KEK Inter-University Research Institute Corporation
High Energy Accelerator Research Organization

Color Entanglement

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- ✧ Quantum Entanglement
- ✧ QCD and Color Entanglement
- ✧ QCD Factorization – Approximation
- ✧ Sign change of the Sivers's effect, Factorization breaking, …
- ✧ Summary and Outlook

Workshop on Progress on Hadron structure functions in 2018
November 18-19, 2018, KEK Tsukuba Campus

Quantum Entanglement

See also Tsutsui's talk

❑ Definition:

A system whose quantum state cannot be factored as a product of states of its local constituents, no matter how far they are separated

That is, all particles in such a system are not individual particles but are an inseparable whole

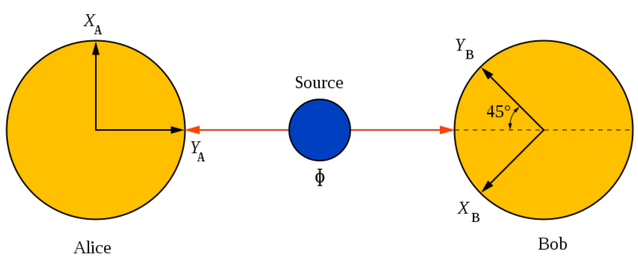
❑ Particle decay – a classical example of the entanglement:

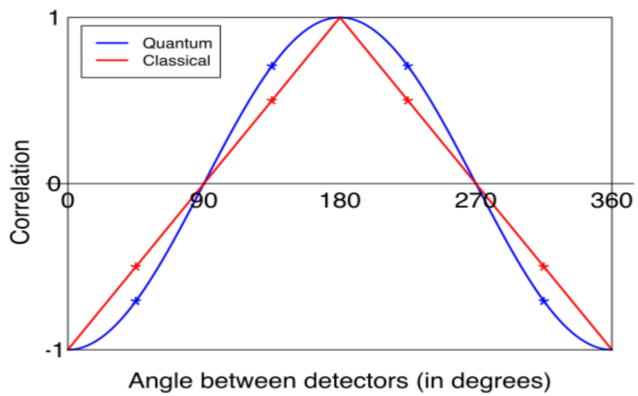
The total number of particles from the decay of a single particle form various systems of entanglement due to various conservation laws, such as momentum, angular momentum, …

A spin-0 particle decays into two spin-1/2 particles:

Entangled quantum state: $\frac{1}{\sqrt{2}}\left[\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle-\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle\right]$

The spin of these two particles are entangled or correlated





QCD and Color Entanglement

❑ QCD as a quantum field theory (QFT):

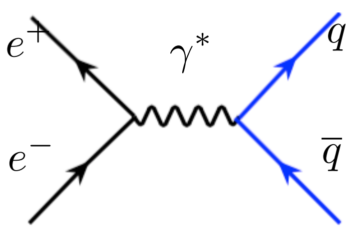
$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f - \frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc}A_{\mu,b}A_{\nu,c}\right]^2 + \text{gauge fixing} + \text{ghost terms}$$

With fields/particles:

- ✧ **Spin-1/2, color triplet quark fields:** $\psi_i^f(x)$ $\quad i = 1, 2, 3 = N_c$, $\quad f = u, d, s, c, b, t$
- ✧ **Spin-1, color octet gluon fields:** $A_{\mu,a}(x)$ $\quad a = 1, 2, ..., 8 = N_c^2 - 1$

Number of quanta in QFT is NOT fixed, key difference from QM!

❑ Microscopic entanglement in QCD:

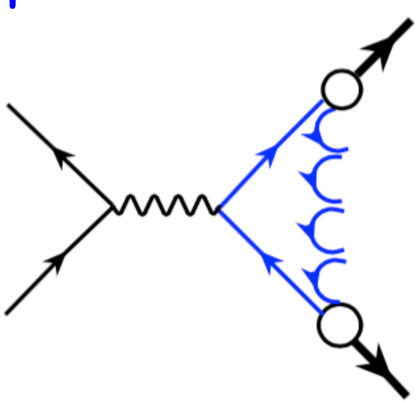


Color of the quark and anti-quark is quantum/color entangled

$$R\overline{R}, B\overline{B}, G\overline{G}$$

❑ Hadronization is an entangled process:

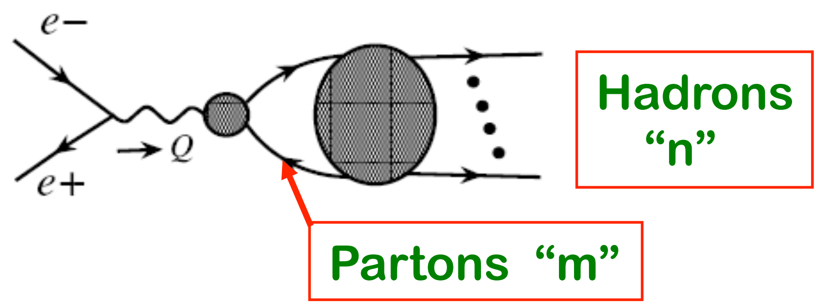
Although hadrons are color singlet, their "distributions" are quantum/color correlated



QCD and Color Entanglement

❑ Unitarity:

$$\sigma^{\rm tot}_{e^+e^-\to \rm hadrons} \propto \Big| \quad \Big|^2$$



✧ Color entanglement between partons affect the hadron distributions

✧ Without asking the details of the "distributions", summing over all final-state, color entanglement is not a direct observable

✧ Unitarity ensures that the "total" cross section is perturbatively calculable for large enough Q

$$\boxed{\sigma^{\rm tot}_{e^+e^-\to \rm hadrons} = \sigma^{\rm tot}_{e^+e^-\to \rm partons}}$$

$$\sigma^{\rm tot}_{e^+e^-\to \rm partons}(s=Q^2) = \sum_n \sigma^{(n)}(Q^2,\mu^2)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n$$

