Transverse Charge Densities of ρ Meson in Light Front Quark Model

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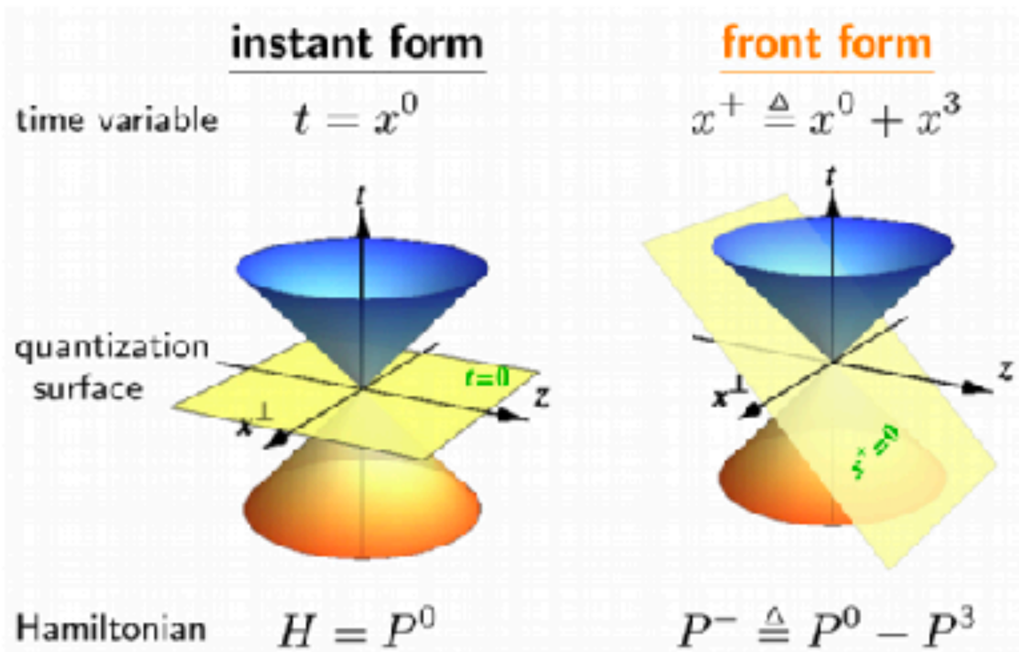
Overview

- What is Transverse Charge Density?
- Electromagnetic Form Factors of Rho Meson
- Transverse Charge Density and Helicity Form Factors
- Results for Unpolarized Rho Meson
- Results for Transversely Polarized Rho Meson
- Generalized Parton Distributions for Rho Meson

Light Front Dynamics



$$p^\mu p_\mu = m^2 \;\Rightarrow\; \begin{cases} p^0 = \sqrt{\vec{p}^2 + m^2}, & \text{equal-time} \\ p^- = (\vec{p}_\perp^2 + m^2)/p^+, & \text{light-front} \end{cases}$$

- light-front energy: p^-

- momenta: (p^+, p^1, p^2), where $p^\mp = p^0 \mp p^3$
- LFWFs = equal-time WFs in IMF $\neq$ equal-time WFs in rest frame
- Light-front wavefunctions (LFWFs) are frame independent and provides intrinsic information of the structure of hadrons:

"Hadron Physics without LFWFs is like Biology without DNA!"

— Stanley J. Brodsky

What is Transverse Charge Density?

- Proper determination of charge density requires measurement of matrix elements of density operators

$$\rho(x^-,\mathbf{b}) = J^+(x^-,\mathbf{b}) = \sum_q e_q \bar{q}(x^-,\mathbf{b})\gamma^+ q(x^-,\mathbf{b})$$

$$\rho(x^-,\mathbf{b}) = \langle p^+, R=0, \lambda | \sum_q q_+^\dagger(x^-,b) q_+(x^-,b) | p^+, \mathbf{R}=\mathbf{0}, \lambda\rangle$$

-D. E. Soper, Phys. Rev. D **15**, 1141 (1977)

- the absolute square of quark field operators, which is the signature of a true density

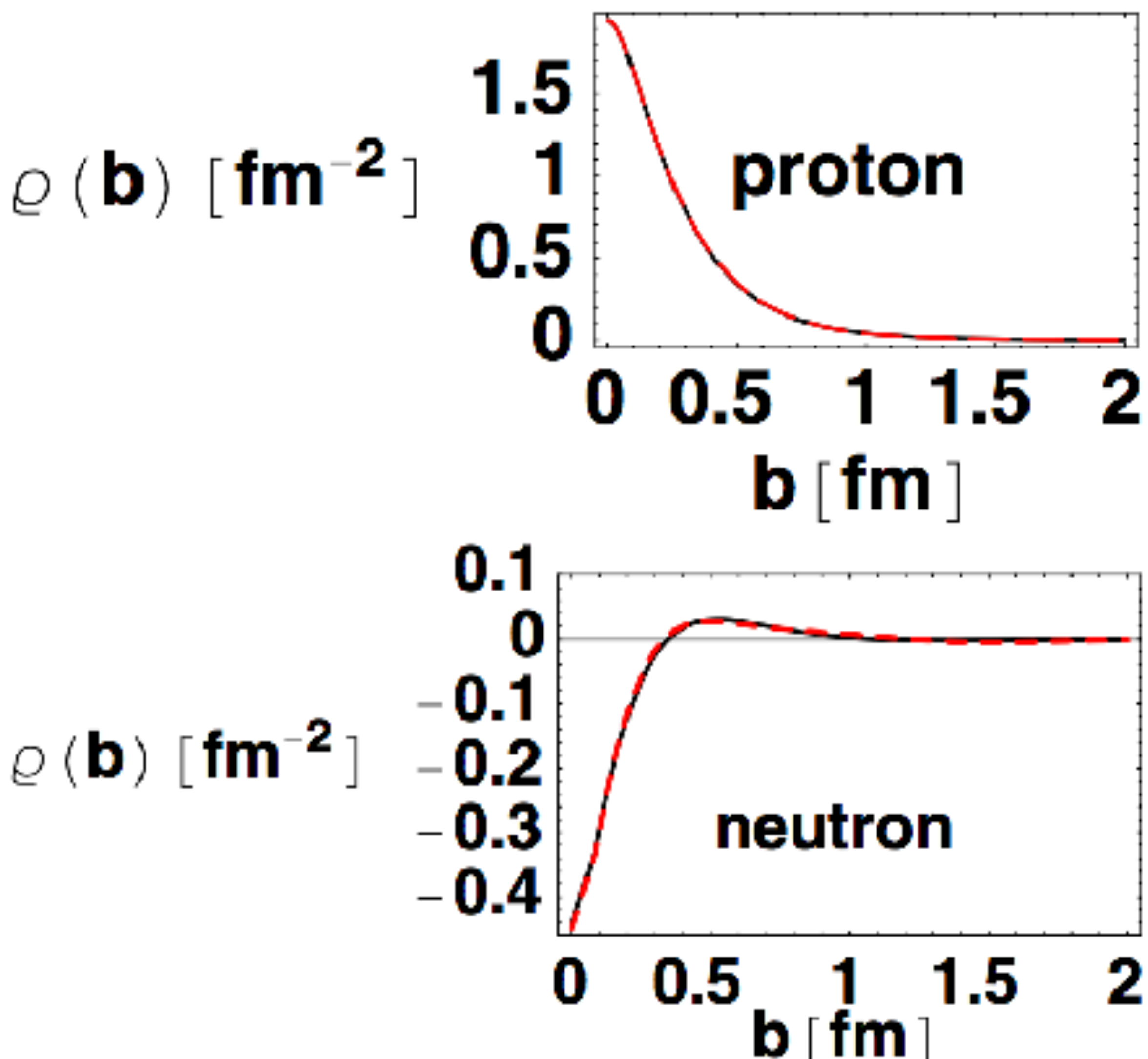
$$F_1 = \langle p^+, p', \lambda | J^+(0) | p^+, p, \lambda\rangle$$

