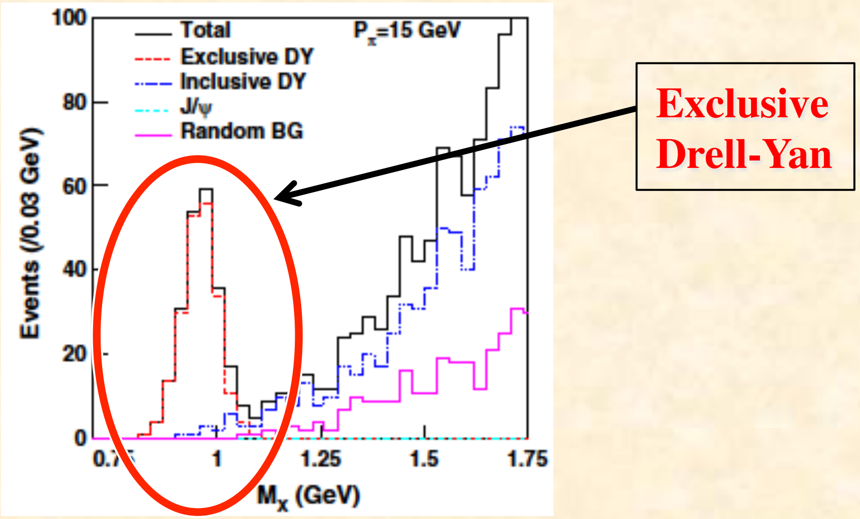
$$Q'^2 = q'^2,\ t = (p - p')^2,\ \tau = \frac{Q'^2}{2p \cdot q_\pi} \simeq \frac{Q'^2}{s - m_N^2}$$

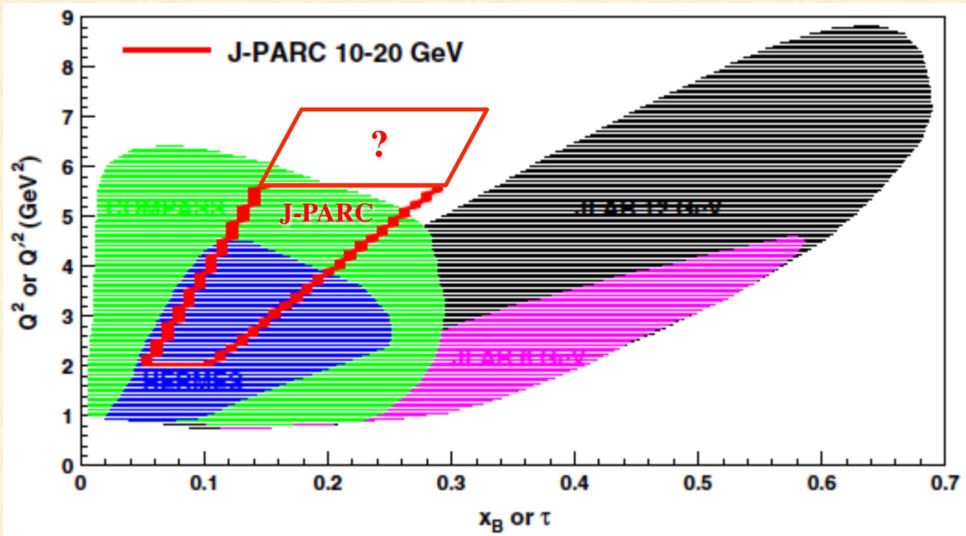
$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27}\frac{\tau^2}{Q'^2} f_\pi^2 \left[(1-\xi^2)\left|\tilde{H}^{du}(-\xi,\xi,t)\right|^2 - 2\xi^2 \mathrm{Re}\left\{\tilde{H}^{du}(-\xi,\xi,t)^* \tilde{E}^{du}(-\xi,\xi,t)\right\} - \xi^2 \frac{t}{4m_N^2}\left|\tilde{E}^{du}(-\xi,\xi,t)\right|^2 \right]$$



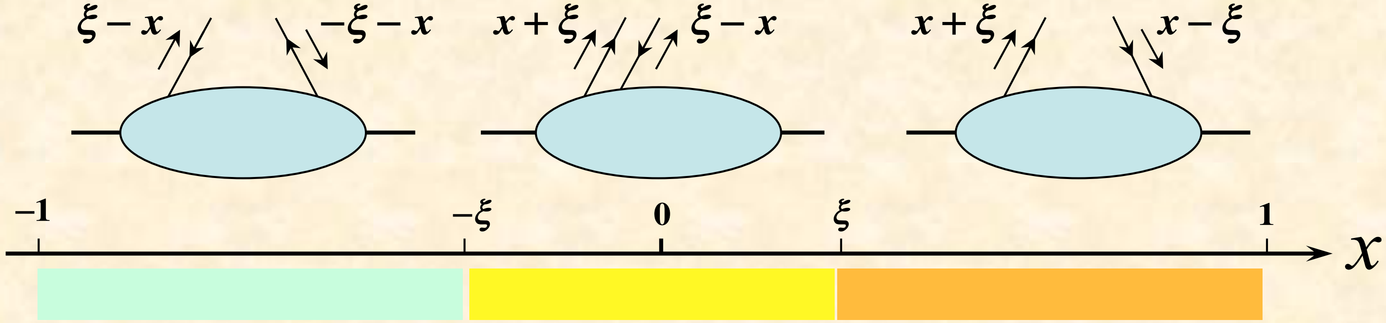


Missing mass





GPDs in different x regions and GPDs at hadron facilities



$-1 < x < \xi \quad (x + \xi < 0,\ x - \xi < 0)$

$\xi < x < 1 \quad (x + \xi > 0,\ x - \xi > 0)$

$-\xi < x < \xi \quad (x + \xi > 0,\ x - \xi < 0)$

Quark distribution

Emission of quark with momentum fraction $x+\xi$

Absorption of quark with momentum fraction $x-\xi$

$q\bar{q}$(meson)-like distribution amplitude

Emission of quark with momentum fraction $x+\xi$

Emission of antiquark with momentum fraction ξ-x

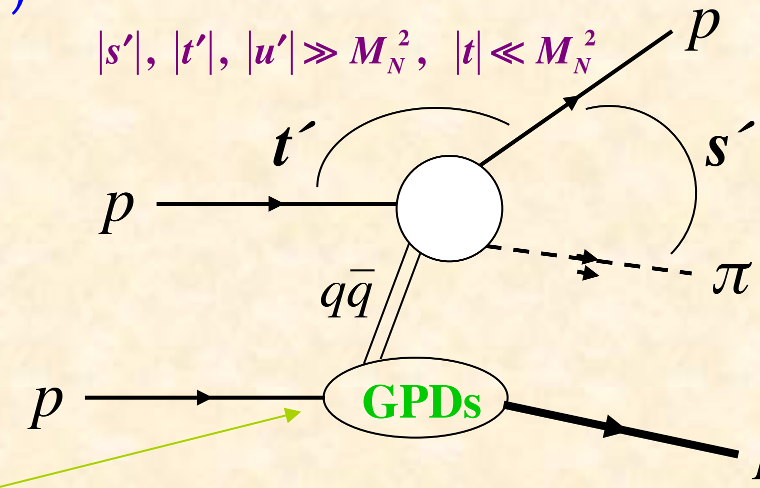
Antiquark distribution

Emission of antiquark with momentum fraction ξ-x

Absorption of antiquark with momentum fraction -ξ-x

Consider a hard reaction with

$|s'|,\ |t'|,\ |u'| \gg M_N^2,\ \ |t| \ll M_N^2$



GPDs at J-PARC: S. Kumano, M. Strikman, and K. Sudoh, PRD 80 (2009) 074003.