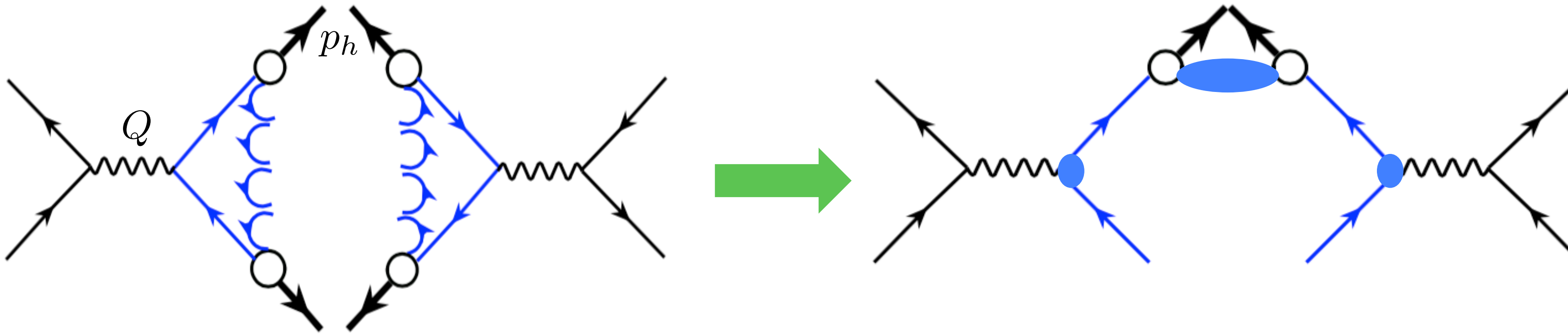
❑ Distribution of an identified hadron(s):

e.g., $E_h \dfrac{d\sigma_{e^+e^-\to h}}{d^3p_h}(\sqrt{s}=Q)$, $E_{h_1}E_{h_2}\dfrac{d\sigma_{e^+e^-\to h_1h_2}}{d^3p_{h_1}d^3p_{h_2}}(\sqrt{s}=Q)$ **are not calculable**

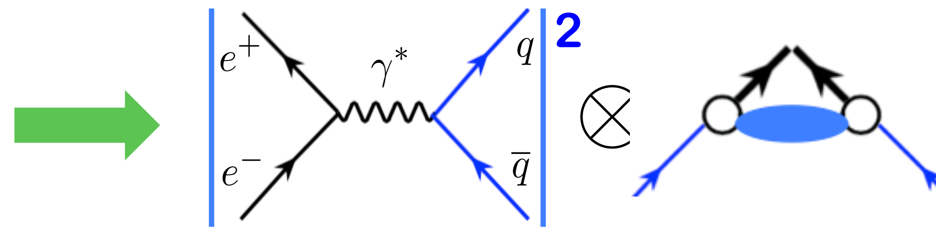
since the hadronization is color entangled and nonperturbative!

QCD Factorization – Approximation

- Single hadron production – collinear factorization:



- QCD factorization – approximation:



When $P_h \gg m_h$ (enhanced by some logarithms from the shower), we "neglect" the color entanglement between the long-distance hadronization processs, while keeping the color entanglement at the short-distance, calculated perturbatively

$$E_h \frac{d\sigma_{e^+e^-\to h}}{d^3 p_h}(\sqrt{s}=Q) \approx \sum_c E_c \frac{d\sigma_{e^+e^-\to c}}{d^3 p_c} \otimes D_{c\to h}(z = p_h/p_c)$$

plus power corrections in $\mathcal{O}(m_h/P_h)$

QCD Factorization – Approximation

- ❑ **Explore the breakdown of QCD factorization:**

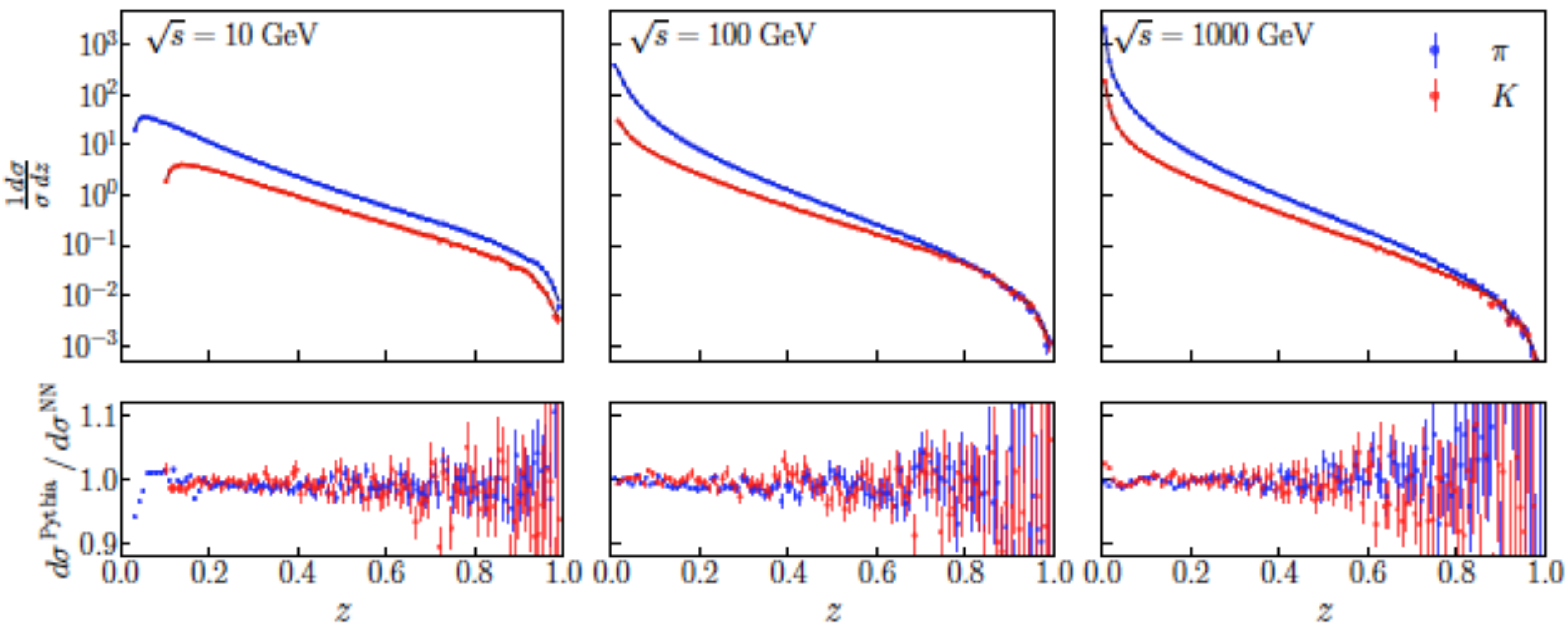
  or the role of color entanglement in hadronization

$$E_h \frac{d\sigma_{e^+e^-\to h}}{d^3p_h}(\sqrt{s}=Q)$$

Can be represented by a 2-D picture for each hadron with *Q* and p_h for the two sides, and density for the rate

- ❑ **NN training and mapping out all $\sqrt{s}=Q$ and $z=2p_h/Q$ dependence of the production rate (without any theory input!):**

