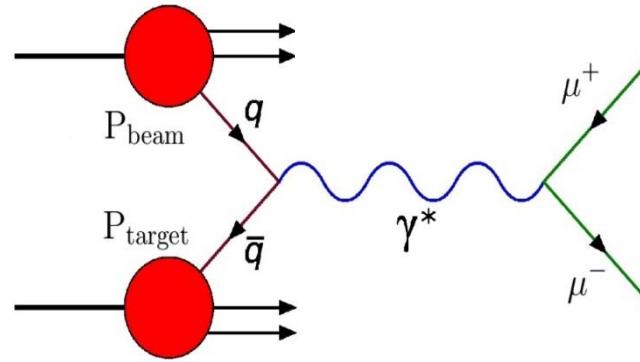


# Exclusive pion-induced Drell-Yan process at J-PARC and soft nonfactorizable mechanism

Kazuhiro Tanaka (Juntendo U/KEK)

# Pion-induced Drell-Yan process

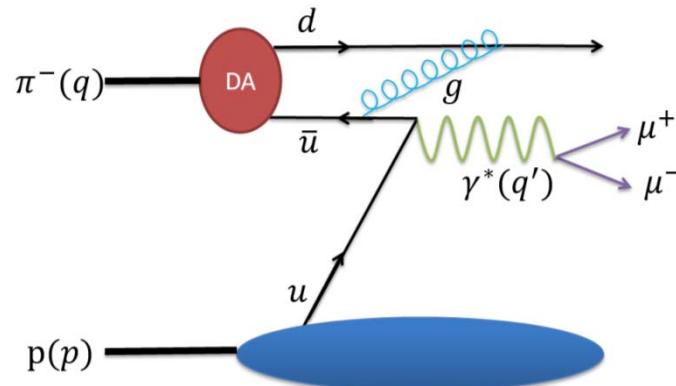
$$\pi N \rightarrow \mu^+ \mu^- X$$



inclusive

# Pion-induced Drell-Yan process

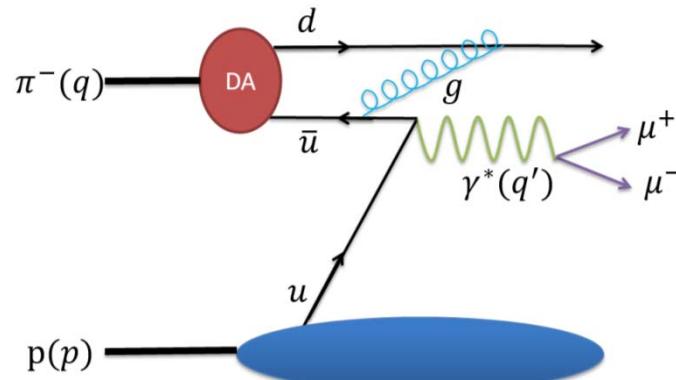
$$\pi N \rightarrow \mu^+ \mu^- X$$



inclusive

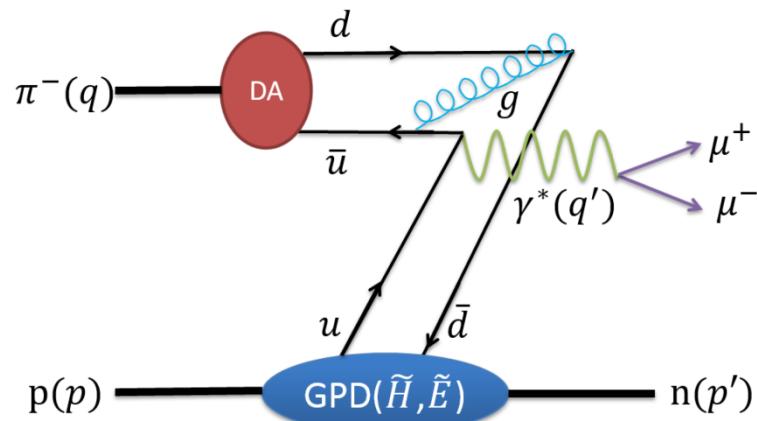
# Pion-induced Drell-Yan process

$$\pi N \rightarrow \mu^+ \mu^- X$$



inclusive

$$\pi N \rightarrow \mu^+ \mu^- N$$



exclusive

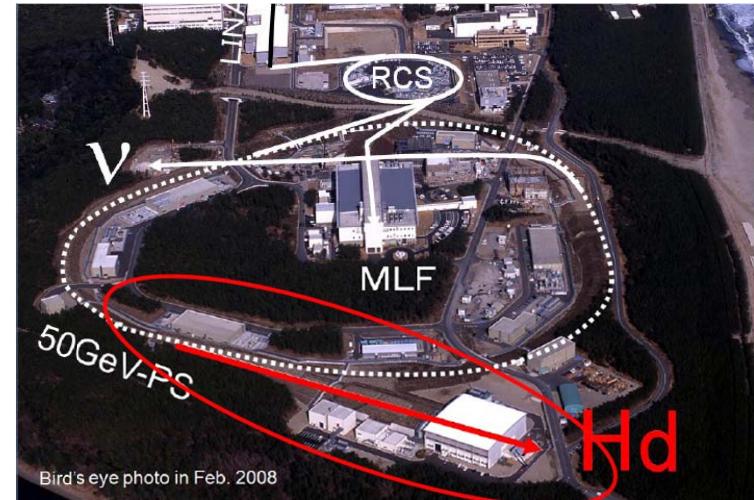
# High momentum beam line at J-PARC

- Primary beam (proton)

$E = 30\text{GeV}$  ( $\rightarrow 50\text{GeV}?$ )

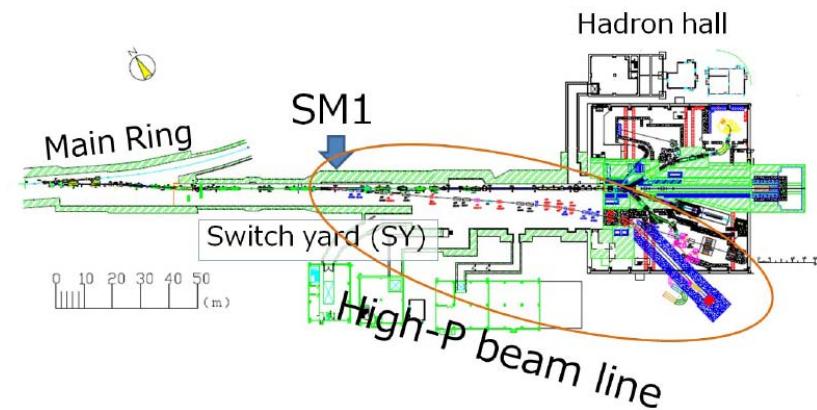
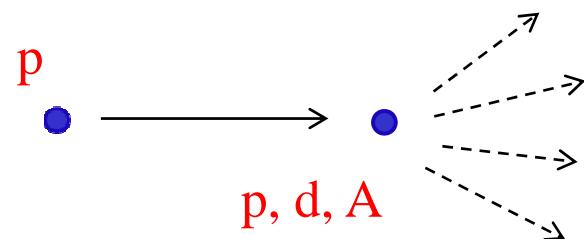
$L = 10^{35} \text{cm}^{-2}\text{s}^{-1}$

Hadron Facility at J-PARC



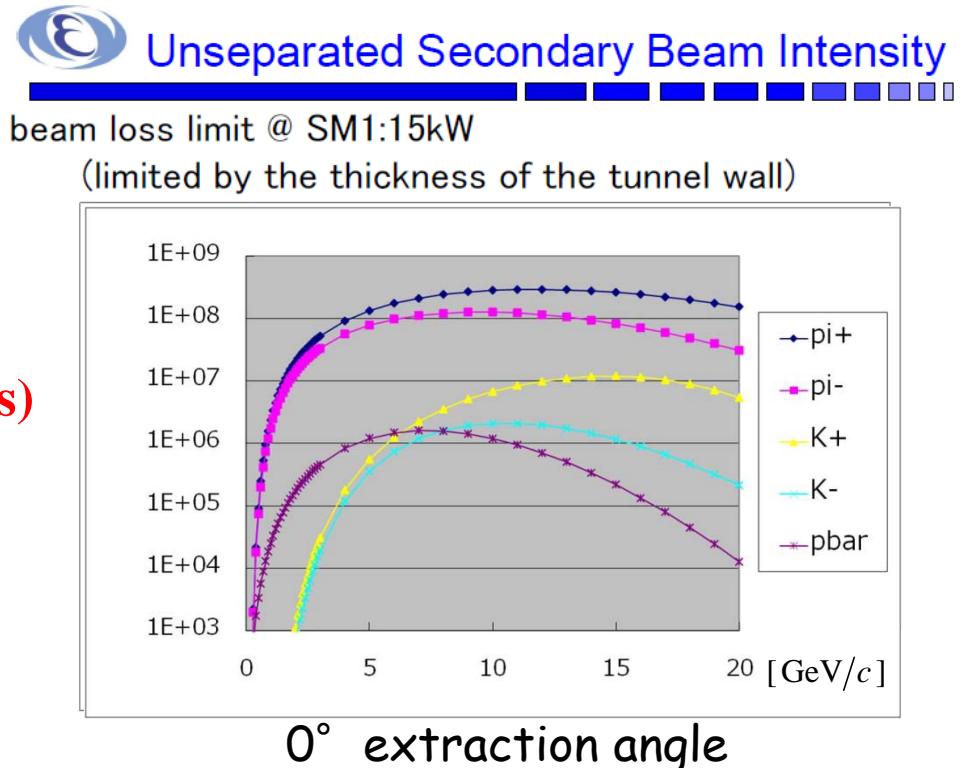
- Secondary beam (pion)

$E = 15\text{-}20\text{GeV}$



## High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)



high intensity

not too high energy

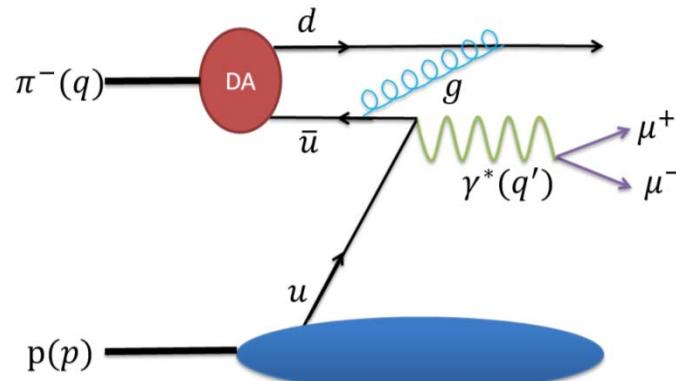
$$d\sigma \sim 1/s^a$$

best suited to study meson-induced  
hard exclusive processes



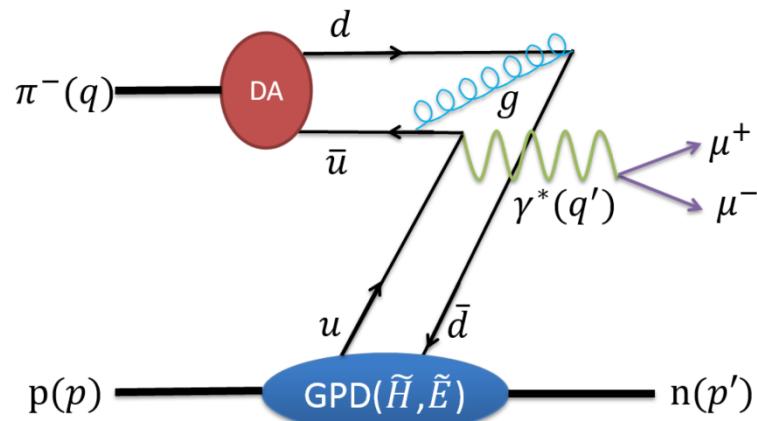
# Pion-induced Drell-Yan process

$$\pi N \rightarrow \mu^+ \mu^- X$$



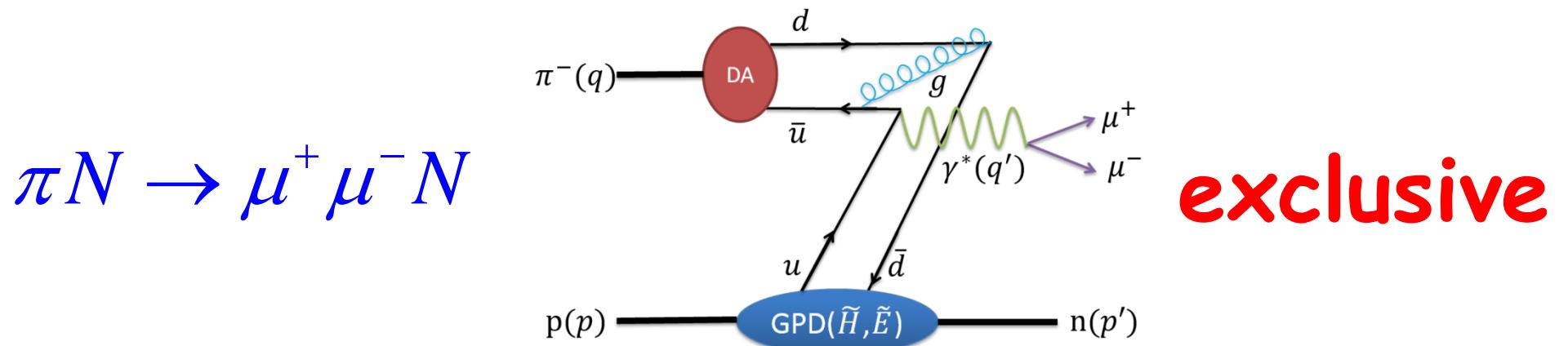
inclusive

$$\pi N \rightarrow \mu^+ \mu^- N$$



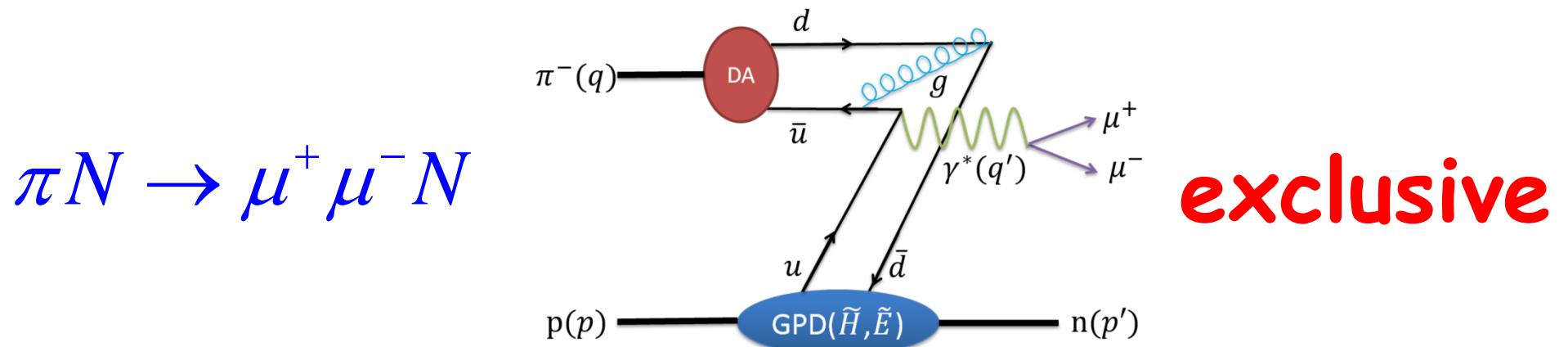
exclusive

# Pion-induced Drell-Yan process

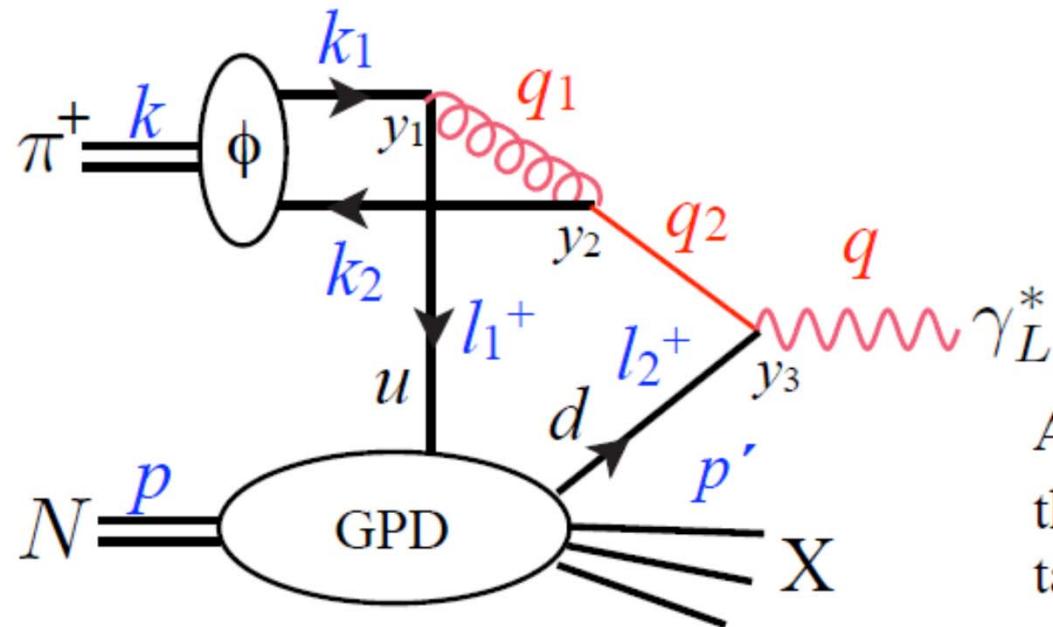


# Pion-induced Drell-Yan process

QCD factorization formula



Hence the stopped quark should be connected to the target:



For each final state  $X$  the target matrix element is given by a **GPD** with skewness

$$l_2^+ - l_1^+ = q^+ = x_B p^+$$

$$\begin{aligned} k_1 &= (0^+, u k^-, \mathbf{k}_\perp) \\ k_2 &= (0^+, (1-u) k^-, -\mathbf{k}_\perp) \end{aligned}$$

Since  $q_1^2 \approx -u k^- l_1^+ \rightarrow \infty$   
the pion wave function contributes through its *distribution amplitude*  $\phi$

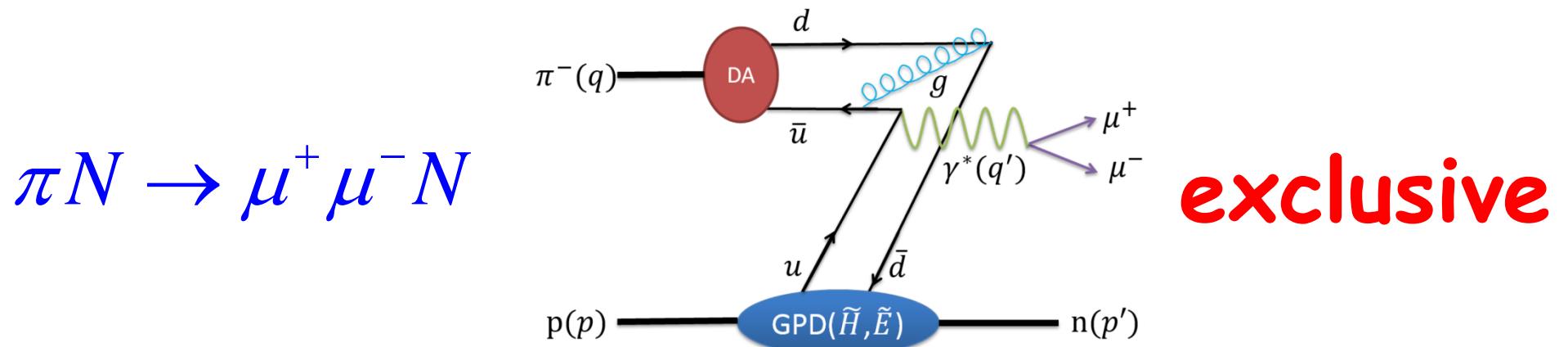
Also  $q_2^2, q_1^-, q_2^- \rightarrow \infty$ , hence the space-time separation of the target interaction points  $y_1, y_3$  is

$$\begin{aligned} |y_{1\perp} - y_{3\perp}| &= \mathcal{O}(1/Q) \rightarrow 0 \\ |y_1^+ - y_3^+| &= \mathcal{O}(1/Q^2) \rightarrow 0 \\ |y_1^- - y_3^-| &= \mathcal{O}(1/\ell_1^+) \text{ finite} \end{aligned}$$

Using perturbative propagators for the gluon  $q_1$  and  $d$ -quark  $q_2$  and adding three more diagrams we get

# Pion-induced Drell-Yan process

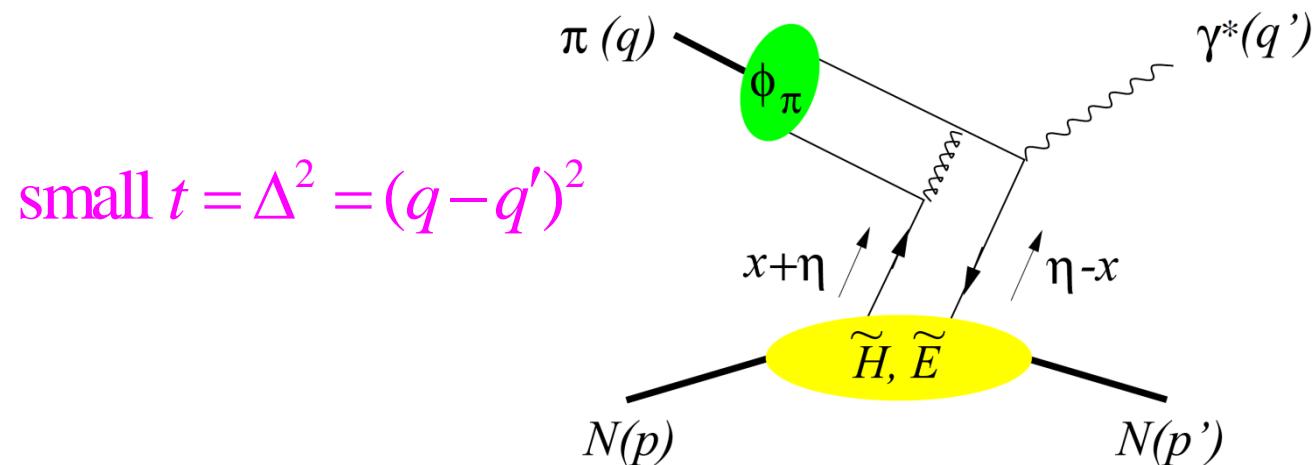
QCD factorization formula



## Exclusive lepton pair production in $\pi N$ scattering

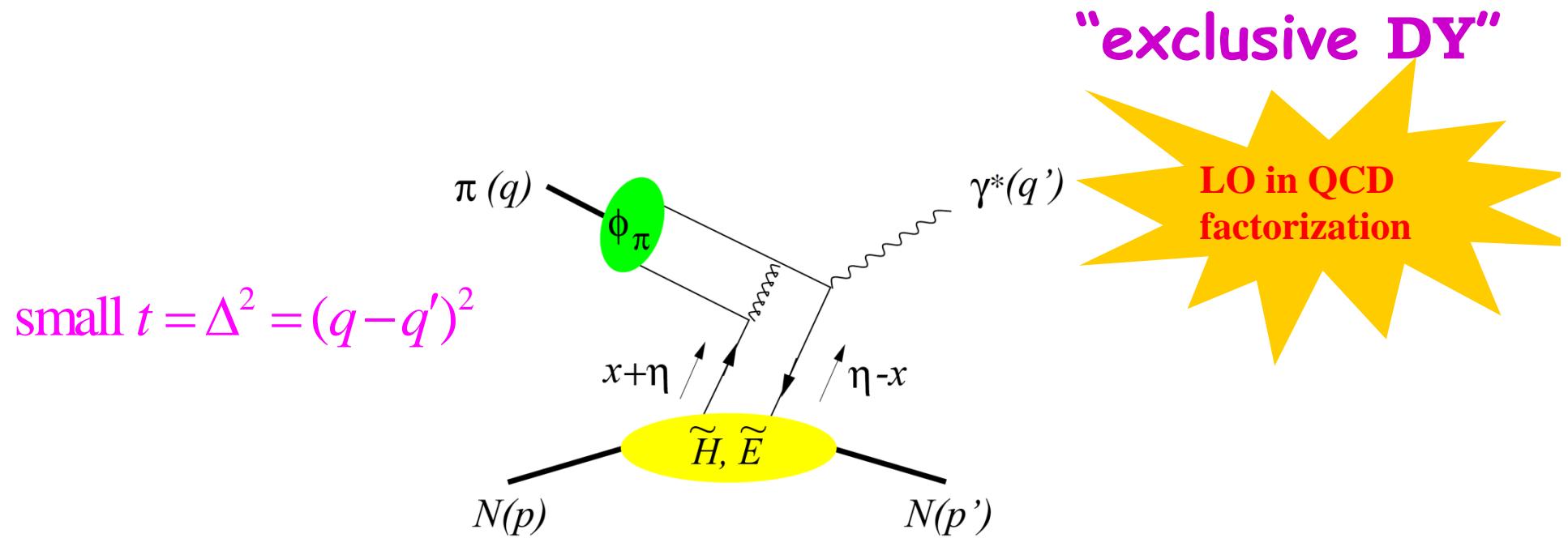
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

“exclusive DY”



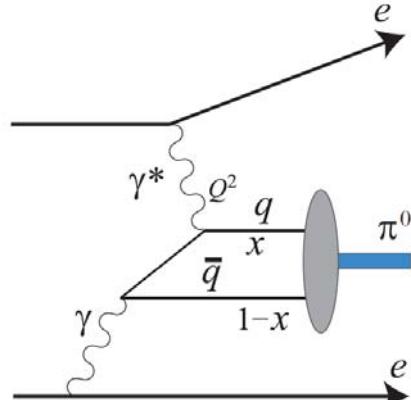
## Exclusive lepton pair production in $\pi N$ scattering

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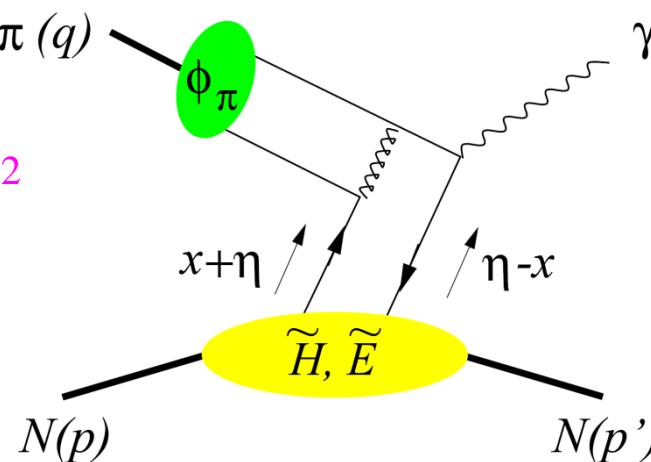
## Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

small  $t = \Delta^2 = (q - q')^2$

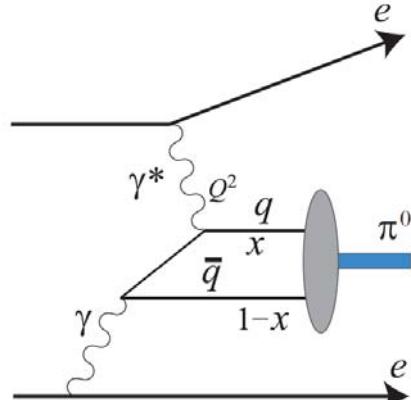


"exclusive DY"



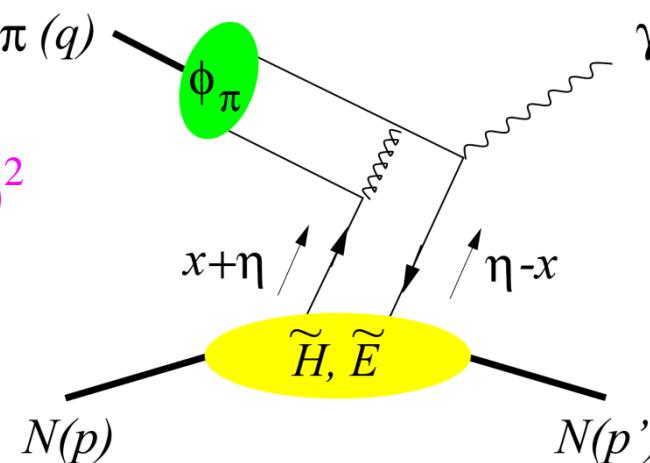
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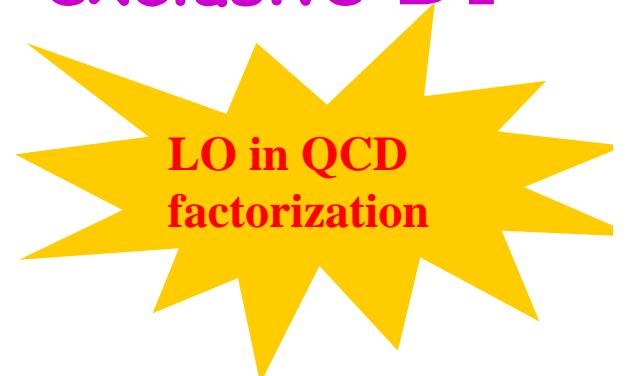


@Belle, Babar

small  $t = \Delta^2 = (q - q')^2$



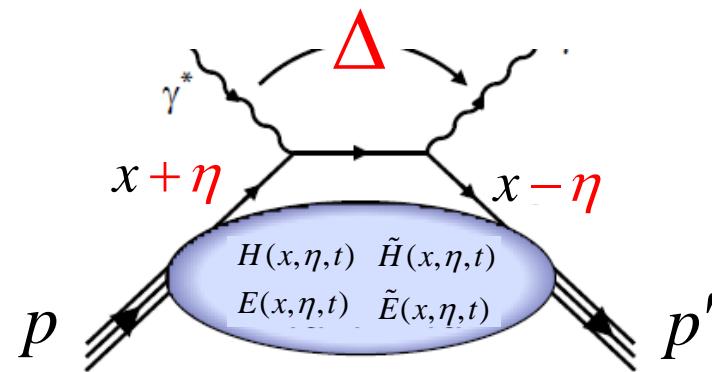
"exclusive DY"



$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta}) \cdot \mathbf{p} \mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

GPD

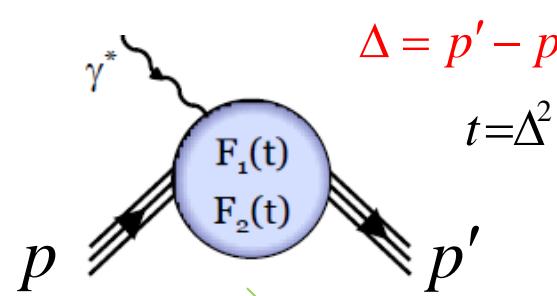
**GPD**



$$-2\eta \bar{P} = \Delta$$

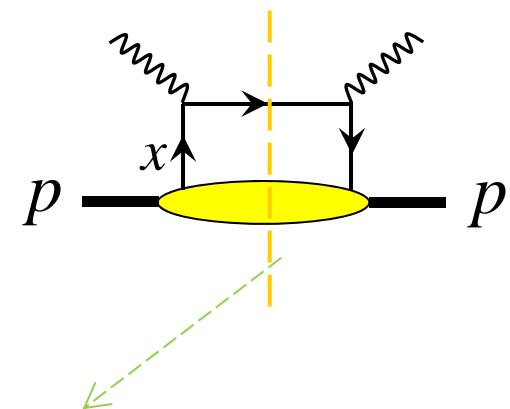
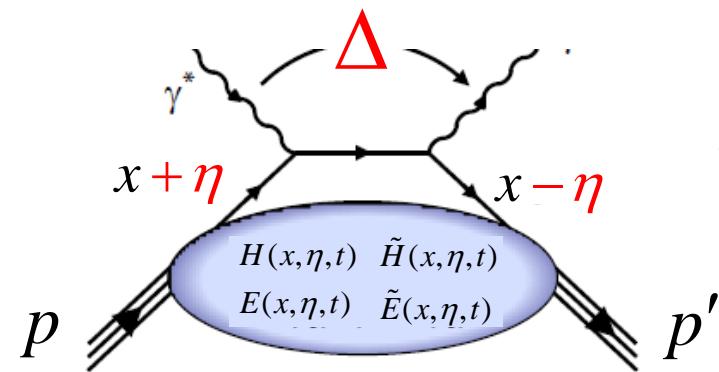
$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta})p\mathbf{z}} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$



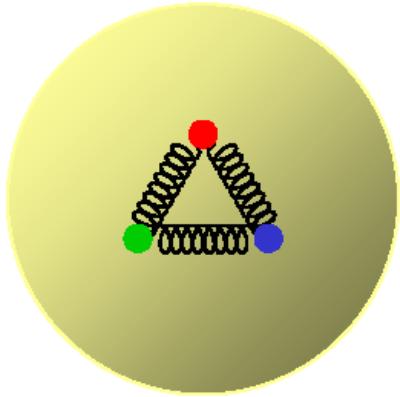
**GPD**

$$\int d\mathbf{z}^- e^{i\mathbf{x} p \mathbf{z}} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$

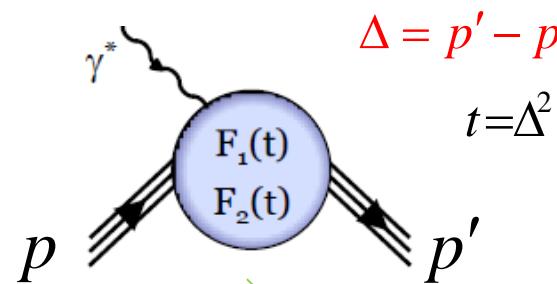


$$-2\eta \bar{P} = \Delta$$

$$\int d\mathbf{z}^- e^{i(\mathbf{x} + \boldsymbol{\eta}) p \mathbf{z}} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

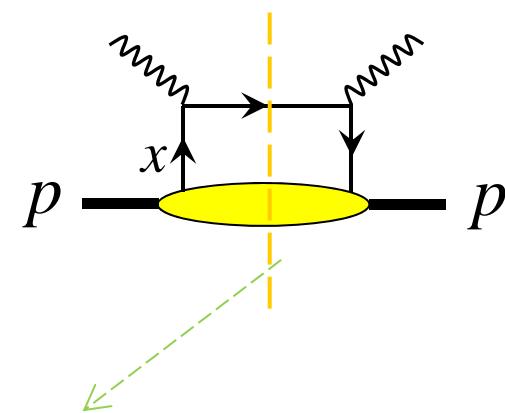
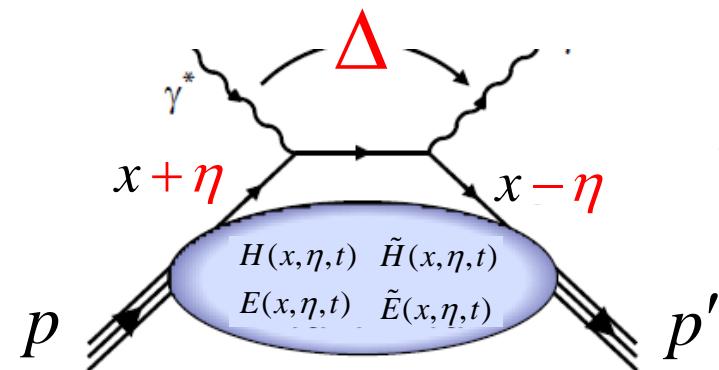


$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$



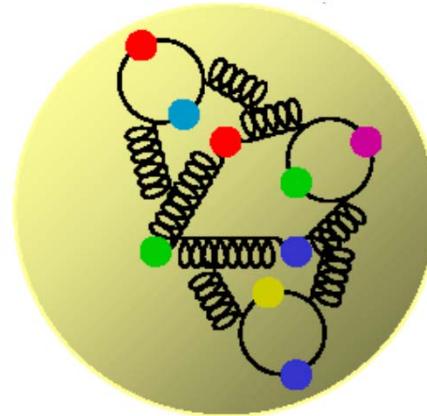
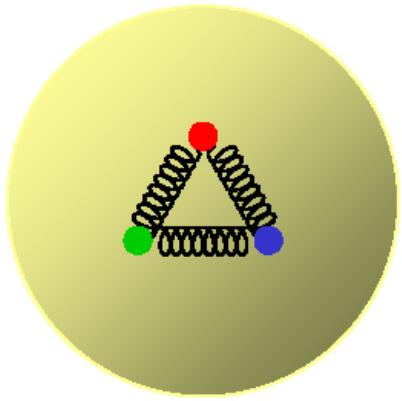
**GPD**

$$\int d\mathbf{z}^- e^{i\mathbf{x} p \mathbf{z}} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$