Efremov-Radyushkin -Brodsky-Lepage (ERBL) region

Generalized Distribution Amplitudes (GDAs)

and KEKB/ILC project

H. Kawamura and S. Kumano,
Phys. Rev. D 89 (2014) 054007.

S. Kumano, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

GPD $H_q^h(x,\xi,t)$ and GDA $\Phi_q^{hh}(z,\zeta,W^2)$

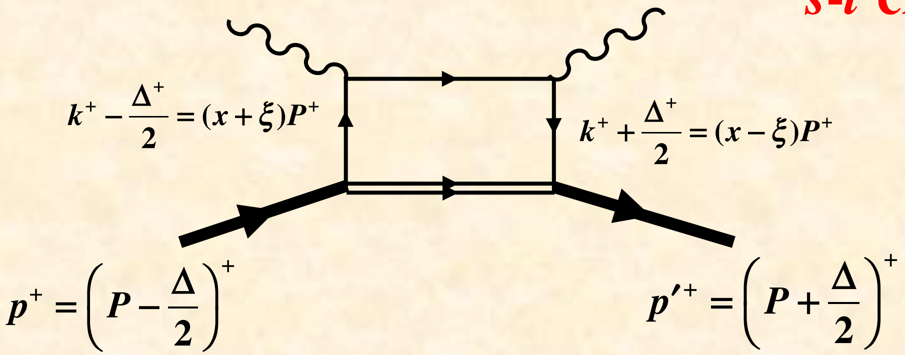
GPD: $H_q(x,\xi,t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | h(p) \rangle \big|_{y^+=0,\vec{y}_\perp=0}$, $\quad P^+ = \frac{(p+p')^+}{2}$

GDA: $\Phi_q(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p)\bar{h}(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | 0 \rangle \big|_{y^+=0,\vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2)\gamma^+\gamma_5\psi(y/2) | 0 \rangle \big|_{y^+=0,\vec{y}_\perp=0}$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

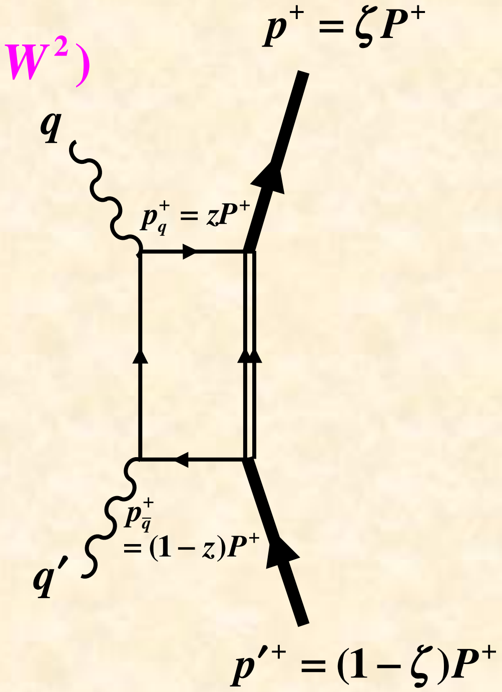
***s*-*t* crossing**



$z \Leftrightarrow \frac{1-x/\xi}{2}$

$\zeta \Leftrightarrow \frac{1-1/\xi}{2}$

$W^2 \Leftrightarrow t$



JLab / COMPASS

KEKB

$P = \frac{p+p'}{2}, \quad \Delta = p' - p$

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewdness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

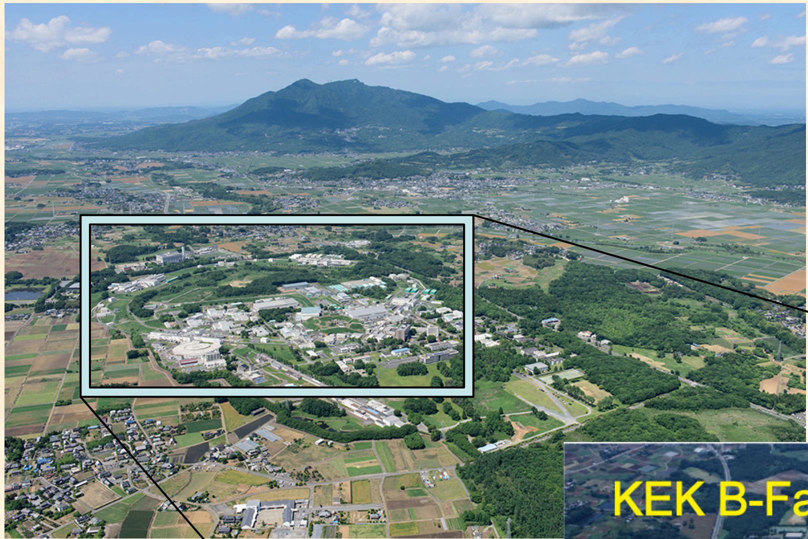
Bjorken variable for $\gamma\gamma^*$: $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1+\beta\cos\theta}{2}$

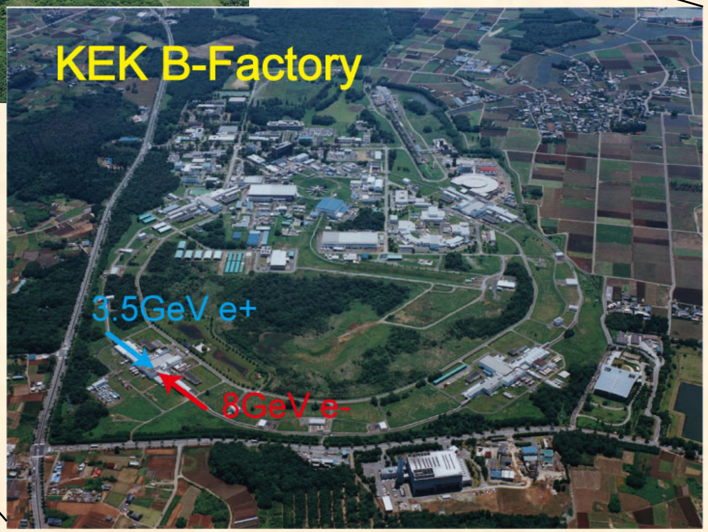
Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Experimental studies of GDAs in future

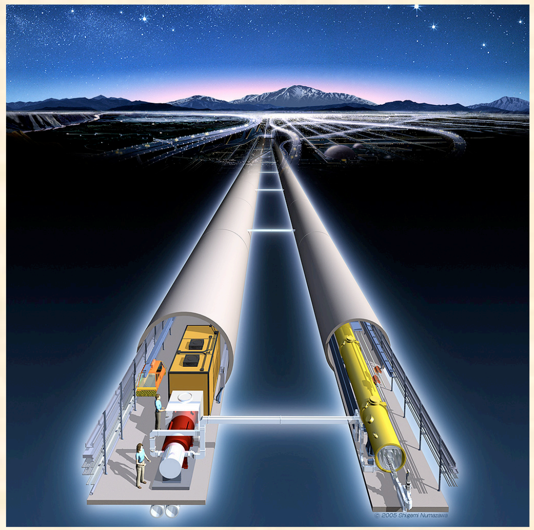
$\gamma\gamma \to h\bar{h}$ for internal structure of exotic hadron candidate h



KEK B-factory



Linear Collider ?



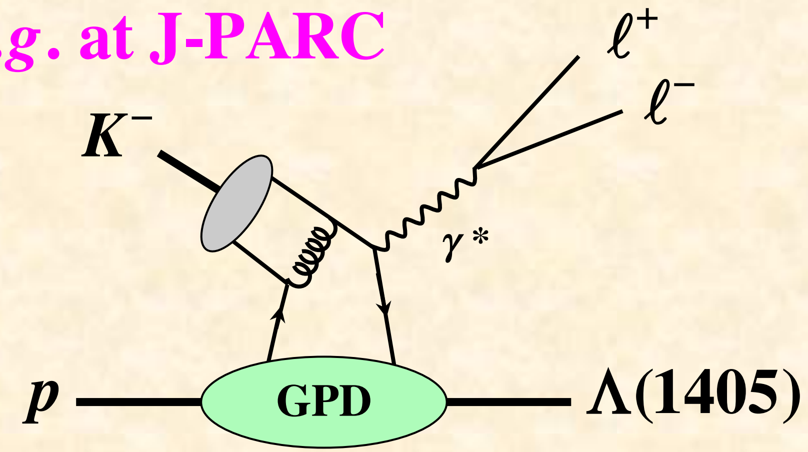
GPDs for exotic hadrons !?