- In the Drell-Yan frame, no momentum is transferred in the plus direction, so information regarding the x^- dependence of the density is not accessible

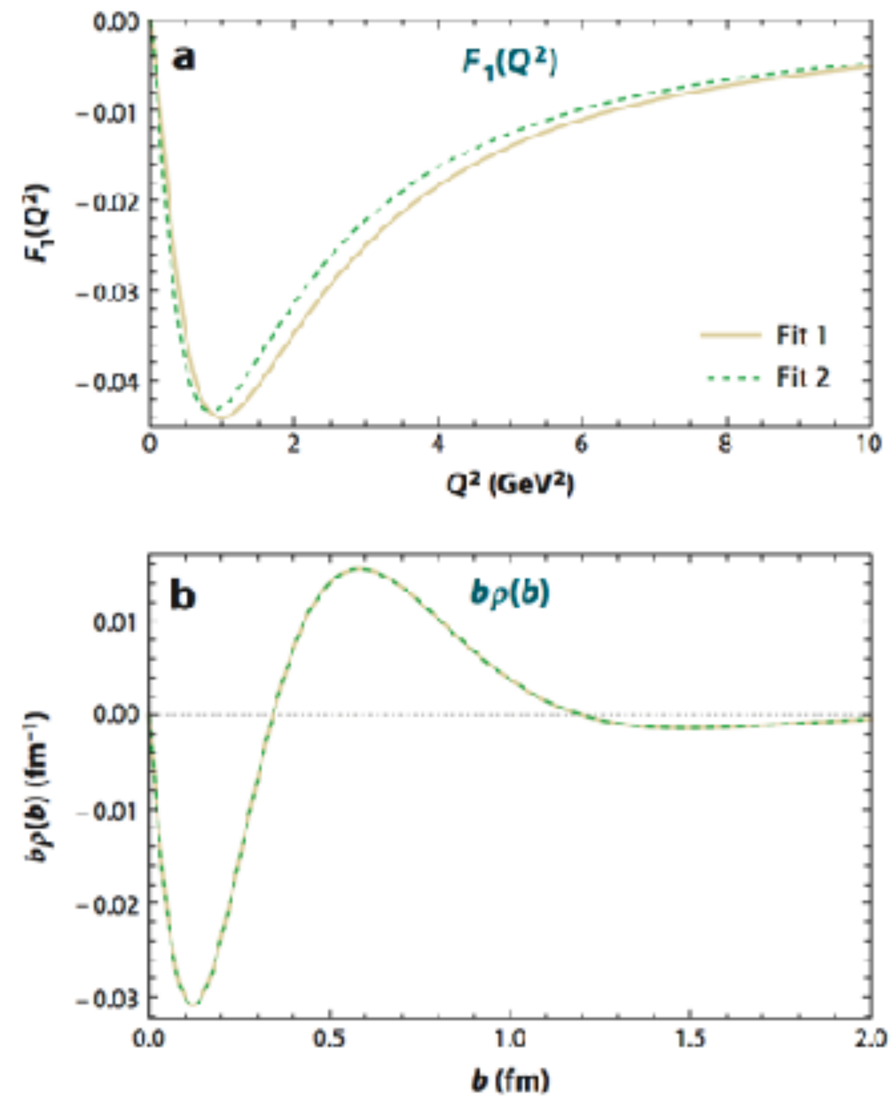
$$\rho(b) = \int \frac{QdQ}{2\pi} F_1(Q^2) J_0(Qb)$$

- Miller, Ann. Rev. Nucl. Part. Sci. 2010. 60:1–25

Transverse Charge Densities for Proton and Neutron



See G A Miller PRL 99 112001



F_1 is negative, so is the central charge density

At very large distances from the centre, again suggesting the existence of the long- ranged pion cloud

See Strikman and Weiss PRC 82, 042201(R)

Light Front Quark Model (LFQM)

- LFQM is quite successful in explaining the various electroweak properties of heavy mesons compared with experimental data.

-PLB 349 393 (1995)
-PRD 59 074015 (1999)
-PRD 65 116001 (2002)

- Successful in obtaining distribution amplitudes, decay constant and radiative decays for mesons.

-PRD 75 034019 (2007)
-PRD 68 054026 (2003)

- In this model, meson is represented as

$$|M\rangle = \psi^M_{q\bar{q}} |q\bar{q}\rangle$$

$$\psi^M_{q\bar{q}} = \sqrt{\frac{\partial k_z}{\partial x}} \phi(x, \mathbf{k}_\perp) \mathcal{R}(x, \mathbf{k}_\perp, \lambda_q, \lambda_{\bar{q}})$$

- Here $\phi(x, \mathbf{k}_\perp)$ is the radial wave function, $\frac{\partial k_z}{\partial x}$ is the Jacobian factor and $\mathcal{R}$ is the spin-orbit wave function obtained from the interaction-independent Melosh transformation .