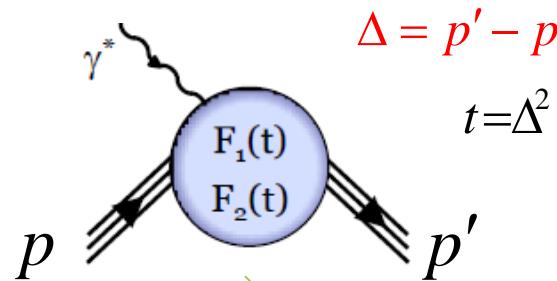


$$-2\eta \bar{P} = \Delta$$

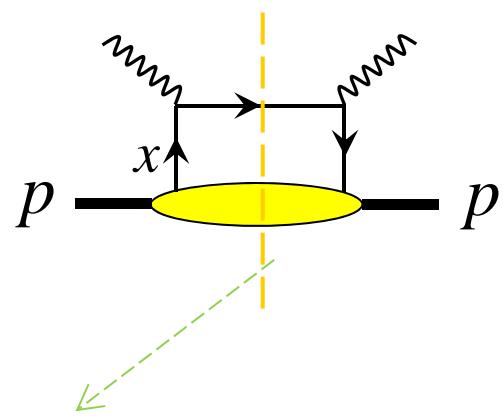
$$\int d\mathbf{z}^- e^{i(\mathbf{x} + \boldsymbol{\eta}) p \mathbf{z}} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



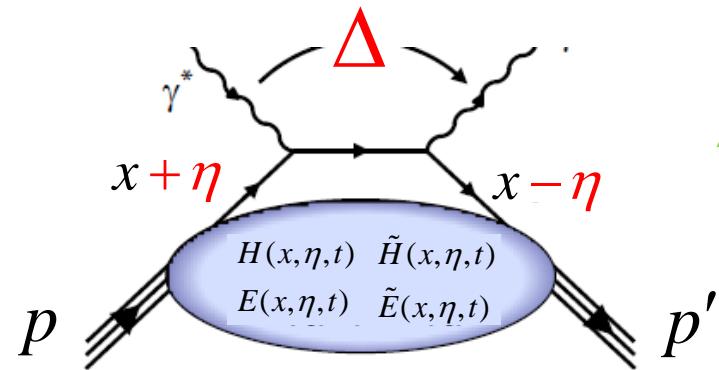
$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$



$$\int d\mathbf{z}^- e^{i\mathbf{x} p \mathbf{z}} \langle N(p) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(p) \rangle$$



**GPD**



$$-2\eta \bar{P} = \Delta$$

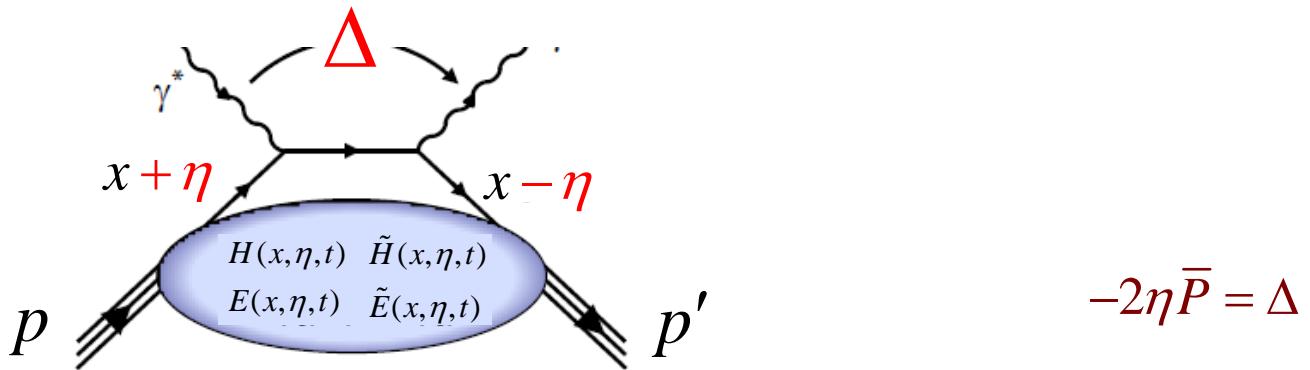
$$\int d\mathbf{z}^- e^{i(\mathbf{x} + \boldsymbol{\eta}) p \mathbf{z}} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

$$\overline{P}=\frac{\textcolor{blue}{p}+p'}{2}$$

$$\int \frac{d\textcolor{red}{z}^-}{2\pi} e^{i(\textcolor{blue}{x}+\eta)\bar{P}\textcolor{red}{z}} \langle p'|\bar{\psi}(0)\gamma^+\psi(\textcolor{red}{z}^-)|p\rangle = \frac{1}{\overline{P}^+}\Bigg[\textcolor{violet}{H}(x,\eta,t)\overline{u}(p')\gamma^+ u(p) + \textcolor{violet}{E}(x,\eta,t)\overline{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\Bigg]$$

$$\int \frac{d\textcolor{red}{z}^-}{2\pi} e^{i(\textcolor{blue}{x}+\eta)\bar{P}\textcolor{red}{z}} \langle p'|\bar{\psi}(0)\gamma^+\gamma_5\psi(\textcolor{red}{z}^-)|p\rangle = \frac{1}{\overline{P}^+}\Bigg[\tilde{H}(x,\eta,\textcolor{violet}{t})\overline{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\eta,\textcolor{violet}{t})\overline{u}(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\Bigg]$$

**GPD**



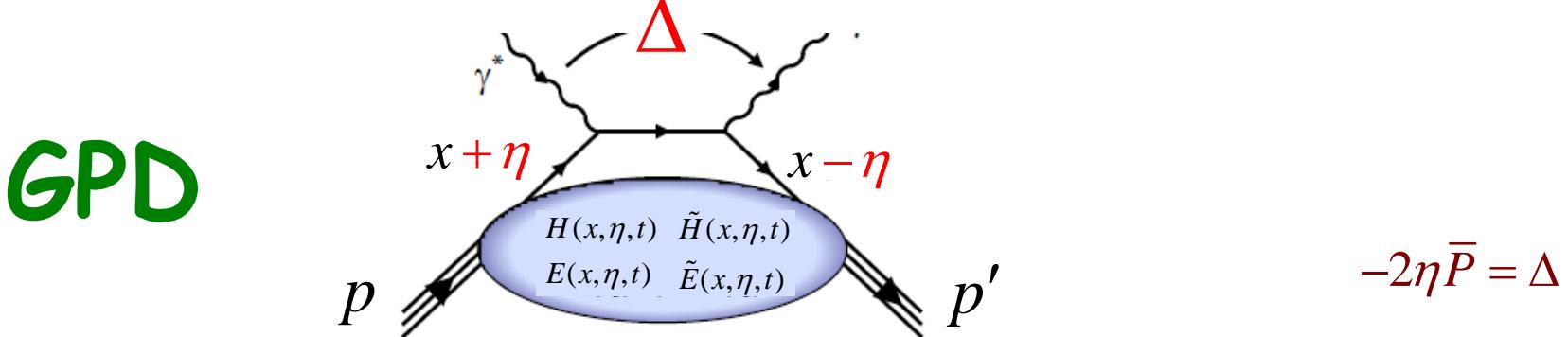
$$\int d\textcolor{red}{z}^- e^{i(\textcolor{blue}{x}+\eta)p\textcolor{red}{z}} \langle N(\textcolor{red}{p}')|\psi^\dagger(0)\psi(\textcolor{red}{z}^-)|N(\textcolor{red}{p})\rangle$$

$$\overline{P}=\frac{\textcolor{blue}{p}+p'}{2}$$

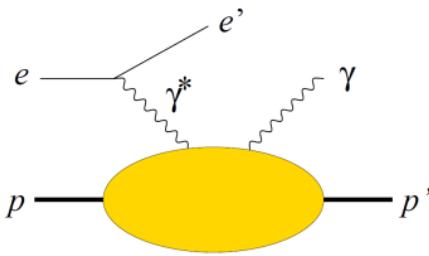
$$\int \frac{d\textcolor{red}{z}^-}{2\pi} e^{i(\textcolor{blue}{x}+\eta)\bar{P}\textcolor{red}{z}} \langle p'|\overline{\psi}(0)\gamma^+\psi(\textcolor{red}{z}^-)|\,p\rangle = \frac{1}{\overline{P}^+}\Bigg[\textcolor{violet}{H}(x,\eta,t)\overline{u}(p')\gamma^+ u(p) + \textcolor{violet}{E}(x,\eta,t)\overline{u}(p')\frac{i\sigma^{+\alpha}(p^{\,\prime}-p)_\alpha}{2M}u(p)\Bigg]$$

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$$J_{\mathfrak{q}}=\frac{1}{2}\int_{-1}^1dx x\big(H(x,\eta,0)+E(x,\eta,0)\big)$$



$$\int d\textcolor{red}{z}^- e^{i(\textcolor{blue}{x}+\eta)p\textcolor{red}{z}} \big\langle N(\textcolor{red}{p}') \Big| \psi^\dagger(0) \psi(\textcolor{red}{z}^-) \Big| N(\textcolor{red}{p}) \big\rangle$$



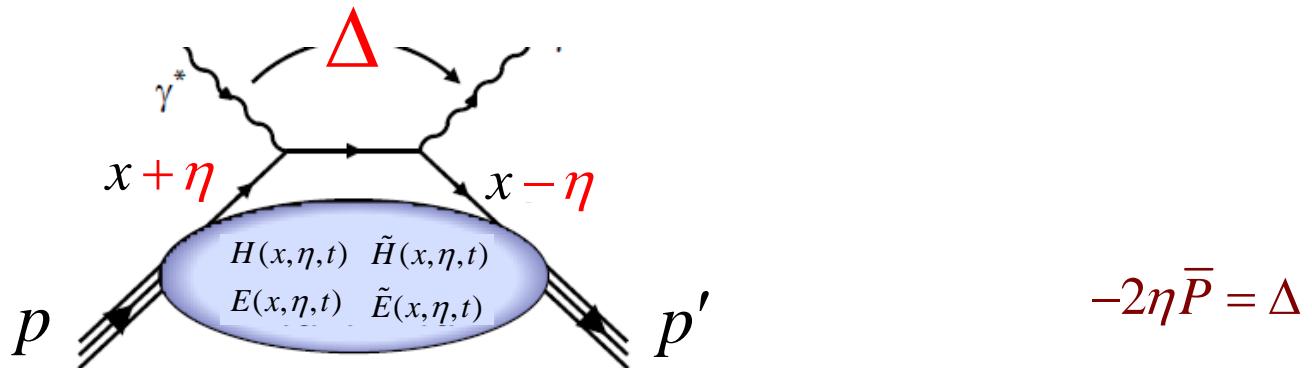
$$\bar{P} = \frac{p + p'}{2}$$

JLab, HERMES, COMPASS, ...

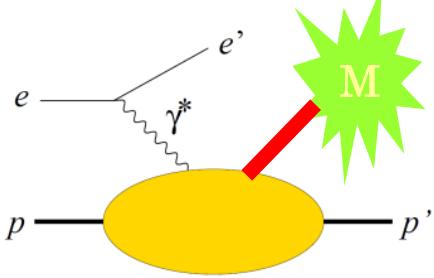
$$\int \frac{d\textcolor{red}{z}^-}{2\pi} e^{i(\textcolor{blue}{x}+\eta)\bar{P}_z} \langle p' | \bar{\psi}(0) \gamma^+ \psi(\textcolor{red}{z}^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \textcolor{magenta}{H}(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + \textcolor{magenta}{E}(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

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**GPD**



$$\int d\textcolor{red}{z}^- e^{i(\textcolor{blue}{x}+\eta)p_z} \langle N(\textcolor{red}{p}') | \psi^\dagger(0) \psi(\textcolor{red}{z}^-) | N(\textcolor{red}{p}) \rangle$$



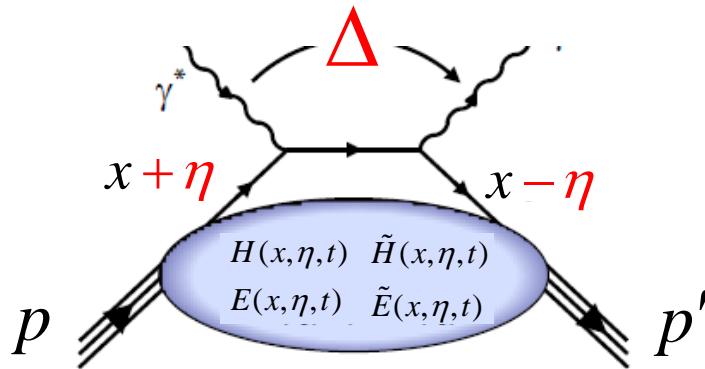
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JLab, HERMES, COMPASS,...

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**GPD**

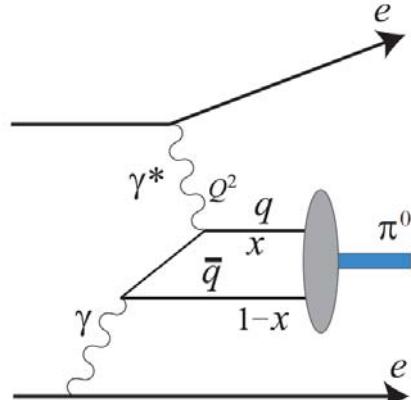


$$-2\eta \bar{P} = \Delta$$

$$\int d\textcolor{red}{z}^- e^{i(\textcolor{blue}{x}+\eta)p_z} \langle N(\textcolor{red}{p}') | \psi^\dagger(0) \psi(\textcolor{red}{z}^-) | N(\textcolor{red}{p}) \rangle$$

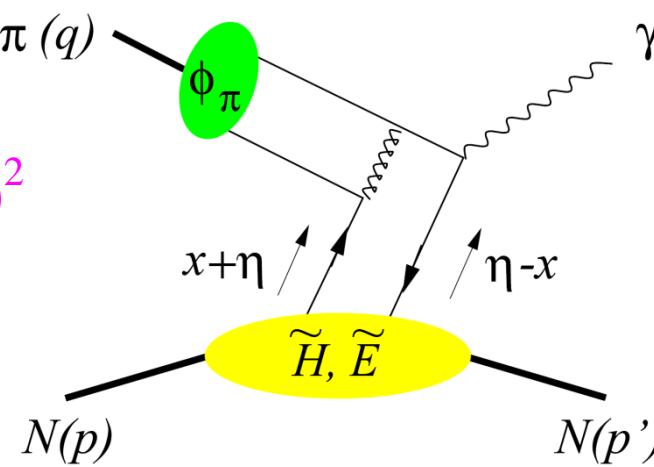
## Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

small  $t = \Delta^2 = (q - q')^2$



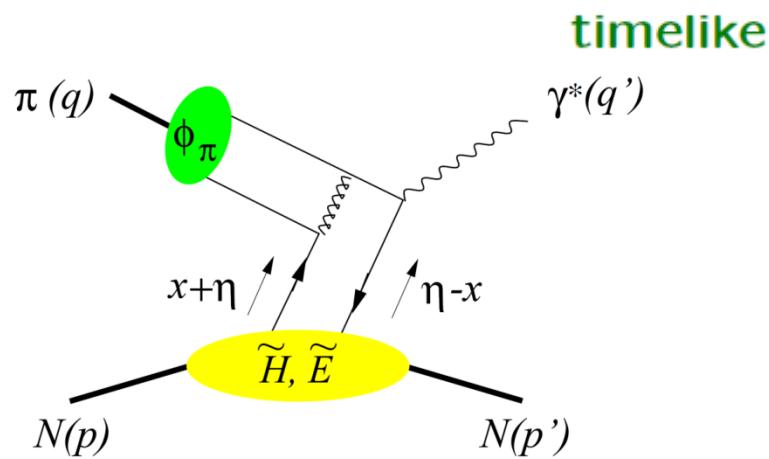
"exclusive DY"



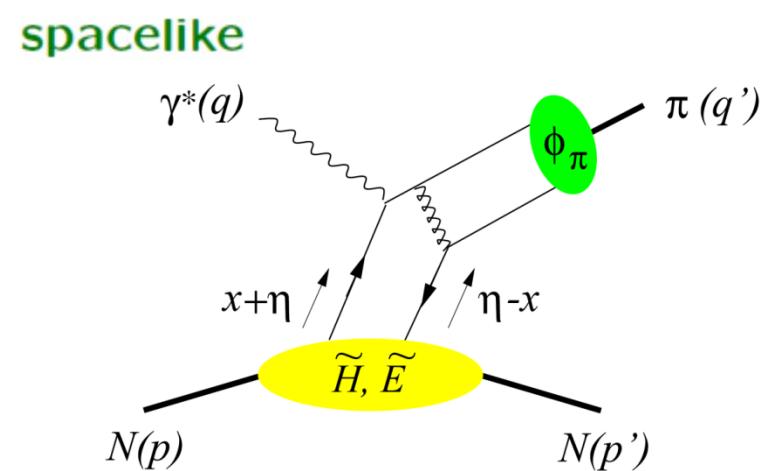
$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta}) \cdot \mathbf{p} \mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

GPD

# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions

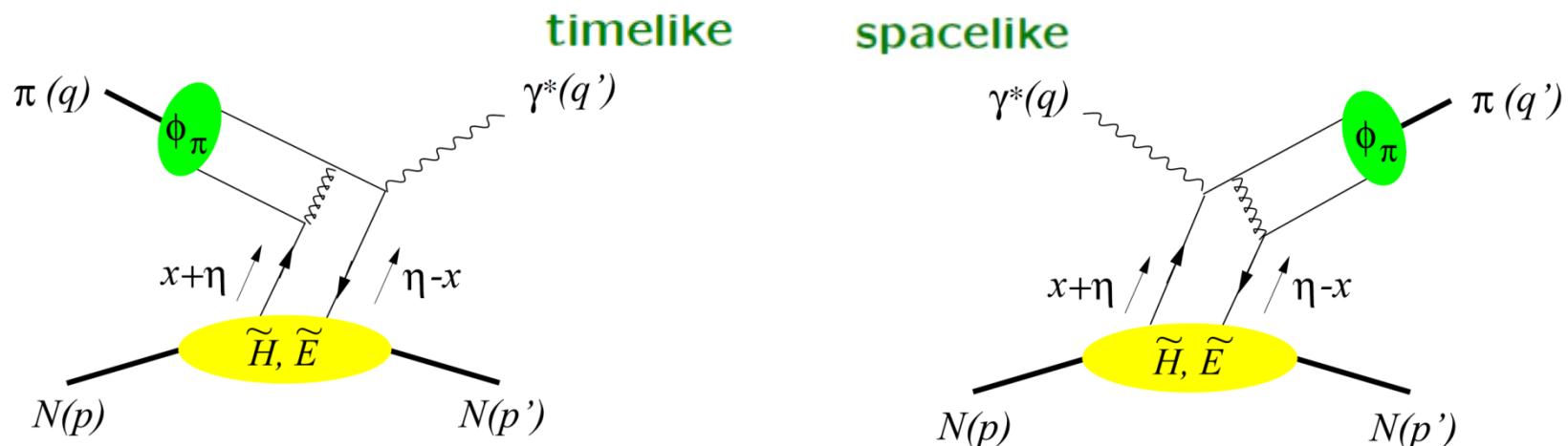


**exDY@J-PARC**



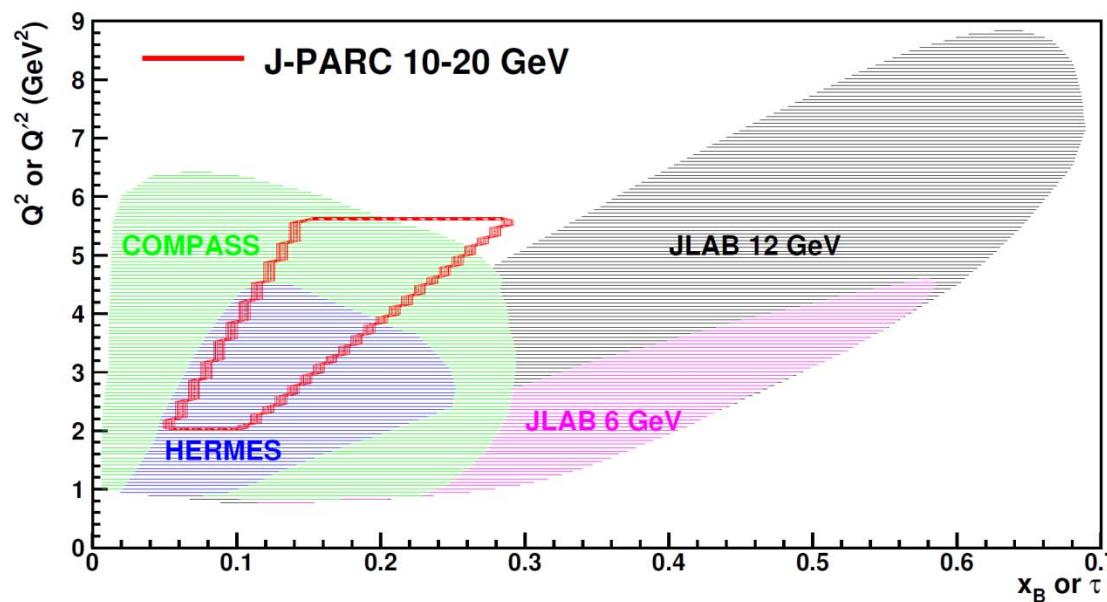
**DVMP@JLab**

# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions

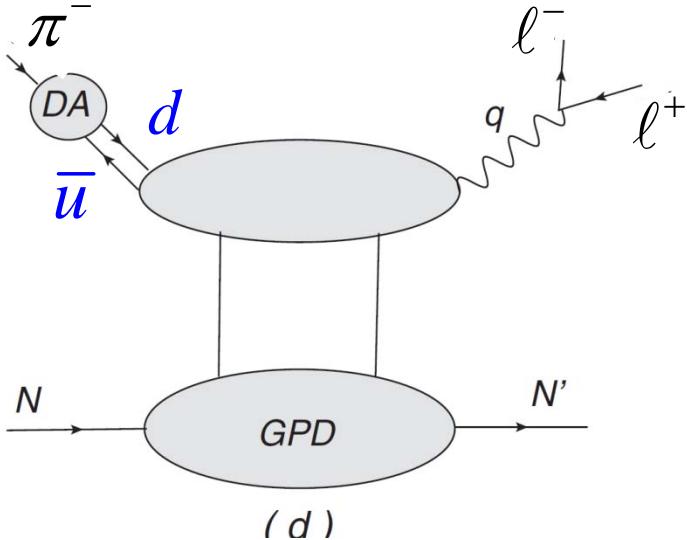


**exDY@J-PARC**

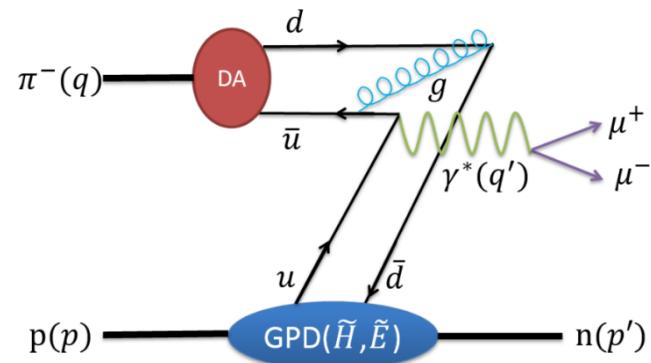
**DVMP@JLab**



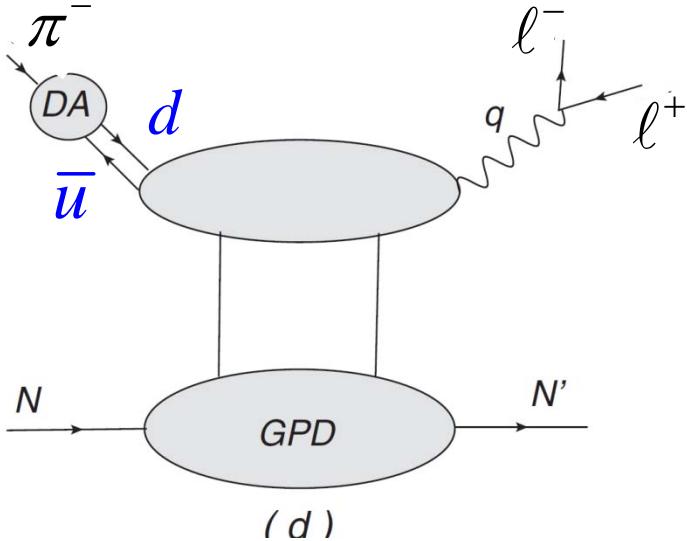
T. Sawada et al.,  
PRD93, 114034



**exDY@J-PARC**



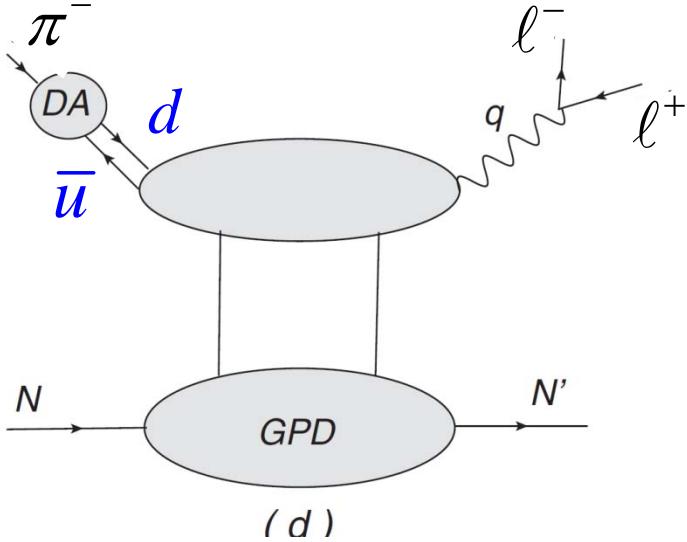
$$\int \frac{dz^-}{4\pi} e^{ix\bar{p}_z} \langle n(p') \left| \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) \right| p(p) \rangle = \frac{1}{2P^+} \bar{u}(p') \left[ \tilde{H}^{du}(x, \eta, t) \gamma^+ \gamma_5 + \tilde{E}^{du}(x, \eta, t) \frac{\gamma_5 \Delta^+}{2m_N} \right] u(p)$$



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$$\int \frac{dz^-}{4\pi} e^{ix\bar{p}_z} \langle \textcolor{violet}{n}(p') \left| \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{z^-}{2}) \right| \textcolor{violet}{p}(p) \rangle = \frac{1}{2P^+} \bar{u}(p') \left[ \tilde{H}^{\textcolor{teal}{du}}(x, \eta, \textcolor{blue}{t}) \gamma^+ \gamma_5 + \tilde{E}^{\textcolor{teal}{du}}(x, \eta, t) \frac{\gamma_5 \Delta^+}{2m_N} \right] u(p)$$

$$\langle \textcolor{violet}{p}(p') \left| \bar{u}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{z^-}{2}) - \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \textcolor{red}{d}(\frac{z^-}{2}) \right| \textcolor{violet}{p}(p) \rangle$$

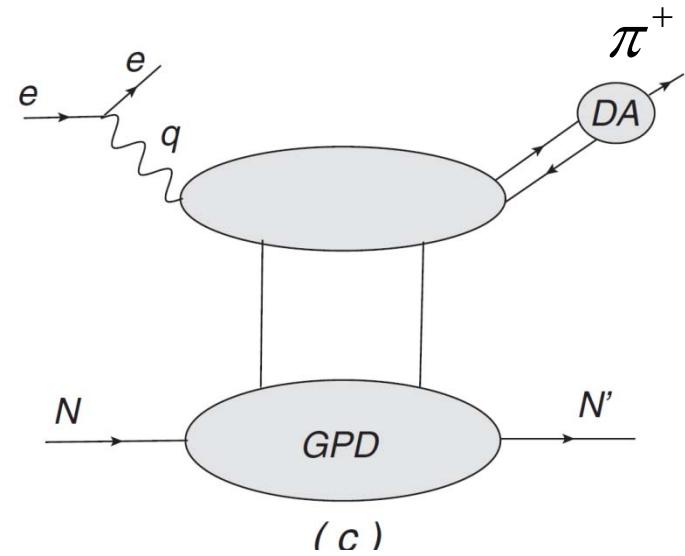
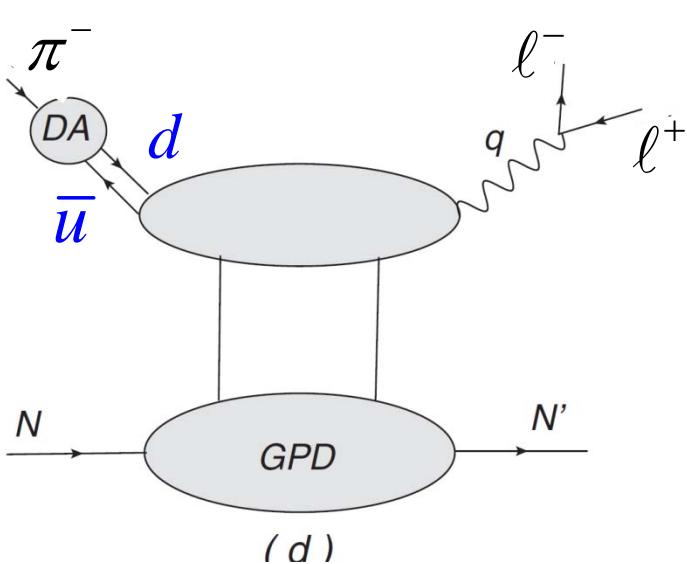


**exDY@J-PARC**

$$\int \frac{dz^-}{4\pi} e^{ix\bar{p}_z} \langle \textcolor{violet}{n}(p') \left| \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{z^-}{2}) \right| \textcolor{violet}{p}(p) \rangle = \frac{1}{2P^+} \bar{u}(p') \left[ \tilde{H}^{\textcolor{teal}{du}}(x, \eta, t) \gamma^+ \gamma_5 + \tilde{E}^{\textcolor{teal}{du}}(x, \eta, t) \frac{\gamma_5 \Delta^+}{2m_N} \right] u(p)$$

$$\langle \textcolor{violet}{p}(p') \left| \bar{u}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{z^-}{2}) - \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{d}(\frac{z^-}{2}) \right| \textcolor{violet}{p}(p) \rangle = \tilde{H}^{\textcolor{teal}{du}}(x, \eta, t) = \tilde{H}^{\textcolor{blue}{u}}(x, \eta, t) - \tilde{H}^{\textcolor{blue}{d}}(x, \eta, t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x, \eta, t) = \tilde{E}^{\textcolor{blue}{u}}(x, \eta, t) - \tilde{E}^{\textcolor{blue}{d}}(x, \eta, t)$$



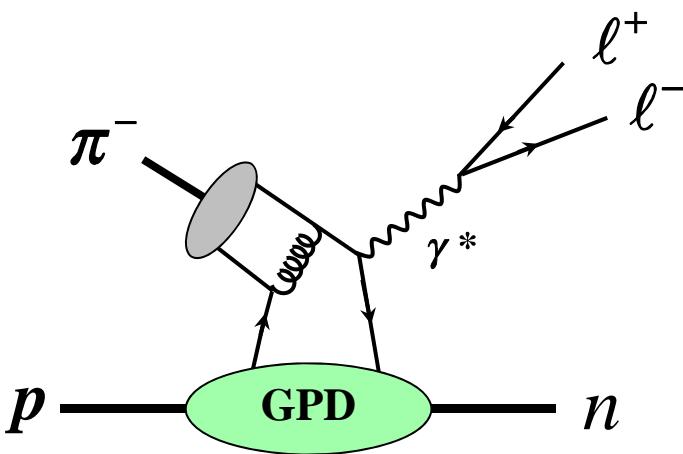
**exDY@J-PARC**

**DVMP@HERA, JLab**

$$\int \frac{dz^-}{4\pi} e^{ix\bar{p}_z} \langle n(p') \left| \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) \right| p(p) \rangle = \frac{1}{2P^+} \bar{u}(p') \left[ \tilde{H}^{du}(x, \eta, t) \gamma^+ \gamma_5 + \tilde{E}^{du}(x, \eta, t) \frac{\gamma_5 \Delta^+}{2m_N} \right] u(p)$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

$$\tilde{E}^{du}(x, \eta, t) = \tilde{E}^u(x, \eta, t) - \tilde{E}^d(x, \eta, t)$$



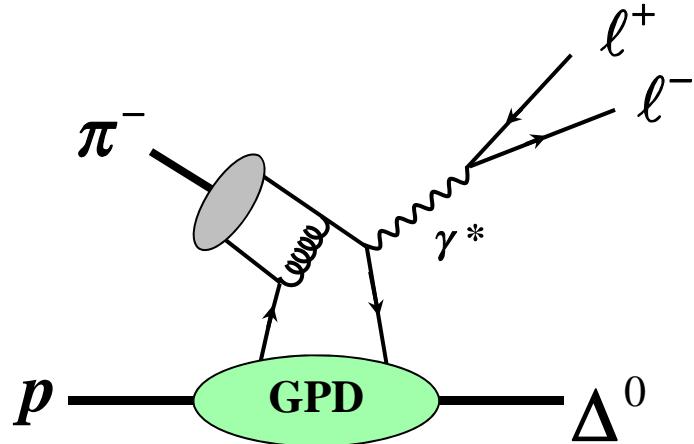
exDY@J-PARC

$$\int\frac{dz^-}{4\pi}e^{i\cancel{x}\bar{P}_z}<\textcolor{violet}{n}(p')\left|\overline{\textcolor{green}{d}}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{blue}{u}(\frac{z^-}{2})\right| \textcolor{violet}{p}(p)>=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{\textcolor{blue}{H}}^{\textcolor{teal}{du}}(x,\eta,\textcolor{blue}{t})\gamma^+\gamma_5+\tilde{\textcolor{blue}{E}}^{\textcolor{teal}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2m_N}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\pi^- + p \rightarrow \gamma^* + \Delta^0 \rightarrow \mu^+ + \mu^- + \Delta^0$$



**exDY@J-PARC**

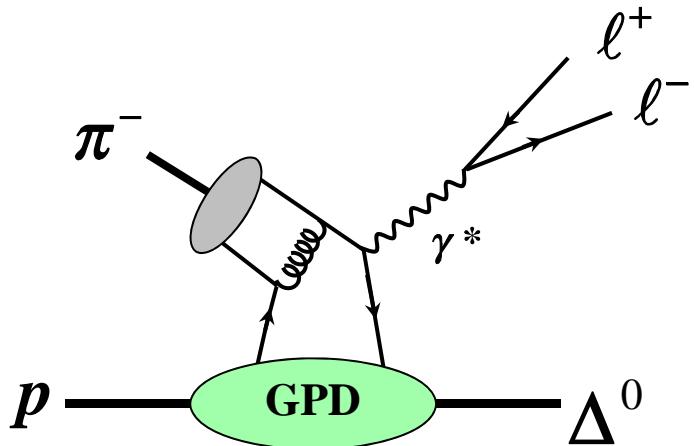
$$\int\frac{dz^-}{4\pi}e^{i\textcolor{blue}{x}\bar{P}_z}<\textcolor{violet}{n}(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{green}{u}(\frac{z^-}{2})\right| \textcolor{violet}{p}(p)>=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{\textcolor{teal}{H}}^{\textcolor{violet}{du}}(x,\eta,\textcolor{blue}{t})\gamma^+\gamma_5+\tilde{\textcolor{red}{E}}^{\textcolor{violet}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{violet}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{violet}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\pi^- + p \rightarrow \gamma^* + \Delta^0 \rightarrow \mu^+ + \mu^- + \Delta^0$$

$$\int\frac{d\textcolor{red}{z}^-}{4\pi}e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}}<\textcolor{violet}{\Delta}^0(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{green}{u}(\frac{z^-}{2})\right|p(p)\>>$$



**exDY@J-PARC**

$$\int\frac{d\textcolor{red}{z}^-}{4\pi}e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}}<\textcolor{violet}{n}(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{green}{u}(\frac{z^-}{2})\right|p(p)\>>=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{\boldsymbol{H}}^{\textcolor{teal}{du}}(x,\eta,\textcolor{blue}{t})\gamma^+\gamma_5+\tilde{\boldsymbol{E}}^{\textcolor{teal}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