Because stable targets do not exit for exotic hadrons, it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or → $s \leftrightarrow t$ crossed qunatity = GDAs at KEKB, Linear Collider

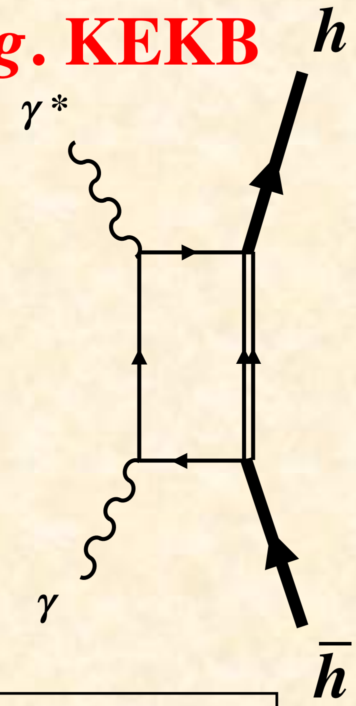
***e.g.* at J-PARC**



$K^-(\bar{u}s) + p(uud) \to \Lambda_{1405}(uud\bar{u}s) + \gamma^*$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate

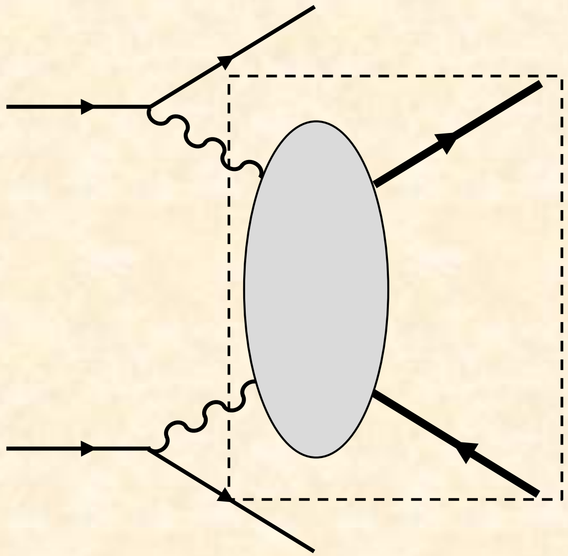
***e.g.* KEKB**



See also H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

Generalized Distribution Amplitudes (GDAs) for pion

from KEKB measurements



$$\gamma\gamma \to h\bar{h}$$

Cross section for $\gamma^* \gamma \to \pi^0 \pi^0$

$$d\sigma = \frac{1}{4\sqrt{(q\cdot q')^2 - q^2 q'^2}} (2\pi)^4 \delta^4(q+q'-p-p') \overline{\sum_{\lambda,\lambda'}} |\mathcal{M}|^2 \frac{d^3 p}{(2\pi)^3 2E} \frac{d^3 p'}{(2\pi)^3 2E'}$$

$$q = (q^0,\ 0,\ 0,\ |\vec{q}|),\ \ q' = (|\vec{q}|,\ 0,\ 0,\ -|\vec{q}|),\ \ q'^2 = 0 \ \text{(real photon)}$$

$$p = (p^0,\ |\vec{p}|\sin\theta,\ 0,\ |\vec{p}|\cos\theta),\ \ p = (p^0,\ -|\vec{p}|\sin\theta,\ 0,\ -|\vec{p}|\cos\theta)$$

$$\beta = \frac{|\vec{p}|}{p^0} = \sqrt{1 - \frac{4m_\pi^2}{W^2}}$$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1-\frac{4m_\pi^2}{s}} \overline{\sum_{\lambda,\lambda'}} |\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu}, \quad T^{\mu\nu} = i\int d^4\xi\, e^{-i\xi\cdot q} \langle \pi(p)\pi(p') | T J_{em}^\mu(\xi) J_{em}^\nu(0) | 0 \rangle$$

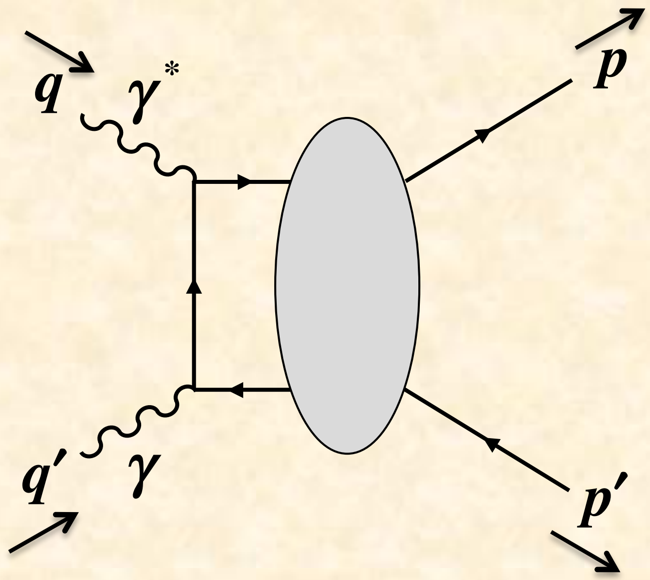
$$\mathcal{M} = e^2 A_{\lambda\lambda'} = 4\pi\alpha A_{\lambda\lambda'}$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2)$$

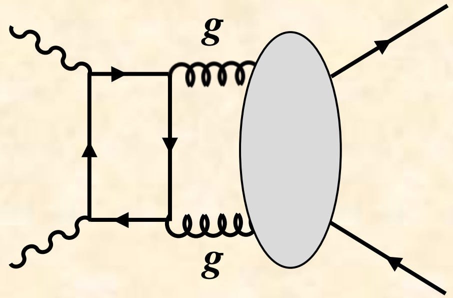
GDA: $$\Phi_q^{\pi\pi}(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p)\pi(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | 0 \rangle \Big|_{y^+=0,\vec{y}_\perp=0}$$

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2), \quad \varepsilon_\mu^+(q)\varepsilon_\nu^+(q') g_T^{\mu\nu} = -1$$

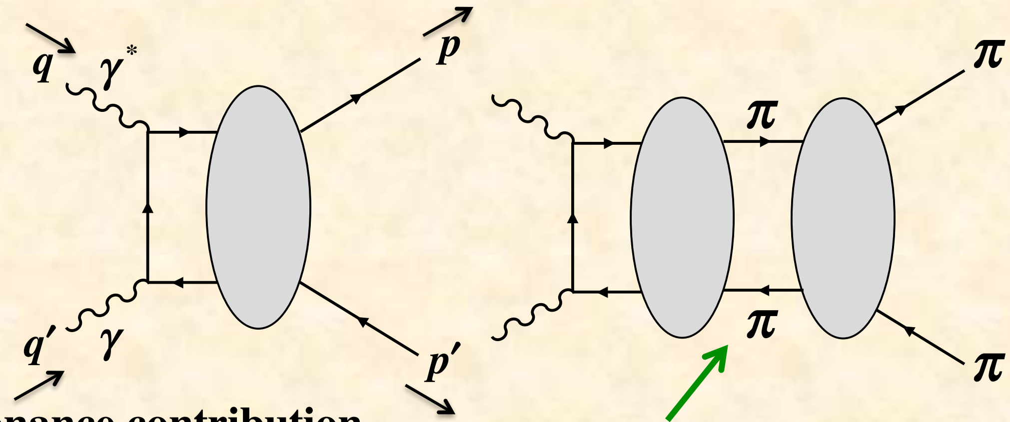
$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1-\frac{4m_\pi^2}{s}} |A_{++}|^2$$



Gluon GDA is higher-order term,
and it is not included in our analysis,



GDA parametrization for pion



Including intermediate resonance contributions

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m^2}{s}}\,|A_{++}|^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

- **Continuum: GDAs without intermediate-resonance contribution**

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{\left[1+(s-4m_\pi^2)/\Lambda^2\right]^{n-1}},$$ $n=2$ **according to constituent counting rule**