Electromagnetic Form Factors of ρ Meson

- Form factors of meson can be expressed as

$$\langle P', \Lambda' | J^\mu | P, \Lambda \rangle = \quad -\epsilon^*_{\Lambda'} \cdot \epsilon_\Lambda (P + P')^\mu F_1(Q^2) + (\epsilon^\mu_{\Lambda'} q \cdot \epsilon^*_{\Lambda'} - \epsilon^{*\mu}_{\Lambda'} q \cdot \epsilon_\Lambda) F_2(Q^2) + \frac{(\epsilon^*_{\Lambda'} \cdot q)(\epsilon_\Lambda \cdot q)}{2M^2} (P + P')^\mu F_3(Q^2),$$

-Berger, PRL 87 142302

$q = p - p'$ and $\epsilon_h[\epsilon'_h]$ is the polarization vector of the initial[final] meson with physical mass M_v

- The co-variant form factors of spin-1 hadron can be determined by the plus component of current

$$I^+_{h'h}(0) = \langle P', h' | J^+ | P, h \rangle$$

- Current matrix element is constrained by the invariance under the LF parity and time reversal and therefore reduce to four elements

$$I^+_{++}, I^+_{+-}, I^+_{+0} \text{ and } I^+_{00}$$

- In practical computation, instead of Lorentz invariant form factors $F_i(Q^2)$, the $G_C(Q^2)$ physical charge, $G_M(Q^2)$ magnetic and $G_Q(Q^2)$ quadrupole form factors are often used.

$$\begin{aligned} G_C &= F_1 + \frac{2}{3}\kappa G_Q \\ G_M &= -F_2 \\ G_Q &= F_1 + F_2 + (1+\kappa)F_3 \\ \kappa &= \frac{Q^2}{4M_v^2} \end{aligned}$$

- At zero momentum transfer

$$eG_C(0) = e, \quad \text{(Charge)}$$
$$eG_M(0) = 2M_v\mu, \quad \text{(Magnetic Moment)}$$
$$-eG_Q(0) = M_v^2 Q. \quad \text{(Quadrupole Moment)}$$

- In literature, there are two types of prescription are available, for example Grach and Kondrayutak (GK) and Brodsky and Hiller (BH).

$$G_C^{GK} = \frac{1}{2P^+}\left[\frac{(3-2\kappa)}{3}I^+_{++} + \frac{4\kappa}{3}\frac{I^+_{+0}}{\sqrt{2\kappa}} + \frac{1}{3}I^+_{+-}\right]$$

$$G_M^{GK} = \frac{2}{2P^+}\left[I^+_{++} - \frac{1}{\sqrt{2\kappa}}I^+_{+0}\right]$$

$$G_Q^{GK} = \frac{1}{2P^+}\left[-I^+_{++} + 2\frac{I^+_{+0}}{\sqrt{2\kappa}} - \frac{I^+_{+-}}{\kappa}\right]$$

$$G_C^{BH} = \frac{1}{2P^+(1+2\kappa)}\left[\frac{(3-2\kappa)}{3}I^+_{00} + \frac{16\kappa}{3}\frac{I^+_{+0}}{\sqrt{2\kappa}} + \frac{2}{3}(2\kappa-1)I^+_{+-}\right]$$

$$G_M^{BH} = \frac{2}{2P^+(1+2\kappa)}\left[I^+_{00} + \frac{2\kappa-1}{\sqrt{2\kappa}}I^+_{+0} - I^+_{+-}\right]$$

$$G_Q^{BH} = \frac{-1}{2P^+(1+2\kappa)}\left[I^+_{00} - 2\frac{I^+_{+0}}{\sqrt{2\kappa}} + I^+_{+-}\frac{1+\kappa}{\kappa}\right]$$

- However, in this work we choose **BH-prescription** as it has (0,0) component which gives the most dominant contribution in the high momentum perturbative QCD (PQCD) region.

- In LFQM, the physical form factors are obtained from the $I^{+}_{\Lambda'\Lambda}$ which is defined as

$$I^{+}_{\Lambda'\Lambda} = \int \frac{dx}{2(1-x)} \int d^2\mathbf{k}_{\perp} \sqrt{\frac{\partial k'_z}{\partial x}\frac{\partial k_z}{\partial x}} \phi^*(x, \mathbf{k}_{\perp f})\phi(x, \mathbf{k}_{\perp i}) \frac{(S^{+}_{\Lambda'\Lambda})}{M_{oi}M_{of}}$$

- $S^{+}_{\Lambda'\Lambda}$ is defined in PRD 70 053015.
- Radial wave function is defined as

$$\phi(x, \mathbf{k}^2) = \sqrt{\frac{1}{\pi^3/2\beta^3}} exp(-\mathbf{k}^2/2\beta^2)$$

$$\mathbf{k}^2 = \mathbf{k}_{\perp}{}^2 + k_z^2, k_z = (x - 1/2)M_o$$

$$M_{oi}^2 = M_{of}^2 = M_o^2 = \frac{\mathbf{k}_{\perp}{}^2 + m^2}{x(1-x)}$$

- The model parameters used in this study are m=0.22 GeV, β =0.3659 GeV and M_v =0.77 GeV.
- These model parameters were obtained from the linear confining potential of QCD motivated Hamiltonian in LFQM.

Transverse Charge Density and Helicity Form Factors

- Charge density in transverse plane as a standard interpretation can be obtained by two- dimensional Fourier transform of form factor

$$\rho^{\rho}_{\lambda}(b) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp\cdot\mathbf{b}_\perp} G^{+}_{\lambda'\lambda}(Q^2), \\ = \int_0^\infty \frac{dQ\ Q}{2\pi} J_0(Qb) G^{+}_{\lambda'\lambda}(Q^2),$$

$G^{+}_{\lambda'\lambda}(Q^2)$ is the form factor related with the matrix elements of electromagnetic current $J^+(0)$ sandwich between two ρ meson states.

Here λ and λ' are the initial and final ρ meson states respectively.

$$\langle P^+, \frac{\mathbf{q}_\perp}{2}, \lambda' | J^+(0) | P^+, -\frac{\mathbf{q}_\perp}{2}, \lambda \rangle = 2P^+ G^{+}_{\lambda'\lambda}(Q^2)$$

- One can define the helicity- conserving form factor(G^{+}_{++} , G^{+}_{00}) and helicity non-conserving form factors (G^{+}_{0+}, G^{+}_{-+}) respectively, in terms of G_C, G_M and G_Q

$$G^{+}_{++} = \frac{1}{1+\kappa}\left[G_C + G_M + \frac{\kappa}{3}G_Q\right],$$

$$G^{+}_{00} = \frac{1}{1+\kappa}\left[(1-\kappa)G_C + 2\kappa G_M - \frac{2\kappa}{3}(1+2\kappa)G_Q\right],$$