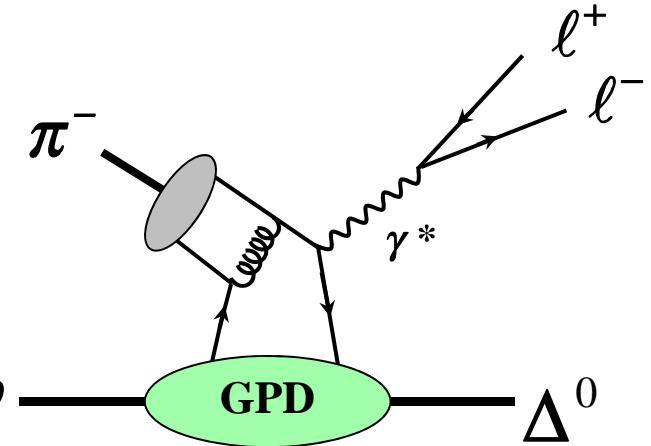
$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\pi^- + p \rightarrow \gamma^* + \Delta^0 \rightarrow \mu^+ + \mu^- + \Delta^0$$

$$\int \frac{dz^-}{4\pi} e^{ix\bar{P}_z} <\Delta^0(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5 u(\frac{z^-}{2})\right|p(p)>$$

$$=\frac{1}{2}\int \frac{dz^-}{4\pi} e^{ix\bar{P}_z}$$

$$\times<\Delta^+(p')\left|\overline{u}(-\frac{z^-}{2})\gamma^+\gamma_5 u(\frac{z^-}{2})-\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5 d(\frac{z^-}{2})\right|p(p)>$$



$$\int \frac{dz^-}{4\pi} e^{ix\bar{P}_z} <\textcolor{violet}{n}(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5 \textcolor{green}{u}(\frac{z^-}{2})\right|\textcolor{violet}{p}(p)> = \frac{1}{2P^+} \overline{u}(p') \left[ \tilde{H}^{\textcolor{teal}{du}}(x,\eta,\textcolor{blue}{t}) \gamma^+ \gamma_5 + \tilde{E}^{\textcolor{teal}{du}}(x,\eta,t) \frac{\gamma_5 \Delta^+}{2M} \right] u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\pi^- + p \rightarrow \gamma^* + \Delta^0 \rightarrow \mu^+ + \mu^- + \Delta^0$$

$$\int \frac{dz^-}{4\pi} e^{ix\bar{P}_z} < \Delta^0(p') \left| \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) \right| p(p) >$$

$$= \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ix\bar{P}_z}$$

$$\times < \Delta^+(p') \left| \bar{u}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) - \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 d(\frac{z^-}{2}) \right| p(p) >$$

$$= \frac{1}{\sqrt{2}} \int \frac{dz^-}{4\pi} e^{ix\bar{P}_z} < p(p') \left| \bar{u}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) - \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 d(\frac{z^-}{2}) \right| p(p) > + O\left(\frac{1}{N_c}\right)$$

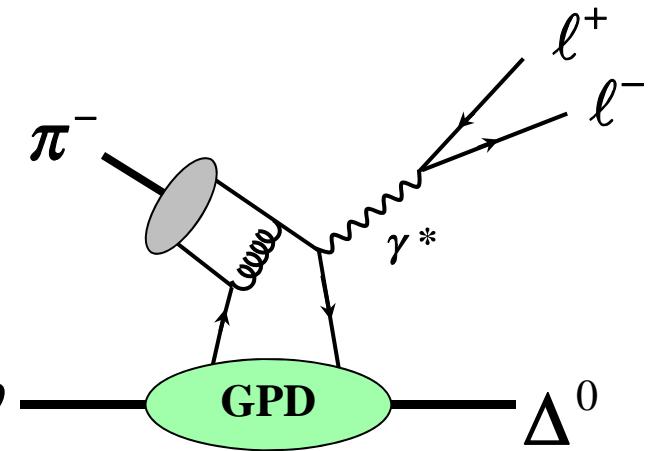
$$N_c \rightarrow \infty \quad \Delta \quad \longleftrightarrow \quad p$$

**Goeke, Polyakov, Vanderhaeghen, Prog. Part. Nucl. Phys. 47 ('01) 401**

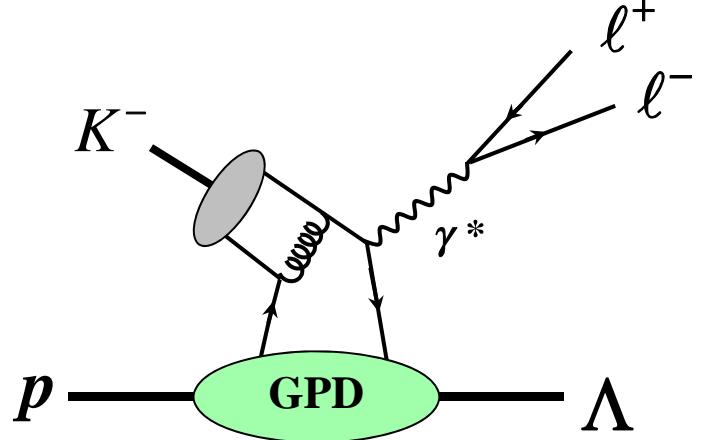
$$\int \frac{dz^-}{4\pi} e^{ix\bar{P}_z} < n(p') \left| \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) \right| p(p) > = \frac{1}{2P^+} \bar{u}(p') \left[ \tilde{H}^{du}(x, \eta, t) \gamma^+ \gamma_5 + \tilde{E}^{du}(x, \eta, t) \frac{\gamma_5 \Delta^+}{2M} \right] u(p)$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

$$\tilde{E}^{du}(x, \eta, t) = \tilde{E}^u(x, \eta, t) - \tilde{E}^d(x, \eta, t)$$



$$K^- + p \rightarrow \gamma^* + \Lambda \rightarrow \mu^+ + \mu^- + \Lambda$$



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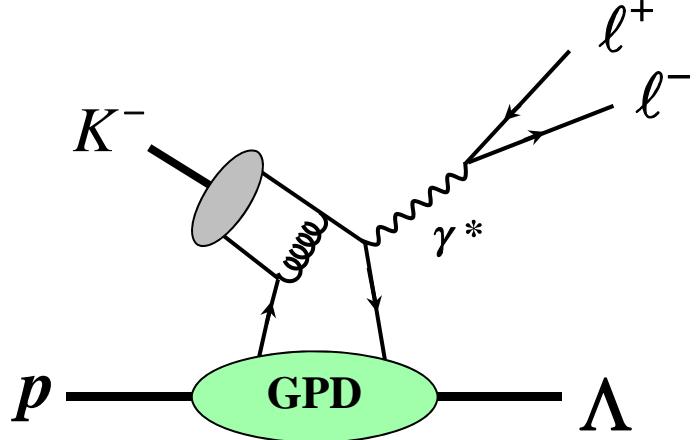
$$\int\!\frac{dz^-}{4\pi}e^{i\textcolor{blue}{x}\bar{P}_z}<\textcolor{violet}{n}(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{green}{u}(\frac{z^-}{2})\right| \textcolor{violet}{p}(p)>=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{\textcolor{blue}{H}}^{\textcolor{teal}{du}}(x,\eta,\textcolor{blue}{t})\gamma^+\gamma_5+\tilde{\textcolor{blue}{E}}^{\textcolor{teal}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$K^-+p\rightarrow \gamma^*+\Lambda\rightarrow \mu^++\mu^-+\Lambda$$

$$\int \frac{d\textcolor{red}{z}^-}{4\pi} e^{i \textcolor{blue}{x}\bar P_{\textcolor{red}{z}}} <\!\! \Lambda(p') \left| \overline{s}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) \right| \!\!> \!\! \textcolor{purple}{p}(p)$$



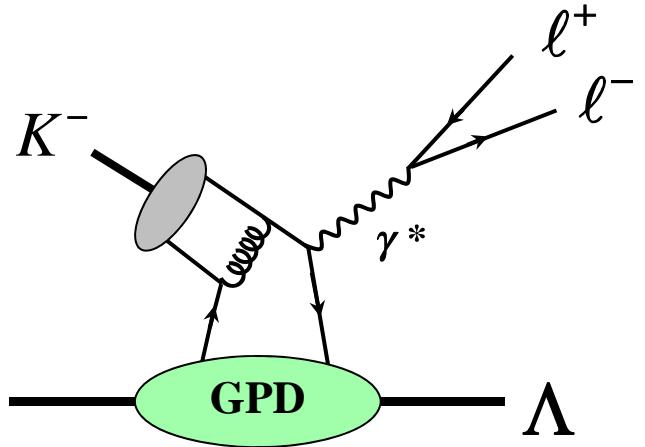
**exDY@J-PARC**

$$\int \frac{d\textcolor{red}{z}^-}{4\pi} e^{i \textcolor{blue}{x}\bar P_{\textcolor{red}{z}}} <\!\! \textcolor{purple}{n}(p') \left| \overline{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{z^-}{2}) \right| \!\!> \!\! \textcolor{purple}{p}(p) = \frac{1}{2P^+} \overline{u}(p') \Bigg[ \tilde{\boldsymbol{H}}^{\textcolor{teal}{du}}(x,\eta,\textcolor{blue}{t}) \gamma^+ \gamma_5 + \tilde{\boldsymbol{E}}^{\textcolor{teal}{du}}(x,\eta,t) \frac{\gamma_5 \Delta^+}{2M} \Bigg] u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$K^-+p\rightarrow \gamma^*+\Lambda\rightarrow \mu^++\mu^-+\Lambda$$



$$\int \frac{d\textcolor{red}{z}^-}{4\pi} e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}} <\!\! \Lambda(p') \left| \overline{s}(-\frac{\textcolor{red}{z}^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{\textcolor{red}{z}^-}{2}) \right| \textcolor{violet}{p}(p) \!>$$

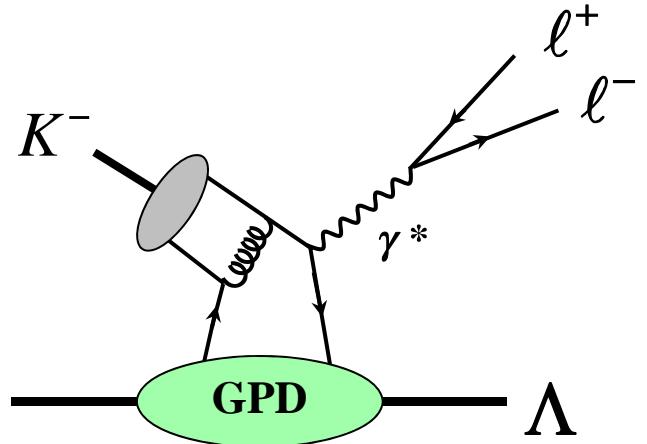
$$=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{H}^{\textcolor{green}{su}}_{\Lambda p}(x,\eta,t)\gamma^+\gamma_5+\tilde{E}^{\textcolor{green}{su}}_{\Lambda p}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

$$\int \frac{d\textcolor{red}{z}^-}{4\pi} e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}} <\!\! \textcolor{violet}{n}(p') \left| \overline{d}(-\frac{\textcolor{red}{z}^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{\textcolor{red}{z}^-}{2}) \right| \textcolor{violet}{p}(p) \!> =\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{H}^{\textcolor{green}{du}}(x,\eta,t)\gamma^+\gamma_5+\tilde{E}^{\textcolor{green}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{green}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{green}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$K^-+p\rightarrow \gamma^*+\Lambda\rightarrow \mu^++\mu^-+\Lambda$$



$$\int \frac{d\textcolor{red}{z}^-}{4\pi} e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}} <\!\! \Lambda(p') \left| \overline{s}(-\frac{\textcolor{red}{z}^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{\textcolor{red}{z}^-}{2}) \right| \textcolor{violet}{p}(p) \!>$$

$$=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{H}^{\textcolor{green}{su}}_{\Lambda p}(x,\eta,t)\gamma^+\gamma_5+\tilde{E}^{\textcolor{green}{su}}_{\Lambda p}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

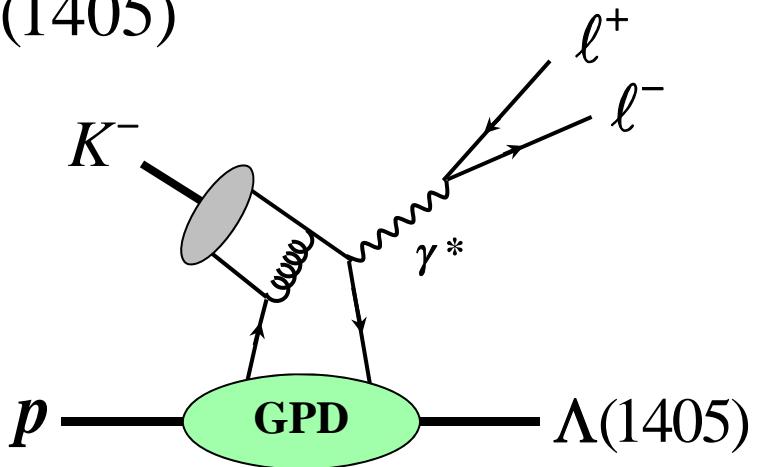
$$\tilde{H}^{\textcolor{green}{su}}_{\Lambda p}=\frac{-1}{\sqrt{6}}\Big(2\tilde{H}^{\textcolor{blue}{u}}-\tilde{H}^{\textcolor{blue}{d}}-\tilde{H}^{\textcolor{blue}{s}}\Big)$$

$$\int \frac{d\textcolor{red}{z}^-}{4\pi} e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}} <\!\! \textcolor{violet}{n}(p') \left| \overline{d}(-\frac{\textcolor{red}{z}^-}{2}) \gamma^+ \gamma_5 \textcolor{green}{u}(\frac{\textcolor{red}{z}^-}{2}) \right| \textcolor{violet}{p}(p) \!> =\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)\gamma^+\gamma_5+\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$K^- + p \rightarrow \gamma^* + \Lambda(1405) \rightarrow \mu^+ + \mu^- + \Lambda(1405)$$

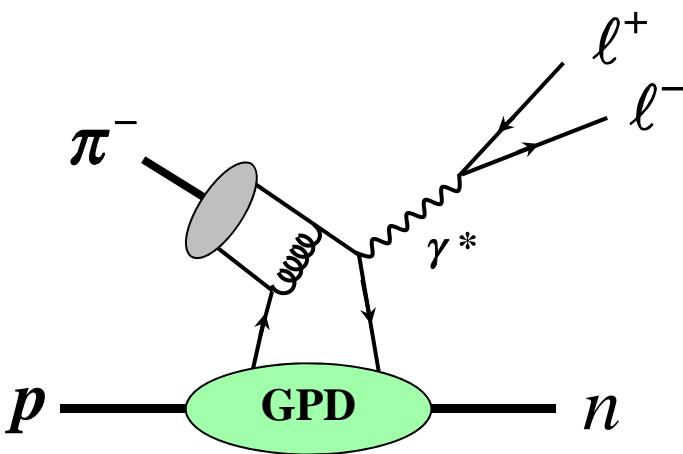


$$\int\frac{d\textcolor{red}{z}^-}{4\pi}e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}}<\Lambda_{1405}(p')\left|\overline{s}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{green}{u}(\frac{z^-}{2})\right| \textcolor{violet}{p}(p)\ >$$

$$\int\frac{d\textcolor{red}{z}^-}{4\pi}e^{i\textcolor{blue}{x}\bar{P}_{\textcolor{red}{z}}}<\textcolor{violet}{n}(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{green}{u}(\frac{z^-}{2})\right| \textcolor{violet}{p}(p)\ >=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{\boldsymbol{H}}^{\textcolor{teal}{du}}(x,\eta,\textcolor{blue}{t})\gamma^+\gamma_5+\tilde{\boldsymbol{E}}^{\textcolor{teal}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2M}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$

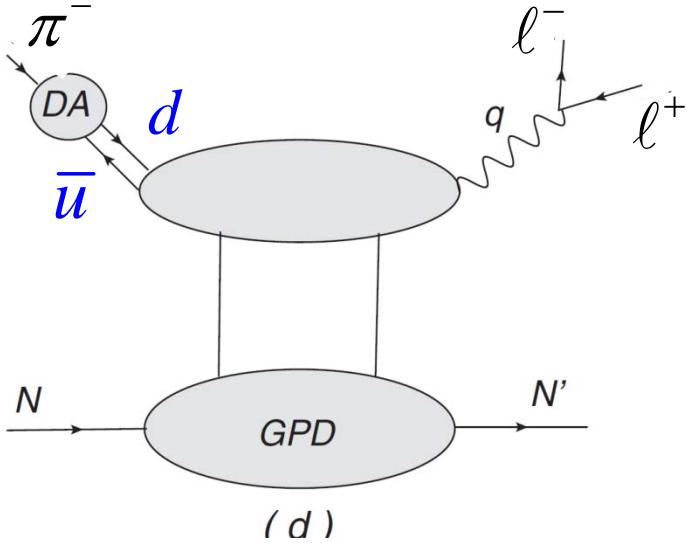


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$$\int\frac{dz^-}{4\pi}e^{i\textcolor{blue}{x}\bar{P}_z}<\textcolor{violet}{n}(p')\left|\overline{d}(-\frac{z^-}{2})\gamma^+\gamma_5\textcolor{green}{u}(\frac{z^-}{2})\right| \textcolor{violet}{p}(p)>=\frac{1}{2P^+}\overline{u}(p')\Bigg[\tilde{\textcolor{teal}{H}}^{\textcolor{teal}{du}}(x,\eta,\textcolor{blue}{t})\gamma^+\gamma_5+\tilde{\textcolor{blue}{E}}^{\textcolor{teal}{du}}(x,\eta,t)\frac{\gamma_5\Delta^+}{2m_N}\Bigg]u(p)$$

$$\tilde{H}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{H}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{H}^{\textcolor{blue}{d}}(x,\eta,t)$$

$$\tilde{E}^{\textcolor{teal}{du}}(x,\eta,t)=\tilde{E}^{\textcolor{blue}{u}}(x,\eta,t)-\tilde{E}^{\textcolor{blue}{d}}(x,\eta,t)$$



# exDY@J-PARC

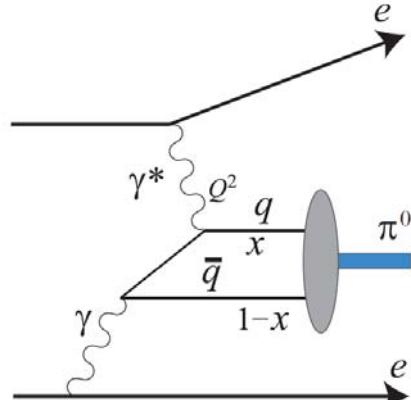
$$\int \frac{dz^-}{4\pi} e^{ix\bar{p}_z} \langle n(p') \left| \bar{d}(-\frac{z^-}{2}) \gamma^+ \gamma_5 u(\frac{z^-}{2}) \right| p(p) \rangle = \frac{1}{2P^+} \bar{u}(p') \left[ \tilde{H}^{du}(x, \eta, t) \gamma^+ \gamma_5 + \tilde{E}^{du}(x, \eta, t) \frac{\gamma_5 \Delta^+}{2m_N} \right] u(p)$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

$$\tilde{E}^{du}(x, \eta, t) = \tilde{E}^u(x, \eta, t) - \tilde{E}^d(x, \eta, t)$$

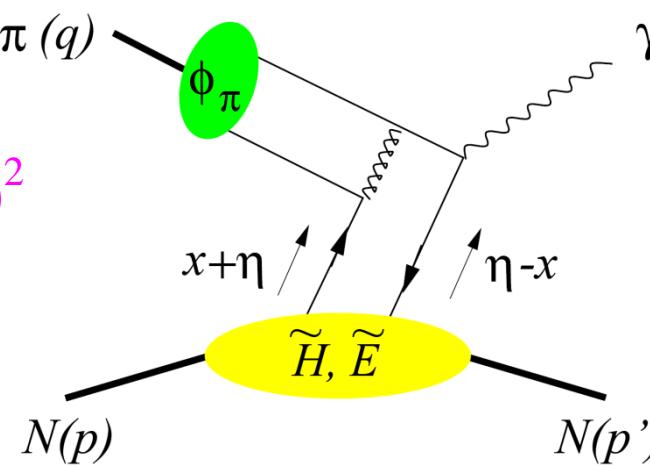
## Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

small  $t = \Delta^2 = (q - q')^2$



"exclusive DY"



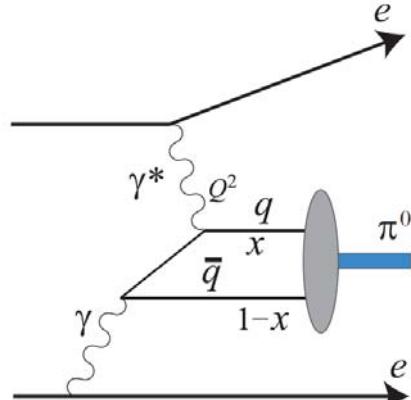
$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta}) \cdot \mathbf{p} \mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

GPD

# Exclusive lepton pair production in $\pi N$ scattering

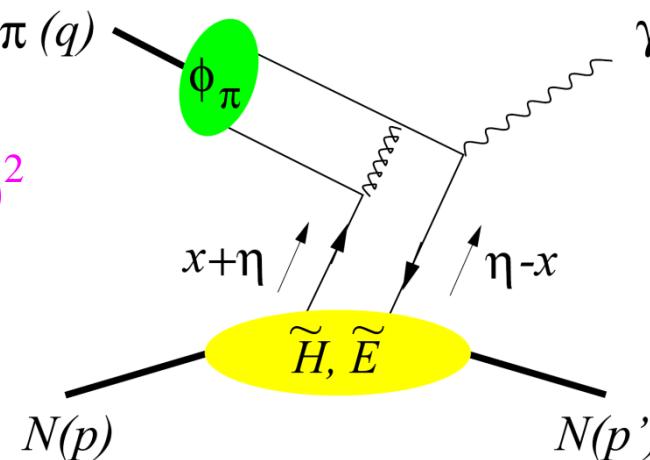
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

Berger, Diehl, Pire, PLB523(2001)265

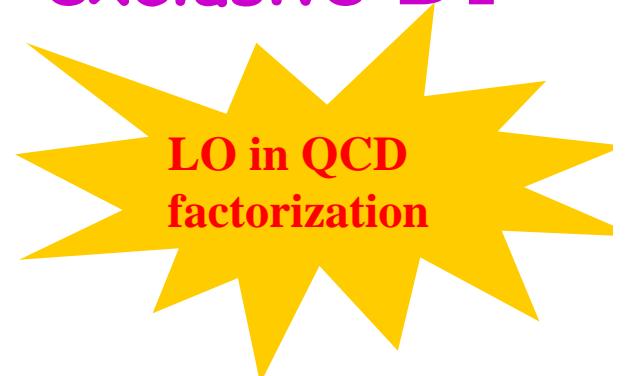


@Belle, Babar

small  $t = \Delta^2 = (q - q')^2$



"exclusive DY"



$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta}) \cdot \mathbf{p} \mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

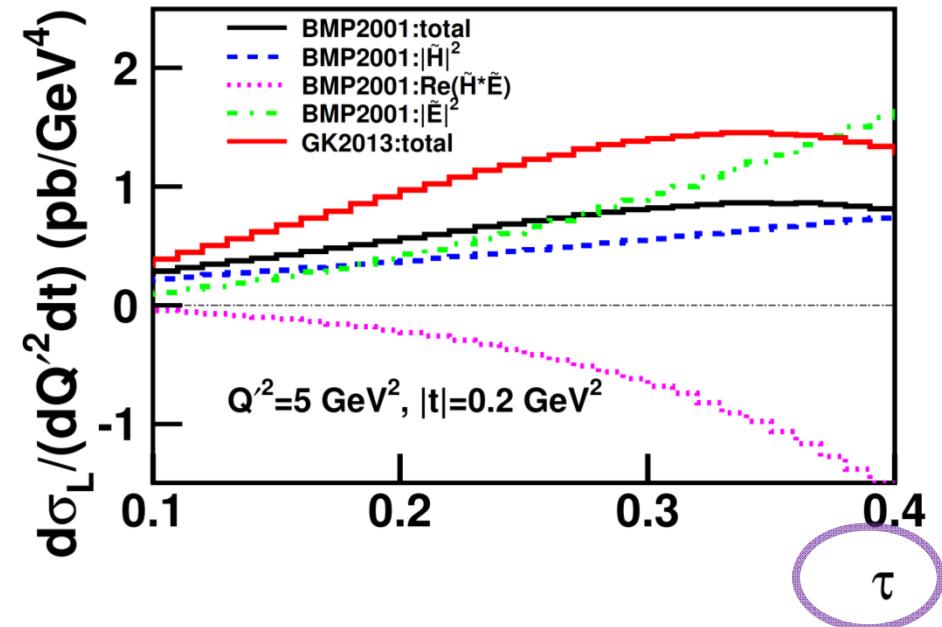
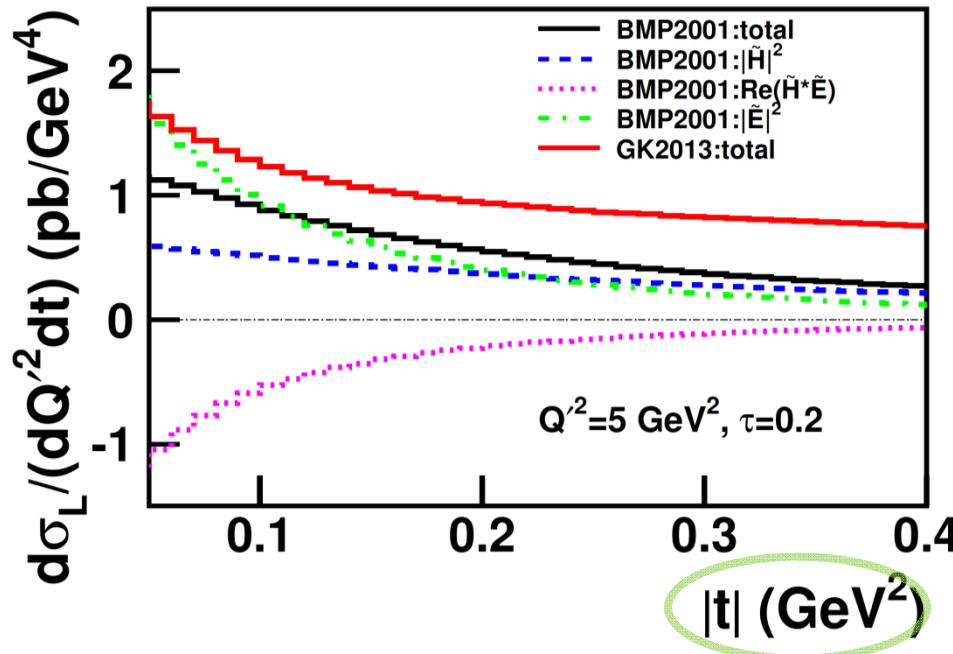
GPD

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,  
PRD93, 114034

Bjorken variable

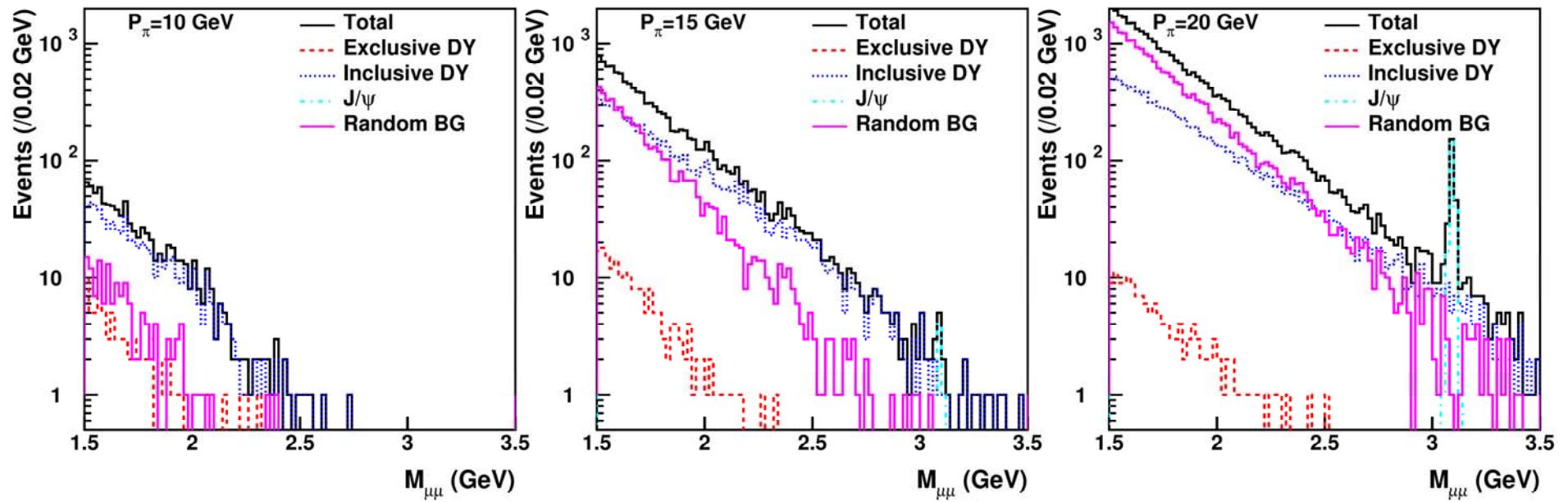
$$\tau = \frac{Q'^2}{s-M^2}$$

$$Q'^2 = 5 \text{ GeV}^2$$



$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \rightarrow \gamma^* n) = \frac{4\pi\alpha_{\text{em}}^2}{27} \frac{\tau^2}{Q'^8} f_\pi^2 \left[ (1-\eta^2) |\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\tilde{\mathcal{H}}^{du*} \tilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\tilde{\mathcal{E}}^{du}|^2 \right]$$

$$\tilde{\mathcal{H}}^{du} = \frac{8\alpha_s}{3} \int_0^1 du \frac{\phi_\pi(u)}{4u(1-u)} \int_{-1}^1 dx \left( \frac{e_d}{-\eta - x - i\epsilon} - \frac{e_u}{-\eta + x - i\epsilon} \right) \left( \tilde{H}^d(x, \eta, t) - \tilde{H}^u(x, \eta, t) \right)$$

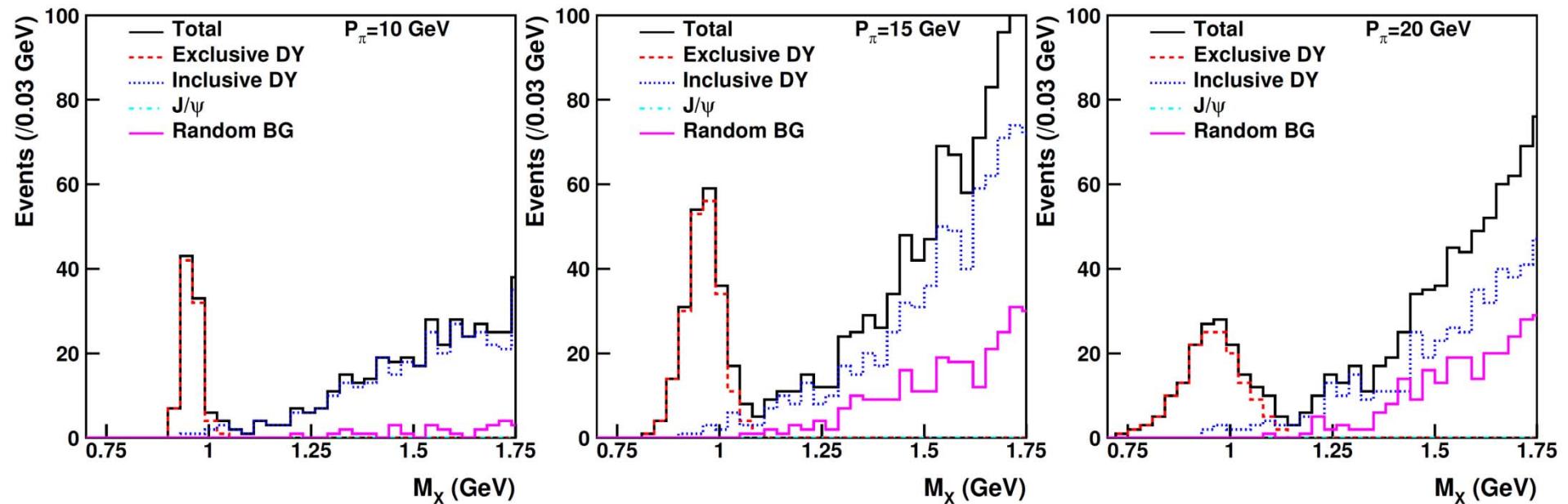


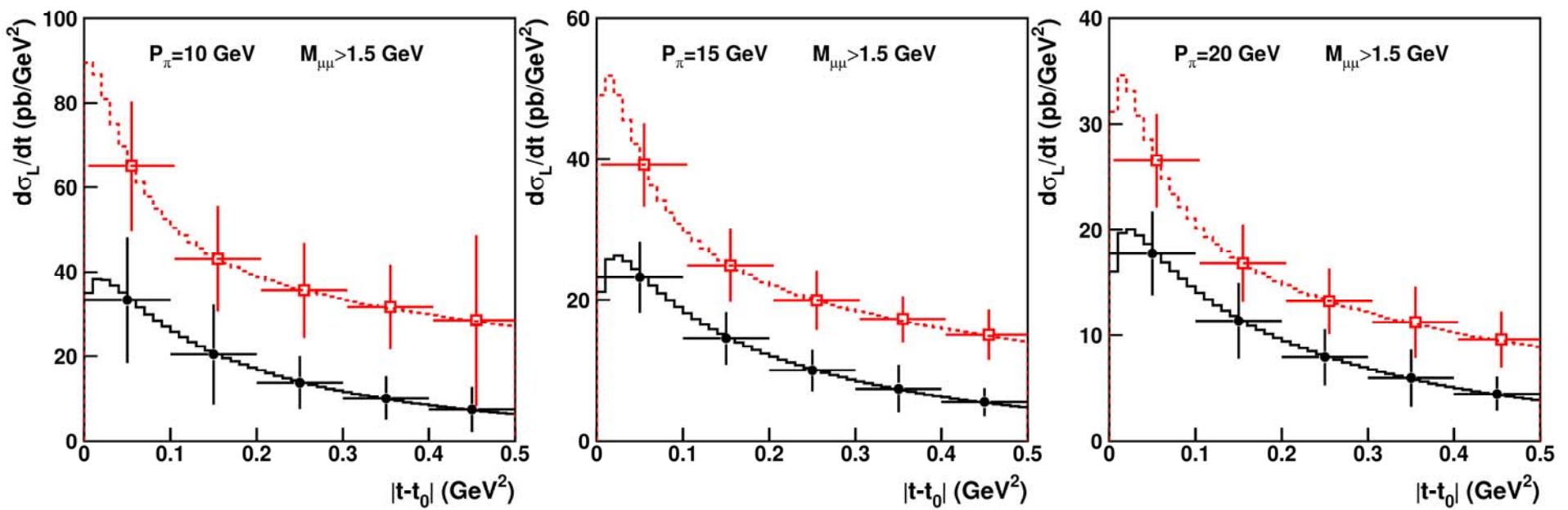
# feasibility with E50 spectrometer at J-PARC

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,  
PRD93, 114034

# feasibility with E50 spectrometer at J-PARC

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,  
PRD93, 114034



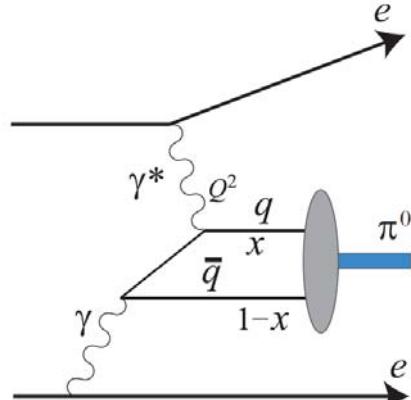


# feasibility with E50 spectrometer at J-PARC

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,  
PRD93, 114034

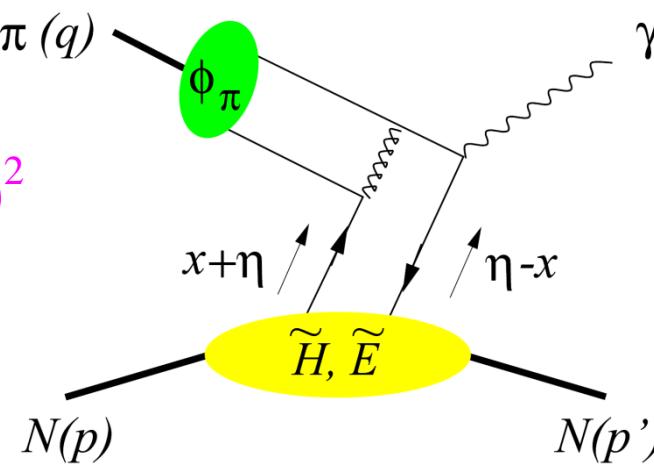
## Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

small  $t = \Delta^2 = (q - q')^2$

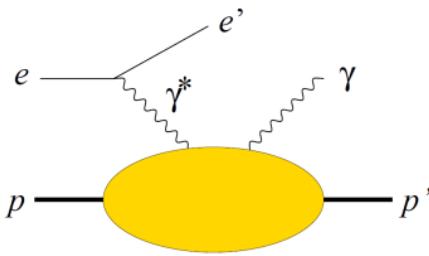


"exclusive DY"



$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta}) \cdot \mathbf{p} \mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