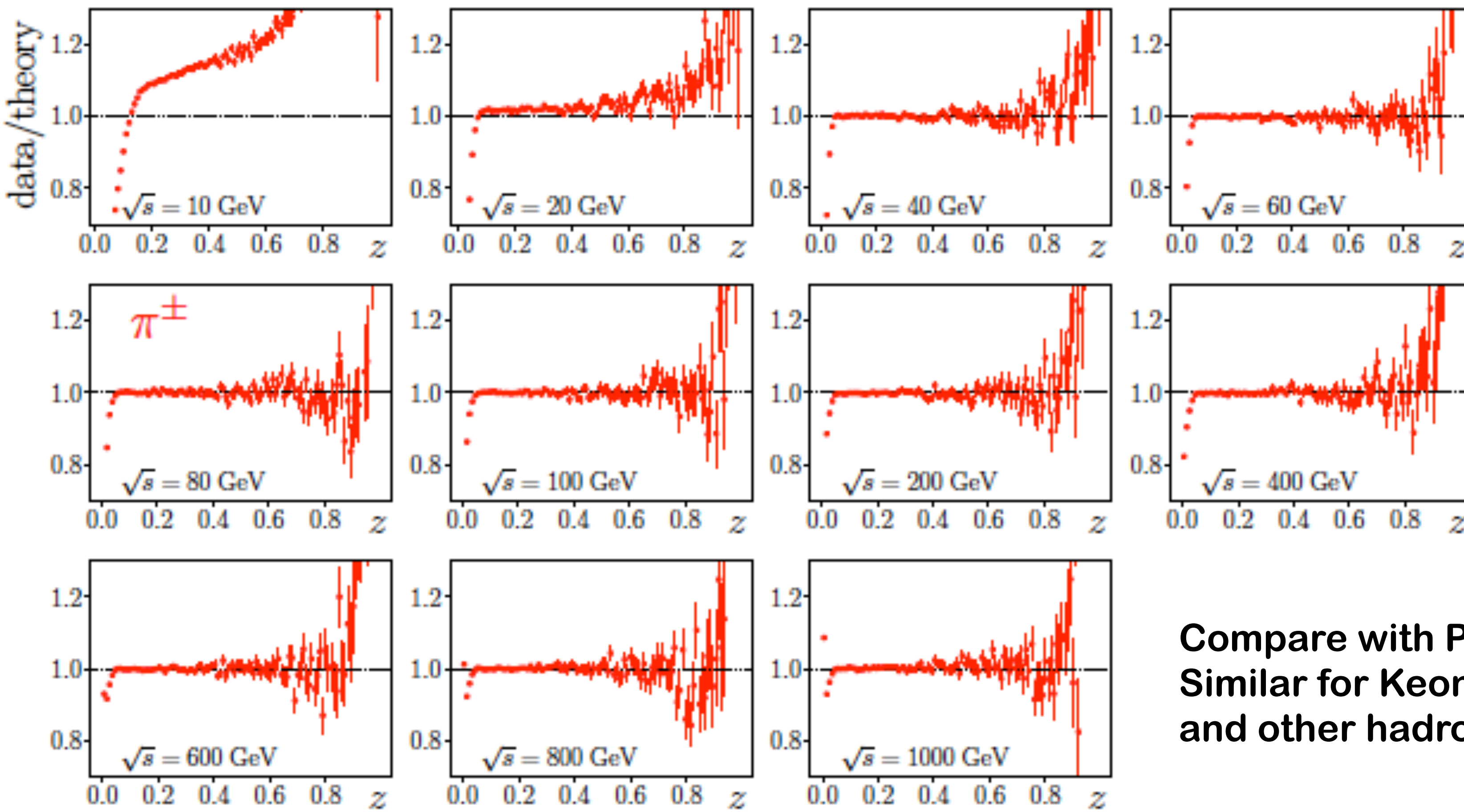
T. Liu, N. Sato et al. @ JLab



QCD Factorization – Approximation

❑ Comparing with pQCD factorization:

T. Liu, N. Sato et al. @ JLab

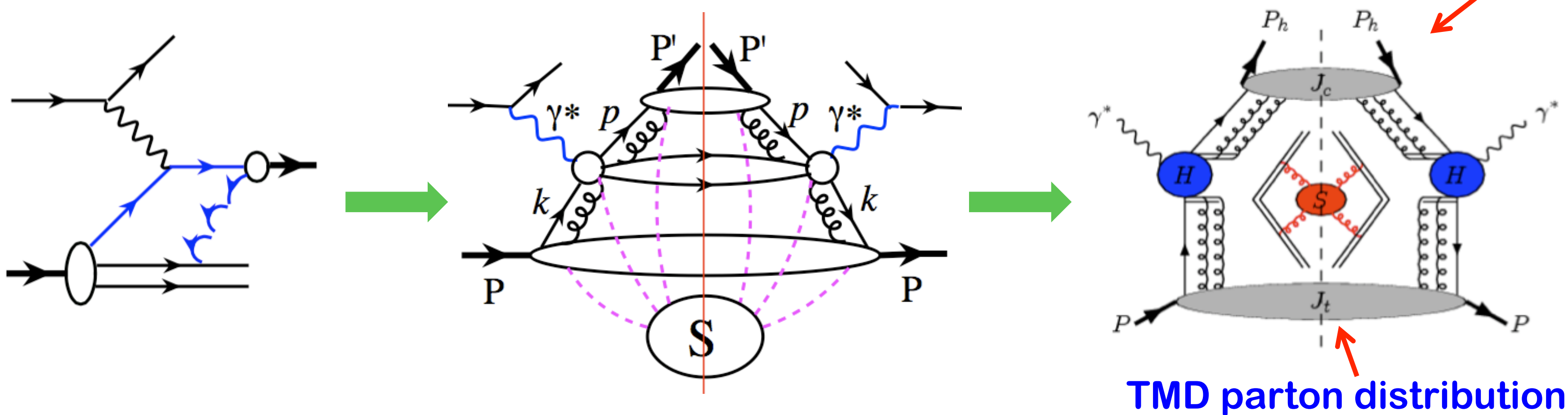


Compare with Pythia
Similar for Keon and other hadrons

Leading power factorization formalism fails at Belle energies, and near the edge of phase-space!