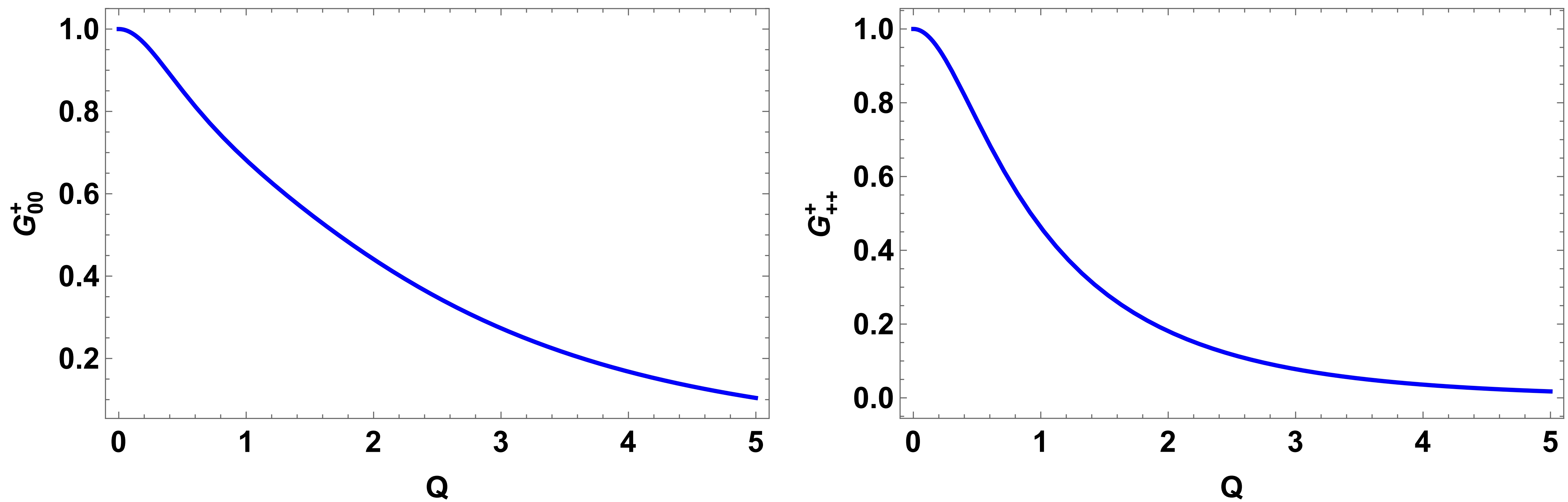
$$G^{+}_{0+} = -\frac{\sqrt{2\kappa}}{1+\kappa}\left[G_C - \frac{1}{2}(1-\kappa)G_M + \frac{\kappa}{3}G_Q\right],$$

$$G^{+}_{-+} = \frac{\kappa}{1+\kappa}\left[G_C - G_M - \left(1+\frac{2\kappa}{3}\right)G_Q\right]$$

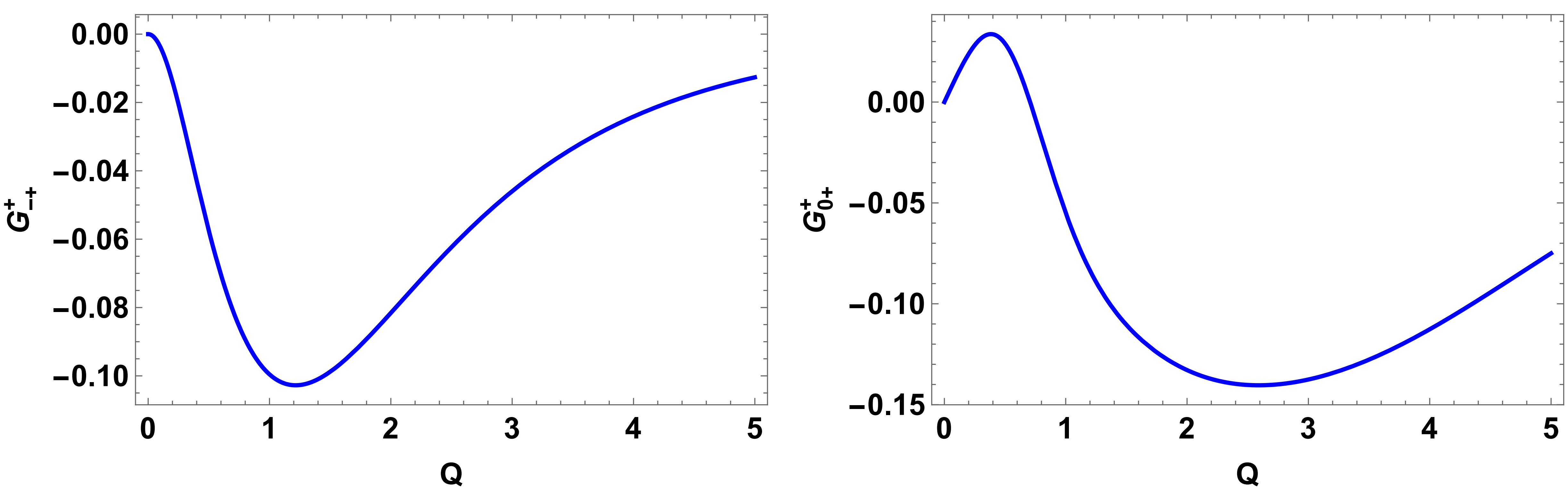
-PRD 70 053015 (2004)

Here, again $\kappa = Q^2/4M_v^2$

- G_C, G_M and G_Q are already calculated by C.R. Ji in PRD 70 053015, we have extracted the helicity form factors and charge density from it.