GPD



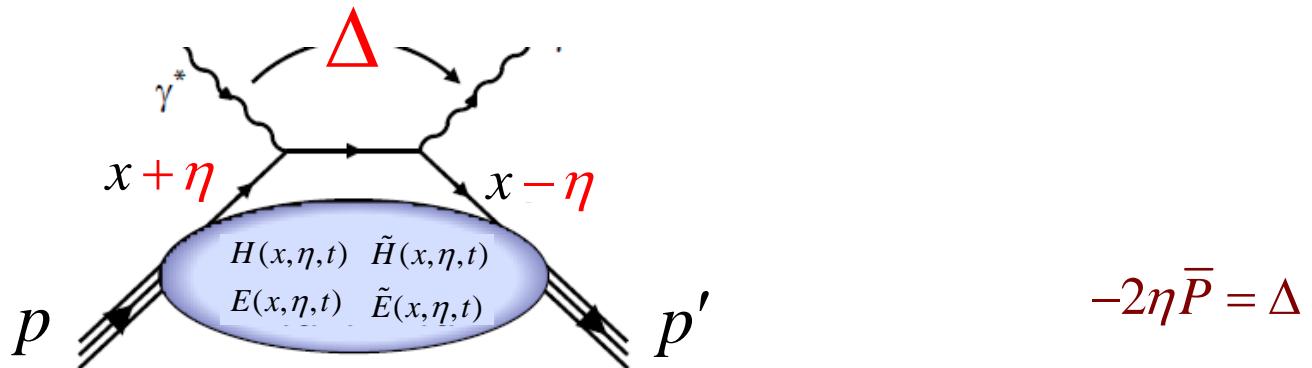
$$\bar{P} = \frac{p + p'}{2}$$

JLab, HERMES, COMPASS, ...

$$\int \frac{d\textcolor{red}{z}^-}{2\pi} e^{i(\textcolor{blue}{x}+\eta)\bar{P}_z} \langle p' | \bar{\psi}(0) \gamma^+ \psi(\textcolor{red}{z}^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \textcolor{magenta}{H}(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + \textcolor{magenta}{E}(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

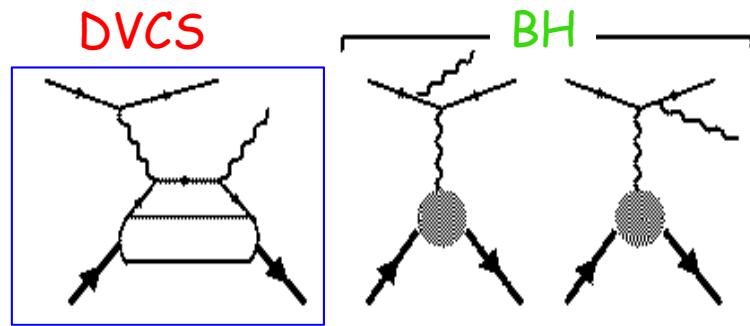
$$\int \frac{d\textcolor{red}{z}^-}{2\pi} e^{i(\textcolor{blue}{x}+\eta)\bar{P}_z} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(\textcolor{red}{z}^-) | p \rangle = \frac{1}{\bar{P}^+} \left[ \tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

**GPD**



$$\int d\textcolor{red}{z}^- e^{i(\textcolor{blue}{x}+\eta)p_z} \langle N(\textcolor{red}{p}') | \psi^\dagger(0) \psi(\textcolor{red}{z}^-) | N(\textcolor{red}{p}) \rangle$$

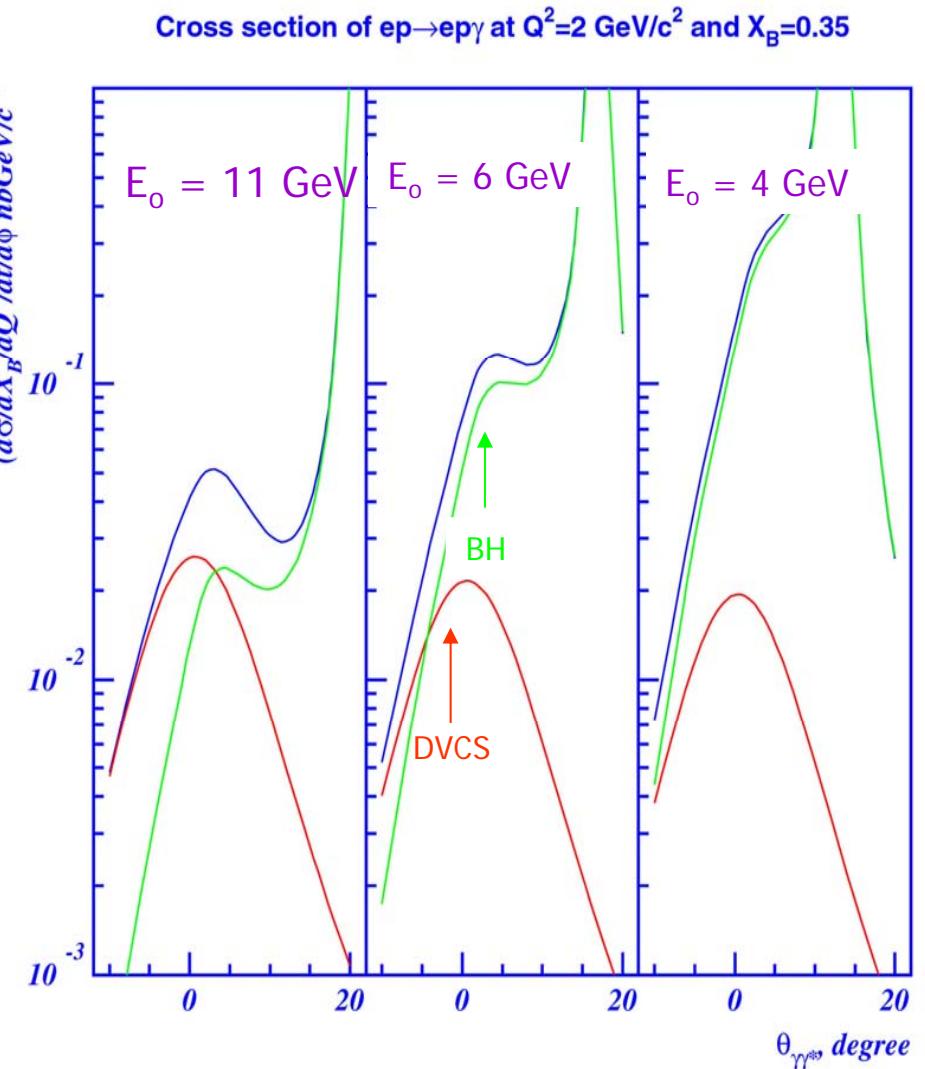
# Accessing GPDs through polarized DVCS



$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} \sim |T_{\text{DVCS}} + T_{\text{BH}}|^2$$

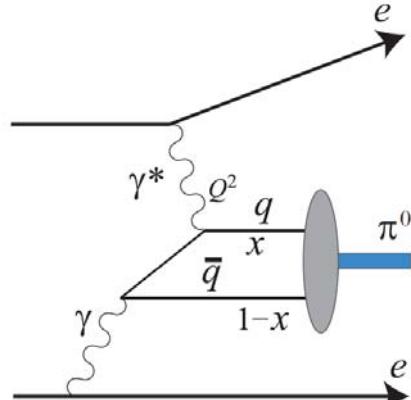
$T_{\text{BH}}$ : real, given by elastic form factors

$T_{\text{DVCS}}$ : complex, determined by GPDs



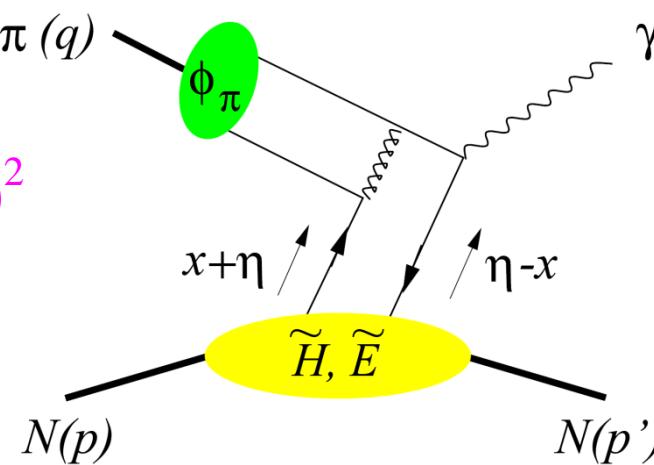
## Exclusive lepton pair production in $\pi N$ scattering

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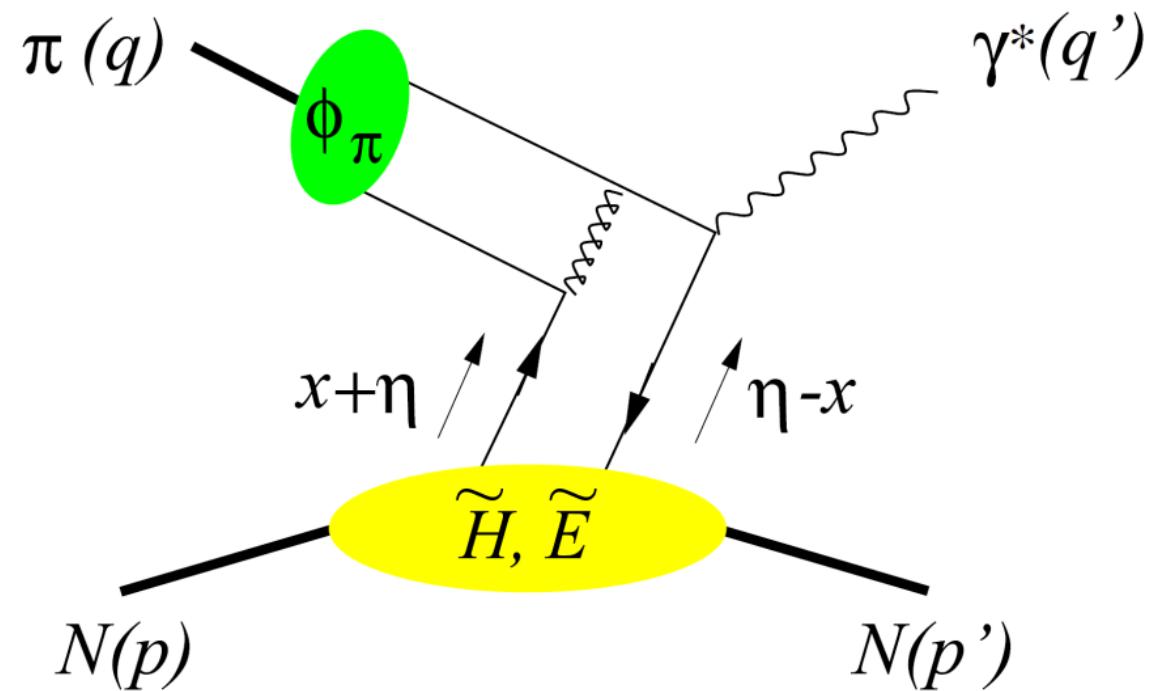


"exclusive DY"

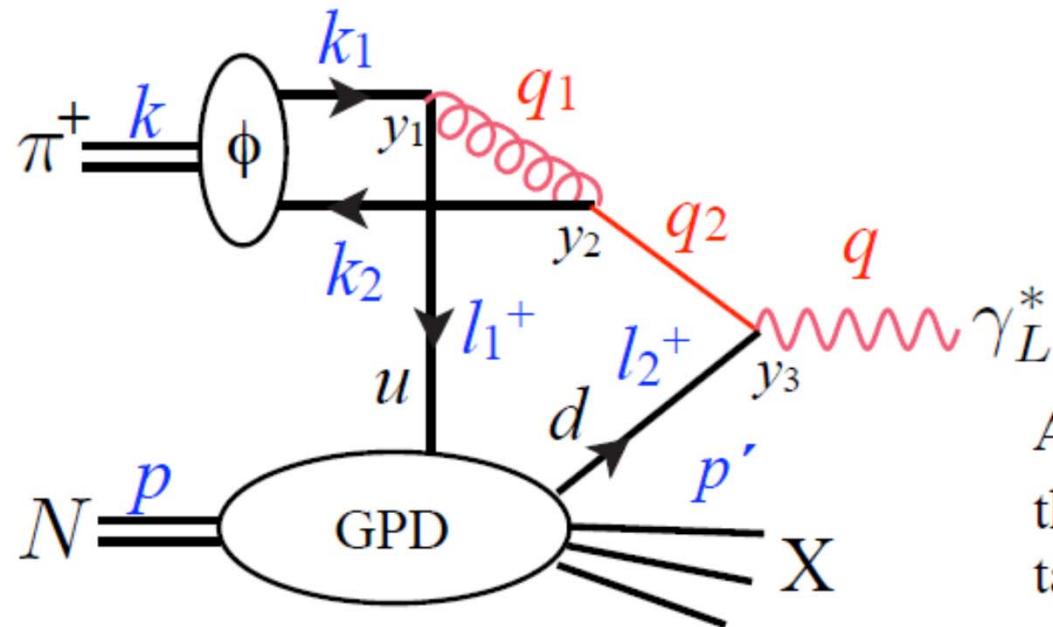


$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta}) \cdot \mathbf{p} \mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

GPD



Hence the stopped quark should be connected to the target:



For each final state  $X$  the target matrix element is given by a **GPD** with skewness

$$l_2^+ - l_1^+ = q^+ = x_B p^+$$

$$\begin{aligned} k_1 &= (0^+, u k^-, \mathbf{k}_\perp) \\ k_2 &= (0^+, (1-u) k^-, -\mathbf{k}_\perp) \end{aligned}$$

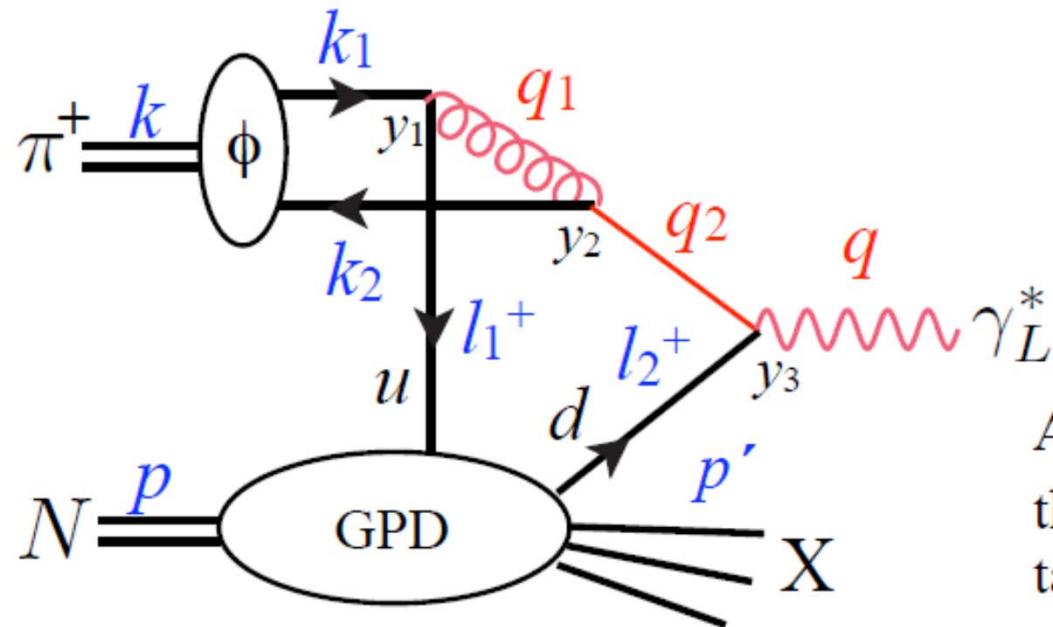
Since  $q_1^2 \approx -u k^- l_1^+ \rightarrow \infty$   
the pion wave function contributes through its *distribution amplitude*  $\phi$

Also  $q_2^2, q_1^-, q_2^- \rightarrow \infty$ , hence  
the space-time separation of the target interaction points  $y_1, y_3$  is

$$\begin{aligned} |y_{1\perp} - y_{3\perp}| &= \mathcal{O}(1/Q) \rightarrow 0 \\ |y_1^+ - y_3^+| &= \mathcal{O}(1/Q^2) \rightarrow 0 \\ |y_1^- - y_3^-| &= \mathcal{O}(1/\ell_1^+) \text{ finite} \end{aligned}$$

Using perturbative propagators for the gluon  $q_1$  and  $d$ -quark  $q_2$  and adding three more diagrams we get

Hence the stopped quark should be connected to the target:



For each final state X the target matrix element is given by a GPD with skewness

**end-point** ( $u \rightarrow 0, 1$ ) **behaviors:**

$$\phi_\pi(u) \sim u(1-u)$$

$$k_1 = (0^+, uk^-, \mathbf{k}_\perp)$$

$$k_2 = (0^+, (1-u)k^-, -\mathbf{k}_\perp)$$

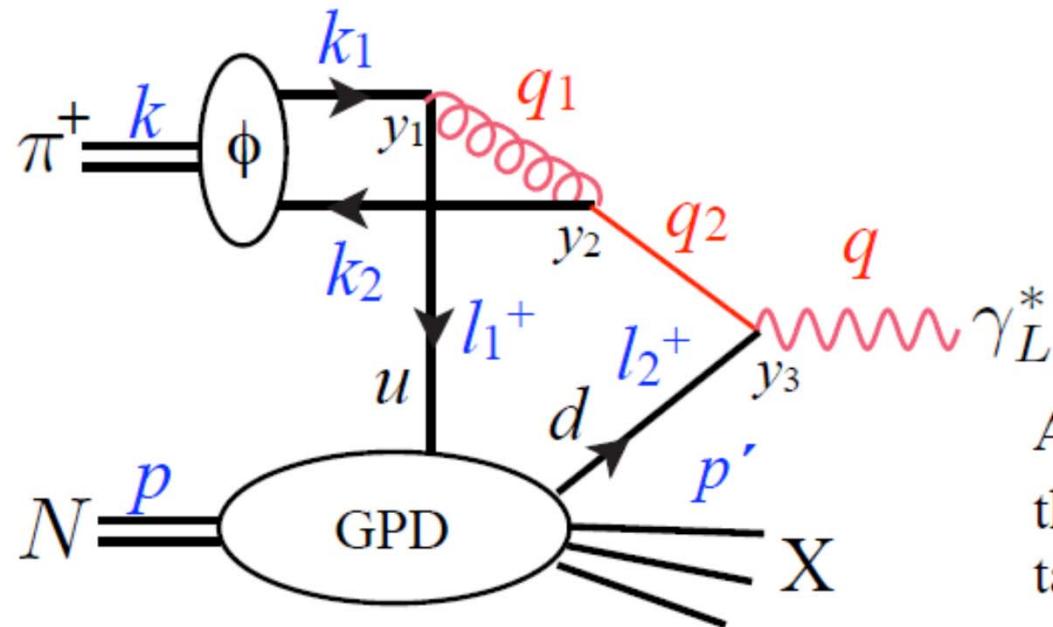
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Hence the stopped quark should be connected to the target:



For each final state  $X$  the target matrix element is given by a **GPD** with skewness

**end-point behaviors:**

$\phi_\pi(u) \sim u(1-u)$        $\phi_{\text{tw.-3}}(u) \sim 1$

$$k_1 = (0^+, uk^-, \mathbf{k}_\perp)$$

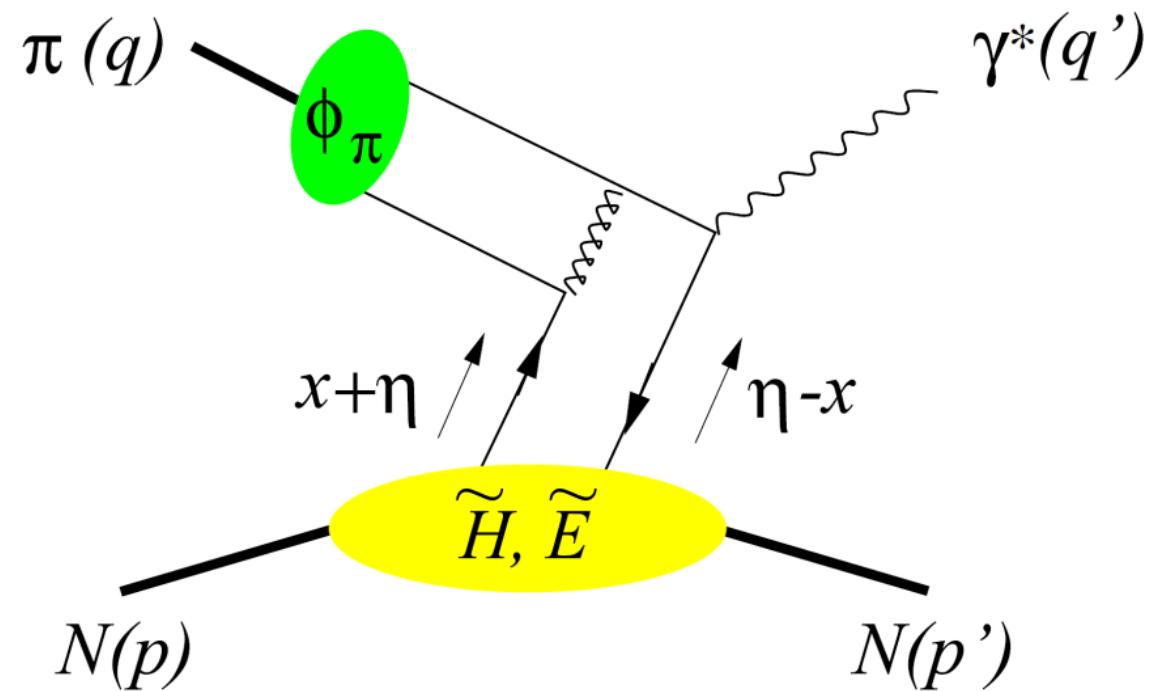
$$k_2 = (0^+, (1-u)k^-, -\mathbf{k}_\perp)$$

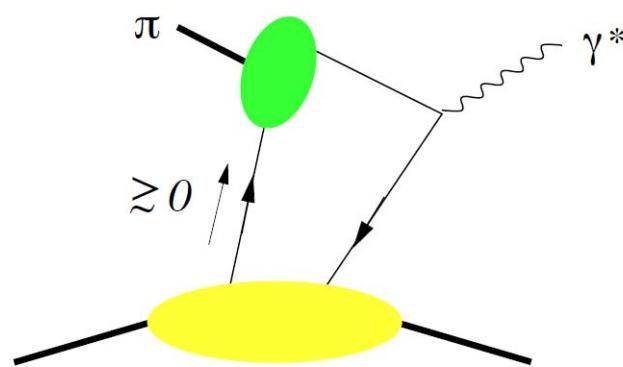
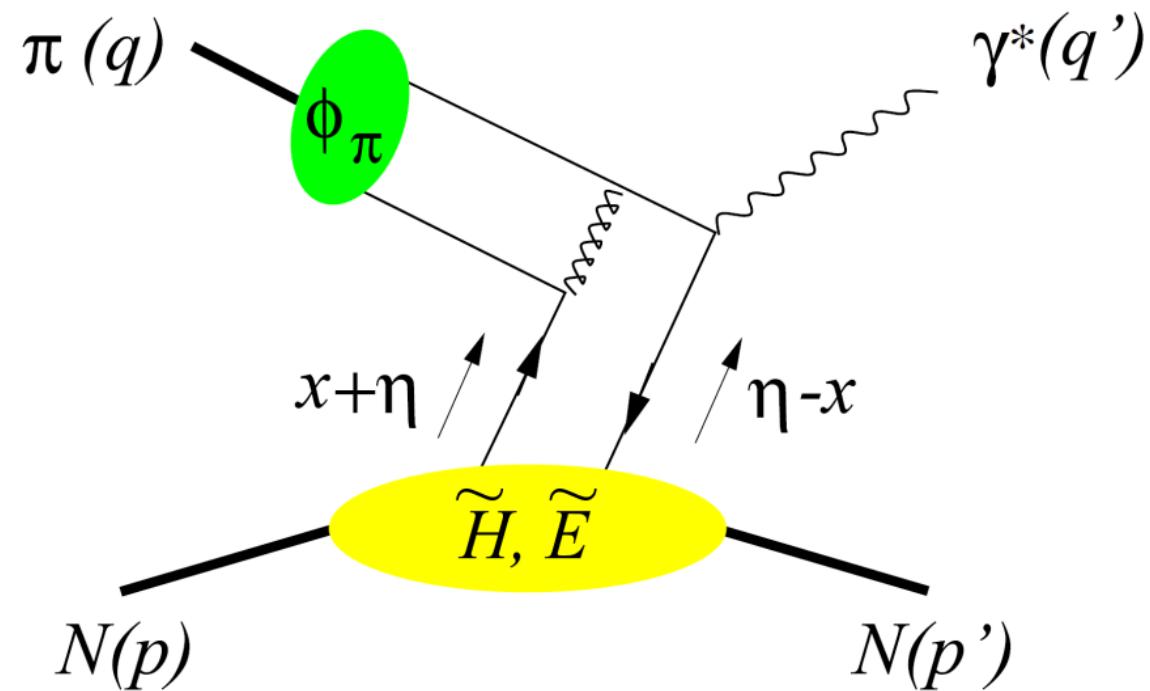
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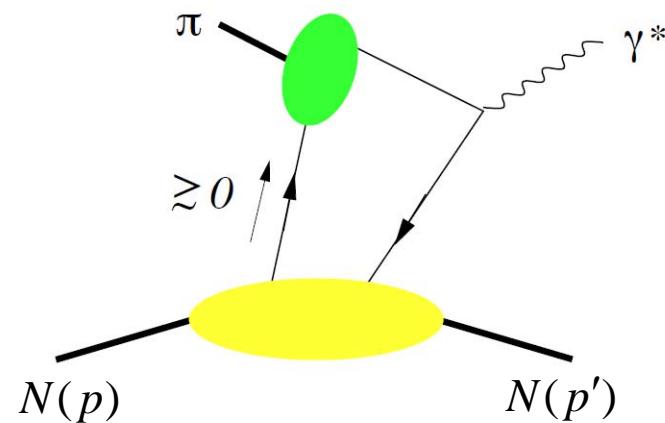
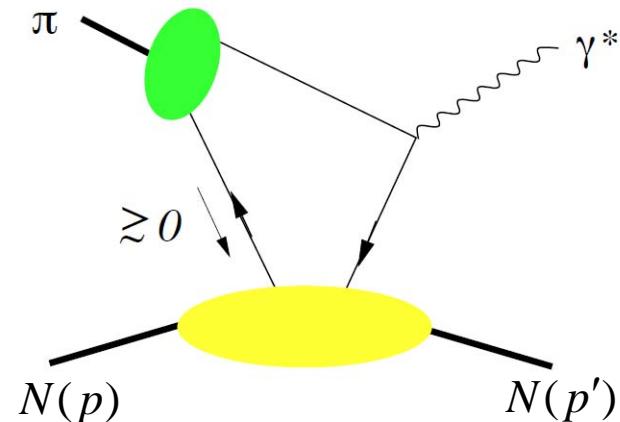
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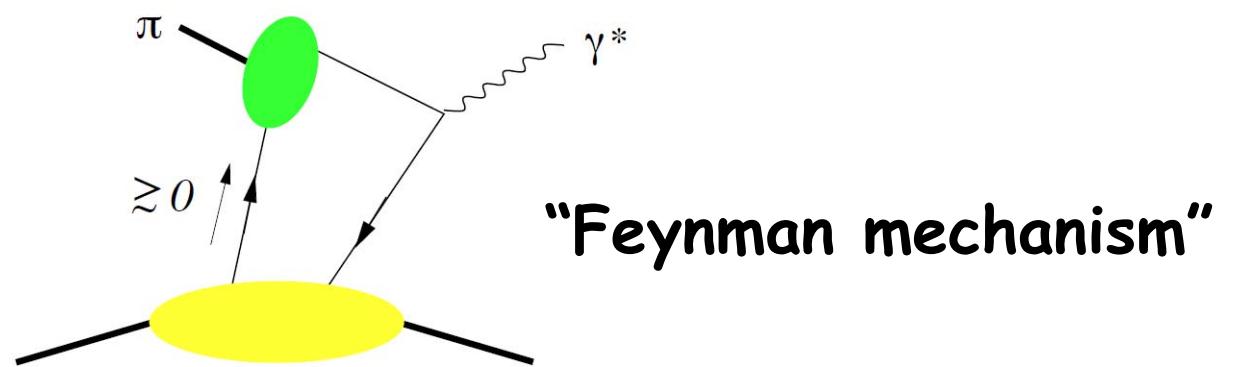
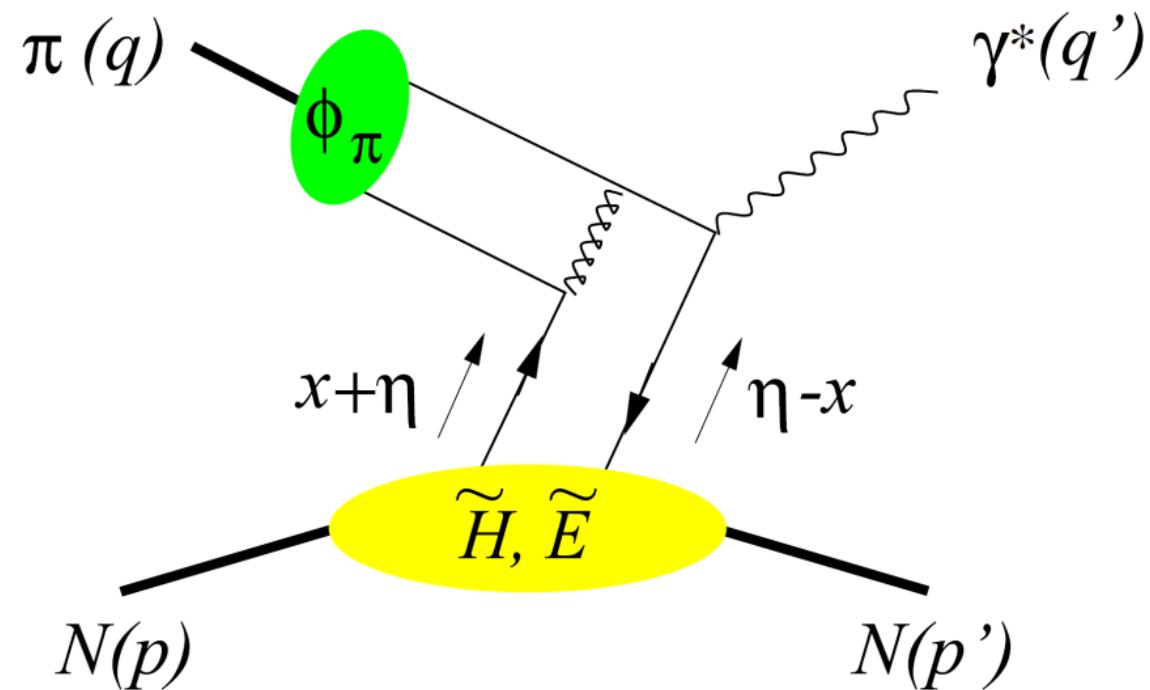
$$|y_{1\perp} - y_{3\perp}| = \mathcal{O}(1/Q) \rightarrow 0$$

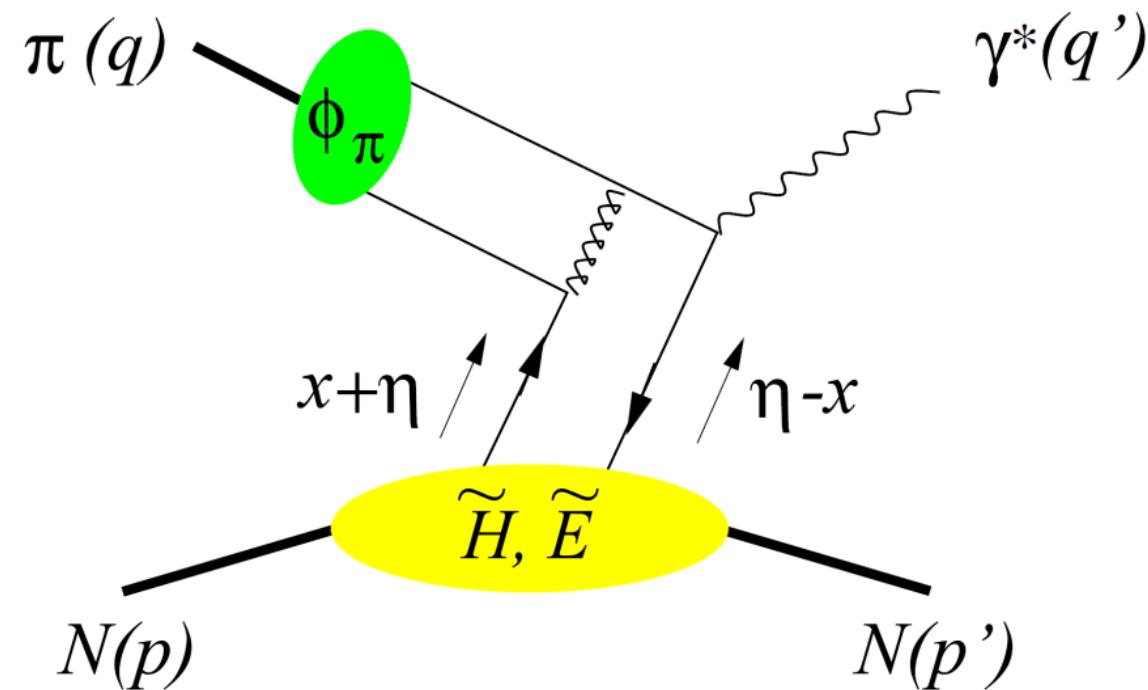
$$|y_1^+ - y_3^+| = \mathcal{O}(1/Q^2) \rightarrow 0$$



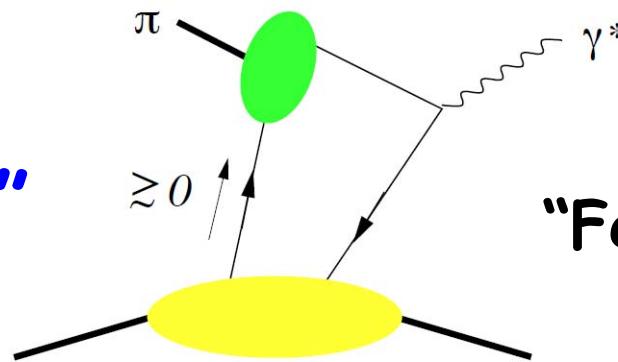




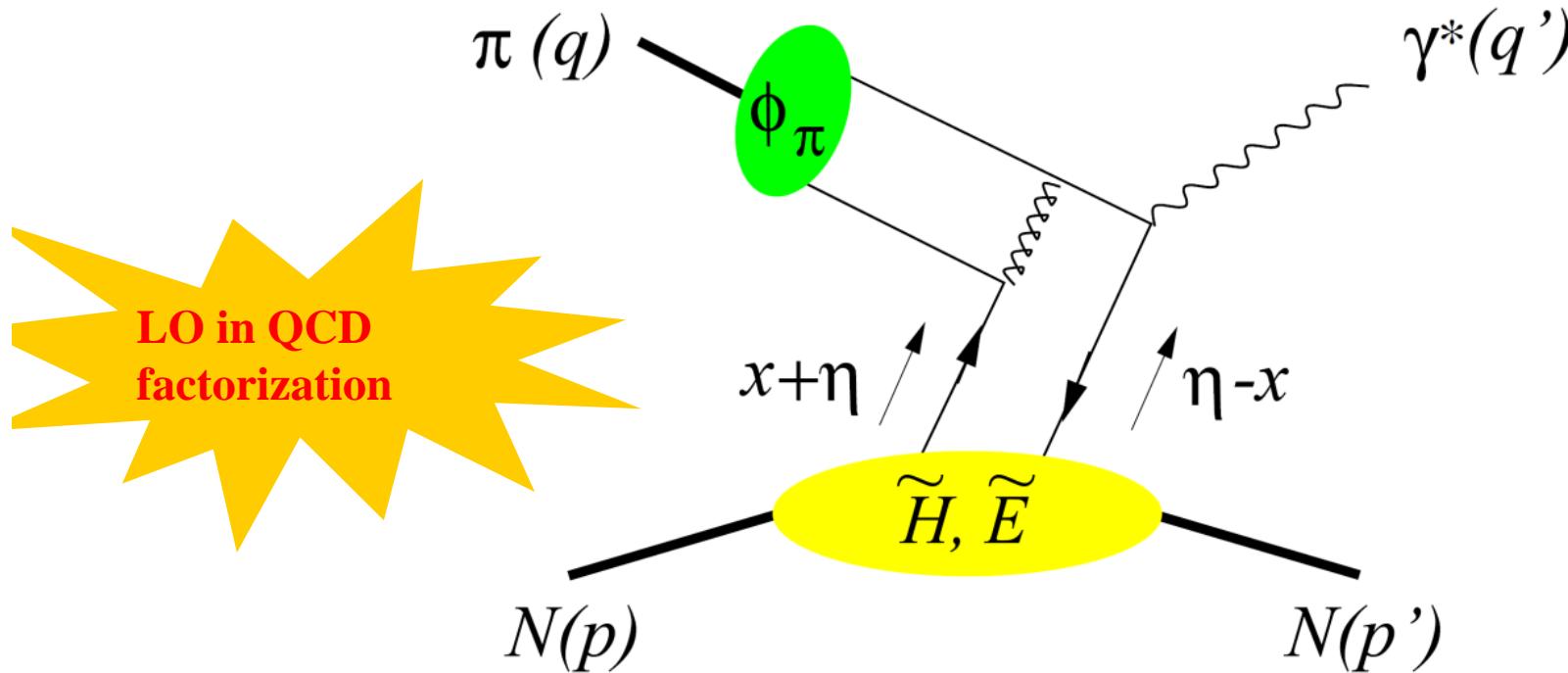




**“nonfactorizable”**

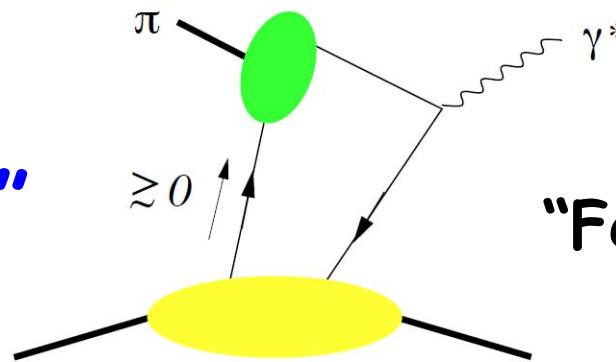


**“Feynman mechanism”**



Berger, Diehl, Pire, PLB523(2001)265

**"nonfactorizable"**



**"Feynman mechanism"**

Nonfactorizable mechanisms should be suppressed at  $Q^2 \rightarrow \infty$

Brodsky, Lepage, Chernyak,...  
Collins, Strikman, ...