Semi-Inclusive DIS

- Color entanglement – factorization:



- Low P_{hT} – TMD factorization:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

- High P_{hT} – Collinear factorization:

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

- P_{hT} Integrated - Collinear factorization:

$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

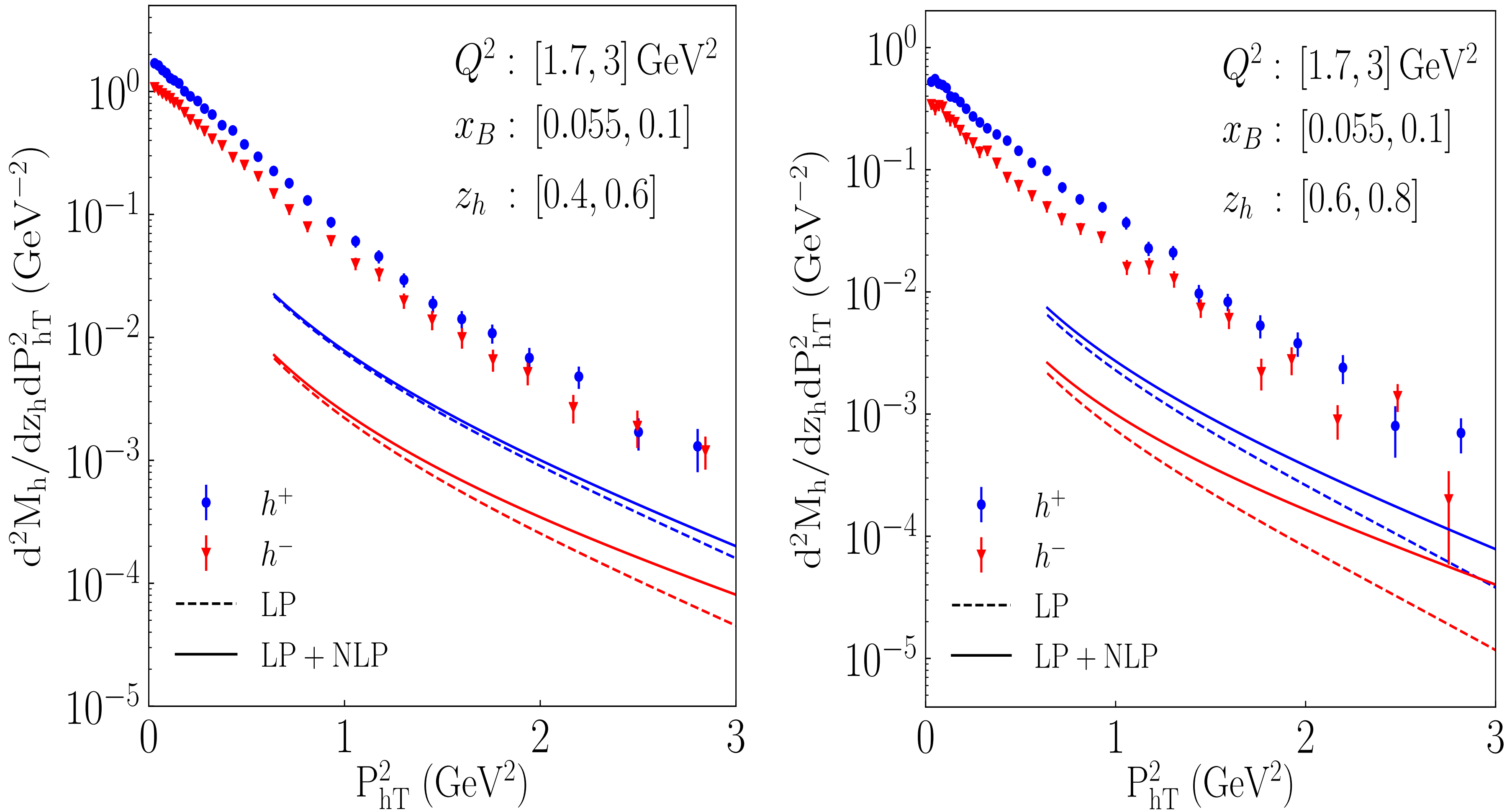
QCD factorization suppress color entanglement between hadrons

Semi-Inclusive DIS

T. Liu, JQ

- Near the edge of phase space:

Much more sensitive to color entanglement of hadronization



Semi-Inclusive DIS

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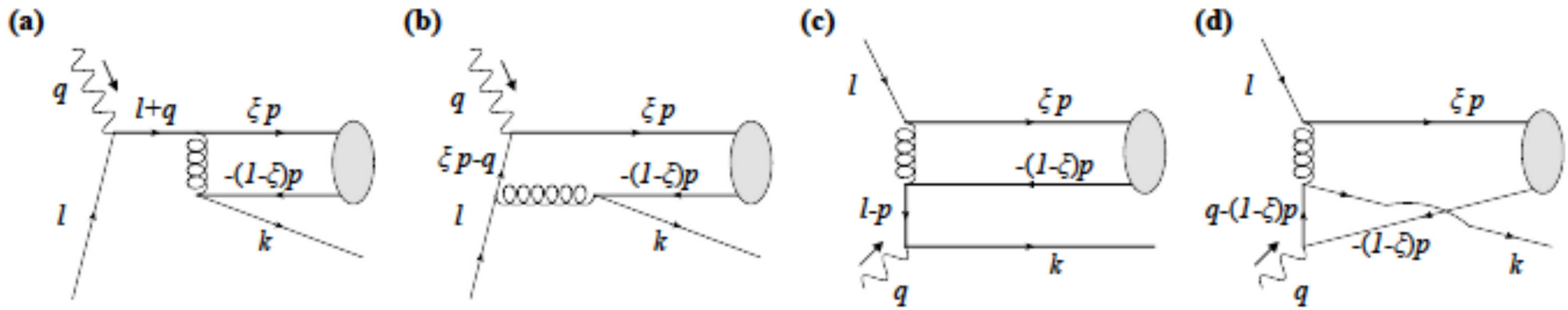
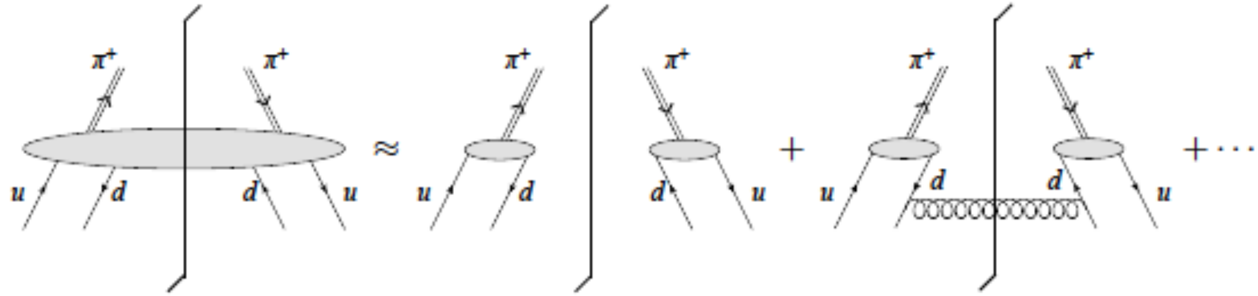
- Power corrections:



- Two-parton fragmentation:



$$D_{[q\bar{q}'(1a)]}(z,\xi,\zeta,\mu_0) = \int \frac{P_h^+ dy^-}{2\pi} \int \frac{P_h^+ dy_1^-}{2\pi} \int \frac{P_h^+ dy_2^-}{2\pi} e^{i(1-\zeta)\frac{P_h^+}{z}y_1^-} e^{-i\frac{P_h^+}{z}y^-} e^{-i(1-\xi)\frac{P_h^+}{z}y_2^-}$$

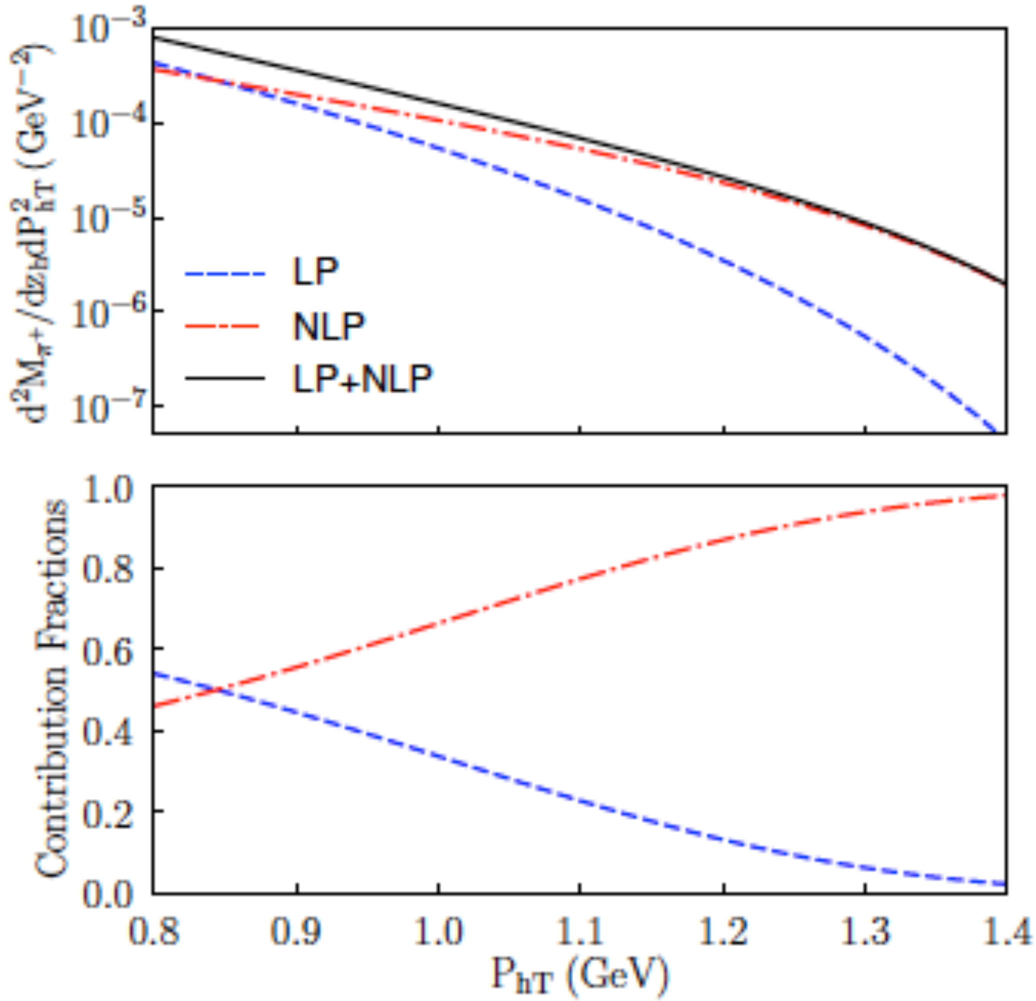
$$\times \frac{1}{4N_c P_h^+} \langle 0|\bar{q}'_{c',k}(y_1^-)(\gamma\cdot n\gamma_5)_{kl} U_{c'd'}(y_1^-,0) q_{d',l}(0)|h(P_h)\rangle$$

$$\approx \frac{f_h^2}{16N_c^2} z\delta(1-z)\phi_h(\zeta,\mu_0)\phi_h(\xi,\mu_0).$$

Semi-Inclusive DIS

T. Liu, JQ

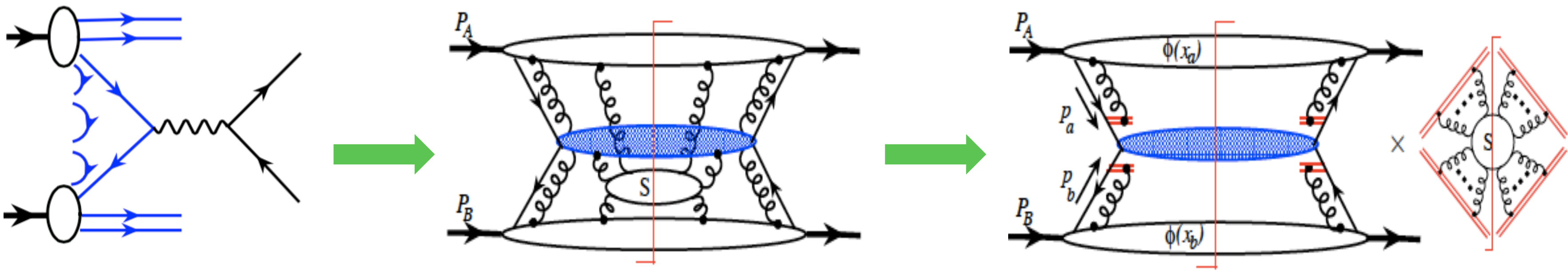
- Prediction for JLab energy – photo production:



$E_{\mathrm{beam}} = 11\,\mathrm{GeV},\ x_B = 0.2,\ Q^2 = 3\,\mathrm{GeV}^2,\ z_h = 0.7.$

Drell-Yan

- Color entanglement – factorization:



- P_{hT} Integrated - Collinear factorization:

$$\frac{d\sigma}{dQ^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') + \mathcal{Q}\left(\frac{1}{Q}\right)$$

- Low P_{hT} – TMD factorization:

$$\frac{d\sigma}{dQ^2 dq_T^2} = \hat{H} \otimes \Phi(x, k_\perp) \otimes \Phi(x', k_{\perp'}) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{Q}\left(\frac{q_T}{Q}\right)$$

- High P_{hT} ~ Q – Collinear factorization:

$$\frac{d\sigma}{dQ^2 dq_T^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') + \mathcal{Q}\left(\frac{1}{Q}, \frac{1}{q_T}\right)$$

- High P_{hT} >> Q – Collinear factorization:

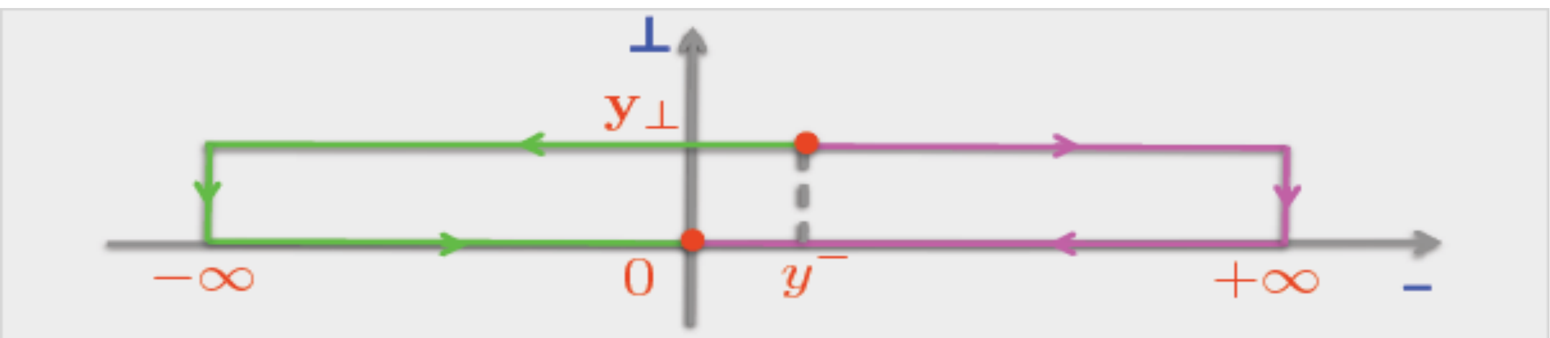
$$\frac{d\sigma}{dQ^2 dq_T^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') \otimes D(z) + \mathcal{Q}\left(\frac{1}{q_T}\right)$$

Sign Change of Sivers Function

- Single transverse spin asymmetry – Sivers effect:

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

- Gauge links:



- Process dependence:

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

Collinear factorized PDFs are process independent

Sign Change of Sivers Function

- Parity – Time reversal invariance:

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- Definition of Sivers function:

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

- Modified universality:

$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x,k_\perp)$$

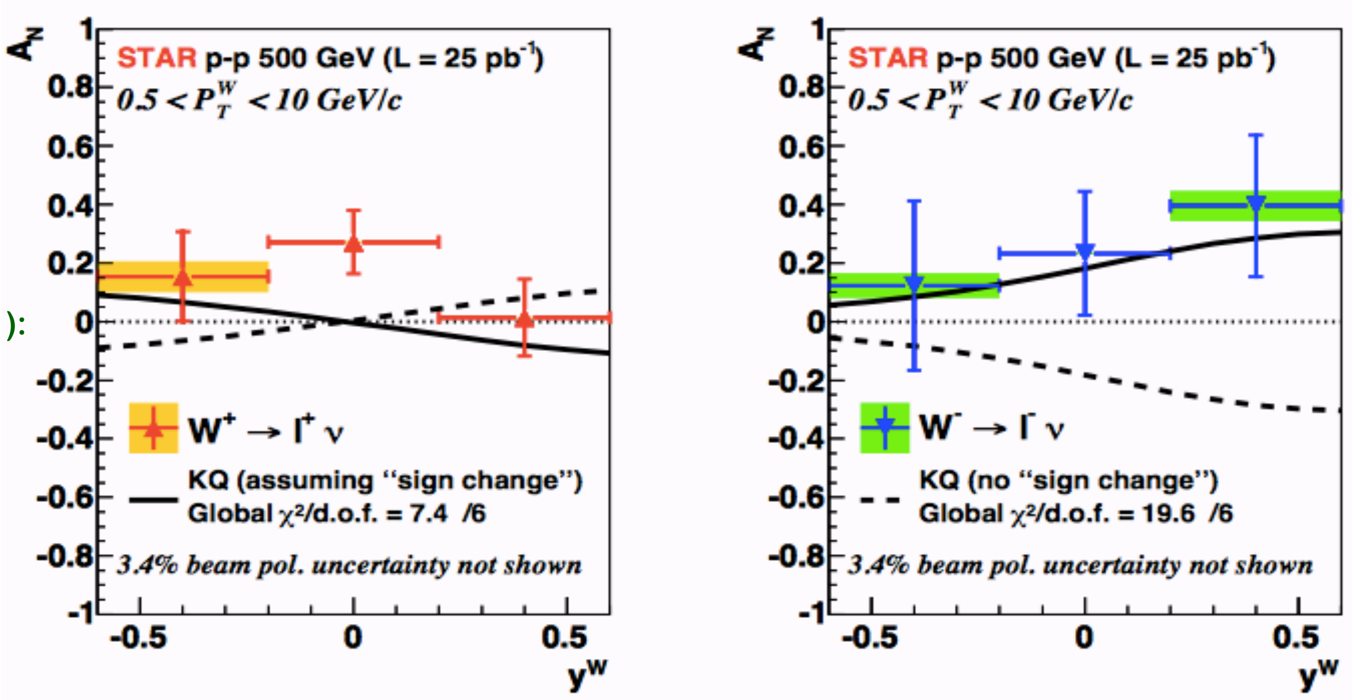
The spin-averaged part of this TMD is process independent,

but, spin-averaged Boer-Mulder's TMD requires the sign change!

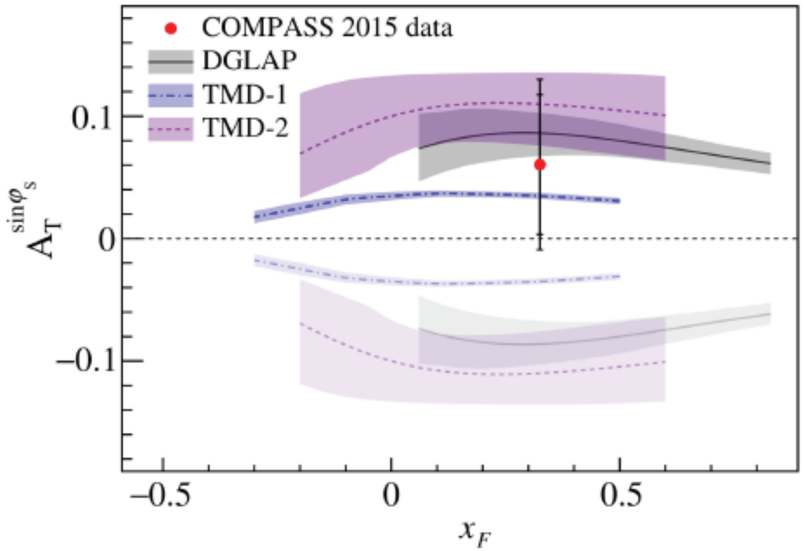
Violation of the "sign" change should be the break of factorization, that is, a much stronger color entanglement!