- **Resonances: Tthere exist resonance contributions to the cross section.**

$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18 N_f z^\alpha (1-z)^\alpha (2z-1)\left[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos\theta)\right]$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$$\tilde{B}_{10}(W) = \text{resonance}\ \left[f_0(500)\equiv\sigma,\ f_0(980)\equiv f_0\right] + \text{continuum}$$

$$\tilde{B}_{12}(W) = \text{resonance}\left[f_2(1270)\right] + \text{continuum}$$

$f_0(500)$ or σ [g]
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400\text{–}550)$ MeV
Full width $\Gamma = (400\text{–}700)$ MeV

$f_0(980)$ [j]

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 10$ to 100 MeV

$f_2(1270)$

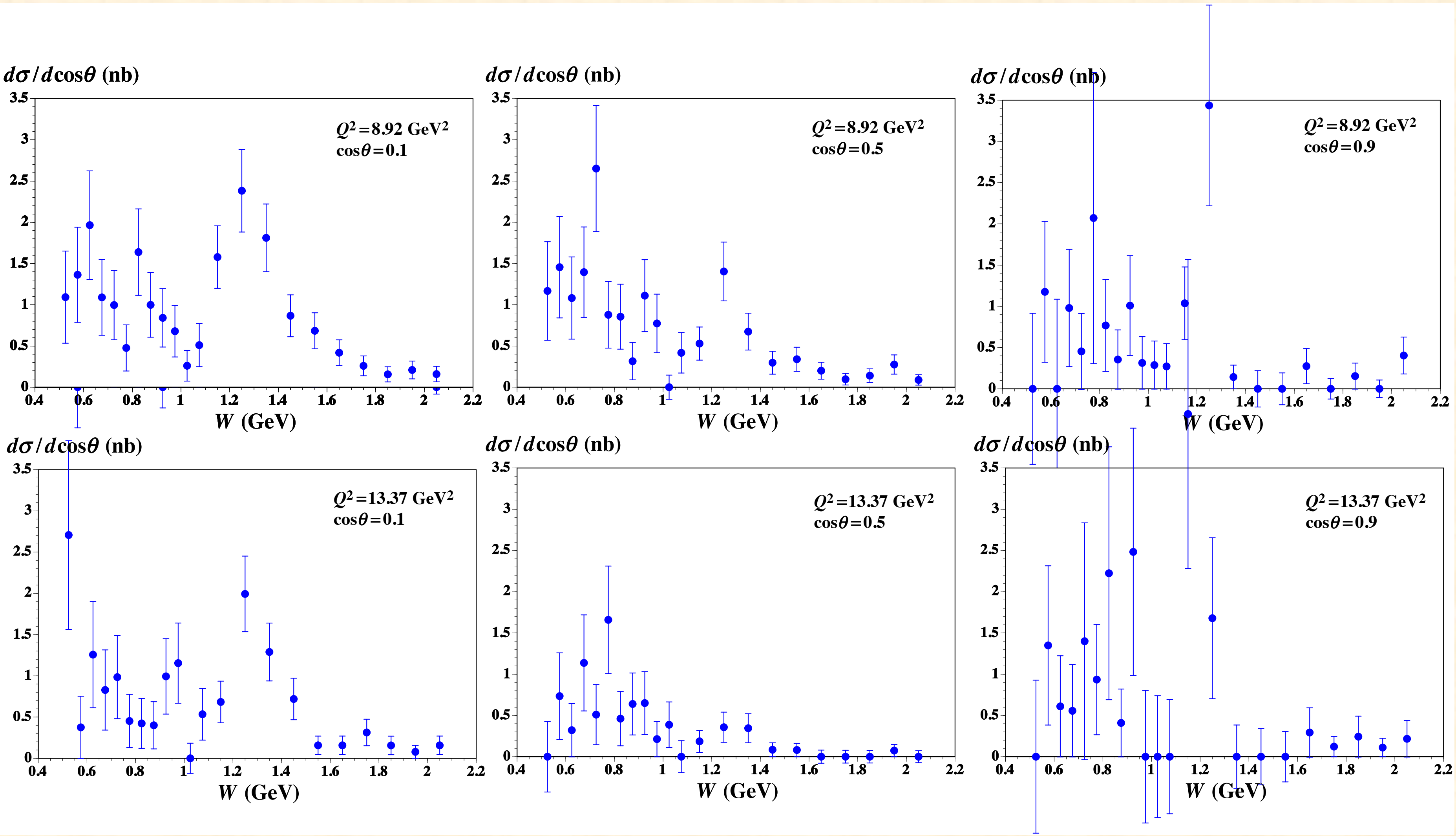
$I^G(J^{PC}) = 0^+(2^{++})$

Mass $m = 1275.5 \pm 0.8$ MeV
Full width $\Gamma = 186.7^{+2.2}_{-2.5}$ MeV (S = 1.4)

Analysis of Belle data on $\gamma\gamma^* \to \pi^0\pi^0$

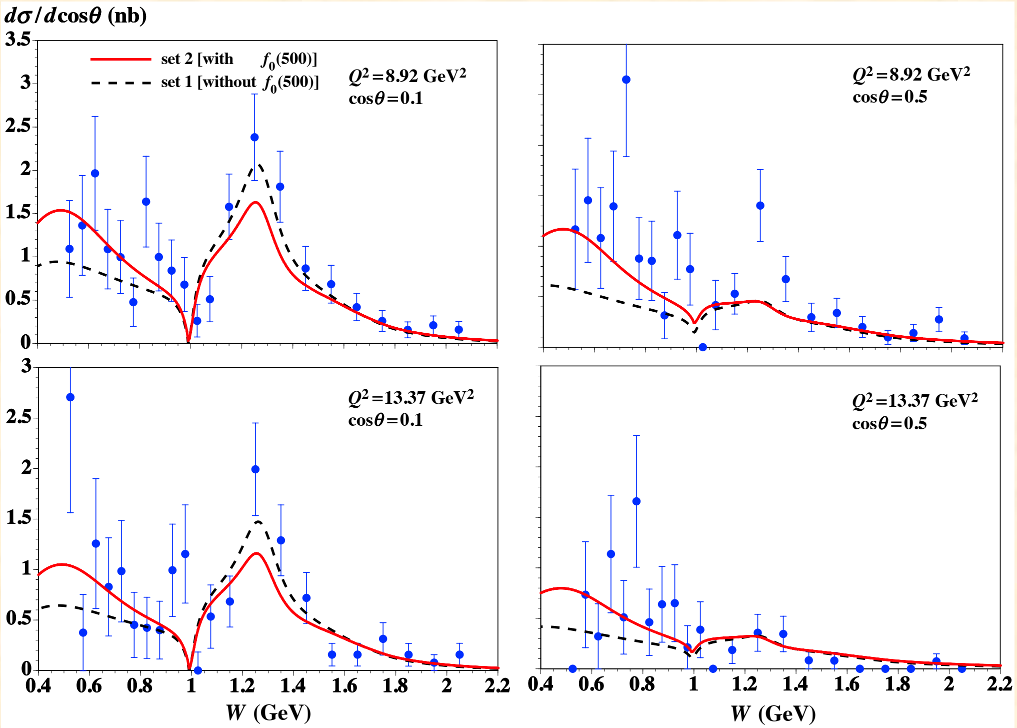
$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$

Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

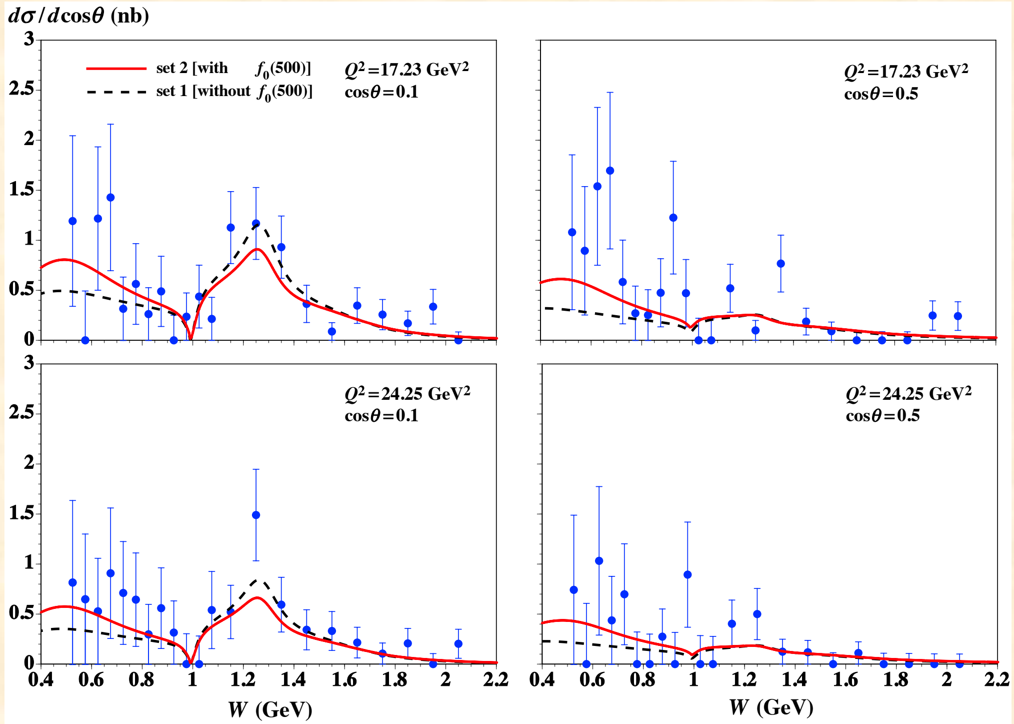


Analysis results for $\cos\theta = 0.1,\ 0.5$

$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$



$Q^2 = 17.23,\ 24.25\ \text{GeV}^2$



Gravitational form factors and radii for pion

$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\langle \pi^0(p)\pi^0(p')|T_q^{++}(0)|0\rangle$$

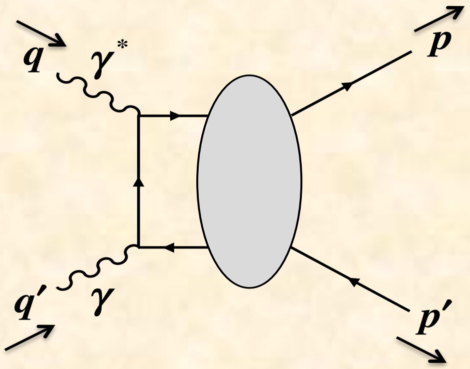
$$\langle \pi^0(p)\pi^0(p')|T_q^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\right]$$

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

See also Hyeon-Dong Son, Hyun-Chul Kim, PRD90 (2014) 111901.

$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$: gravitational form factos for pion



Analyiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section

$\Rightarrow$ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$

$\Rightarrow$ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$

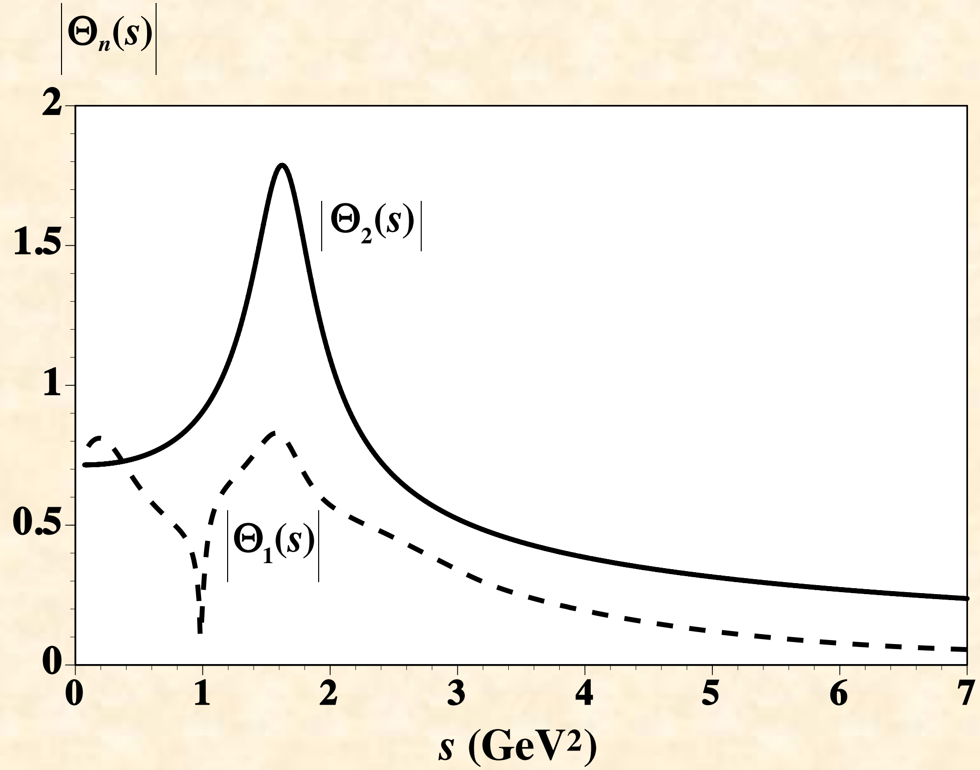
$\Rightarrow$ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$

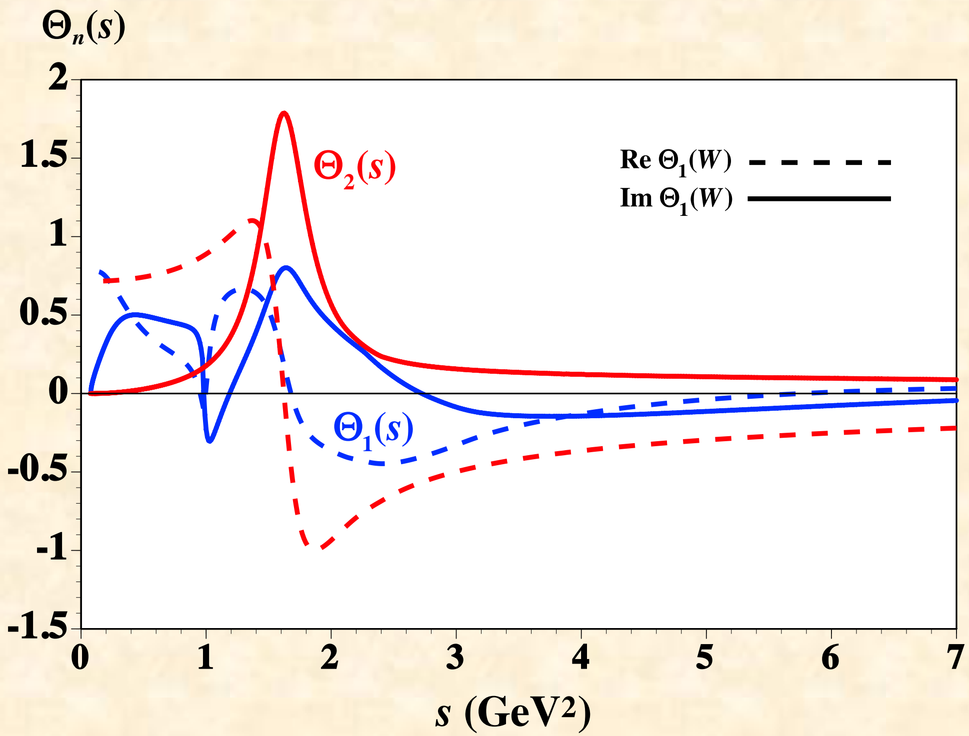
$\Rightarrow$ Gravitational radii of pion

Timelike gravitational form factors for pion

$$\left\langle \pi^a(p)\pi^b(p')\right|T_q^{\mu\nu}(0)\left|0\right\rangle = \frac{\delta^{ab}}{2}\left[(sg^{\mu\nu} - P^\mu P^\nu)\Theta_{1(q)}(s) + \Delta^\mu\Delta^\nu\Theta_{2(q)}(s)\right], \quad P = p + p', \quad \Delta = p' - p$$