# Nonfactorizable mechanisms should be suppressed at $Q^2 \rightarrow \infty$

Brodsky, Lepage, Chernyak,...  
Collins, Strikman, ...

VOLUME 52, NUMBER 13

PHYSICAL REVIEW LETTERS

26 MARCH 1984

## Asymptotic $Q^2$ for Exclusive Processes in Quantum Chromodynamics

Nathan Isgur<sup>(a)</sup> and C. H. Llewellyn Smith

Department of Theoretical Physics, University of Oxford, Oxford OX1 3NP, England, United Kingdom

(Received 19 October 1983)

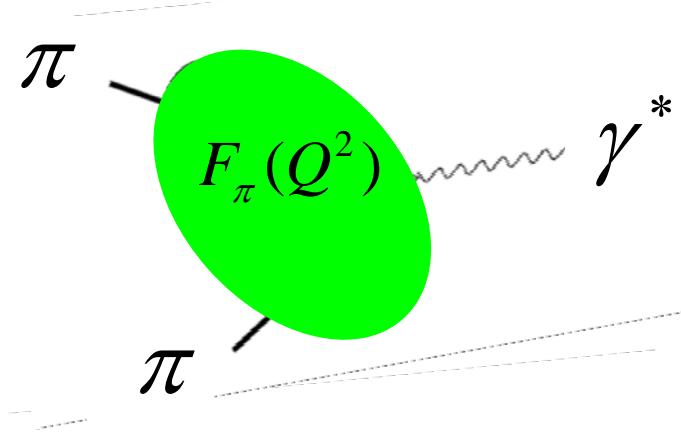
It is found that at available  $Q^2$  the calculable perturbative contributions to the pion electric form factor  $F_\pi(Q^2)$  and the nucleon magnetic form factors  $G_M(Q^2)$  are much smaller than the data, which can probably be explained by soft contributions. Both hard and soft effects are estimated from light-cone/infinite-momentum-frame wave functions suggested by quark models, but the main conclusions have a more general validity.

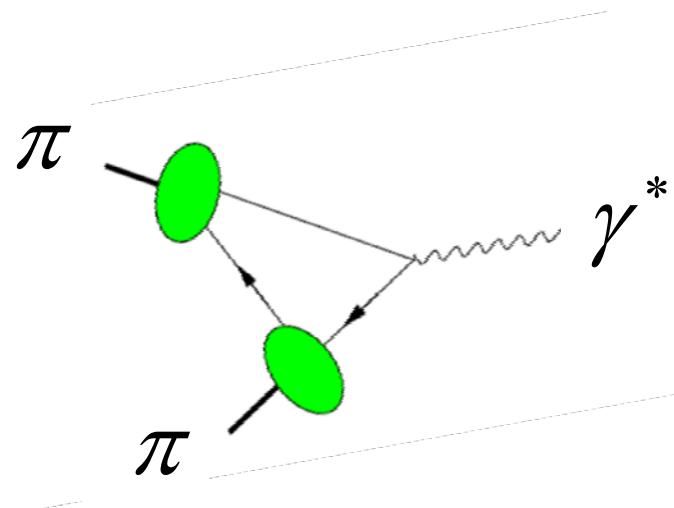
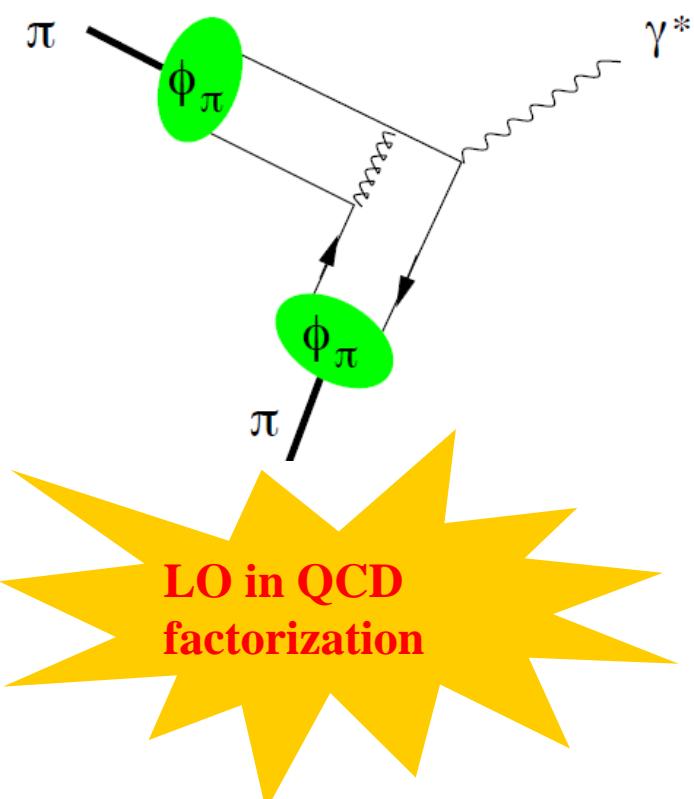
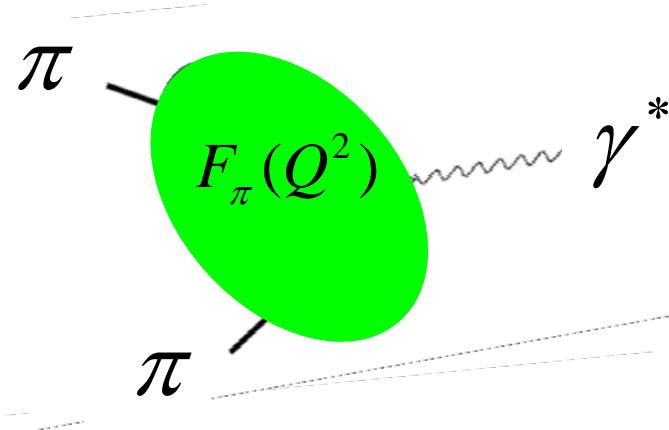
PACS numbers: 12.35.Eq, 13.40.Fn

It has been convincingly argued<sup>1-3</sup> that the asymptotic behavior of many exclusive processes is calculable in perturbative QCD. We show here that in the case of elastic form factors these calculable contributions are unlikely to dominate at available momentum transfers. We therefore wish to sound a note of caution about attempts<sup>2,3</sup> to explain existing exclusive data by perturbative QCD.

functions with  $\langle p_T^2 \rangle^{1/2} \simeq 300$  MeV can naturally generate “soft” nonleading terms which are as large as the data. Similar conclusions hold for the pion.<sup>4</sup> Calculations based on QCD sum rules<sup>5</sup> also generate soft contributions which fit the data for  $F_\pi$  and  $G_M$ .

Our calculations were based on the use of the light-cone quantization formalism<sup>6</sup> adopted by

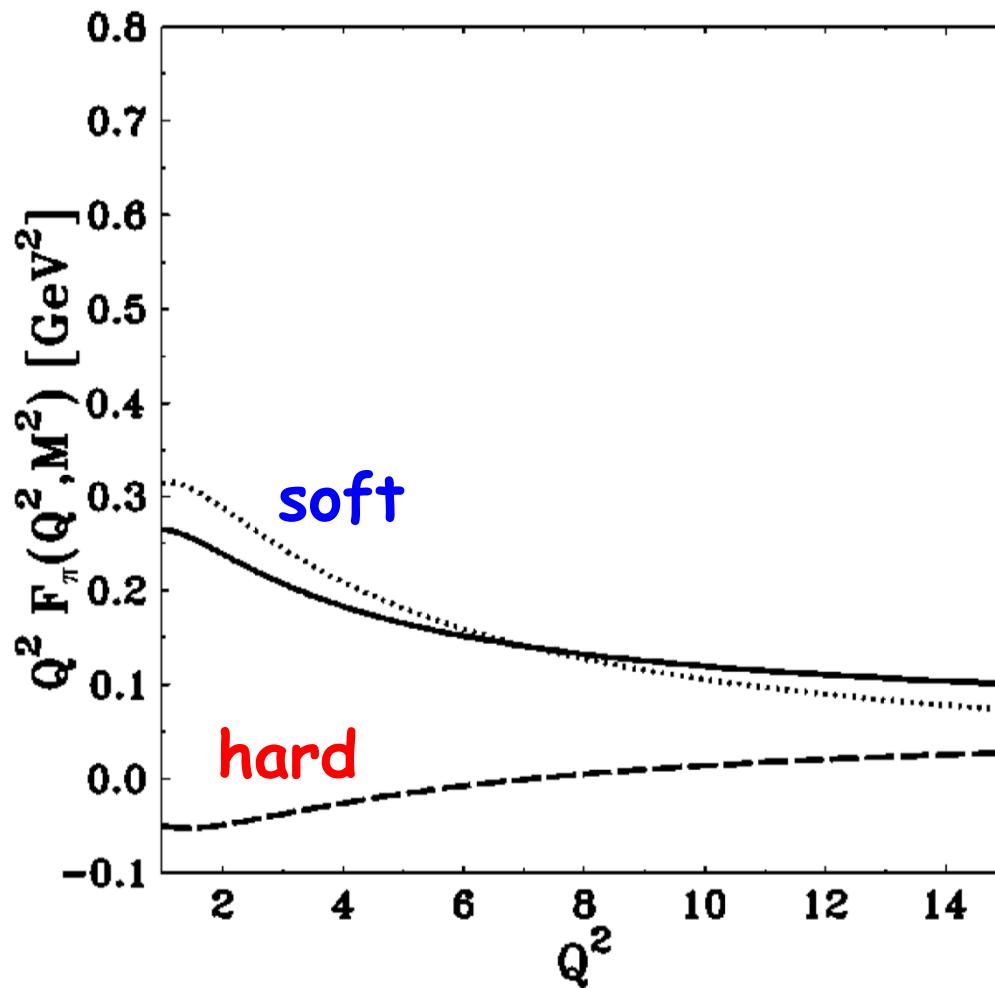




**“nonfactorizable”  
Feynman mechanism**

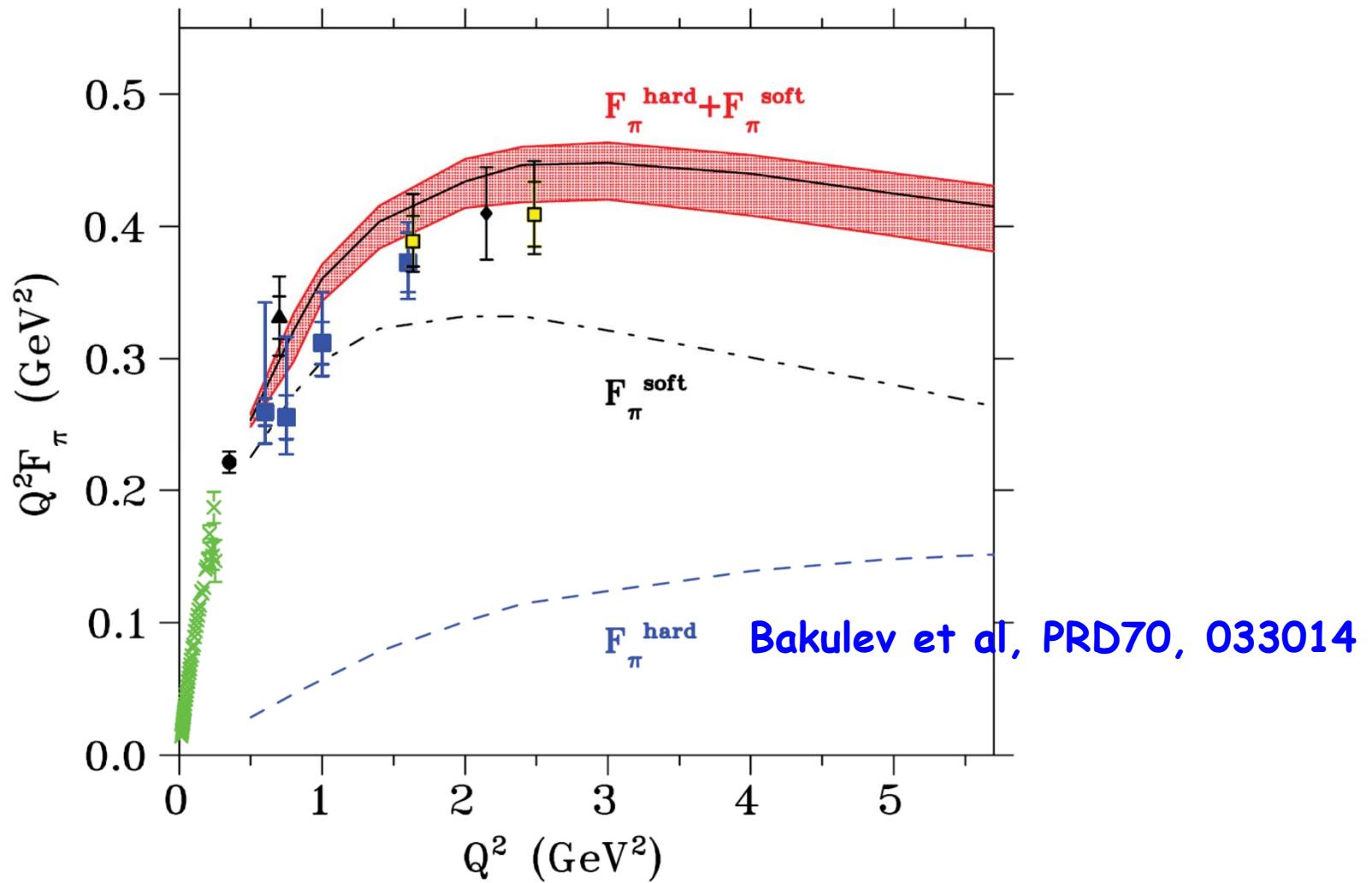
LO in QCD  
factorization

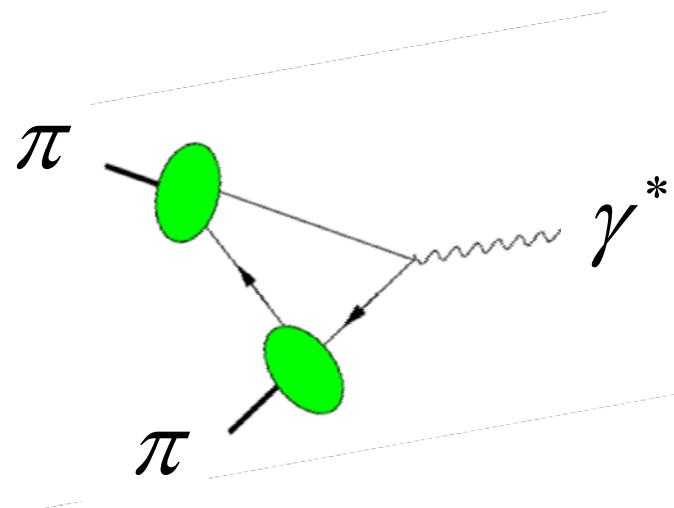
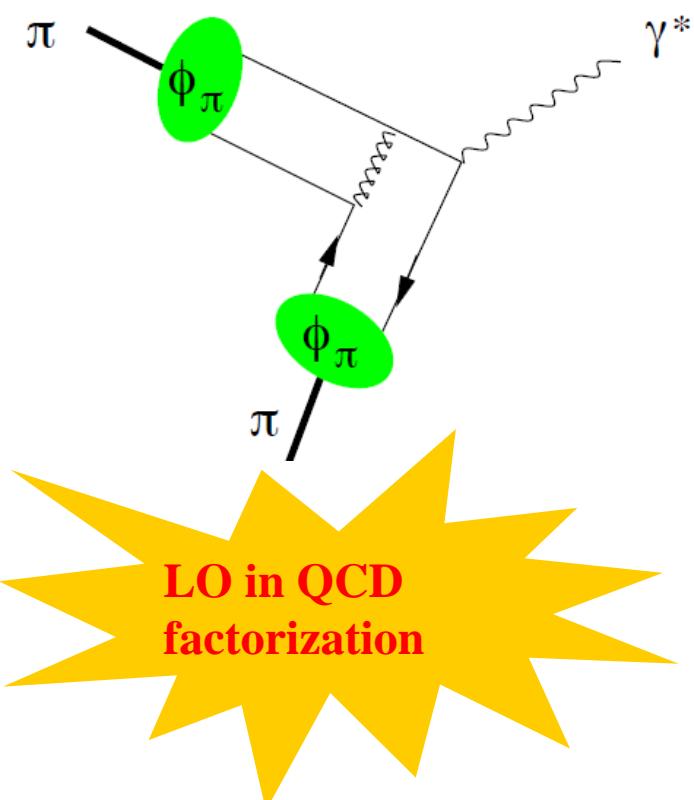
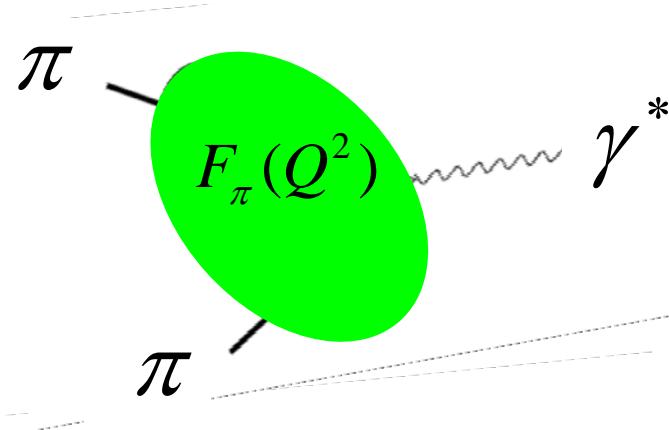
## hard & soft contributions in “light-cone QCD SR (LCSR)”



Braun, Khodjamirian, Maul, PRD61 ('00) 073004

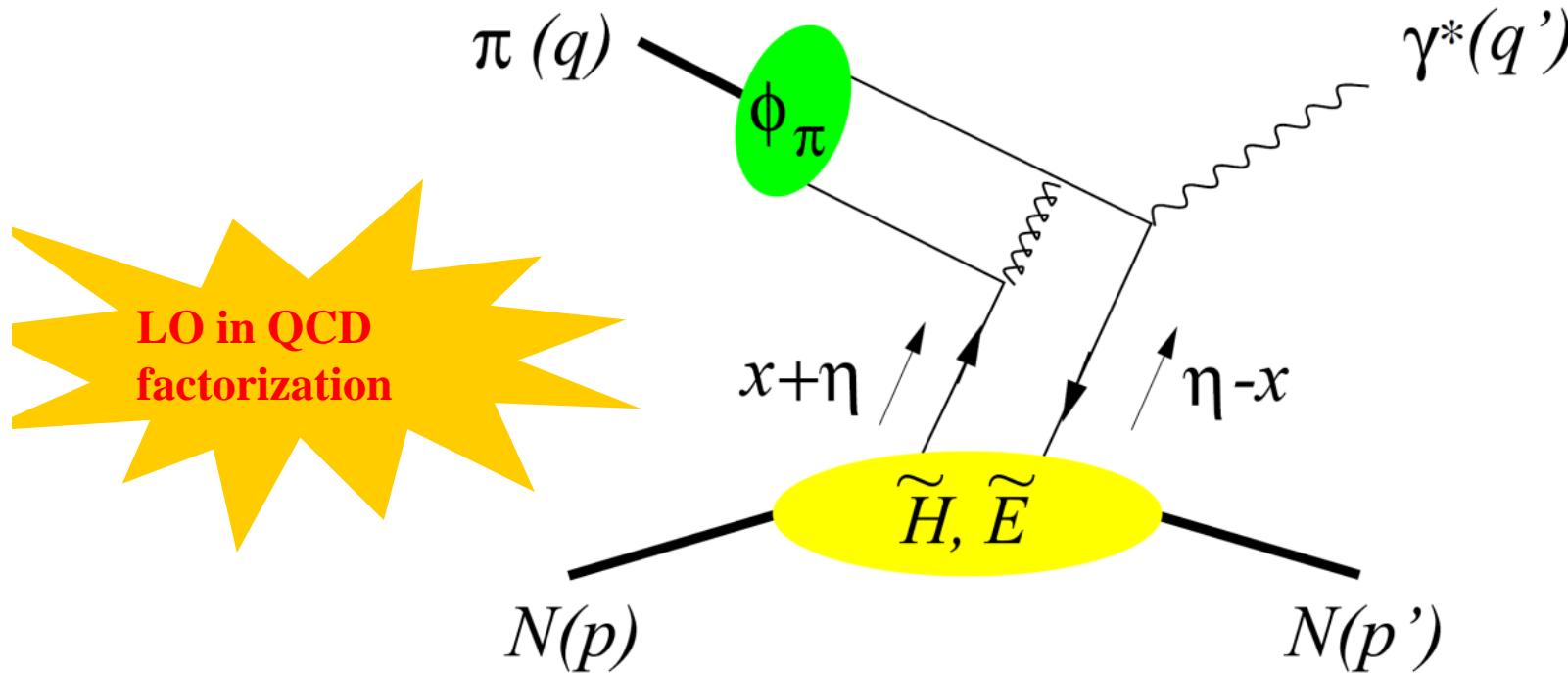
JLab data, PRC78 ('08) 045023



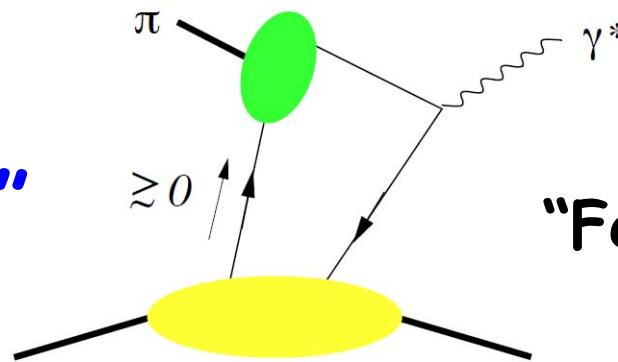


**“nonfactorizable”  
Feynman mechanism**

LO in QCD  
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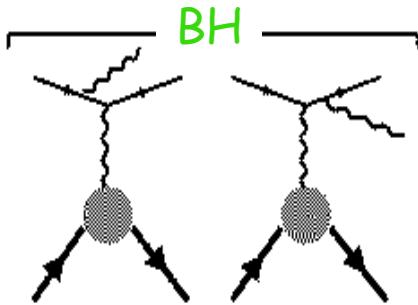
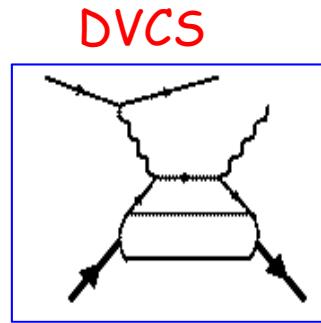


**"nonfactorizable"**



**"Feynman mechanism"**

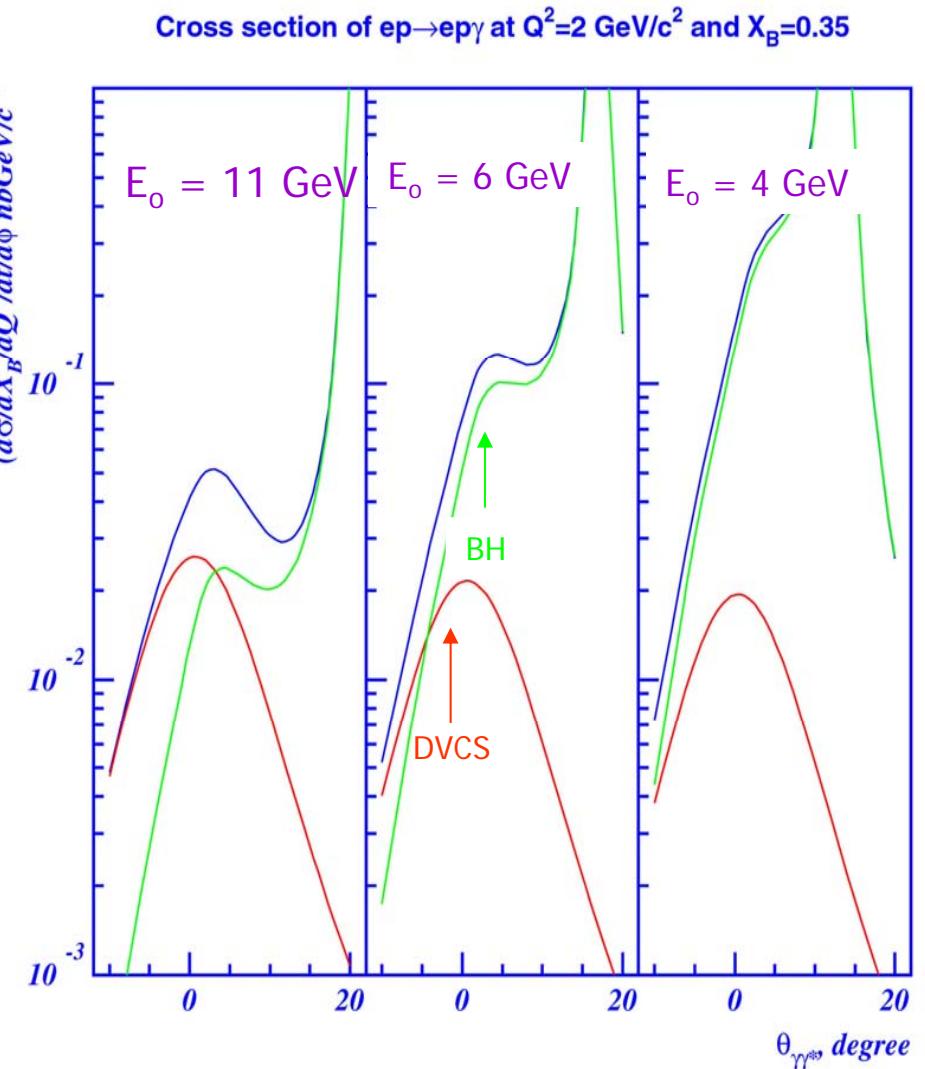
# Accessing GPDs through polarized DVCS



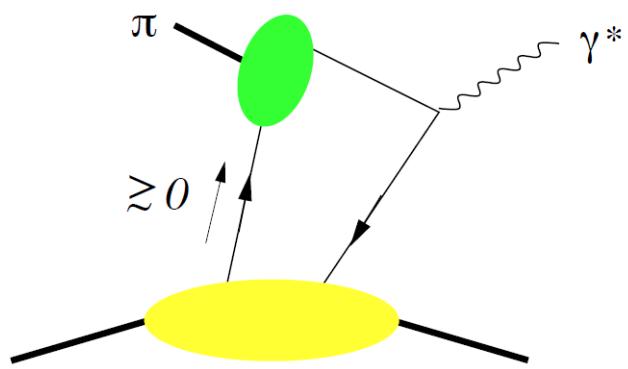
$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} \sim |T_{DVCS} + T_{BH}|^2$$

$T_{BH}$ : real, given by elastic form factors

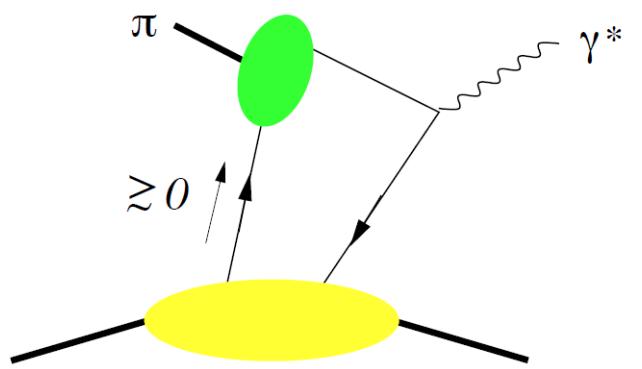
$T_{DVCS}$ : complex, determined by GPDs



# “nonfactorizable” mechanism

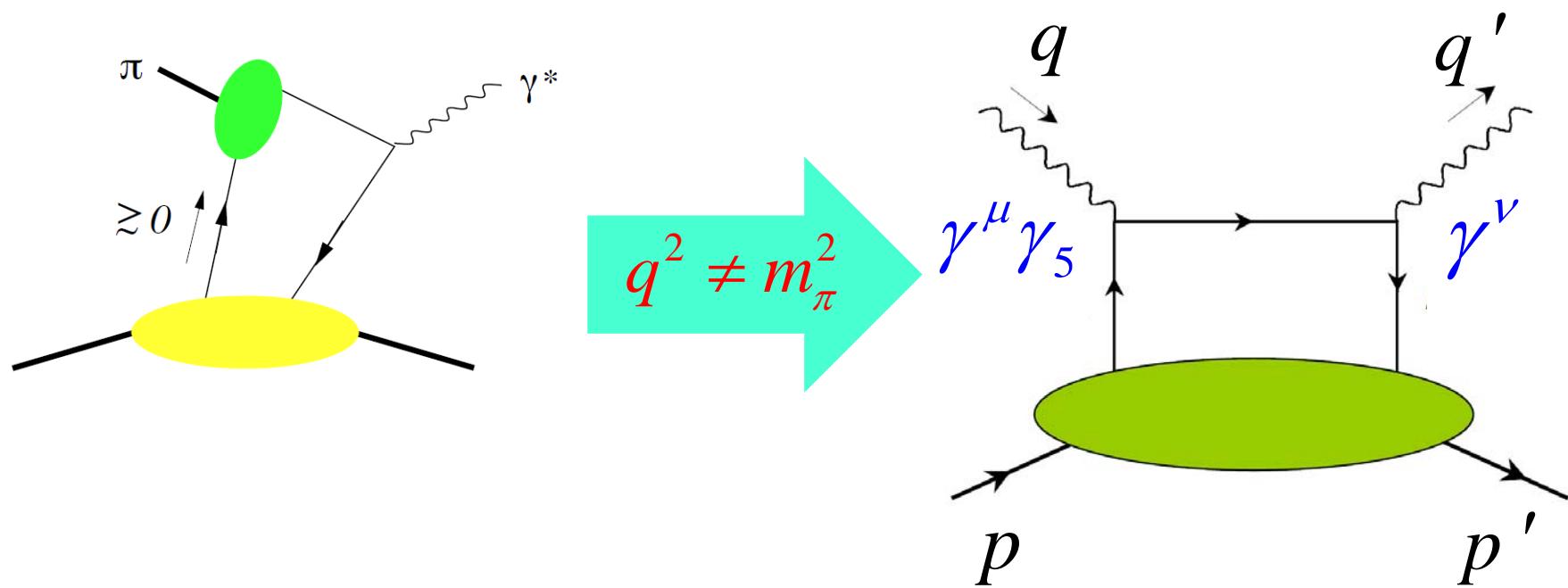


# “nonfactorizable” mechanism

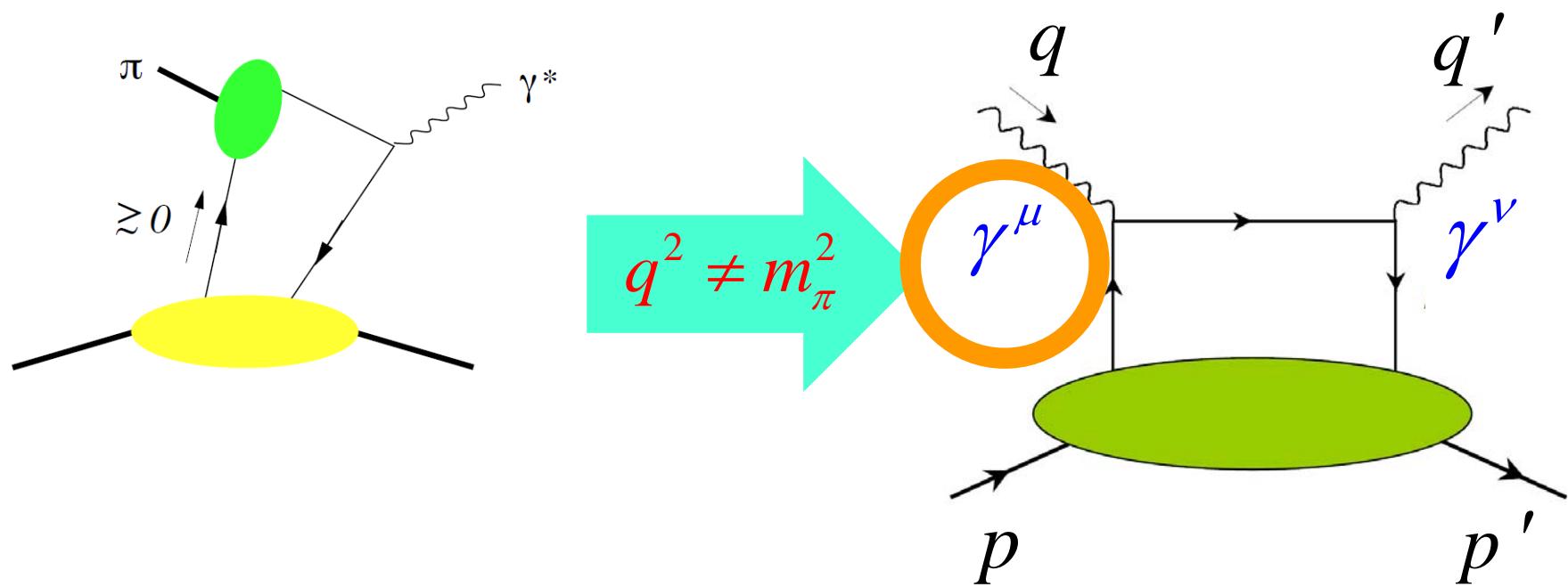


“LCSR”