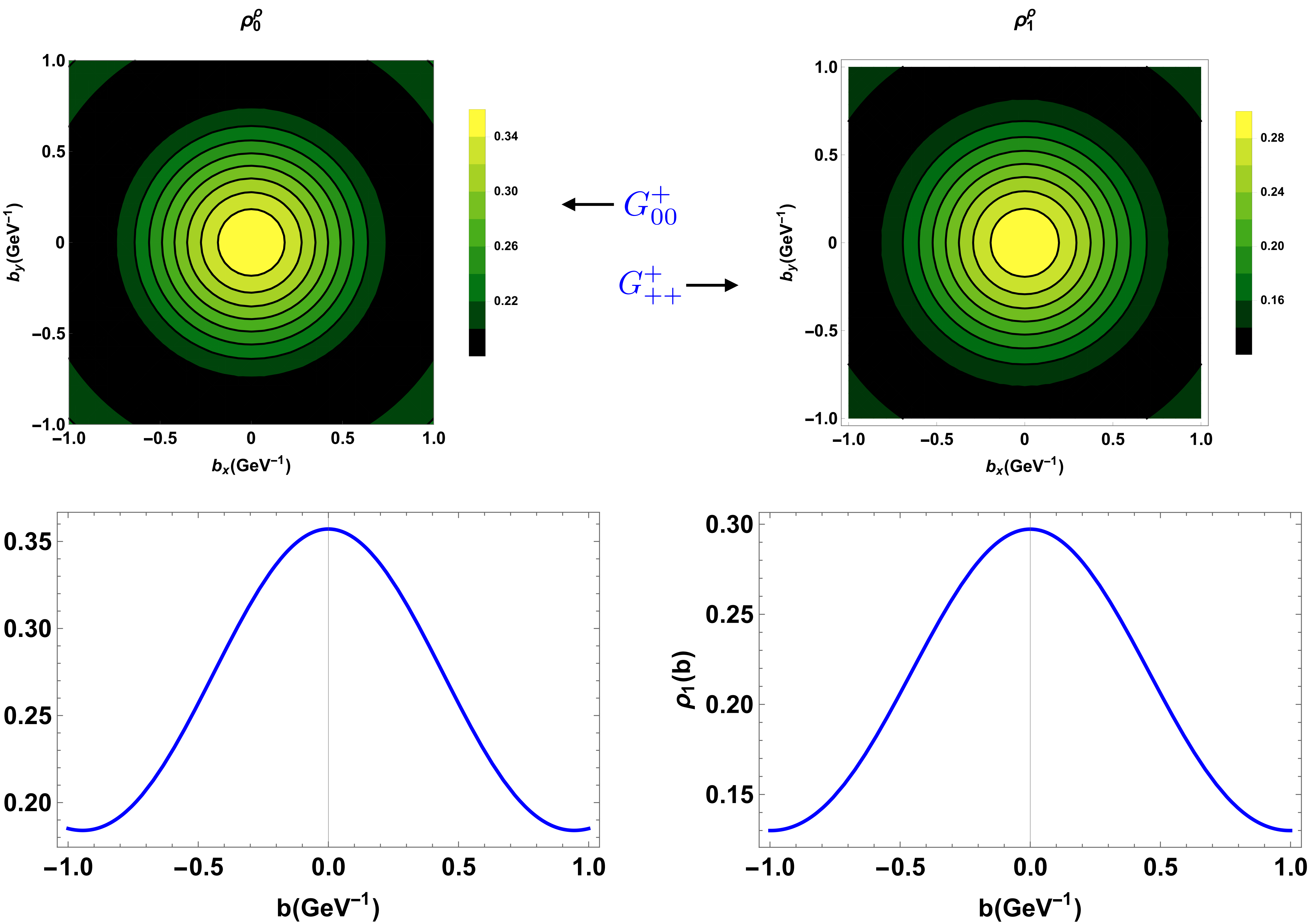


Helicity conserve form factors



Helicity flip form factors

Results for unpolarized ρ meson



Transversely Polarized ρ Meson

- We also consider the transversely polarized ρ meson state which provides information about dipole and quadrupole moments.
- Transverse charge density for transversely polarized meson can be defined as

$$\rho^{\rho}_{s_\perp T}(\mathbf{b}_\perp) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp\cdot\mathbf{b}_\perp} \frac{1}{2P^+} \langle P^+, \frac{\mathbf{q}_\perp}{2}, s_\perp | J^+ | P^+, -\frac{\mathbf{q}_\perp}{2}, s_\perp \rangle,$$

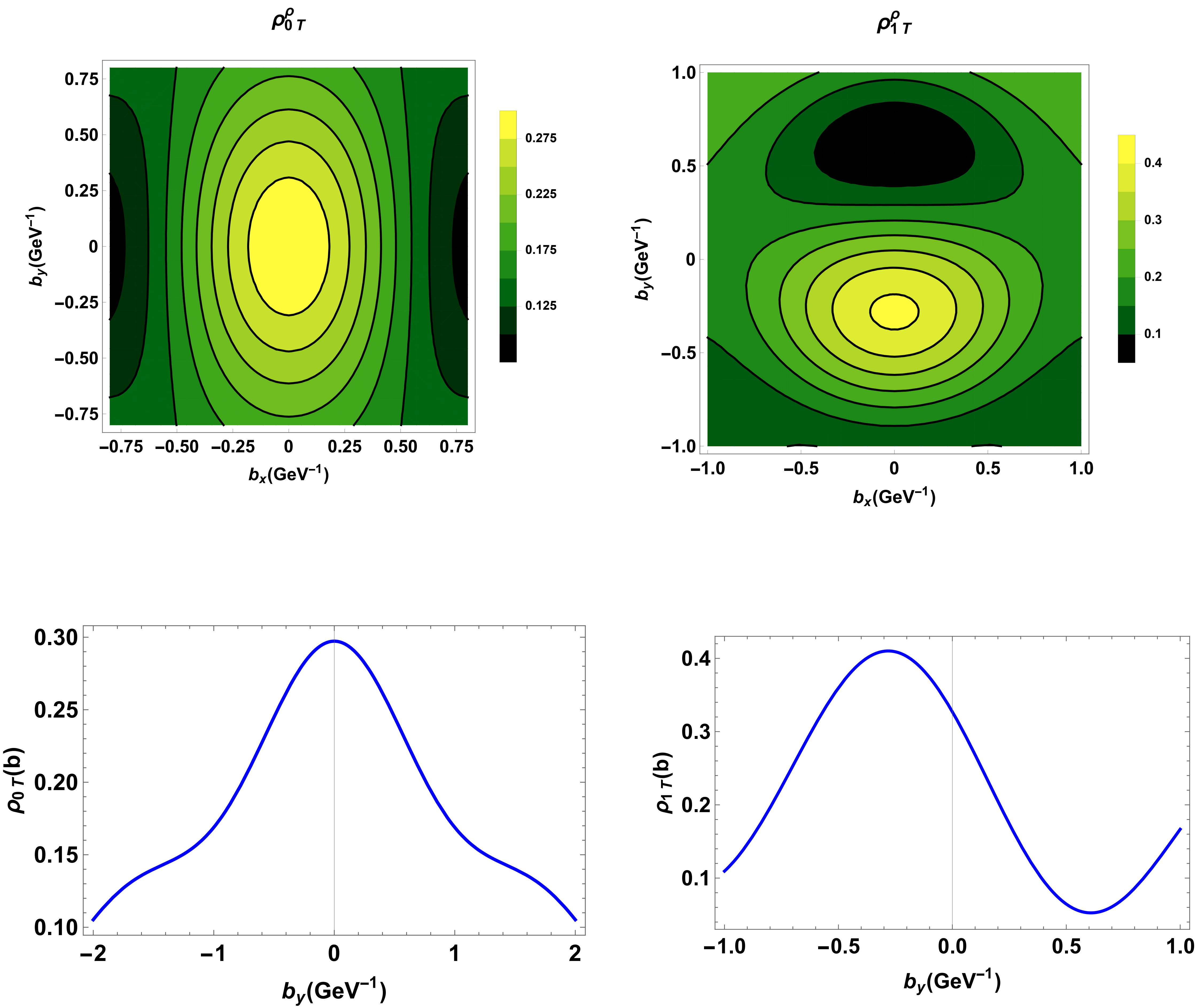
- Here $s_\perp$ is the meson spin projection along the transverse polarization direction $S_\perp = \cos\phi\ \hat{x} + \sin\phi\ \hat{y}$ and for $s_\perp = 0, 1$

$$\rho_{0T}(b) = \int_0^\infty \frac{dQ\ Q}{2\pi} \left[J_0(bQ) G^+_{++} + \cos 2(\phi_b - \phi_s) J_2(bQ) G^+_{+-} \right]$$