- $\Theta_{1(q)}(s) = -\frac{3}{10}\tilde{B}_{10}(W^2) + \frac{3}{20}\tilde{B}_{12}(W^2) = -4B_{(q)}(s)$
- $\Theta_{2(q)}(s) = \frac{9}{20\beta^2}\tilde{B}_{12}(W^2) = A_{(q)}(s)$





Spacelike gravitational form factors and radii for pion

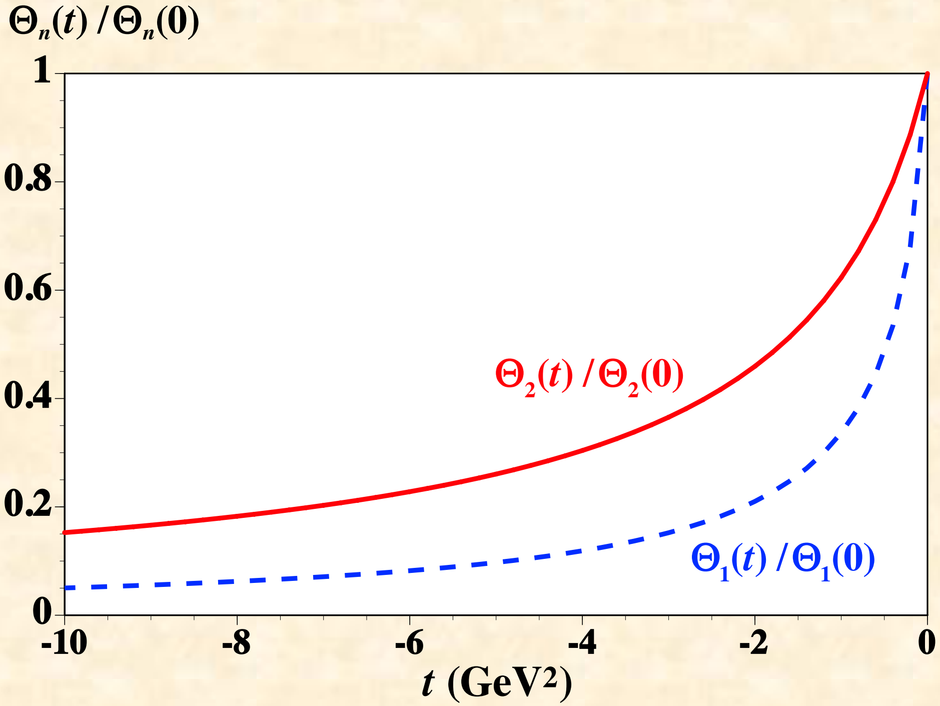
$$F(s) = \Theta_1(s),\ \Theta_1(s),\quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r) = \frac{1}{(2\pi)^3}\int d^3q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\, e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

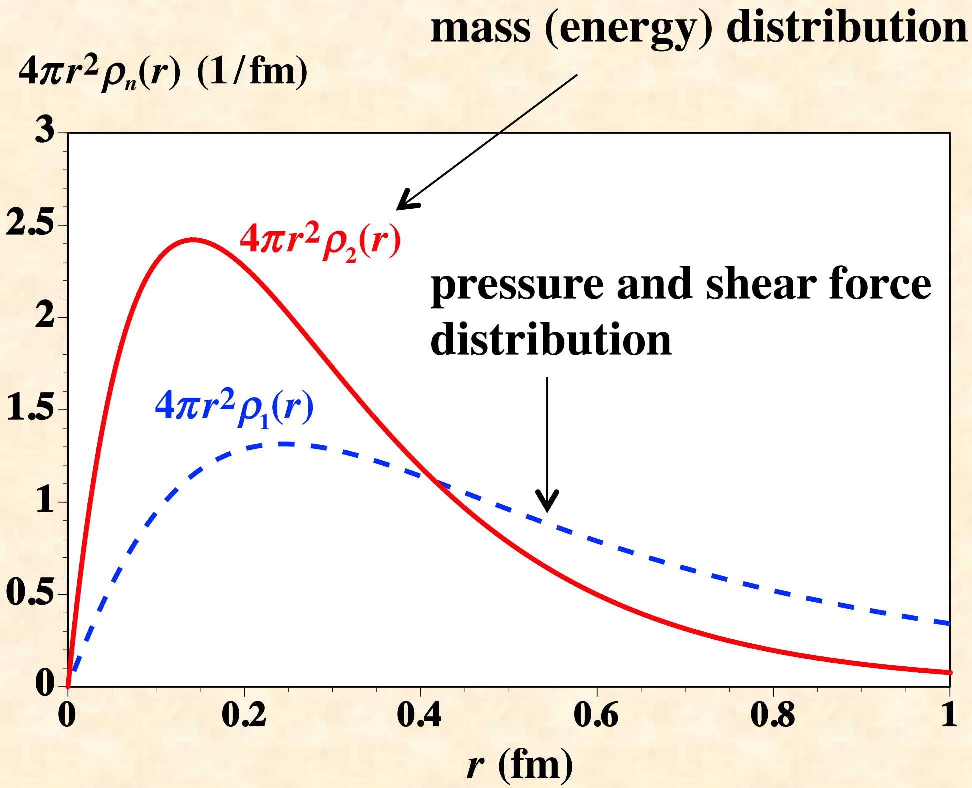
This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.56 \sim 0.69 \text{ fm},\quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 1.45 \sim 1.56 \text{ fm}$$

← First finding on gravitational radius from actual experimental measurements

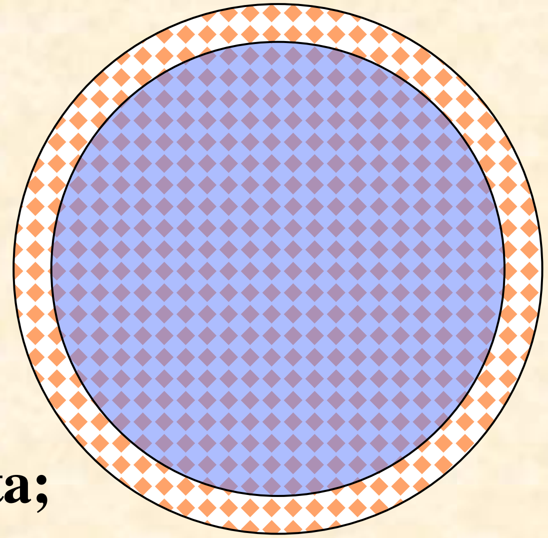
$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$





First finding gravitational radii for pion from experimental data

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.56 \sim 0.69 \text{ fm} \Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$



Comments and prospects:

It is too early to discuss the difference from experimenal data; however, it is interesting to investigate it theoretically.

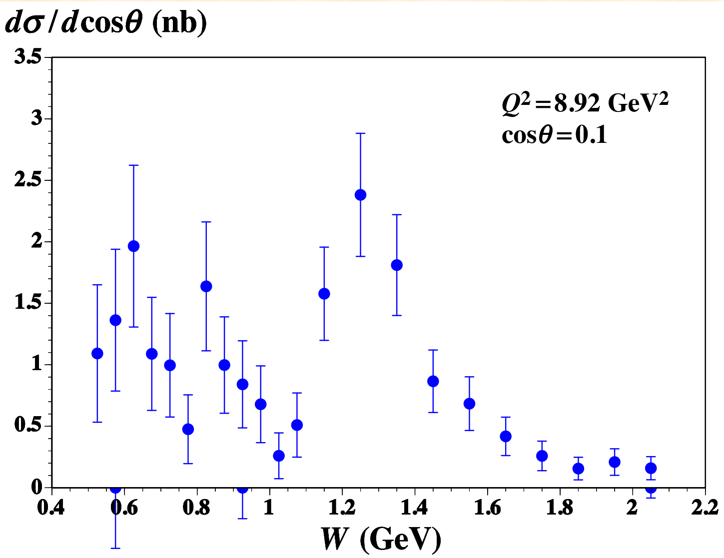
For example, quarks contribute to both charge and mass distributions, but gluons contribute to only the mass distribution.