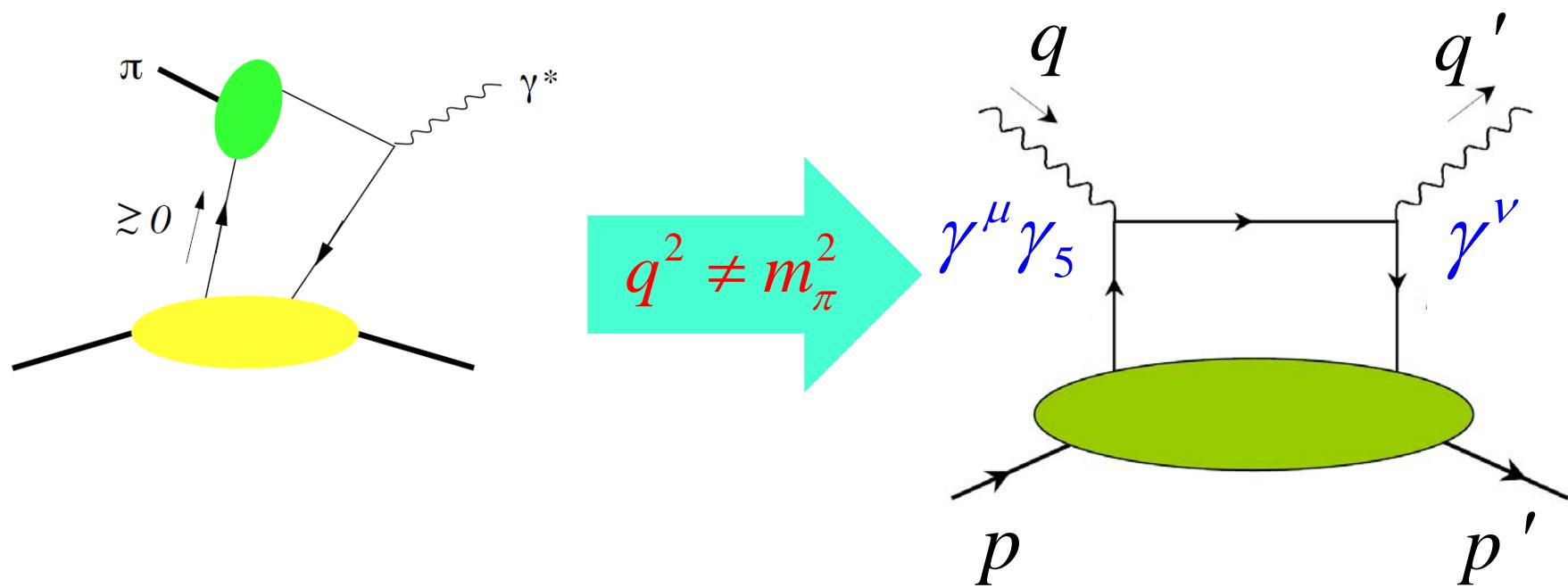
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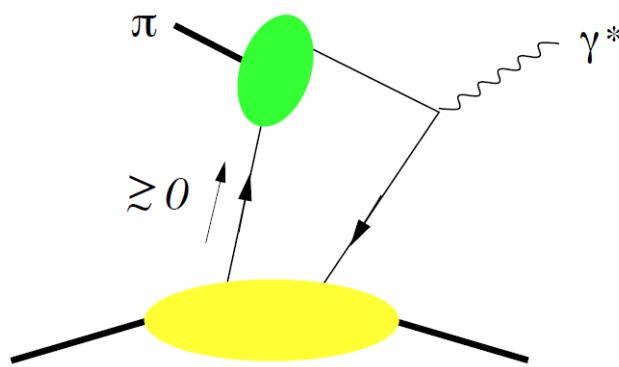
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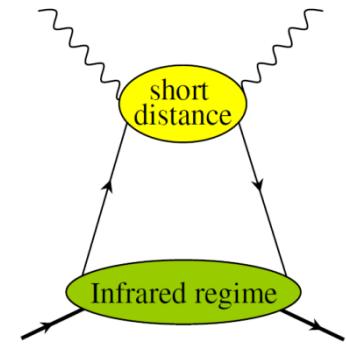
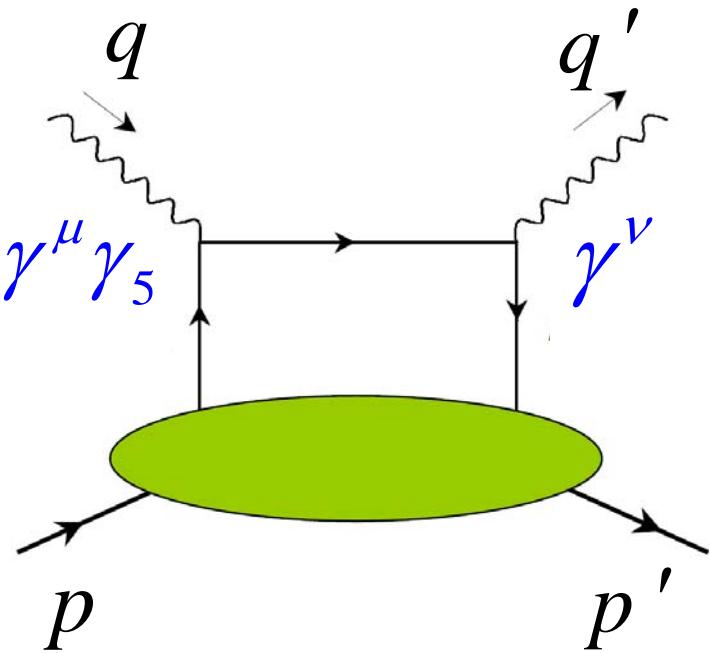
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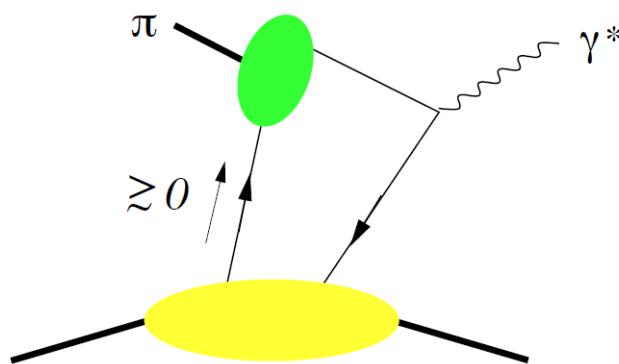
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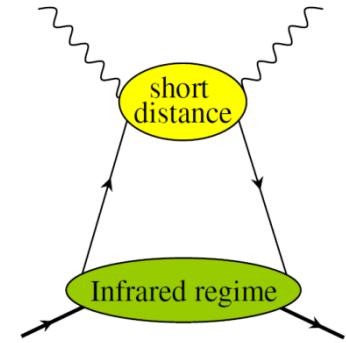
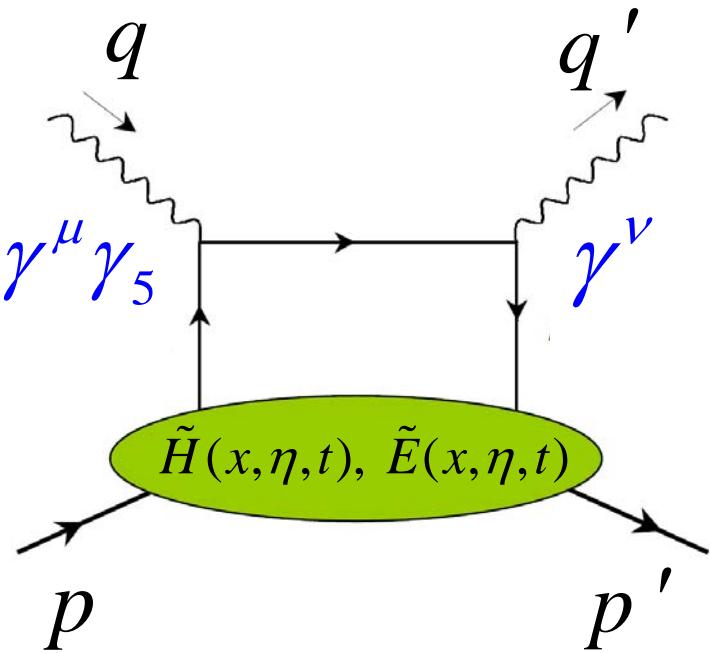
$$q^2 \neq m_\pi^2$$



# “nonfactorizable” mechanism

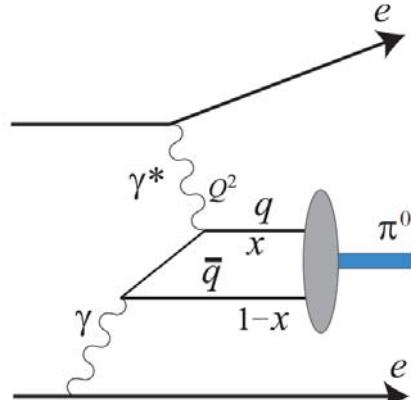


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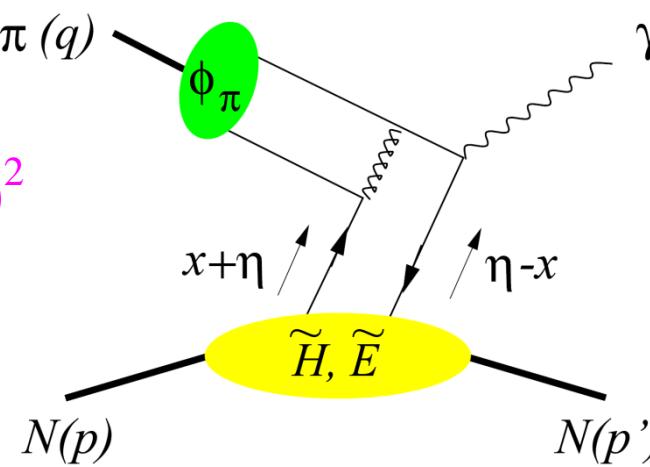
## Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



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small  $t = \Delta^2 = (q - q')^2$



"exclusive DY"



$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta}) \cdot \mathbf{p} \mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

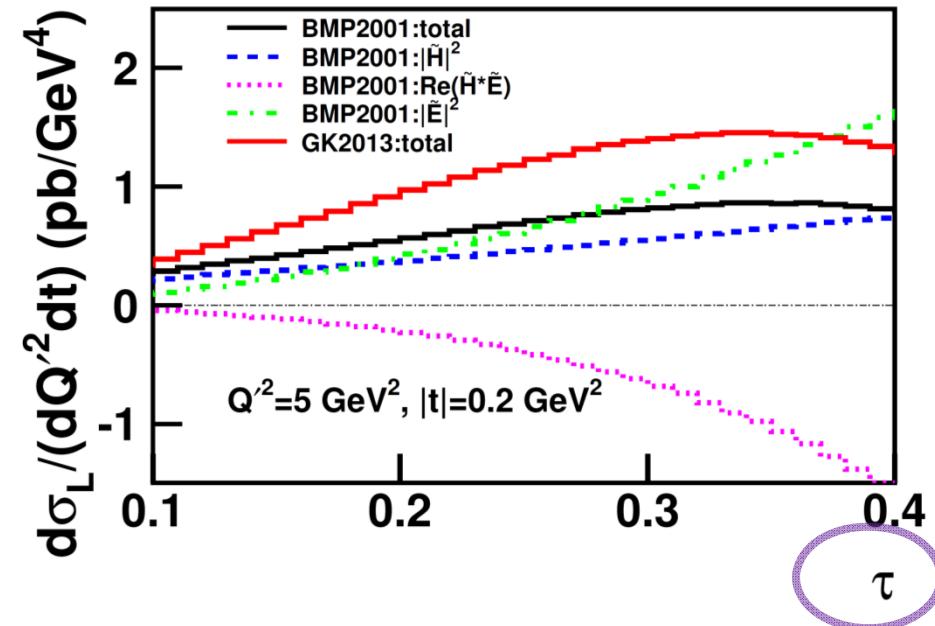
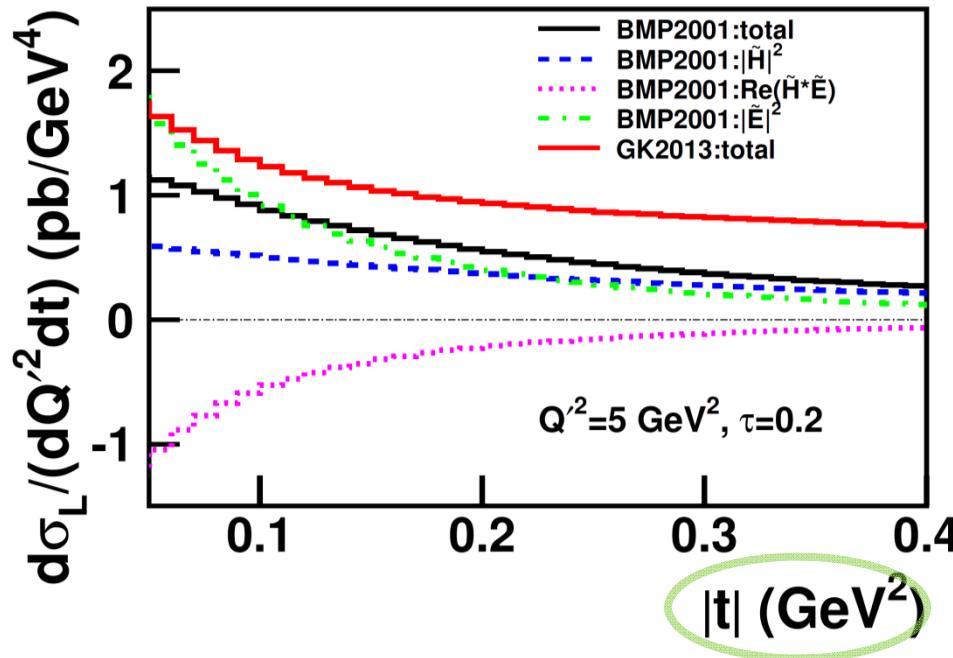
GPD

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PRD93, 114034 ('16)

Bjorken variable

$$\tau = \frac{Q'^2}{s-M^2}$$

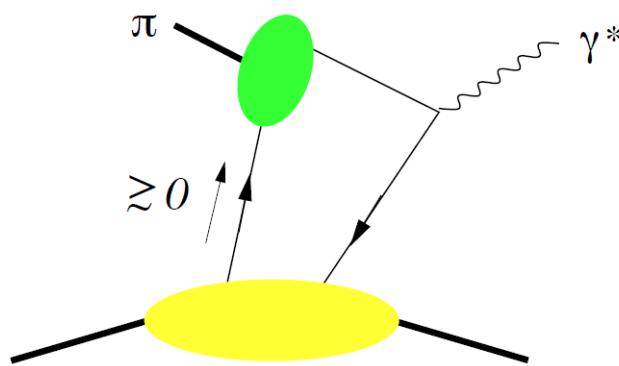
$$Q'^2 = 5 \text{ GeV}^2$$



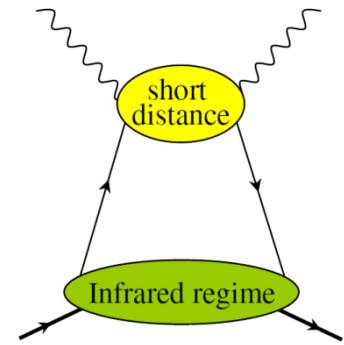
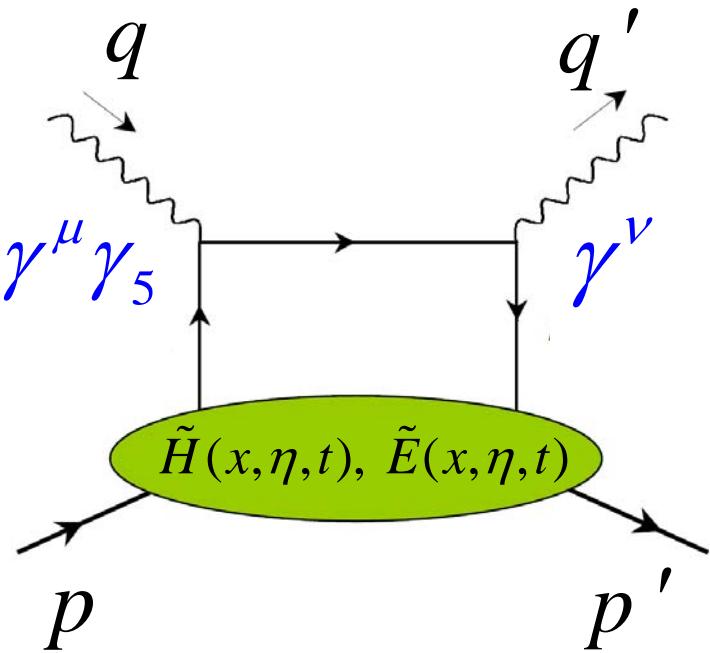
$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \rightarrow \gamma^* n) = \frac{4\pi\alpha_{\text{em}}^2}{27} \frac{\tau^2}{Q'^8} f_\pi^2 \left[ (1-\eta^2) |\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\tilde{\mathcal{H}}^{du*} \tilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\tilde{\mathcal{E}}^{du}|^2 \right]$$

$$\tilde{\mathcal{H}}^{du} = \frac{8\alpha_s}{3} \int_0^1 du \frac{\phi_\pi(u)}{4u(1-u)} \int_{-1}^1 dx \left( \frac{e_d}{-\eta - x - i\epsilon} - \frac{e_u}{-\eta + x - i\epsilon} \right) \left( \tilde{H}^d(x, \eta, t) - \tilde{H}^u(x, \eta, t) \right)$$

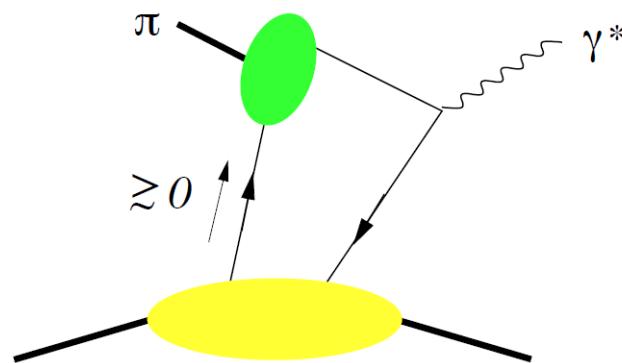
# “nonfactorizable” mechanism



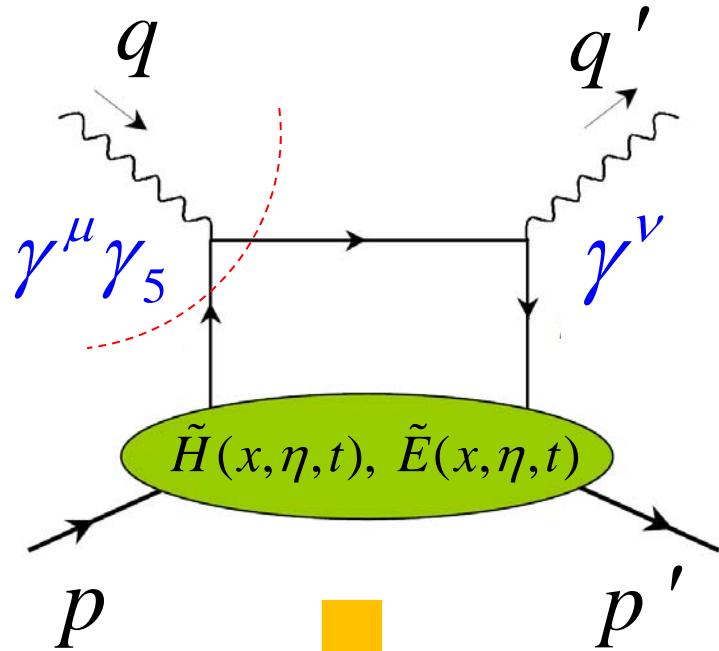
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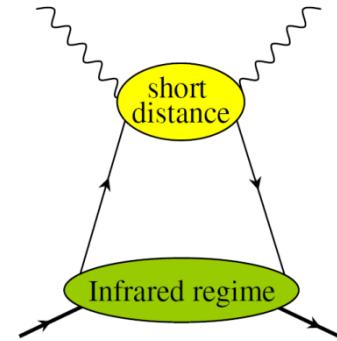
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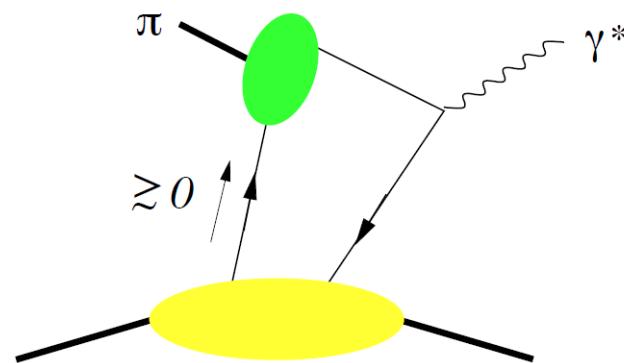
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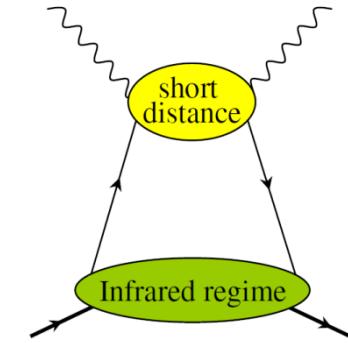
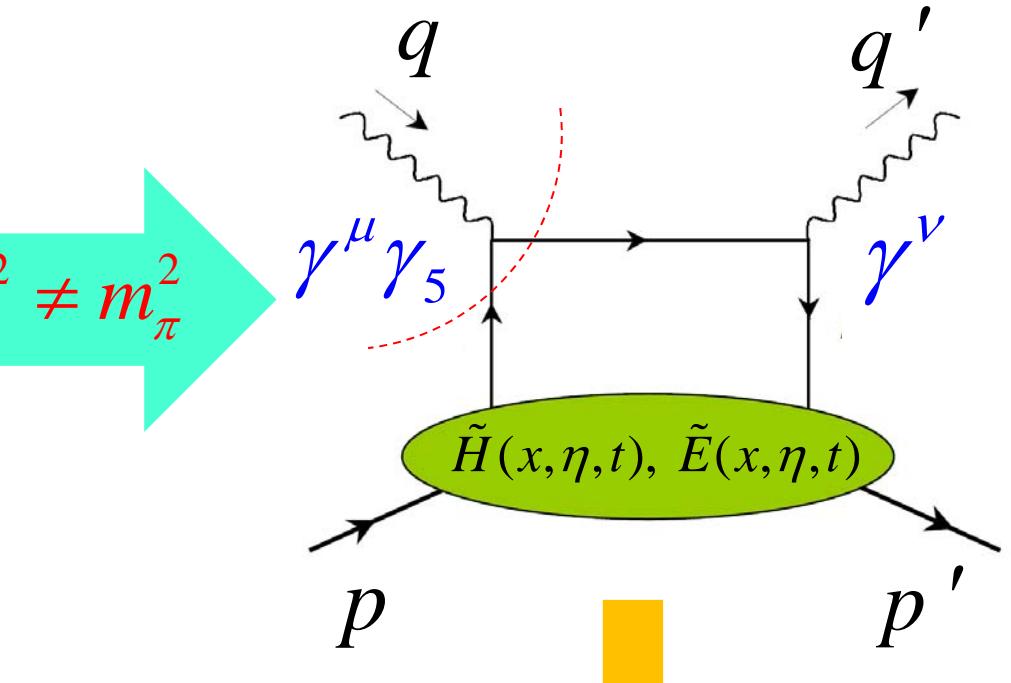
dispersion relation



# “nonfactorizable” mechanism



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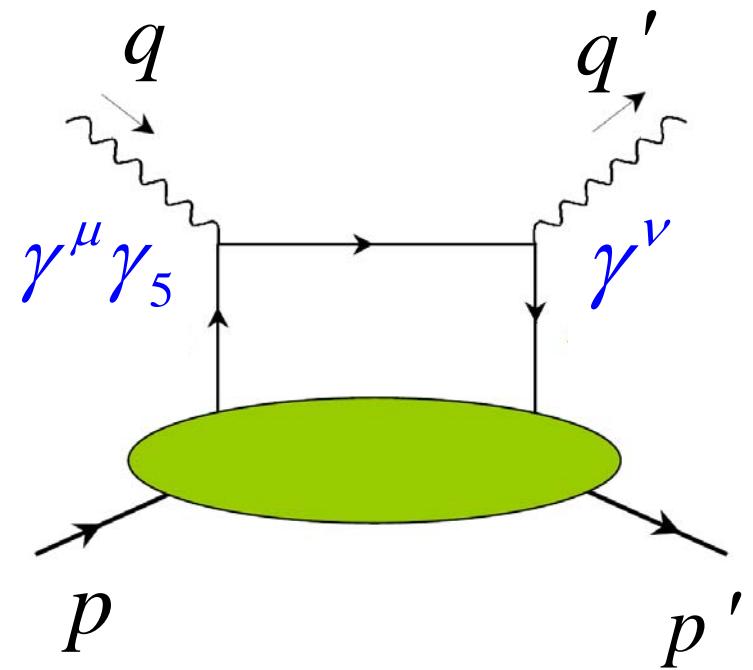
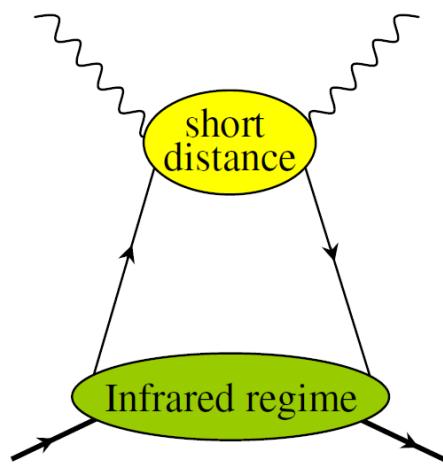


$$\int d^4x e^{iq' \cdot x} \langle p' | T j_\mu^5(0) j_\nu^{\text{em}}(x) | p \rangle \\ \equiv -iT_{\mu\nu}$$

$$j_\mu^5 = \bar{d} \gamma_\mu \gamma_5 u$$

$$j_\nu^{\text{em}} = e_u \bar{u} \gamma_\nu u + e_d \bar{d} \gamma_\nu d$$

$$|q^2|, |q'^2| \gg \Lambda_{\text{QCD}}^2$$



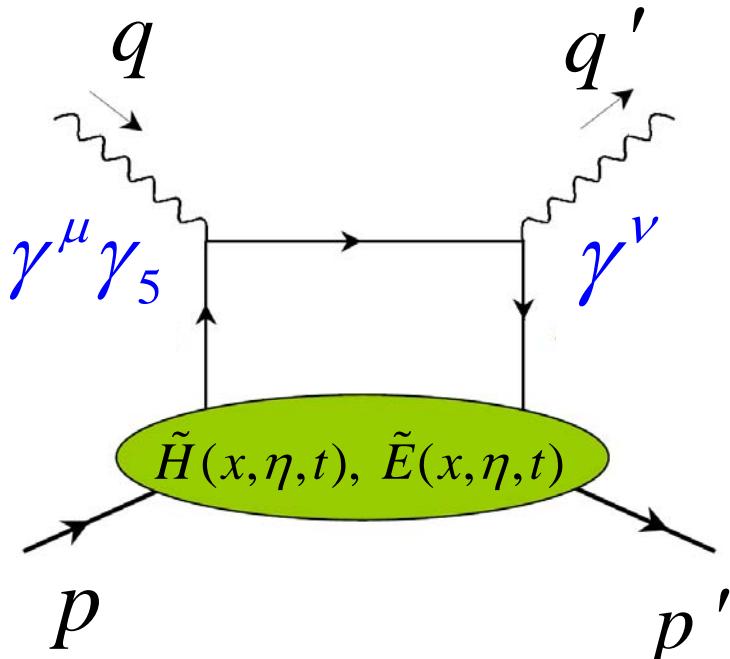
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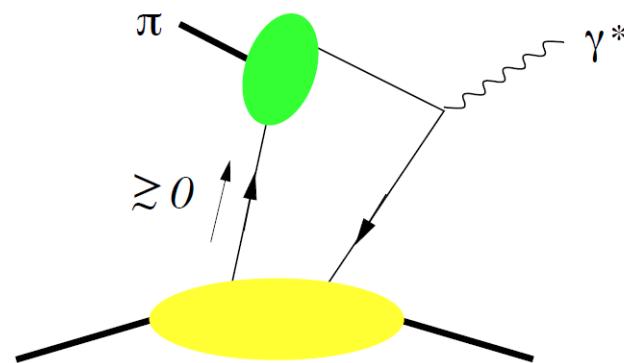
$$T_{\mu\nu}$$

$$= - q_\mu g^-_\nu \int~dx \Big\{ C_H(x,\eta,Q^{\prime 2},q^2) \Big[ e_u \tilde H^{du}(x,\eta,t) - e_d \tilde H^{du}(-x,\eta,t) \Big] \overline u(p') \gamma^+ \gamma_5 u(p)$$

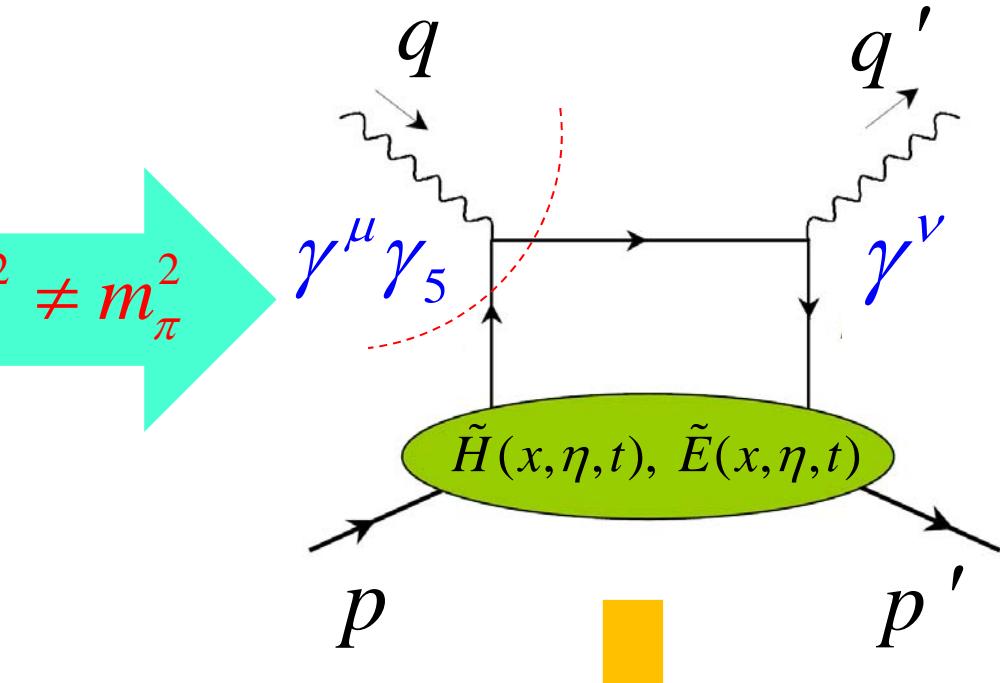
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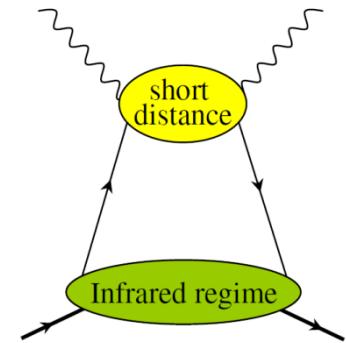
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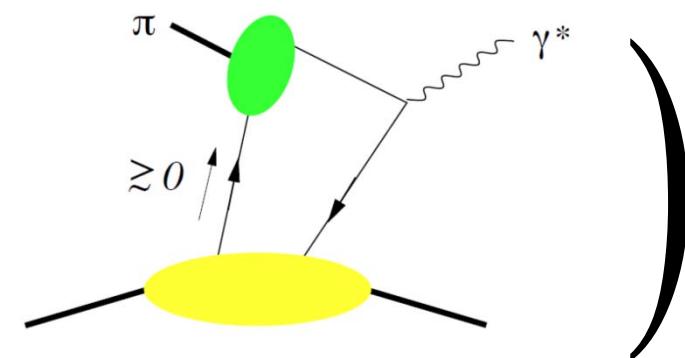
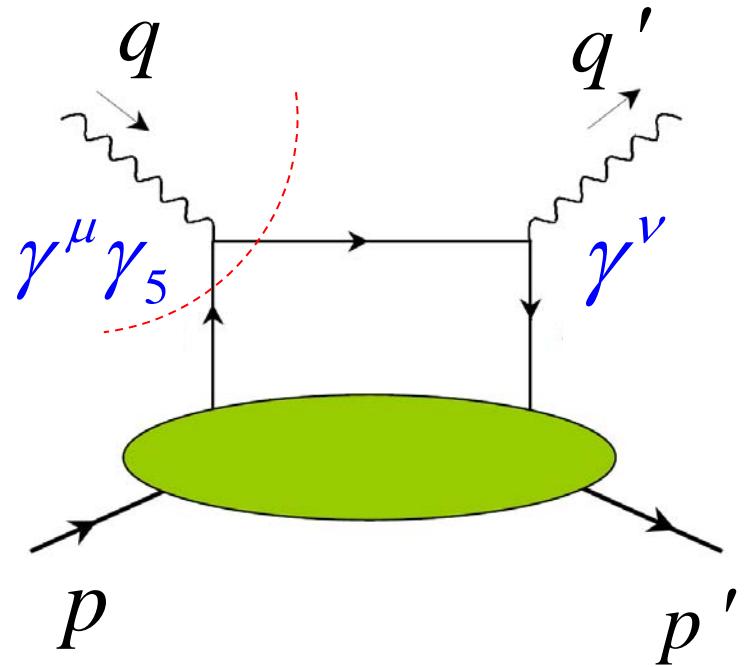
dispersion relation  
quark-hadron duality



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$$\langle 0 | j_\mu^5 | \pi^-(k) \rangle = ik_\mu f_\pi$$

$$T_{\mu\nu} = iq_\mu \left[ f_\pi \frac{1}{q^2 - m_\pi^2} \left( + \int_{q_{\text{th}}^2}^\infty dm^2 \frac{\tilde{a}_\nu(m^2)}{q^2 - m^2} \right) + \dots \right]$$



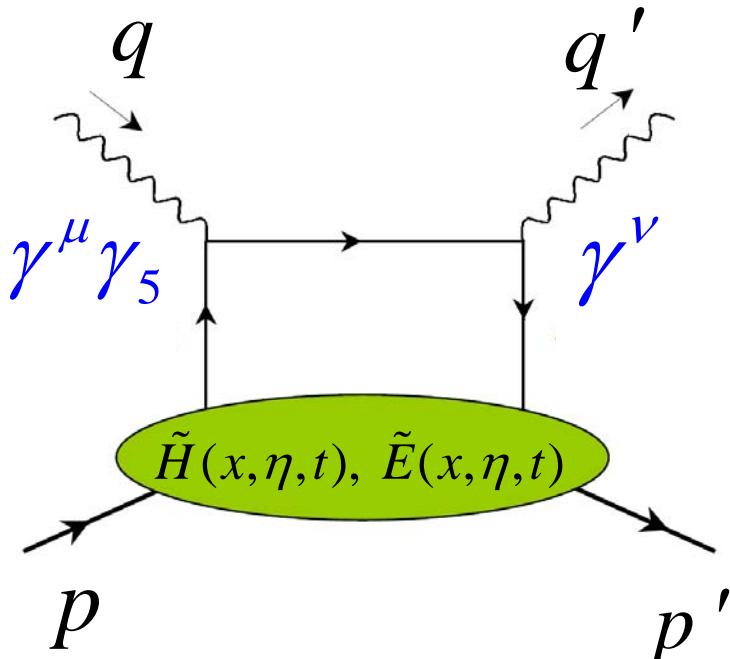
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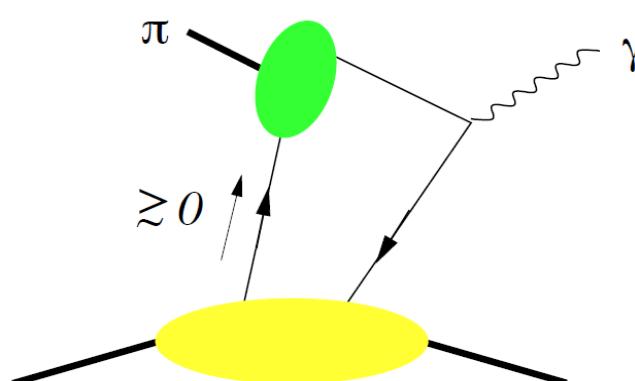
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# Light-cone QCD SR (LCSR)



Feynman diagram showing the annihilation of a pion ( $\pi$ ) into a photon ( $\gamma^*$ ) and a quark-gluon state. The quark-gluon state is represented by a yellow oval with two outgoing lines. A green oval represents the annihilation vertex. A vertical arrow labeled  $\gtrsim 0$  points upwards from the quark-gluon state towards the annihilation vertex.