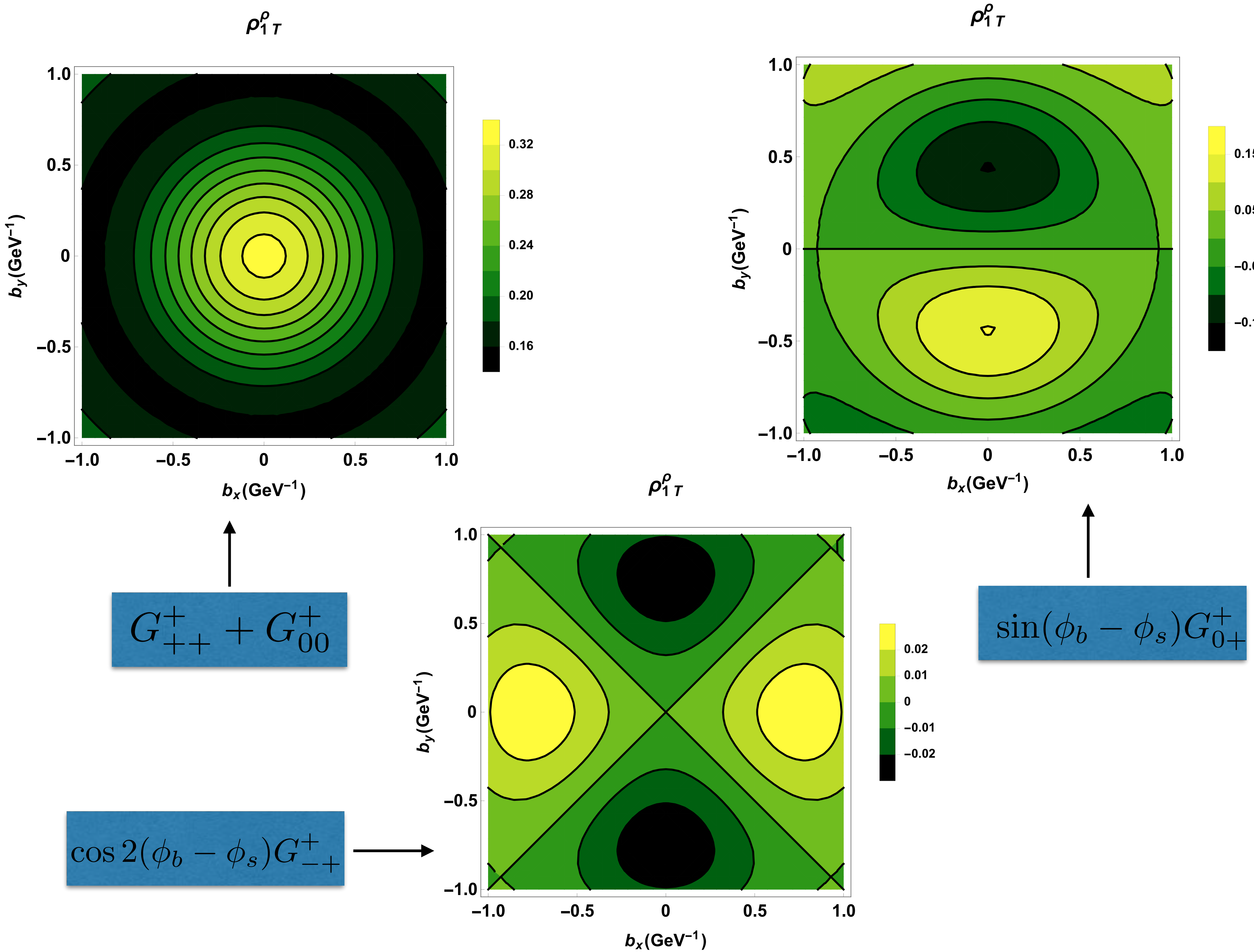
$$\rho_{1T}(b) = \int_0^\infty \frac{dQ\ Q}{2\pi} \left[\frac{J_0(bQ)}{2} (G^+_{++} + G^+_{00}) + \sin(\phi_b - \phi_s) J_1(bQ) \sqrt{2} G^+_{0+} - \cos 2(\phi_b - \phi_s) J_2(bQ) \frac{G^+_{-+}}{2} \right]$$

-Carlson and Vanderhaeghen, PRL 100 032004 (2008)

EPJA 41 1 (2009)



$\rho^{\rho}_{0\,T}$
$\rho^{\rho}_{1\,T}$
b_y(GeV^{-1})
b_x(GeV^{-1})
0.275
0.225
0.175
0.125
0.4
0.3
0.2
0.1
$\rho_{0\,T}$(b)
$\rho_{1\,T}$(b)
b_y(GeV^{-1})



ρ^{ρ}_{1T}
$b_x(\mathrm{GeV}^{-1})$
$b_y(\mathrm{GeV}^{-1})$
0.32
0.28
0.24
0.20
0.16
ρ^{ρ}_{1T}
0.15
0.05
−0.05
−0.15
ρ^{ρ}_{1T}
0.02
0.01
0
−0.01
−0.02
$G^{+}_{++} + G^{+}_{00}$
$\sin(\phi_b - \phi_s)G^{+}_{0+}$
$\cos 2(\phi_b - \phi_s)G^{+}_{-+}$

Generalized Parton Distributions (GPDs)

- Recently, GPDs of the rho meson are obtained in light front constituent quark model.

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ρ meson unpolarized generalized parton distributions with a light-front constituent quark model

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We study ρ meson unpolarized generalized parton distributions based on a light-front constituent quark model where the quark-antiquark-meson vertex is constructed under the symmetric loop momentum convention. The form factors and some other low-energy observables of the ρ meson are calculated. Moreover, the contributions to the form factors and generalized parton distributions from the valence and nonvalence regimes are discussed and analyzed in detail. In the forward limit, the usual structure functions are estimated as well. In addition, by evolving the moments of the obtained structure functions to the scale of the lattice calculation, we give the factorization scale of our quark model. It is found that the present phenomenological model is reasonable to describe the general properties of ρ meson.