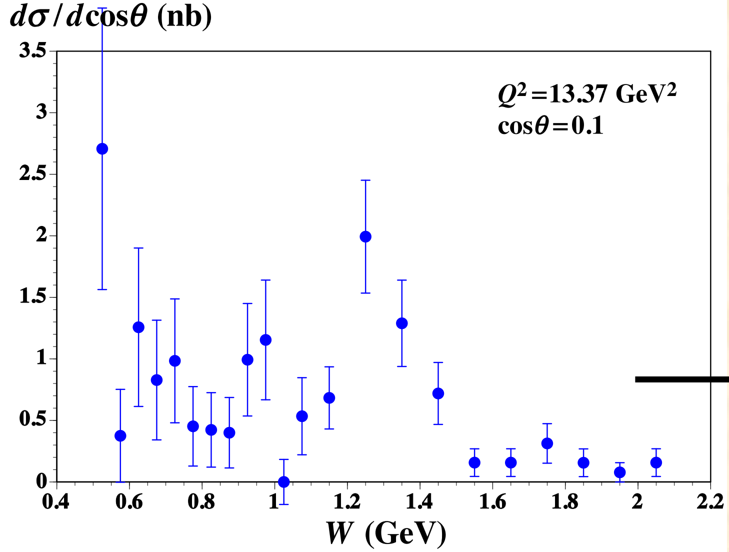
Gravitational physisc has been investigated for macrosopic phenomena. It could be studied also in the microscopic and fundamental quark-gluon level together with the topic of nucleon mass origin.

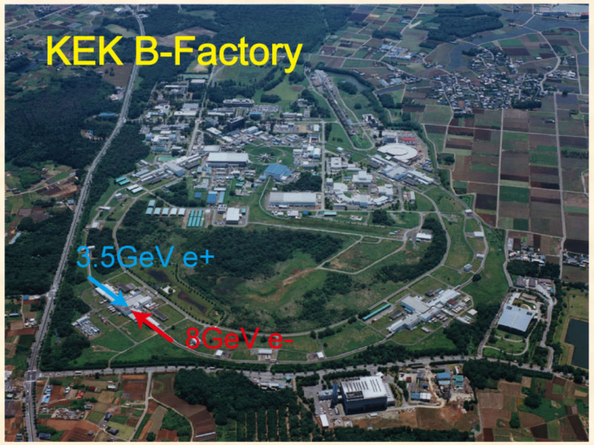
Prospects & Summary

Super KEKB

The errors are dominated by statistical errors, and they will be significantly reduced by super-KEKB.





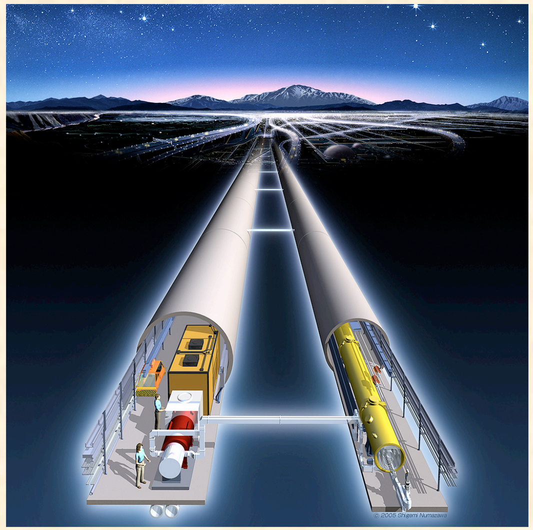


From KEKB to ILC

- **Very Large Q^2**
- **Large W^2**

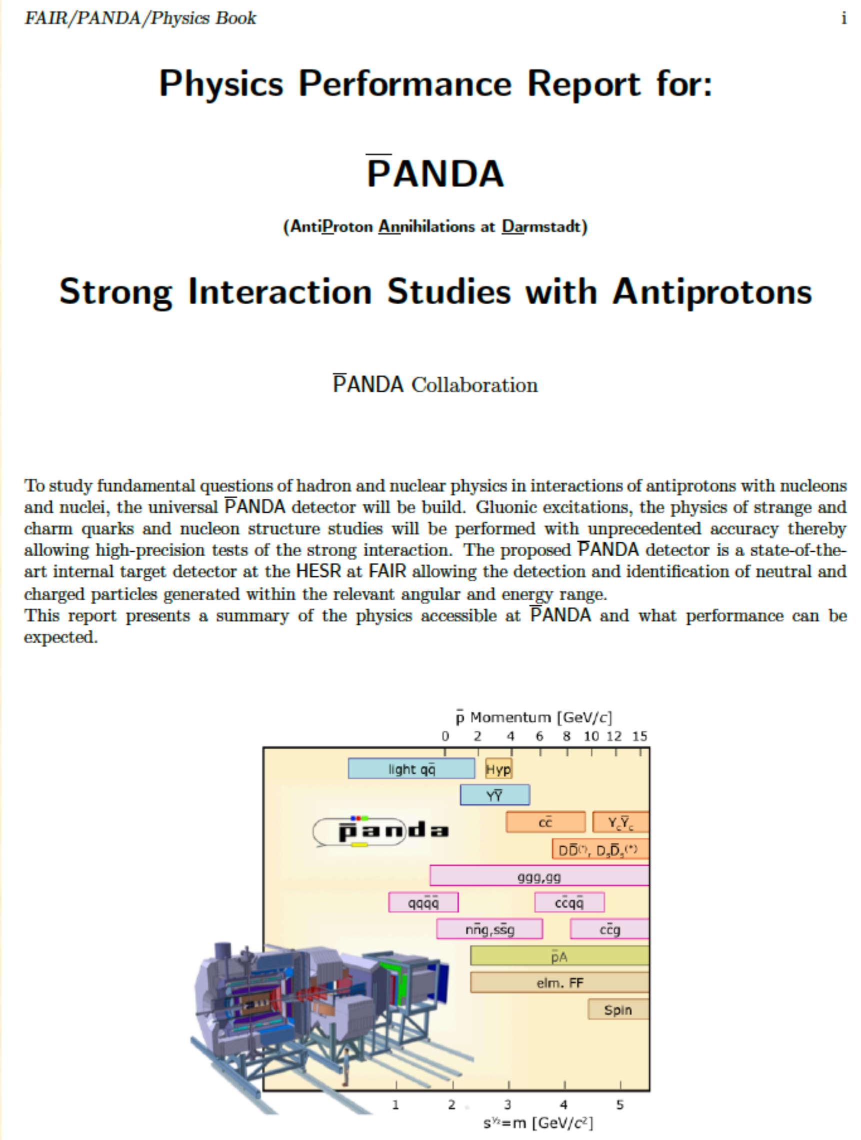
for extracting GDAs

ILC



GSI-FAIR (PANDA)

arXiv:0903.3905 [hep-ex]


FAIR/PANDA/Physics Book i

Physics Performance Report for:

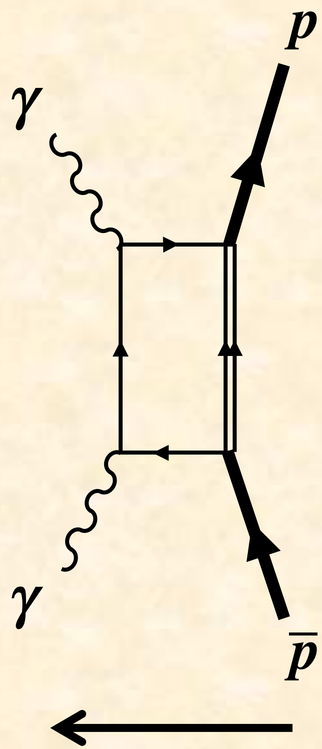
P̄ANDA

(AntiProton Annihilations at Darmstadt)

Strong Interaction Studies with Antiprotons

P̄ANDA Collaboration

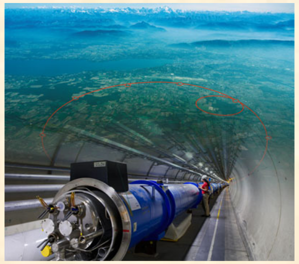
To study fundamental questions of hadron and nuclear physics in interactions of antiprotons with nucleons and nuclei, the universal P̄ANDA detector will be build. Gluonic excitations, the physics of strange and charm quarks and nucleon structure studies will be performed with unprecedented accuracy thereby allowing high-precision tests of the strong interaction. The proposed P̄ANDA detector is a state-of-the-art internal target detector at the HESR at FAIR allowing the detection and identification of neutral and charged particles generated within the relevant angular and energy range.
This report presents a summary of the physics accessible at P̄ANDA and what performance can be expected.