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$\sim g_\nu^- \frac{1}{f_\pi} \int_{\eta}^{x_0} dx \ e^{-\frac{x-\eta}{x+\eta} \frac{Q'^2}{M_B^2}} \tilde{C}_H(x, \eta, Q'^2)$

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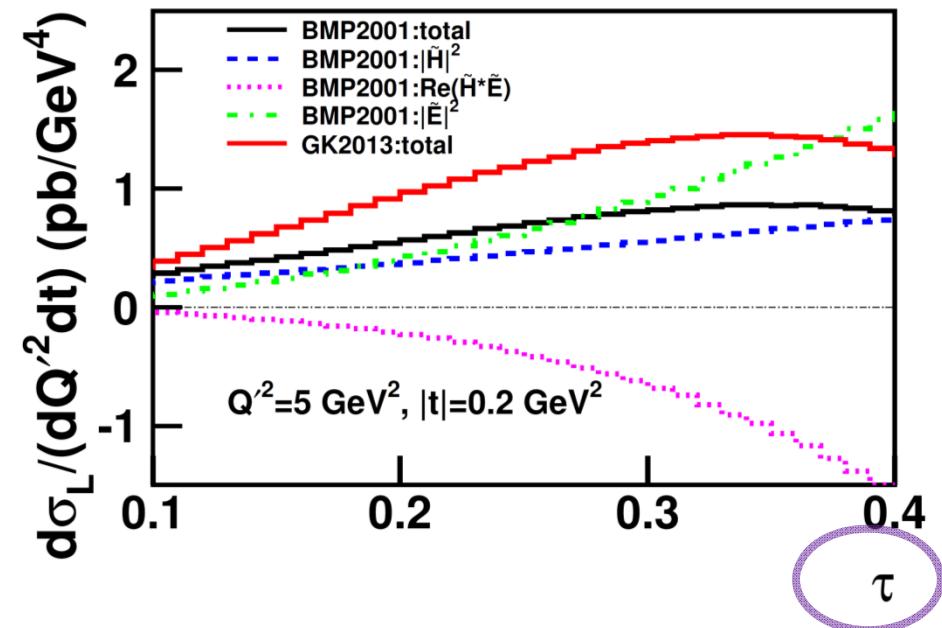
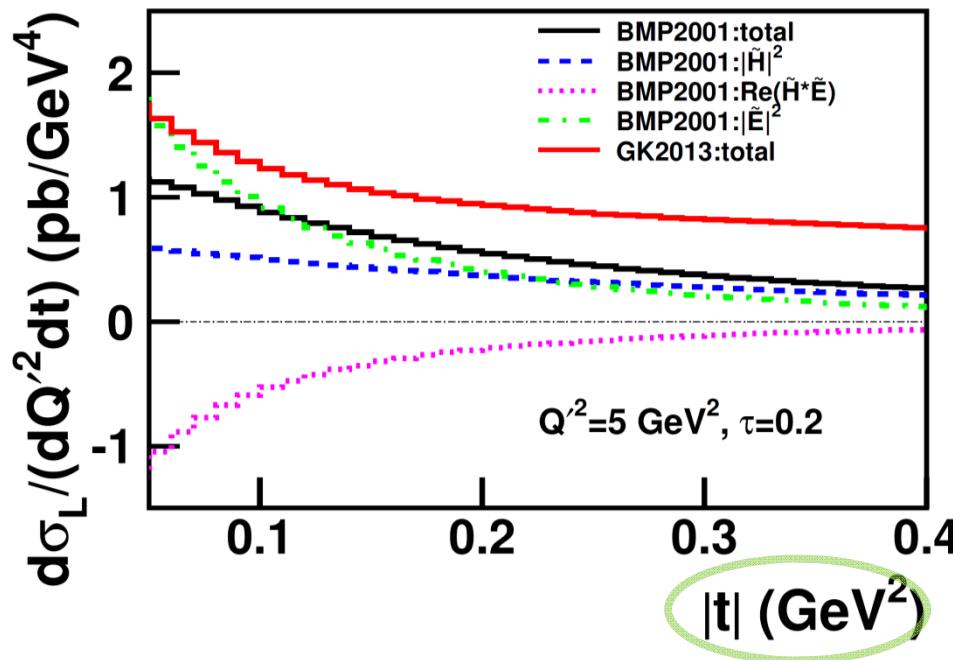
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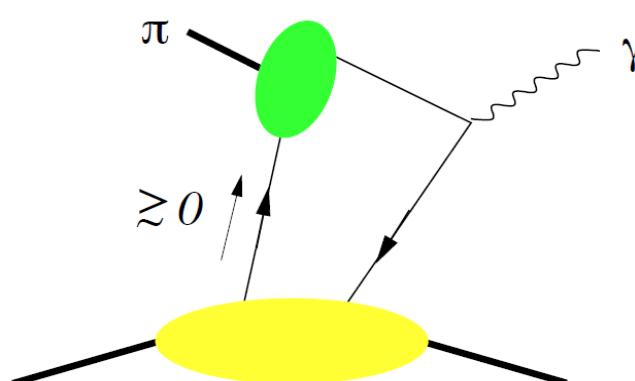
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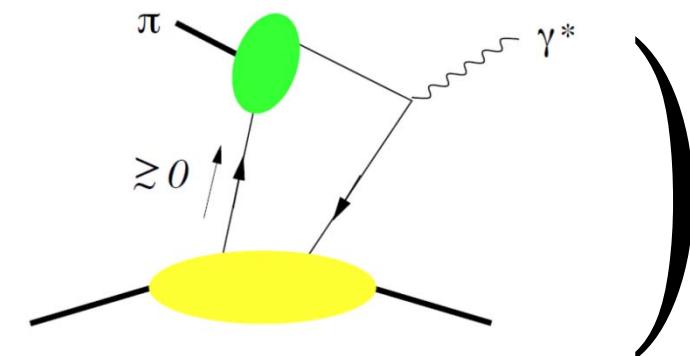
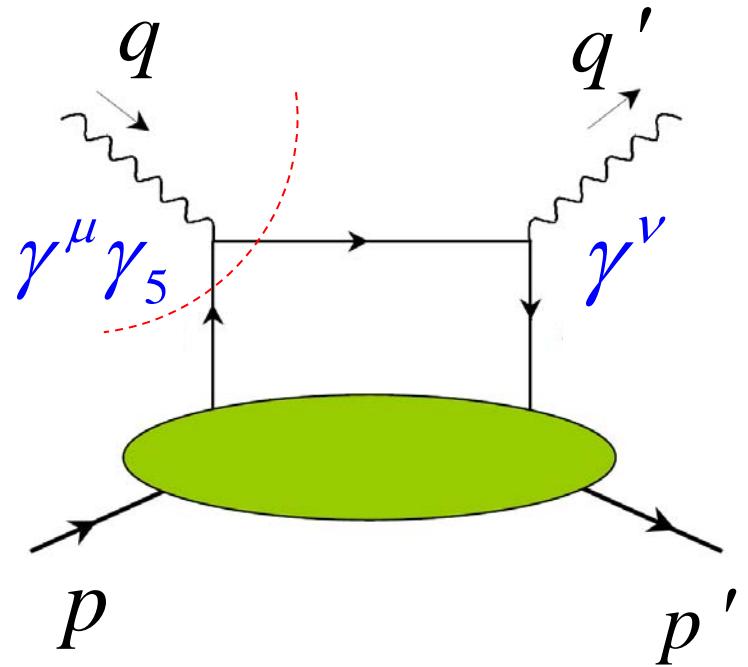
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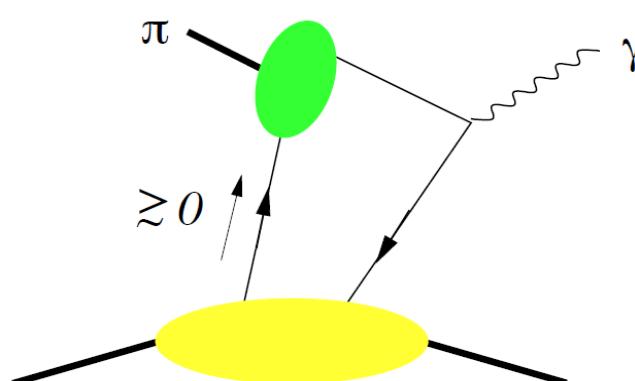
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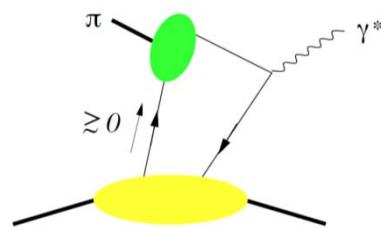
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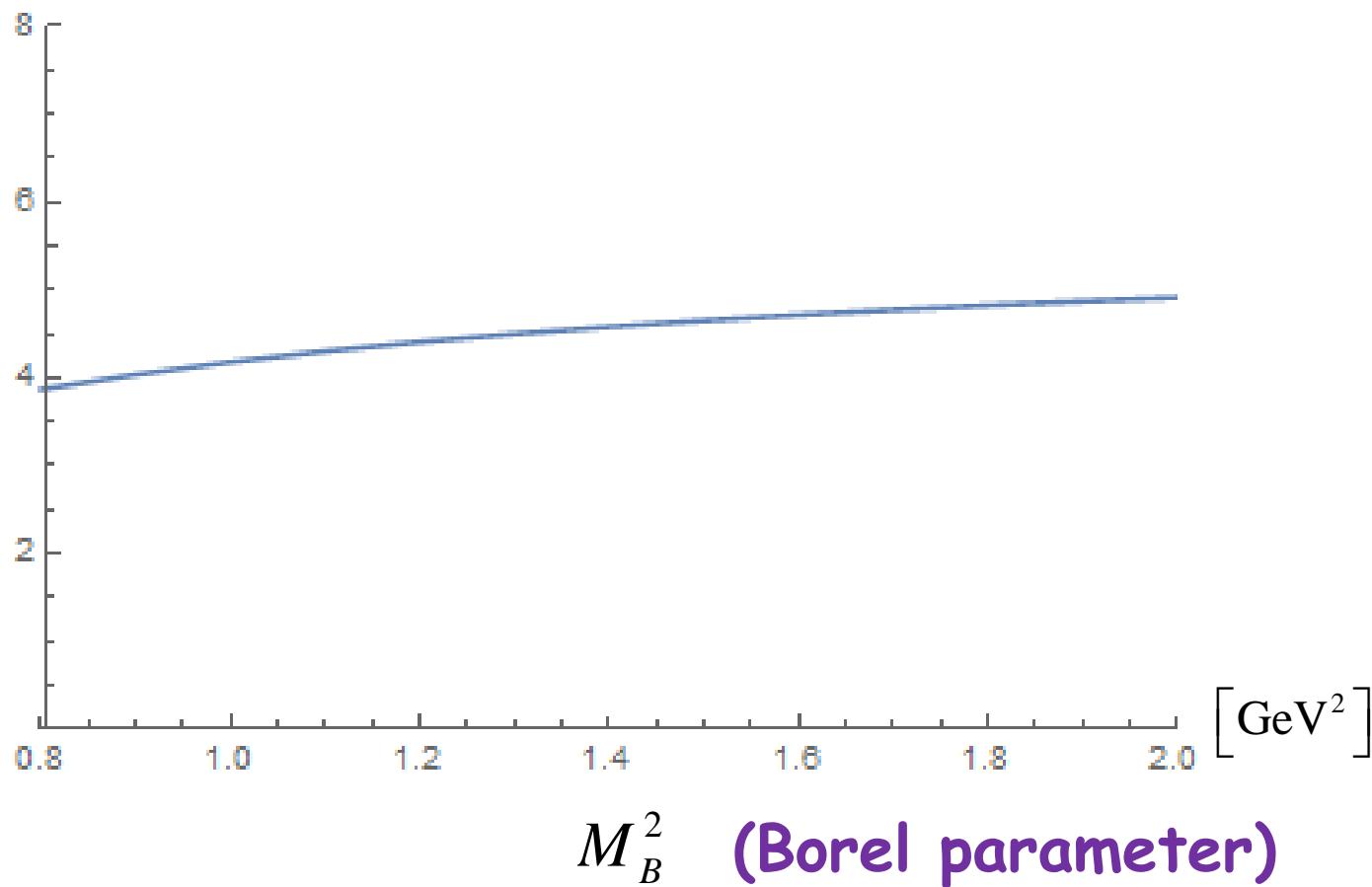
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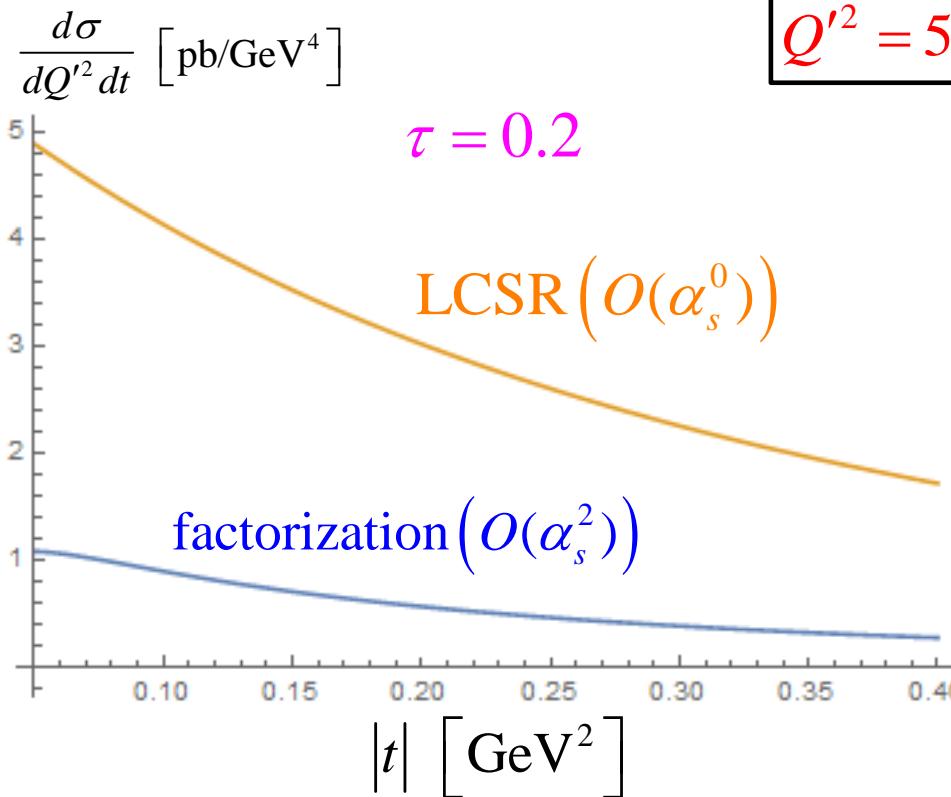
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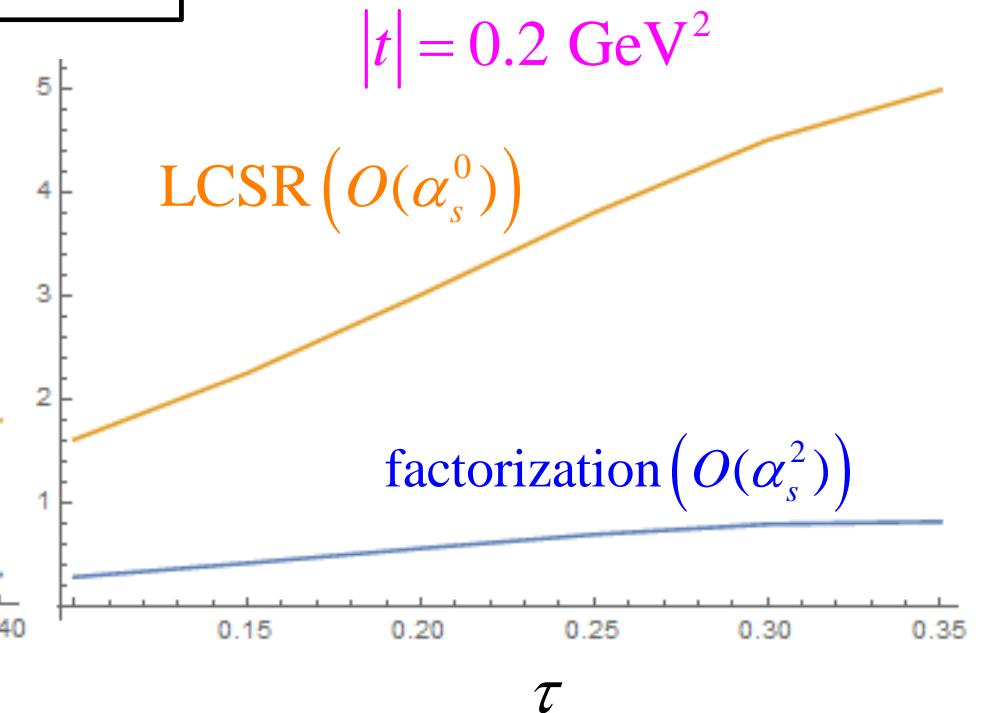


from LCSR





$$Q'^2 = 5 \text{ GeV}^2$$



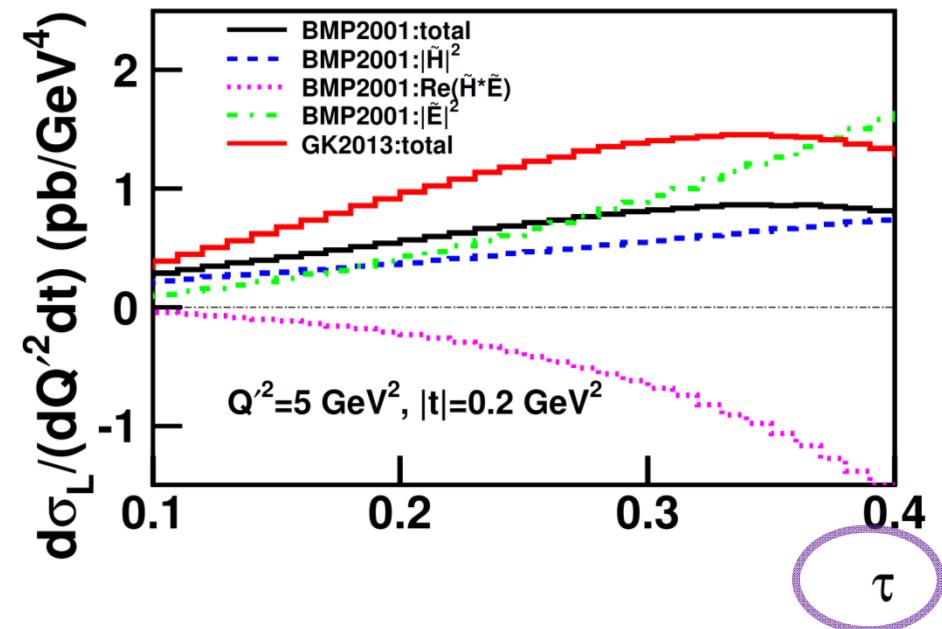
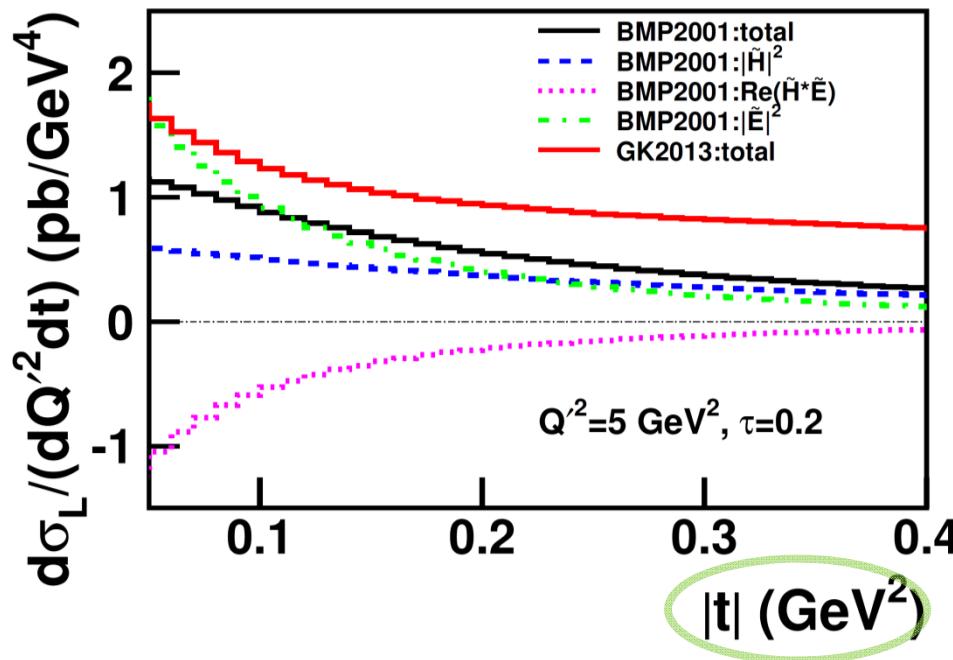
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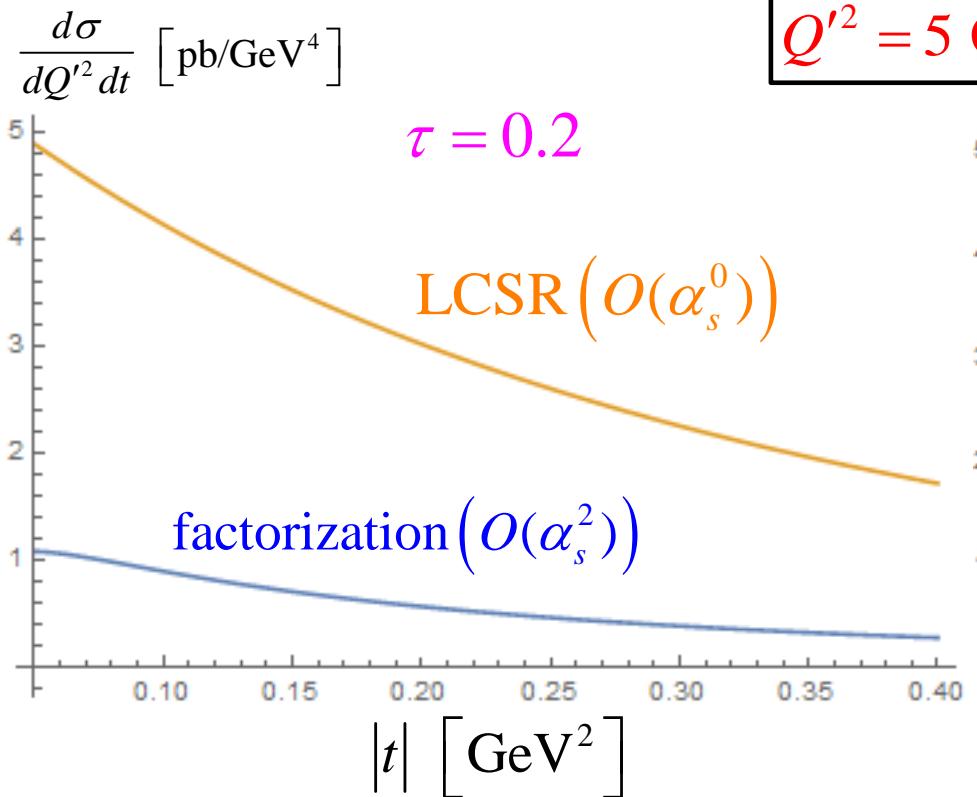
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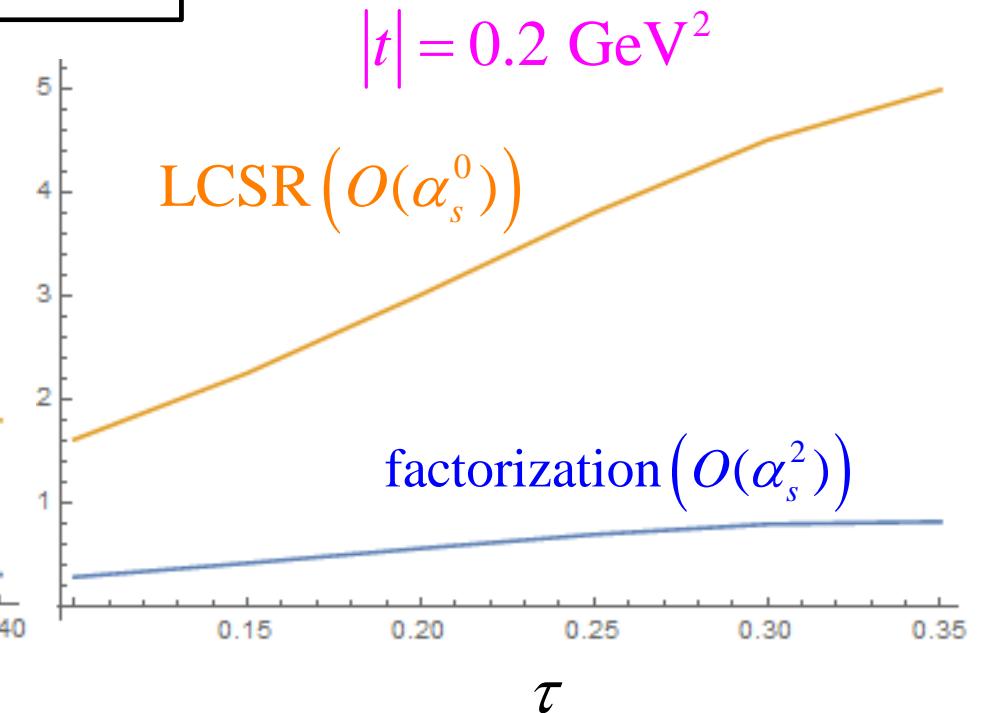


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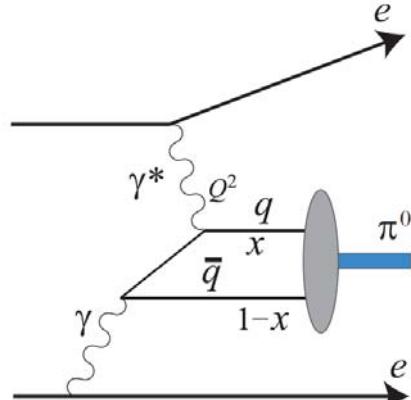
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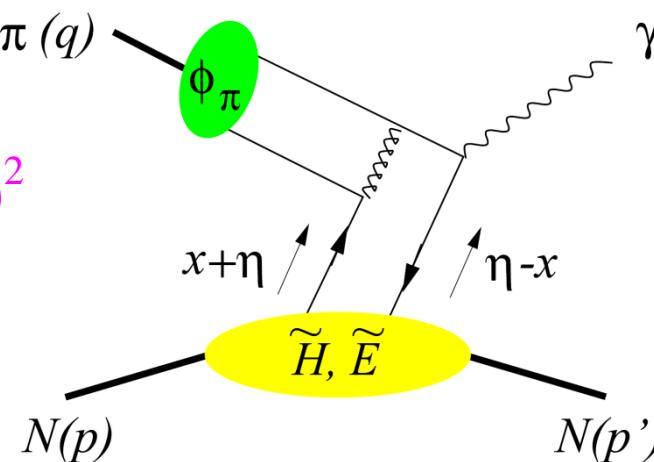
## Exclusive lepton pair production in $\pi N$ scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



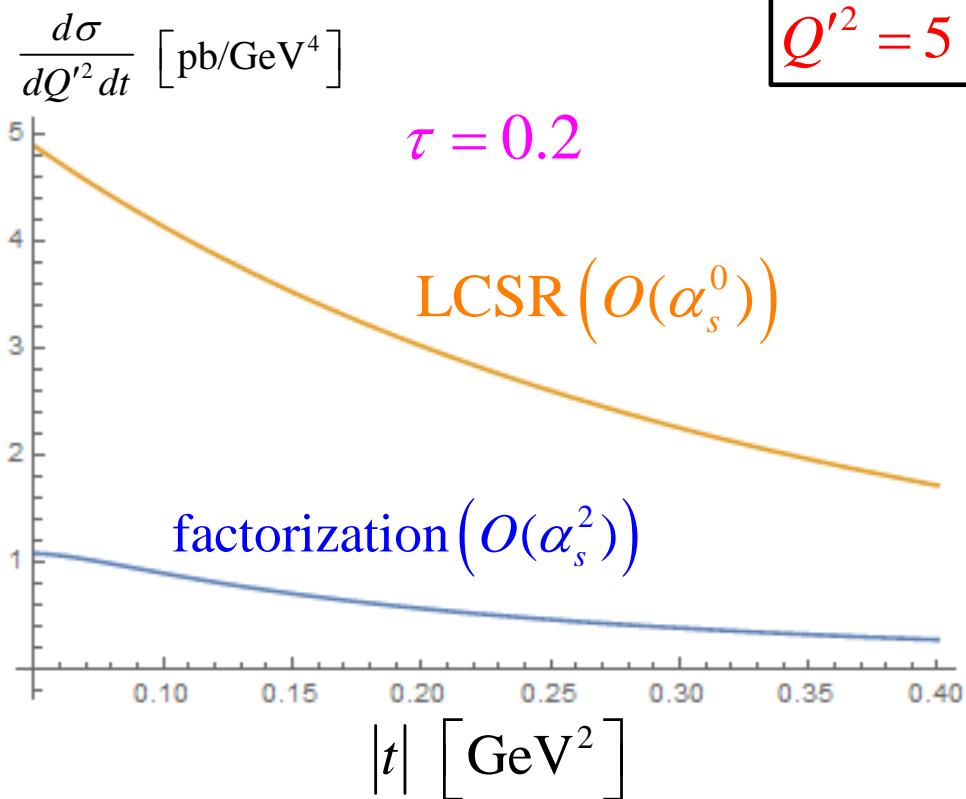
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small  $t = \Delta^2 = (q - q')^2$

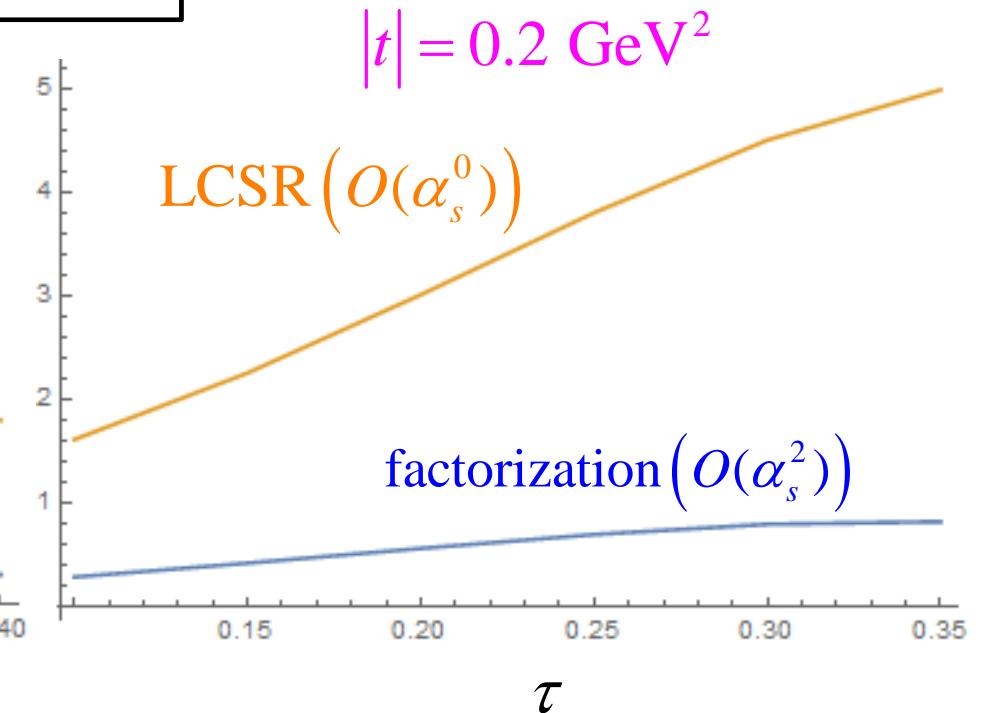


"exclusive DY"

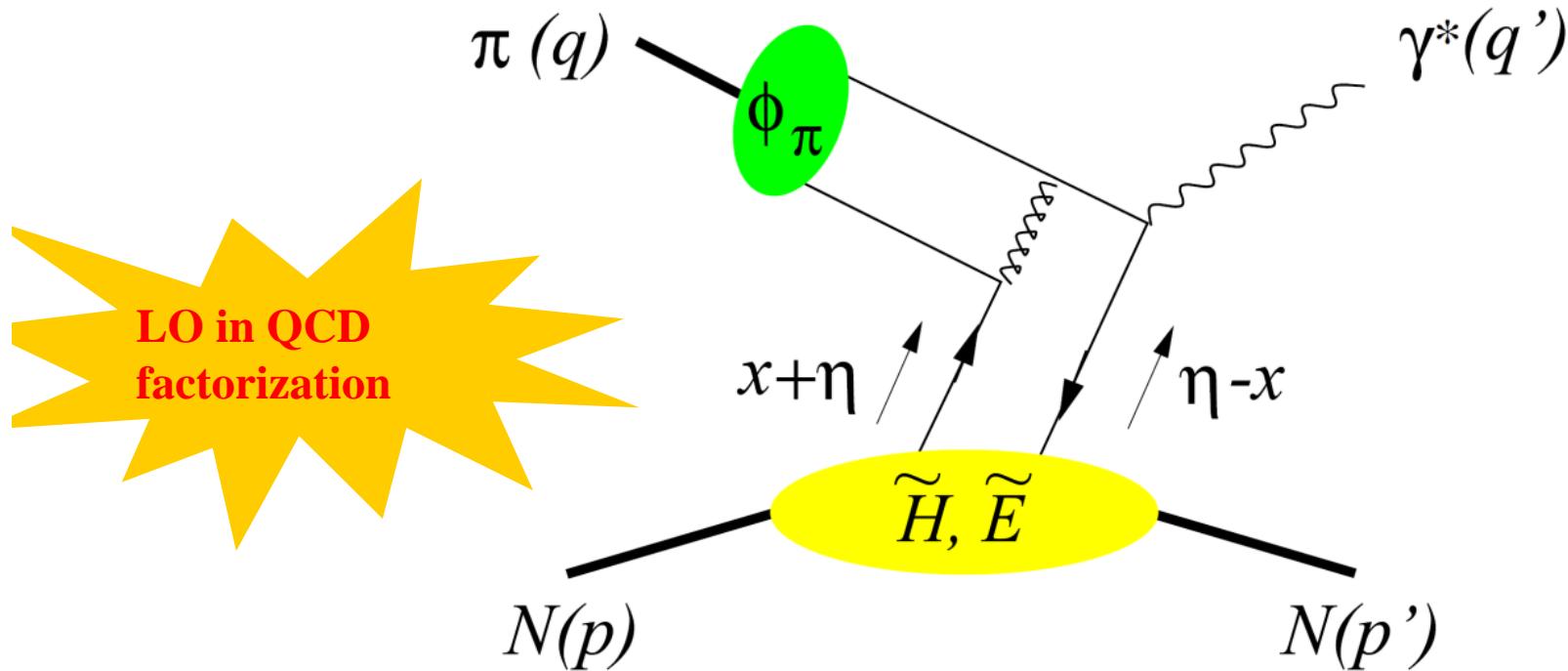




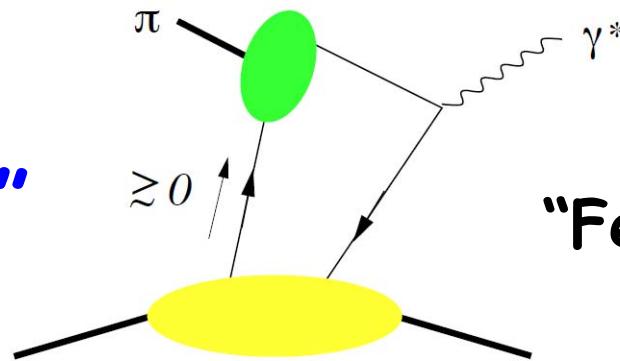
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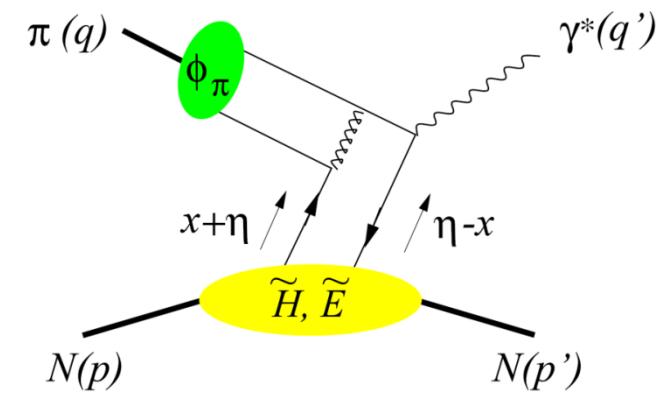
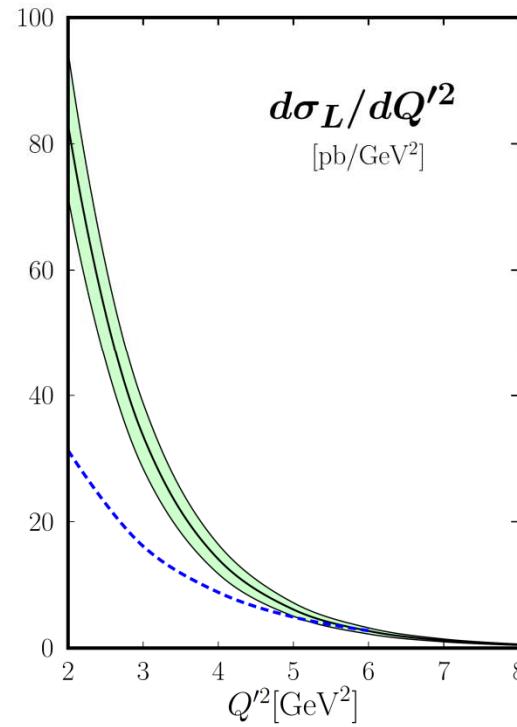
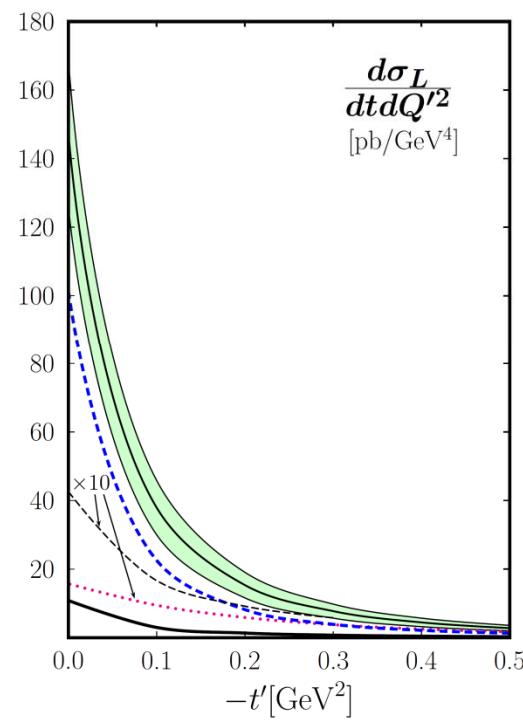
$$\begin{aligned} \frac{d\sigma}{dQ'^2 dt} (\pi^- p \rightarrow \gamma^* n) \\ = \frac{4\pi\alpha_{\text{em}}^2}{27} \frac{\tau^2}{Q'^8} f_\pi^2 \left[ (1 - \eta^2) |\widetilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\widetilde{\mathcal{H}}^{du*} \widetilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\widetilde{\mathcal{E}}^{du}|^2 \right] \end{aligned}$$