DOI: 10.1103/PhysRevD.96.036019

- GPDs encode 3-D structure of the hadrons.
- GPDs of the meson are defined by the correlation

-PRL 87 142302 (2001)

$$V_{\lambda'\lambda} = \frac{1}{2}\int \frac{d\omega}{2\pi} e^{ix(Pz)} \langle P', \lambda' | \bar{q}(-z/2) q(z/2) | P, \lambda \rangle |_{z=\omega n},$$

$$= \sum_i \epsilon^{'*\nu} V^{(i)}_{\nu\mu} \epsilon^{\mu} H_i(x, \xi, t),$$

$$V_{\lambda'\lambda} = -(\epsilon^{*'} \cdot \epsilon) H_1 + \frac{(\epsilon \cdot n)(\epsilon^{*'} \cdot P) + (\epsilon^{*'} \cdot n)(\epsilon \cdot P)}{P \cdot n} H_2 - \frac{2(\epsilon \cdot n)(\epsilon^{*'} \cdot n)}{M^2} H_3$$
$$+ \frac{(\epsilon \cdot n)(\epsilon^{*'} \cdot P) - (\epsilon^{*'} \cdot n)(\epsilon \cdot P)}{P \cdot n} H_4 + \left[\frac{M^2 (\epsilon \cdot n)(\epsilon^{*'} \cdot n)}{(P \cdot n)^2} + \frac{1}{3} (\epsilon^{*'} \epsilon) \right] H_5,$$

- One can predict the physical form factors from the first moment of the GPDs.

$$G_z(Q^2) = \int dx H_i(x, Q^2),$$

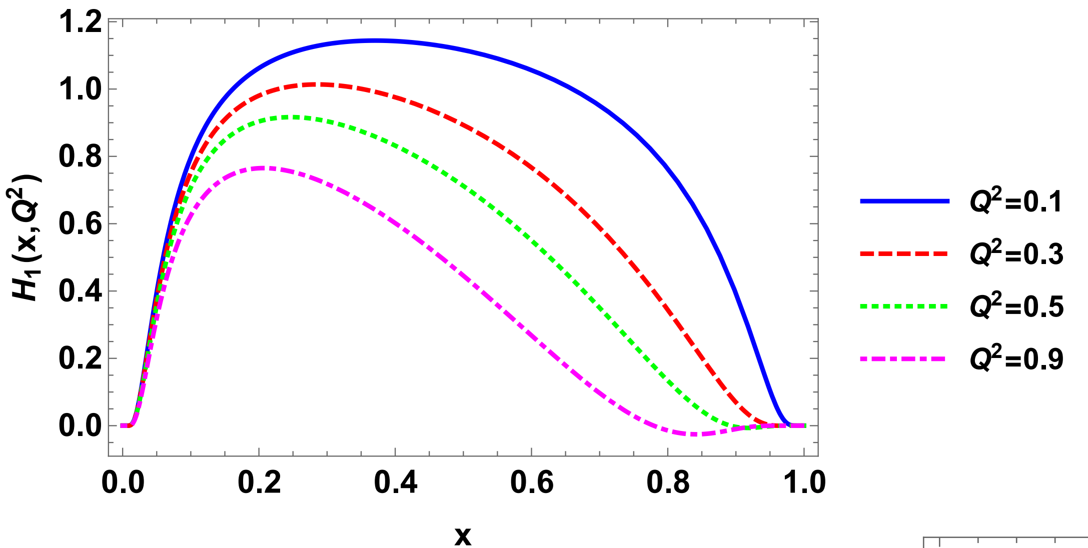
- Corresponding to physical form factors $G_C, G_M \mathrm{and} G_Q$ we have GPDs H_1, H_2 and H_3 respectively.

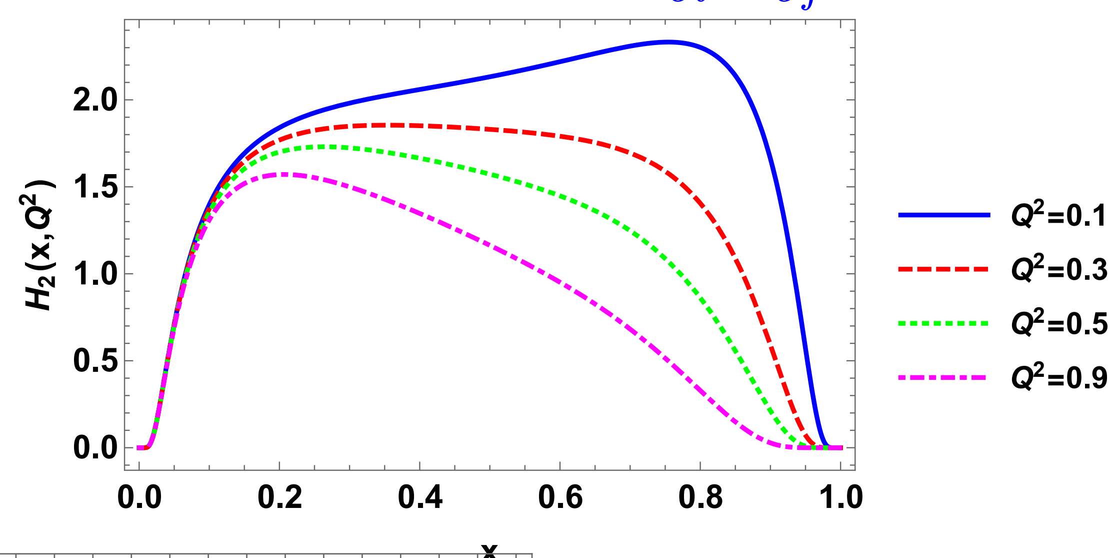
$$G_C^{BH} = \frac{1}{2P^+(1+2\kappa)}\left[\frac{(3-2\kappa)}{3} I_{00}^+ + \frac{16\kappa}{3}\frac{I_{+0}^+}{\sqrt{2\kappa}} + \frac{2}{3}(2\kappa-1)I_{+-}^+\right]$$

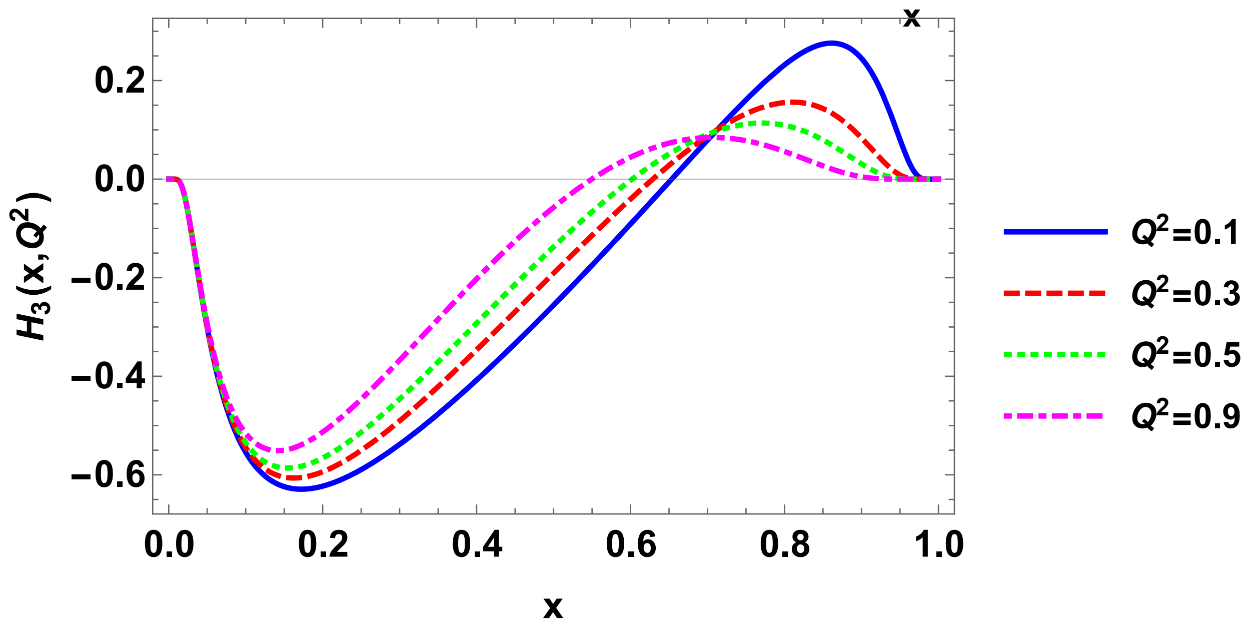
$$G_M^{BH} = \frac{2}{2P^+(1+2\kappa)}\left[I_{00}^+ + \frac{2\kappa-1}{\sqrt{2\kappa}} I_{+0}^+ - I_{+-}^+\right]$$