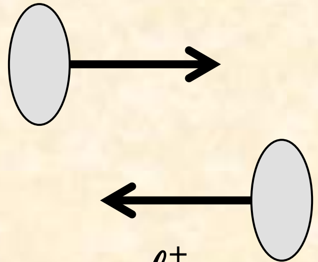
GDAs for the proton!
(super-KEKB?)

Facilities to probe 3D structure functions (GPD, GDA)

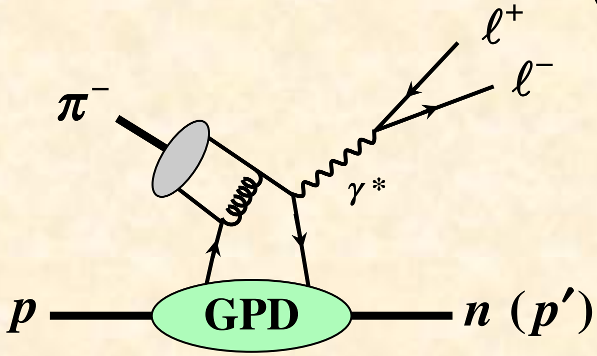
RHIC
LHC



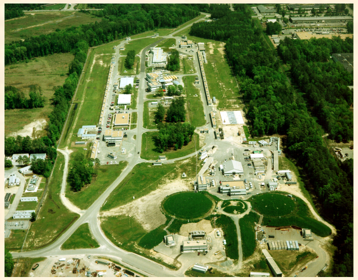
Ultra-peripheral collisions for $\gamma^* \gamma \to h\bar{h}$??

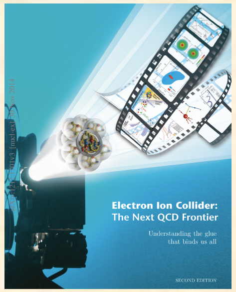


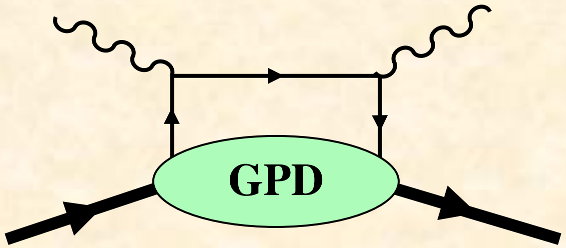
Fermilab
J-PARC
GSI-FAIR



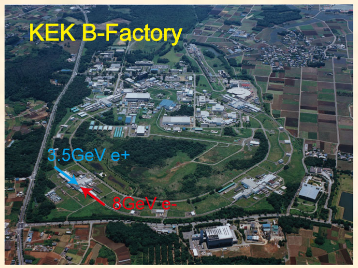
JLab
COMPASS
EIC

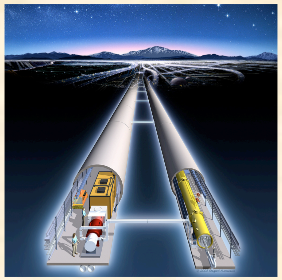




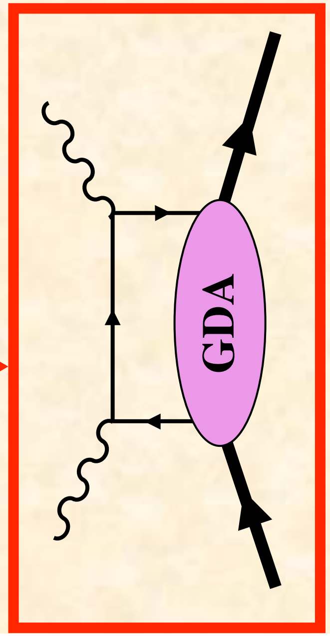


KEKB
ILC



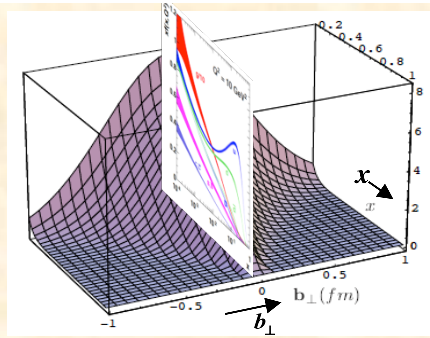


We studied this process.



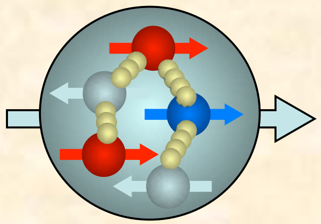
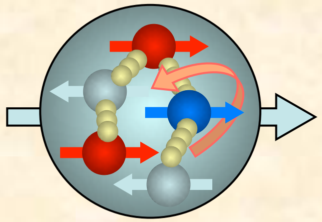
By hadron tomography

3D view of hadrons

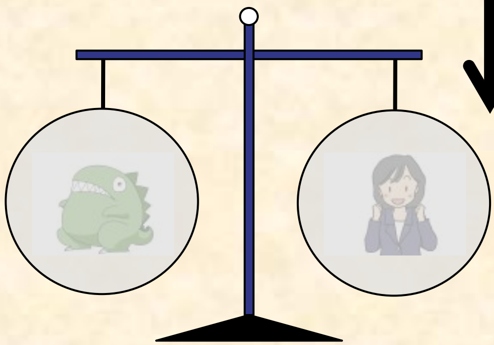


Origin of nucleon spin

By the tomography, we determine

 or  .

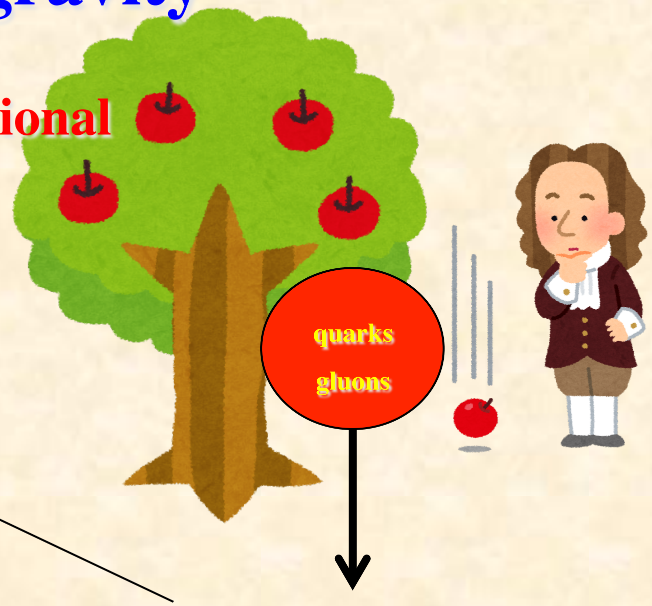
Exotic hadrons



By tomography, we determine  or  .

Origin of gravity

By tomography, we determine gravitational sources in terms of quarks and gluons.



Summary

Hadron tomography studies are important
for solving the origin of the nucleon spin,
for probing internal structure of exotic hadrons,
for probing gravitational sources (masses) in quark/gluon level.

GPDs at J-PARC

GPDs could be measured at hadron facilities such as J-PARC by exclusive Drell-Yan and other exclusive processes.

GDAs at KEKB / ILC

3D structure of hadrons can be investigated by GDAs ($s \Leftrightarrow t$).

Related experimental projects

RHIC, Fermilab, CERN-COMPASS, JLab, BES, ILC, LHC (UPC), GSI, EIC, LHeC, …

Gravitational form factors can be obtained for hadrons by GPDs and GDAs!

The End

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