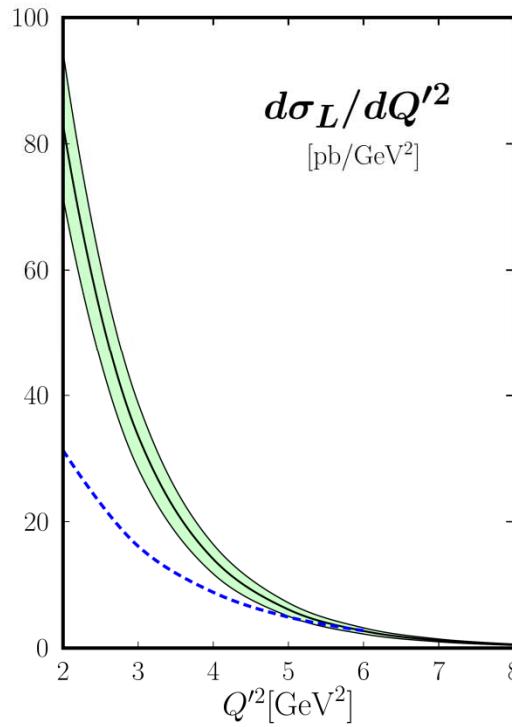
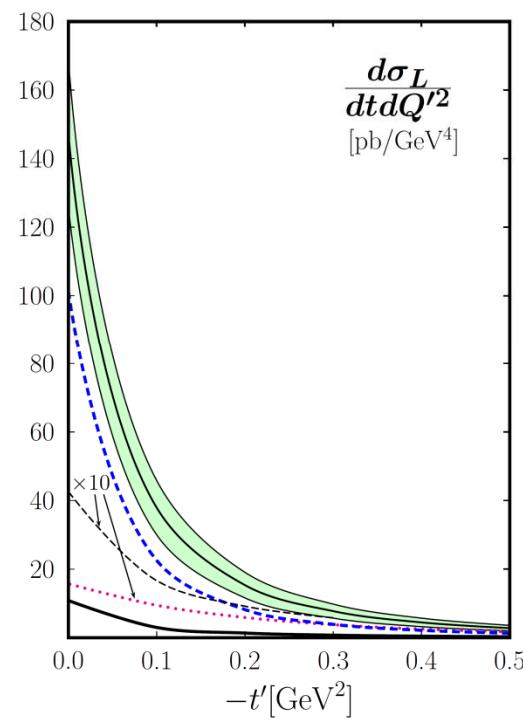


**“nonfactorizable”**

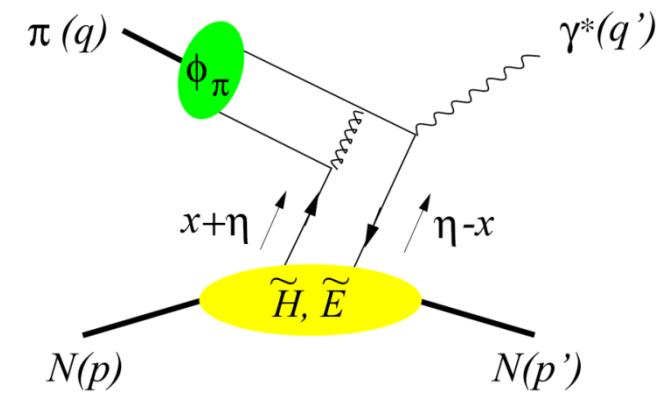


**“Feynman mechanism”**

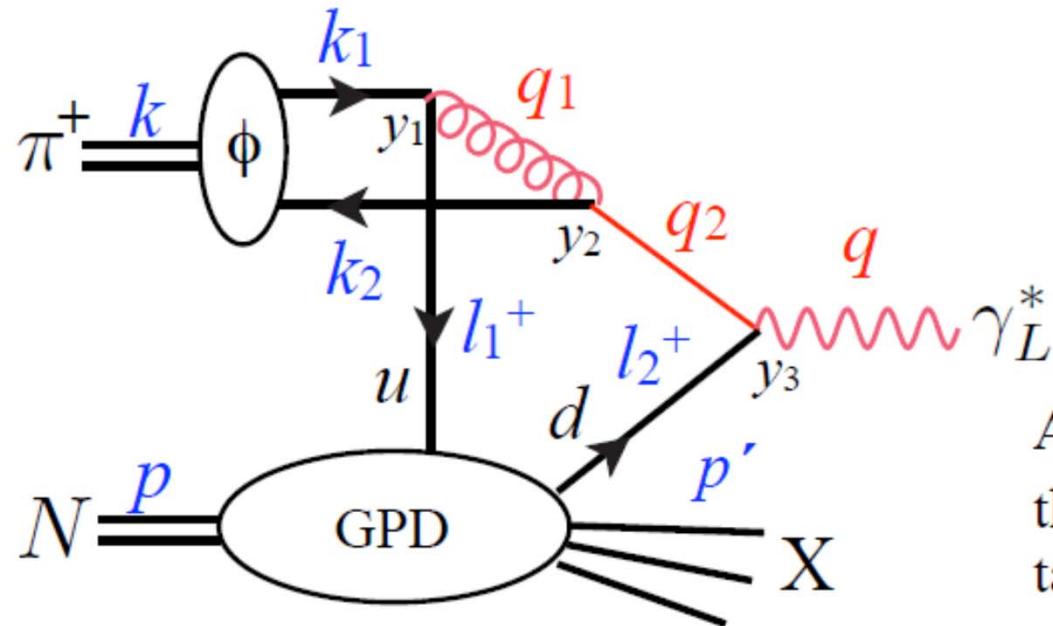




$k_T$  factorization for  $\mathcal{O}(\alpha_s)$



Hence the stopped quark should be connected to the target:



For each final state  $X$  the target matrix element is given by a **GPD** with skewness

**end-point behaviors:**

$\phi_\pi(u) \sim u(1-u)$        $\phi_{\text{tw.-3}}(u) \sim 1$

$$k_1 = (0^+, uk^-, \mathbf{k}_\perp)$$

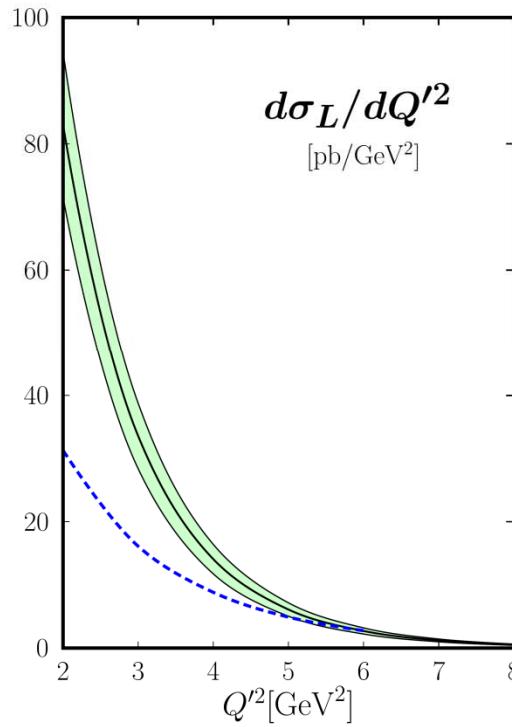
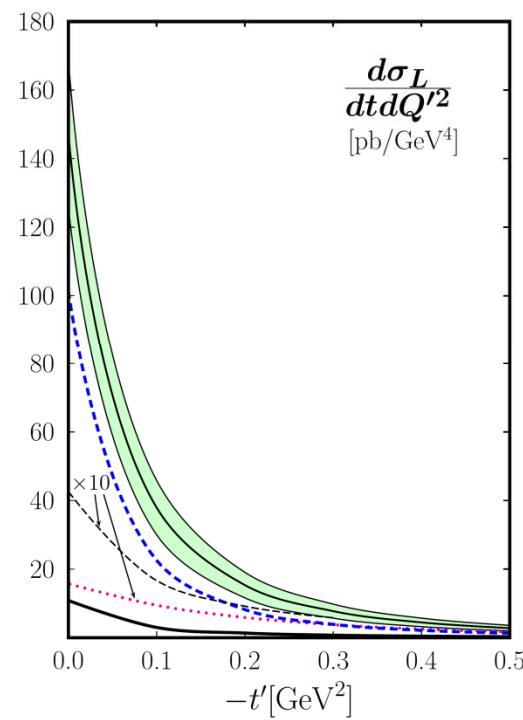
$$k_2 = (0^+, (1-u)k^-, -\mathbf{k}_\perp)$$

Since  $q_1^2 \approx -uk^- l_1^+ \rightarrow \infty$   
the pion wave function contributes through its *distribution amplitude*  $\phi$

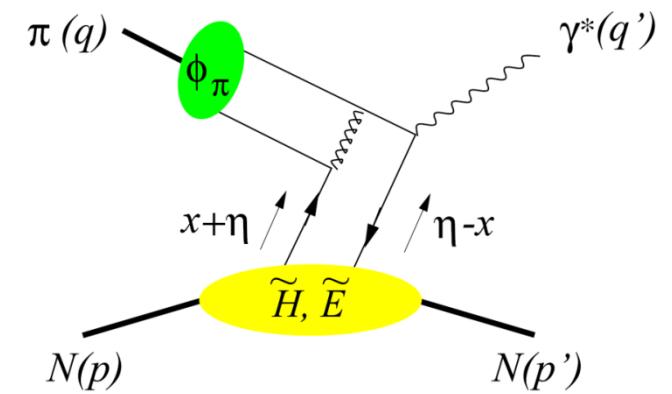
Also  $q_2^2, q_1^-, q_2^- \rightarrow \infty$ , hence the space-time separation of the target interaction points  $y_1, y_3$  is

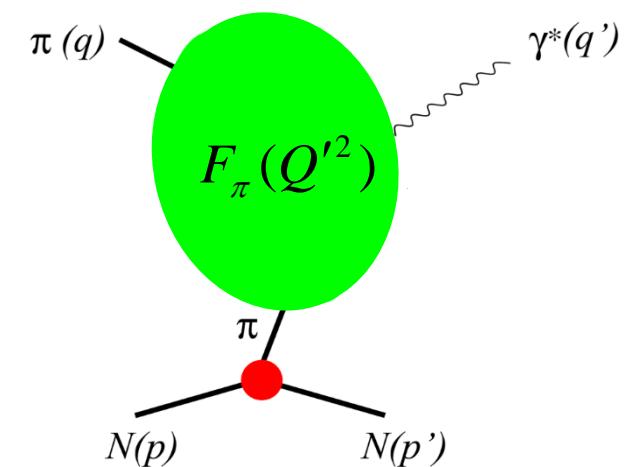
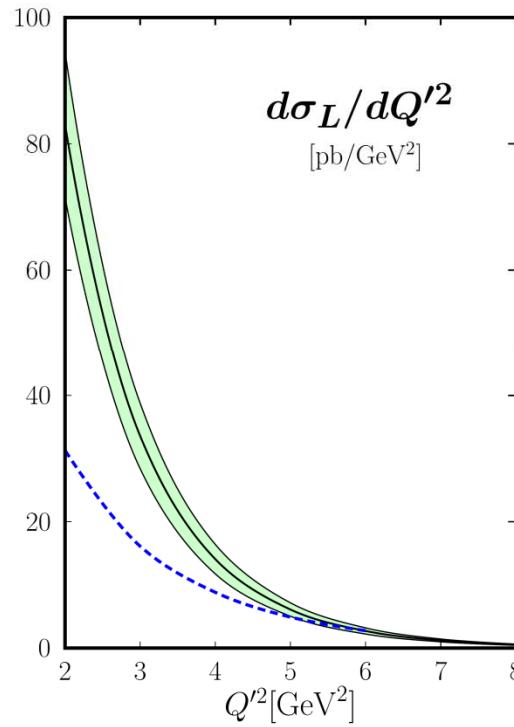
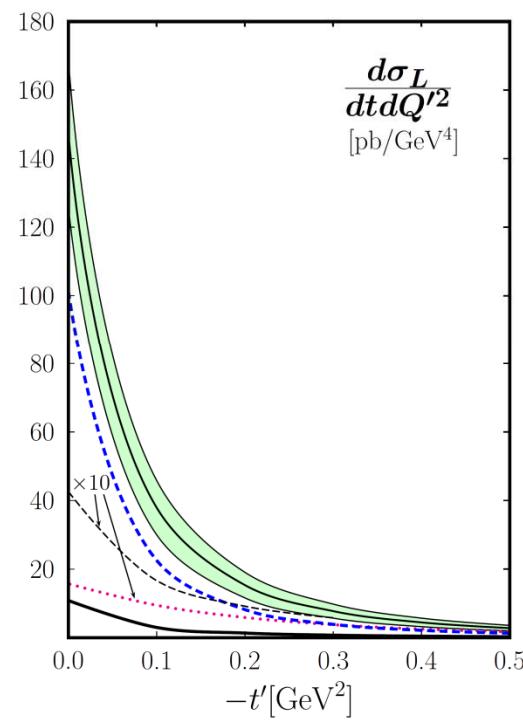
$$|y_{1\perp} - y_{3\perp}| = \mathcal{O}(1/Q) \rightarrow 0$$

$$|y_1^+ - y_3^+| = \mathcal{O}(1/Q^2) \rightarrow 0$$



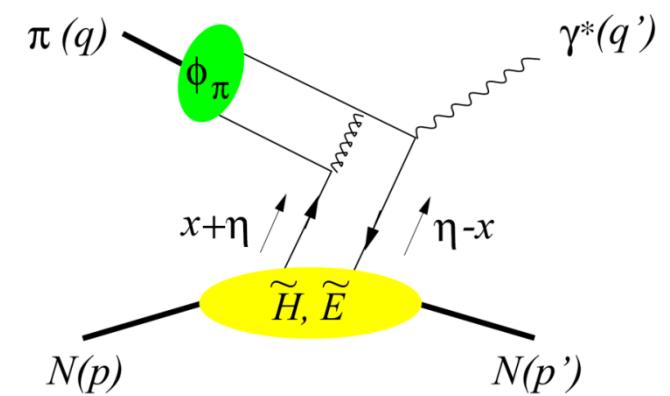
$k_T$  factorization for  $\mathcal{O}(\alpha_s)$

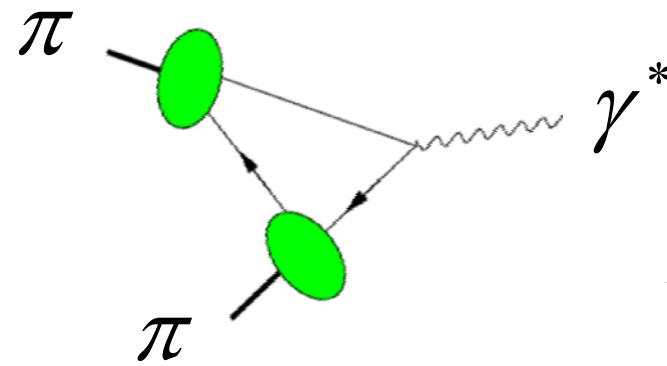
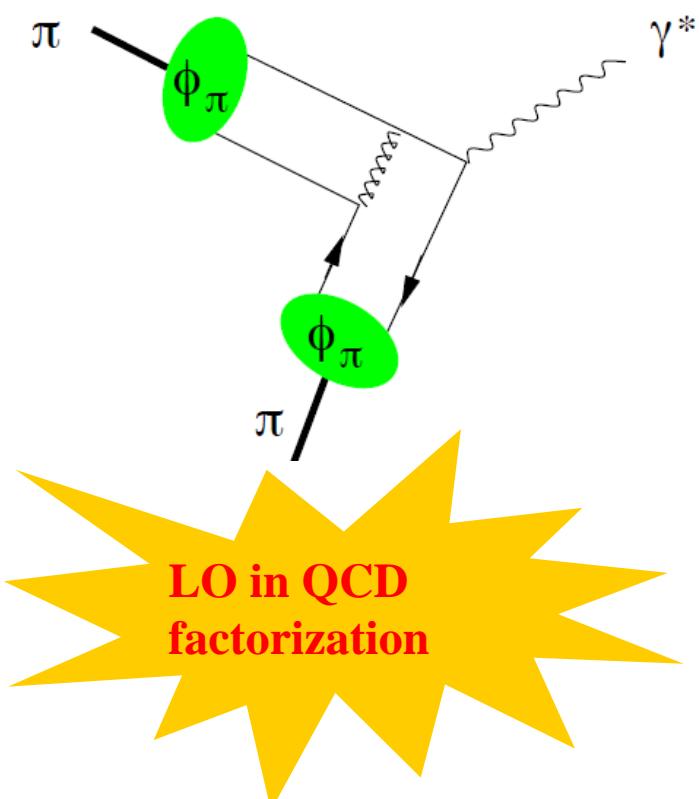
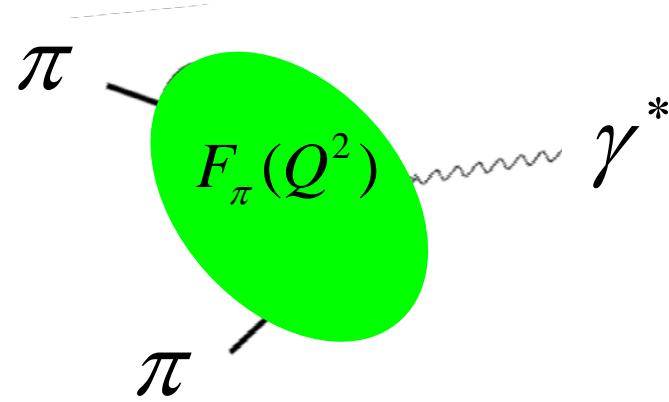




pion exchange for  $O(\alpha_s^0)$   
empirical  $F_\pi(Q'^2)$

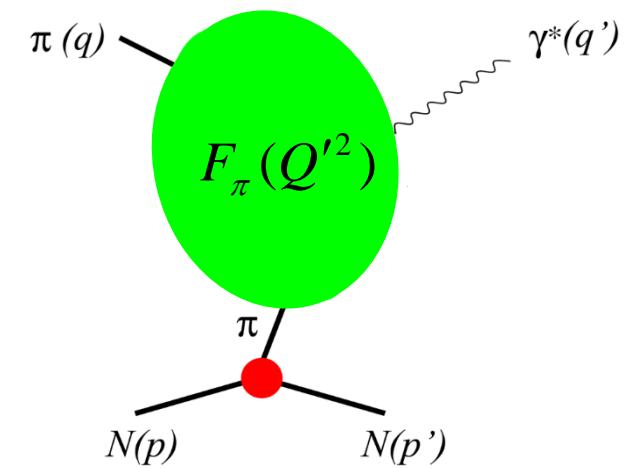
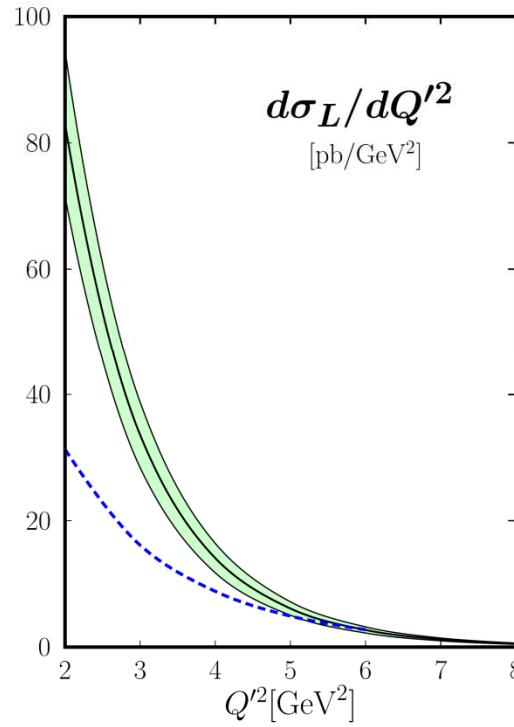
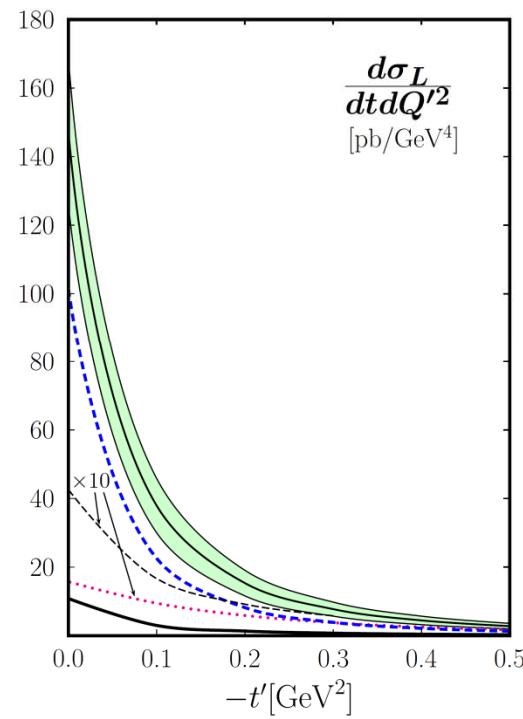
$k_T$  factorization for  $O(\alpha_s)$





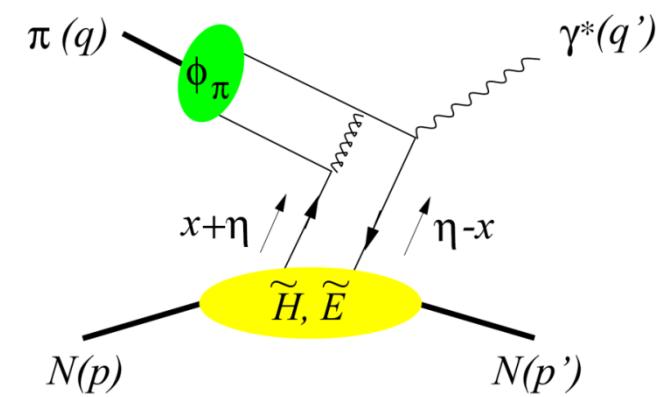
**“nonfactorizable”  
Feynman mechanism**

LO in QCD  
factorization

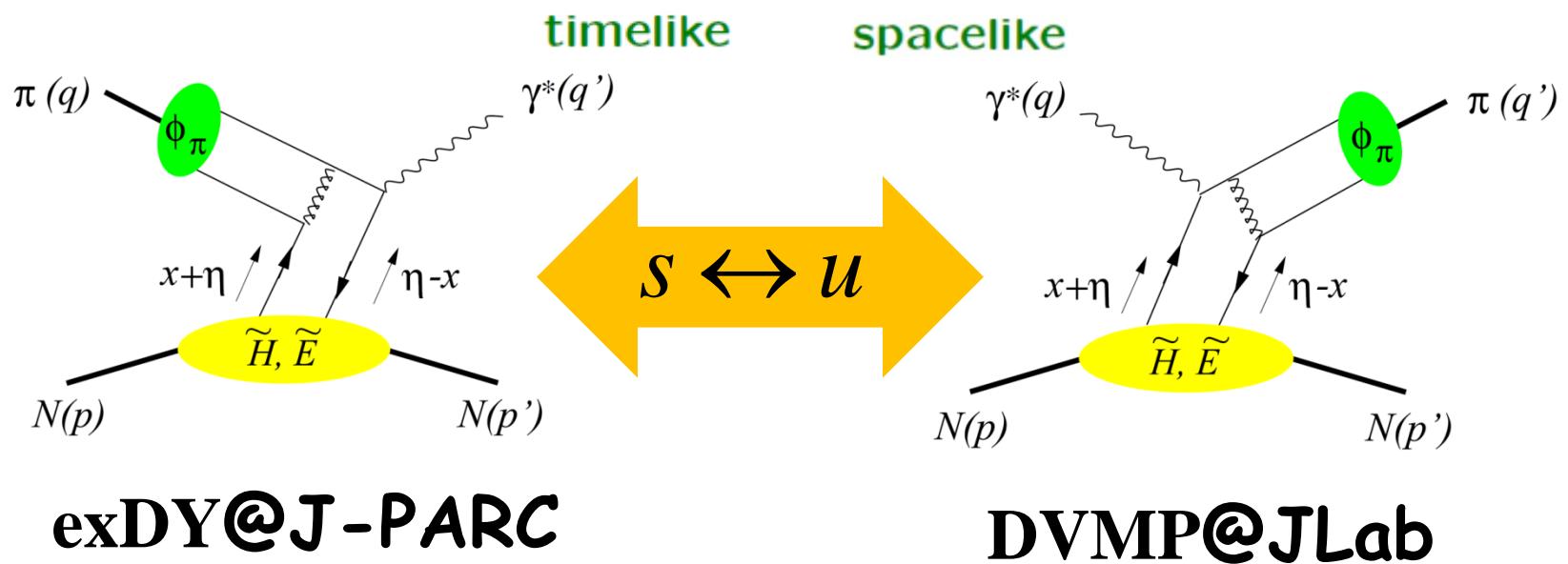


pion exchange for  $O(\alpha_s^0)$   
empirical  $F_\pi(Q'^2)$

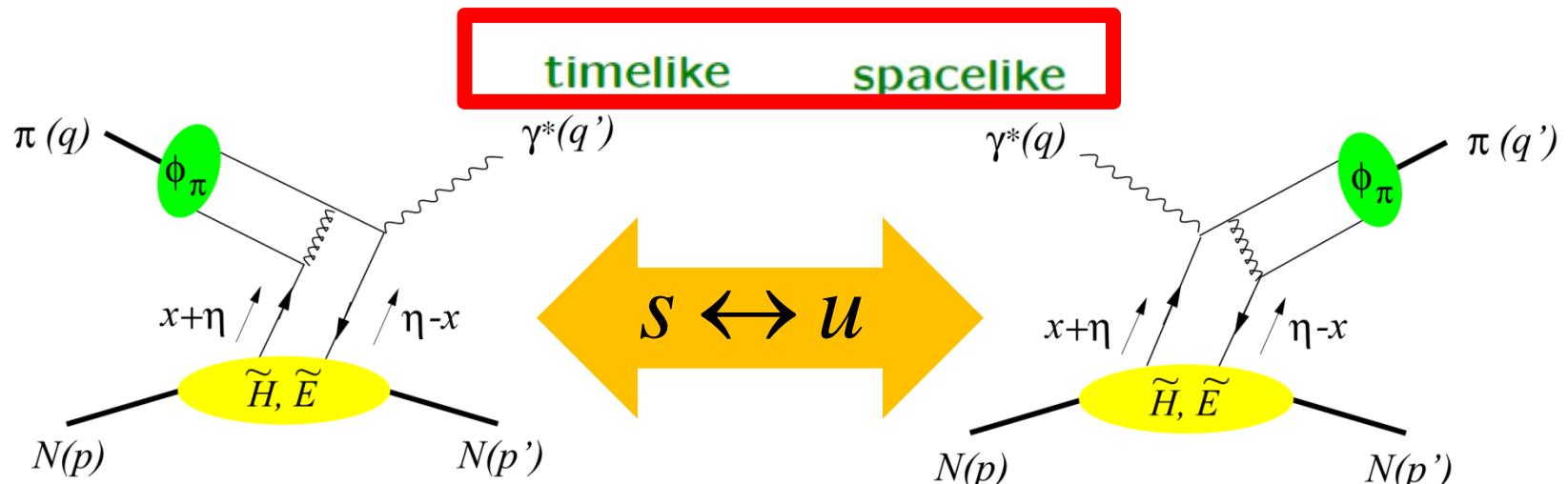
$k_T$  factorization for  $O(\alpha_s)$



# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions



# Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions



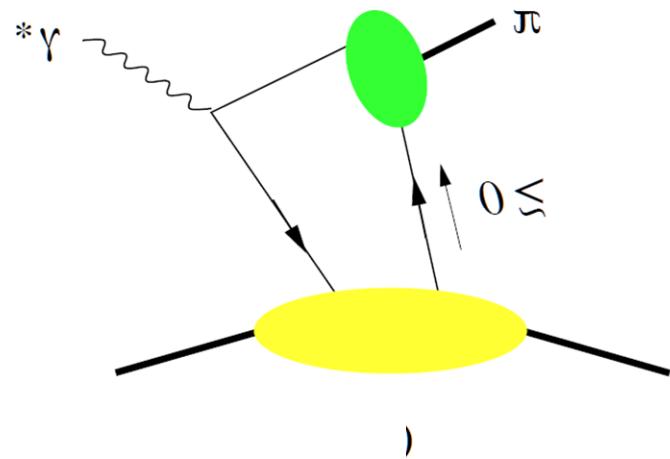
exDY@J-PARC

DVMP@JLab

$$q \leftrightarrow -q'$$

$$S \leftrightarrow u \quad q \leftrightarrow -q'$$

LCSR



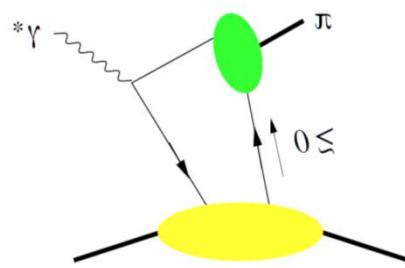
$$\tilde{x}_0 : \frac{q_{\text{th}}^2}{(-Q^2)} \quad \text{quark-hadron duality}$$

$$\hat{L}_{M_B} \left( \frac{1}{m^2 - q^2} \right) = \frac{1}{M_B^2} e^{-\frac{m^2}{M_B^2}}$$

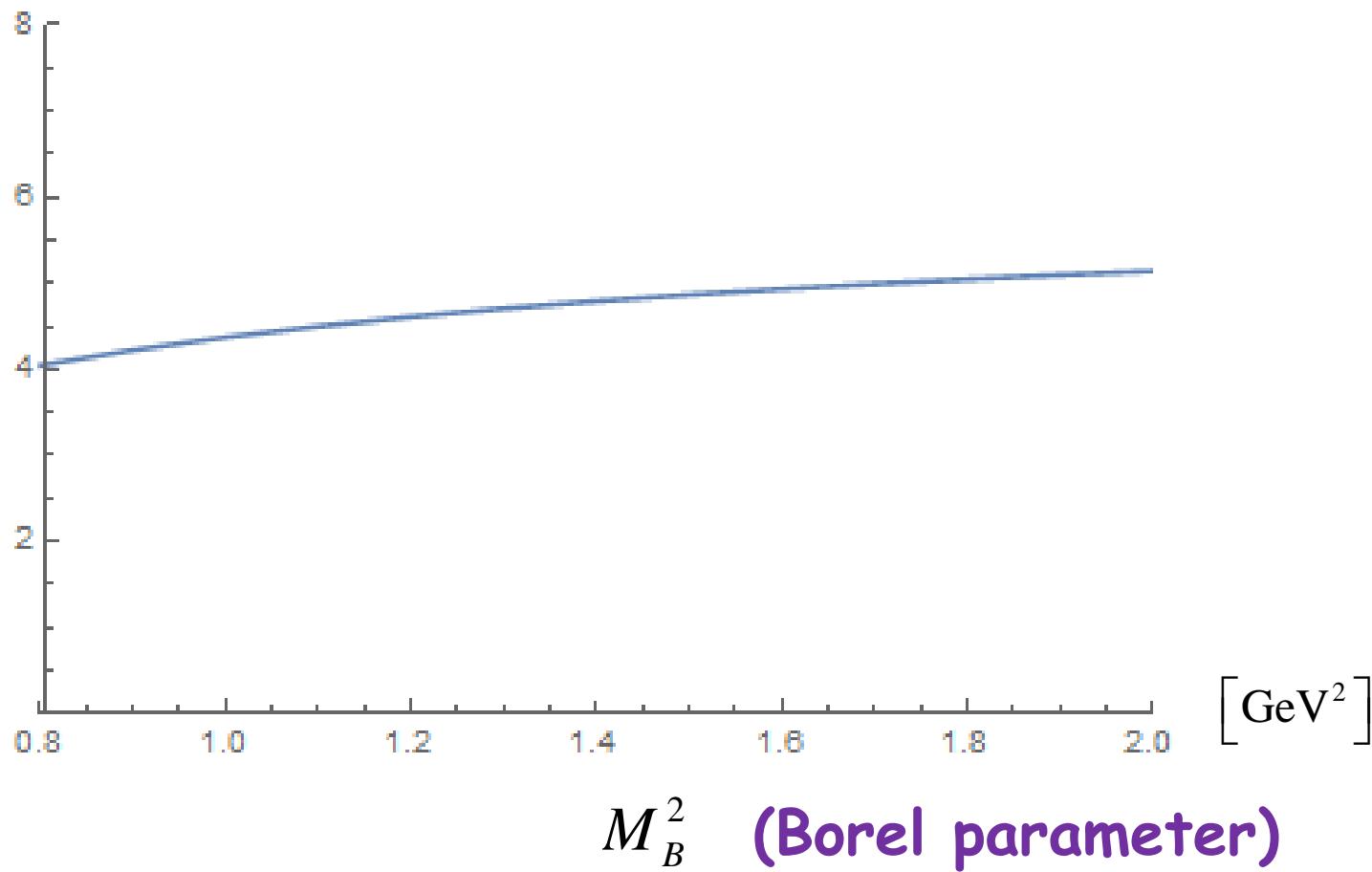
$$\sim g_\nu^- \frac{1}{f_\pi} \int_{\tilde{x}_0}^\eta dx e^{-\frac{x-\eta}{x+\eta} \frac{(-Q^2)}{M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\ \times \left[ e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right]$$

$$\times \bar{u}(p') \gamma^+ \gamma_5 u(p) + \dots$$

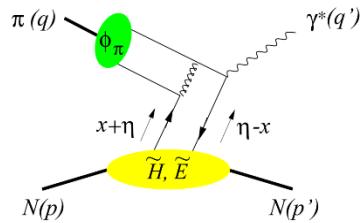
$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$



for  $\ell p \rightarrow \ell' \pi^+ n$  from LCSR



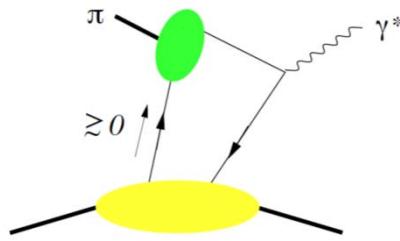
# Summary exDY ( $\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$ ) GPDs



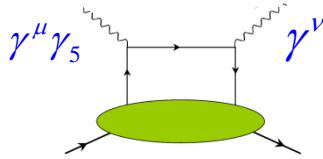
**QCD factorization: measurement at J-PARC is feasible**

T. Sawada, W.C. Chang, S. Kumano,  
J.C. Peng, S. Sawada, KT PRD93, 114034

**soft nonfactorizable mechanism (SNM)**



**LCSR**



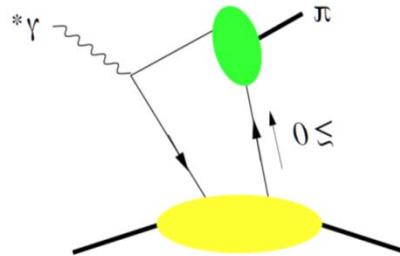
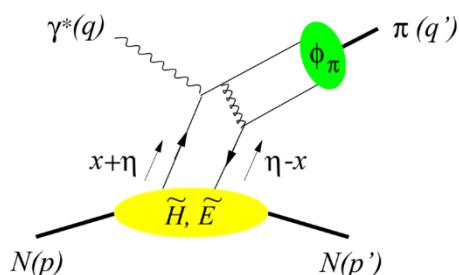
$\tilde{H}, \tilde{E}, q_{\text{th}}^2 (\sim 0.7 \text{ GeV}^2)$

**SNM > QCD factorization**

$\alpha_s^0$

$\alpha_s^2$

**$S \leftrightarrow U$**



**cross section  
for  $\ell p \rightarrow \ell' \pi^+ n$**