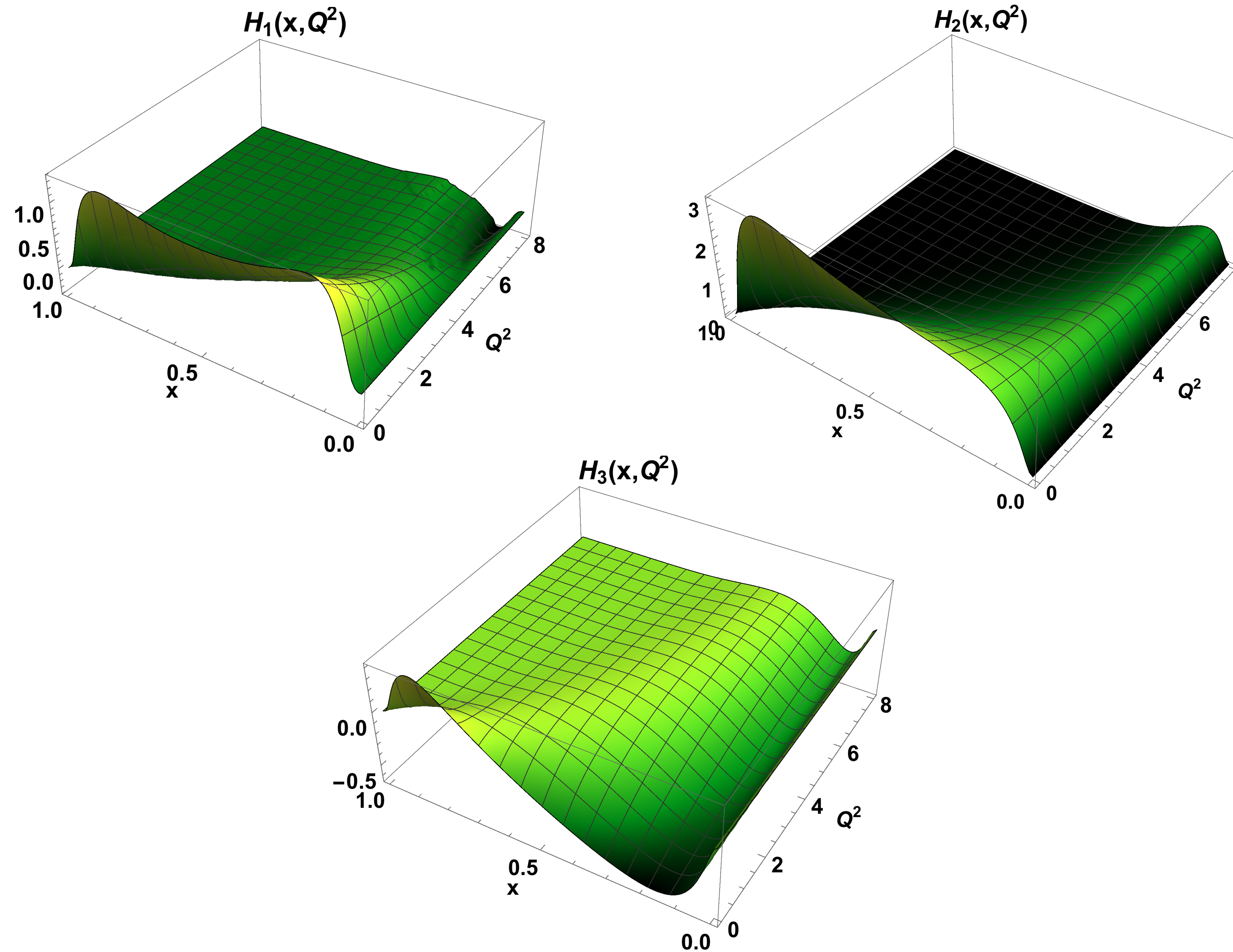
$$G_Q^{BH} = \frac{-1}{2P^+(1+2\kappa)}\left[I_{00}^+ - 2\frac{I_{+0}^+}{\sqrt{2\kappa}} + I_{+-}^+\frac{1+\kappa}{\kappa}\right]$$

$$I_{\Lambda'\Lambda}^+ = \int \frac{dx}{2(1-x)} \int d^2\mathbf{k}_\perp \sqrt{\frac{\partial k_z'}{\partial x}\frac{\partial k_z}{\partial x}}\,\phi^*(x,\mathbf{k}_{\perp f})\phi(x,\mathbf{k}_{\perp i})\frac{(S_{\Lambda'\Lambda}^+)}{M_{oi}M_{of}}$$









H1(x,Q2)
1.0
0.5
0.0
1.0
0.5
x
0.0
0
2
4
Q2
6
8
H2(x,Q2)
3
2
1
0
1.0
0.5
x
0.0
0
2
4
Q2
6
8
H3(x,Q2)
0.0
−0.5
1.0
0.5
x
0.0
0
2
4
Q2
6
8

Conclusion

- Transverse charge densities of rho meson are calculated in LFQM.
- Provides interpretation of quark charge density in the transverse plane for unpolarized and polarized rho meson.
- Helicity form factors are obtained for rho meson.
- We have extracted the charge densities from the helicity form factors.
- Charge density for unpolarized rho meson shows monopole pattern.
- Charge density for polarized rho meson shows monopole pattern together with dipole and quadrupole patterns.
- Results for GPDs of rho meson are also obtained.
- Provides rich information on spatial structure of the rho meson.

Thank You