Sign Change of Sivers Function

- **RHIC:**
  PRL 116(2016)132301):

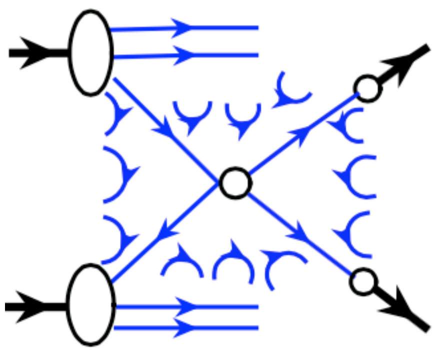


- **COMPASS:**
  PRL 119(2017)112002



Hadronic Scattering

- Color entanglement – factorization:



$$\frac{d\sigma}{dp_T^2} = \hat{H} \otimes \phi(x) \otimes \phi(x') \otimes D(z) + \mathcal{O}\left(\frac{1}{p_T}\right)$$

when $p_T \gg m_h$

- Breaking of TMD factorization for di-jet production:

$$H_1(p_A) + H_2(p_B) \Rightarrow Jet(p_1) + Jet(p_2) + X$$

- ✧ Dominated kinematic region:

$$p_1 = \frac{P}{2} + q \qquad p_2 = \frac{P}{2} - q \qquad \textbf{with } P \gg q$$

- ✧ Proposal: if the TMD factorization is valid in this region, di-jet momentum imbalance is an excellent observable to test the universality of the Sivers function

Boer and Vogelsang

- ✧ Unfortunately, TMD factorization was not valid for this process due to color entanglement

Collins and Qiu
Vogelsang and Yuan

Momentum Imbalance of di-Jet Production

- Single spin-asymmetry could be generated by both initial- and final-state interaction – needs a phase

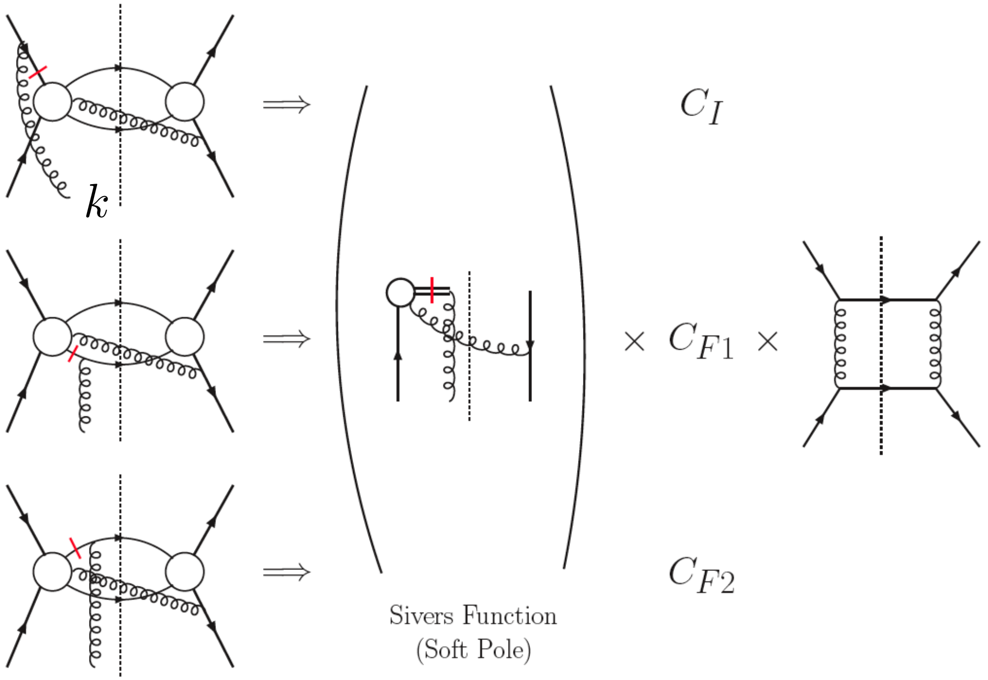
- Very simple representation of qq’ -> qq’ channel:

$$\frac{d\Delta\sigma}{dy_1 dy_2 dP_\perp^2 d^2\vec{q}_\perp} \propto q'(x)\, q_T^{\mathrm{SIDIS}}(x, q_\perp)\, (C_I + C_{F_1} + C_{F_2})\, \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

when k is parallel to the polarized hadron

Perturbatively generated Sivers’ function at g²

Initial- and final-state interactions differ only by a color factor

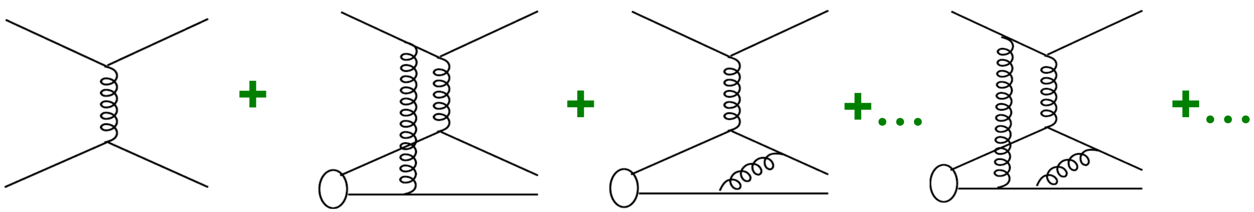


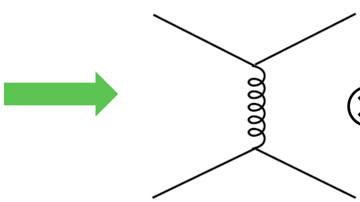
Momentum Imbalance of di-Jet Production

- Test the TMD factorization by studying long-distance physics of partonic scattering cross section:

  If the factorization is valid, all factorized long-distance information should be process independent

- Consider the poles from collinear gluon attachment to the lowest order partonic diagram in the TMD approach





$\otimes$ TMD distribution with the exponentiated gauge link

Qiu, Vogelsang, and Yuan

Initial-state and final-state have different color flow!
If one keeps the color difference in the hard part, one could get the same leading order hard part – necessary, not sufficient!

Momentum Imbalance of di-Jet Production

Collins and Qiu

- A simple model:
  – simplifying the derivation while keeping the same physics

  ✧ Hadron is made of a fermion ψ and a scalar ϕ

  ✧ There are two hadrons, H_1 and H_2

  ✧ Gauge field (Abelian) couples g_i to ψ_i and $-g_i$ to ϕ_i

$$\lambda_i \left(\bar{H}_i \psi_i \, \phi_i^\dagger + \bar{\psi}_i H_i \, \phi_i \right)$$

- Basic idea:

  If the TMD factorization is valid,

  ✧ Gauge link of hadron H_1 should not depend on the property of hadron H2, or any details of the subprocesses

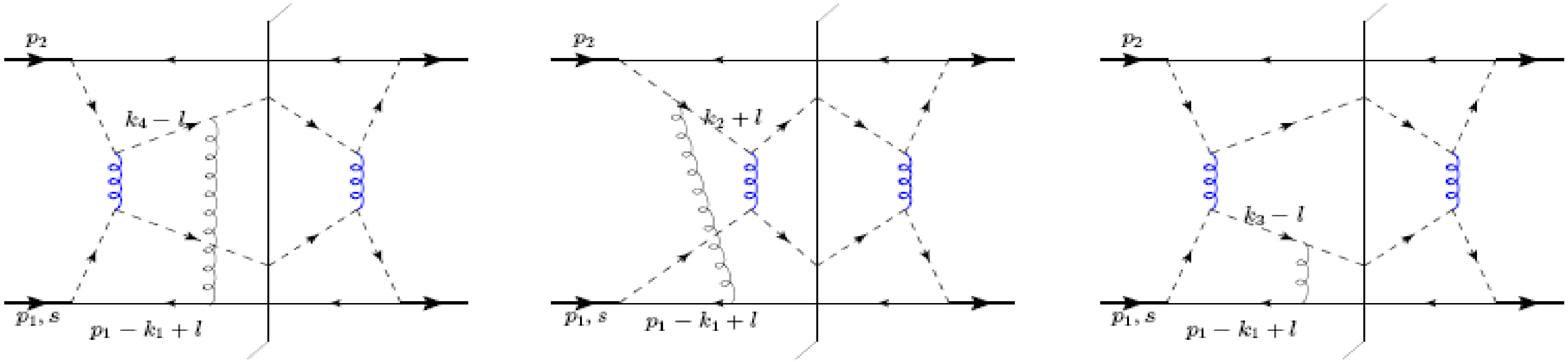
  ✧ Leading contribution from multiple gluon interaction should be expressed in terms of gauge link times the same lowest order hard part

  ✧ Otherwise, the TMD factorization is violated

Momentum Imbalance of di-Jet Production

Collins and Qiu

- Leading contribution to SSA:



Phase from the leading pole: $(i\pi)(g_1 + 2g_2)\delta(l^+)$
depending on the g_2!

- Can we keep the g_2 dependence in the hard part?

We find that the $(i\pi)^2$ from two gluon exchange also depends on g_2 , which cannot be factored into the same lowest order hard part with g_2.

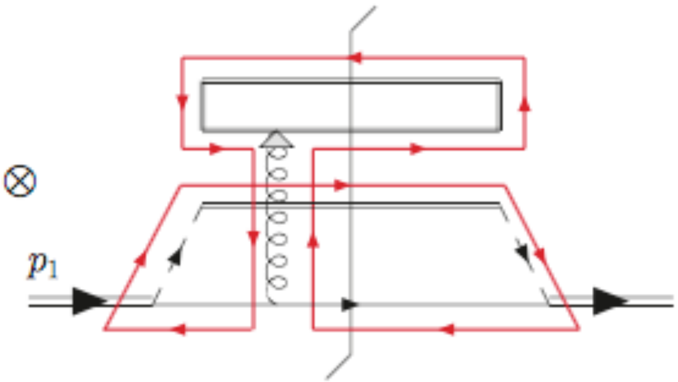
That is, we found an example in which the gauge link of hadron 1 depends on the property of hadron 2, which signals the failure of the TMD factorization

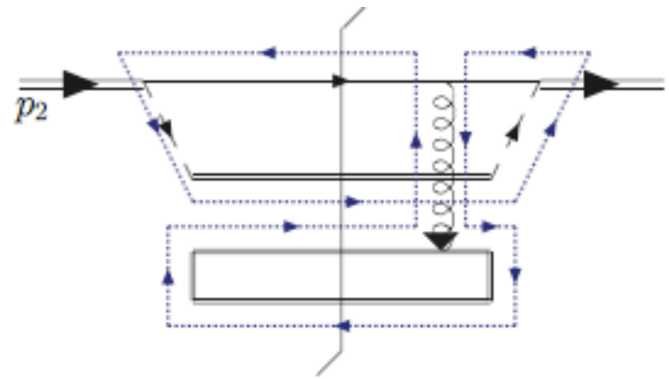
Vogelsang and Yuan, so as Rogers and Mulders obtained the same conclusion

Momentum Imbalance in di-Jet Production

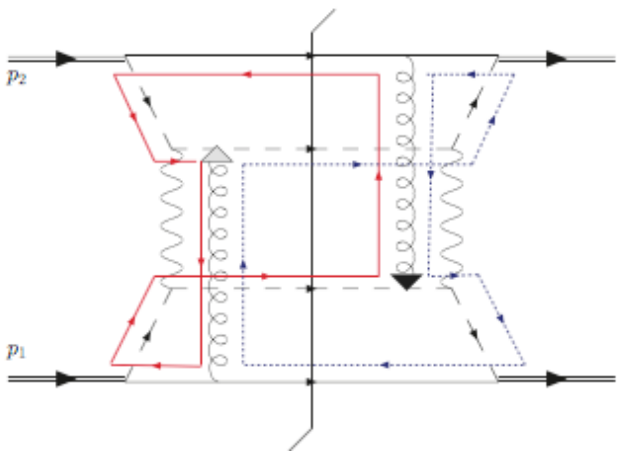
Rogers and Mulders

- Color flow of TMD factorization:

$d\sigma \sim \mathcal{H} \otimes$ 

$\otimes$ 

- Color flow breaks TMD factorization:



The color flow can't be separated into two loops, each of them depends on only one-hadron

Color is entangled!

This is consistent with the general rule that Qiu & Sterman found:

Only the first subleading power term could be factorized when observables involve multiple hadrons

Summary

- Quantum entanglement is a very interesting phenomenon
  - Separates quantum theory from classical ones
- Observables involving multiple identified hadrons could not be calculated perturbatively, without making approximations – leading to the factorization, …
  - QCD factorization is an approximation to suppress the color entanglement
- Breaking of QCD factorization reduces our predictive power, but, might give us new opportunities to explore the even richer phenomena of color entanglement
- QCD dynamics is rich – We only learned very little of it!

Thanks!

Backup slides